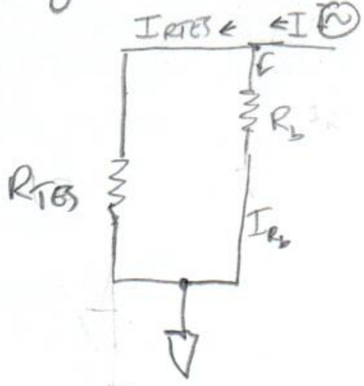
Voltage Bias



Voltage Bias
for TES

Thevenin ↓

$$\frac{1}{R_{tot}} = \frac{1}{R_{TES}} + \frac{1}{R_b}$$

$$R_{total} = \frac{R_b R_{TES}}{R_{TES} + R_b}$$

$$V_{total} = \frac{I R_b R_{TES}}{R_{TES} + R_b}$$

if $R_b \ll R_{TES}$

$$V_{total} = \frac{I R_b R_{TES}}{R_{TES}} \Rightarrow I R_b$$

V_{bias} requirement is set by bolometer properties
$\Rightarrow P_{optical} \rightsquigarrow P_{electrical}$
R_{TES}

SPT-3G example 1Ω bolometer

$P_{optical} \sim 7pW$

$P_{electrical} \sim 7pW \rightarrow V_{bias} \simeq 3\mu V$

$\Rightarrow I_{RTES} \sim 3\mu A$

Assuming R_{TES} fluctuates by 1% rms
in reaction to sky signal (this is big)
→ need to be able to
read out 30nA signal

16 bit ADC $\frac{1V}{2^{16}} \sim 15\mu V$ resolution

⇒ need amplification
w/o introducing <u>noise</u>

The SQUID

②

→ superconducting quantum interference device

→ extremely sensitive to external magnetic flux

→ can detect $\Delta\Phi \sim 10^{-6}\,\Phi_0/\sqrt{Hz}$

$\Phi_0 = \frac{h}{2e}$ → flux quantum

$\sim 2 \times 10^{-15}$ Webers or

$1\,G = 10^{-4}\,T$

$1\,T = 1\,W/m^2$

Earth magnetic field $\sim 10^{-4}\,W/m^2$

→ low noise
→ high forward gain
→ large operational bandwidth

But also drawbacks
→ dynamic range
→ cryogenics

Building block of a SQUID 1) is the josephson junction

2) flux quantization in a superconducting ring

$2\pi n = \oint \nabla\varphi \, dl \Rightarrow \Phi$

magnetic flux through contour

The Josephson Junction

ψ_0 ⇒ cooper pair density

φ_i ⇒ center of mass motion of cooper pairs

$\psi = \psi_0 e^{i\varphi_1}$

$\psi = \psi_0 e^{i\varphi_2}$

Cooper pairs can tunnel through the barrier

2 equations to describe behavior

Cooper pair = pairs of charge carriers in superconductor
ee

1) $I_J = I_c \sin\delta$

current phase relation

I_c = critical current → maximum supercurrent

$\delta = \varphi_1 - \varphi_2$ = phase difference between the wavefunctions of S1 & S2

I = supercurrent across the junction

$\hbar = \frac{h}{2\pi}$

EQN #1 comes from ⇒ phase gradient eqn

→ calculate current as a function of δ

→ rewriting as a Fourier series

→ considering only 1st sine term

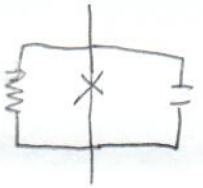
voltage-phase relation 2) $$\frac{d\delta}{dt} = \frac{2eU}{\hbar} = \frac{2\pi}{\Phi_0} U$$

where $\Phi_0 = \frac{h}{2e}$ = magnetic flux quantum

ie if the phase across the junction varies, a potential develops

RCSJ Model

More realistic model



resistively & capacitively shunted junction

→ Stray capacitance due to junction geometry

→ resistive shunt due to non-zero quasiparticles (single electrons) & purposeful to prevent hysteresis

$$\Rightarrow I = I_J + I_R + I_C$$

$$= I_c \sin\delta + \frac{U}{R} + C\frac{dU}{dt}$$ insert eqn #2

Note → ignore noise

$$I - I_c \sin\delta = \frac{\hbar}{2eR}\dot{\delta} + \frac{C\hbar}{2e}\ddot{\delta}$$

Analogous to damped harmonic oscillator w/ non-linear force term

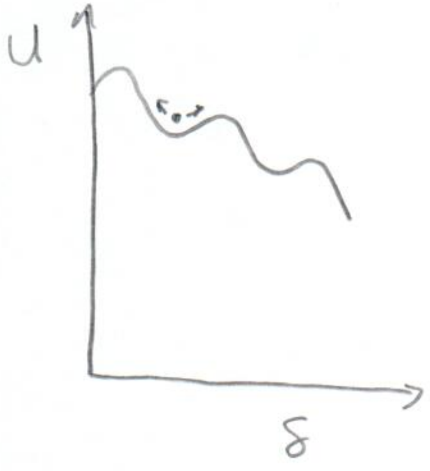
⇒ Solution $U = -I\delta - I_c \cos\delta$

Voltage as a function of phase difference

→ washboard potential

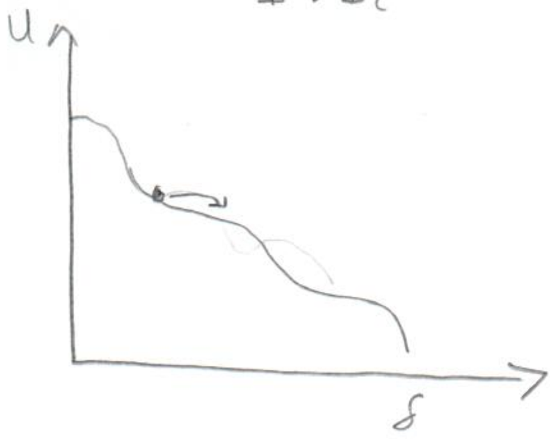
→ bias current tilts it to allow voltage to develop

$I < I_c$

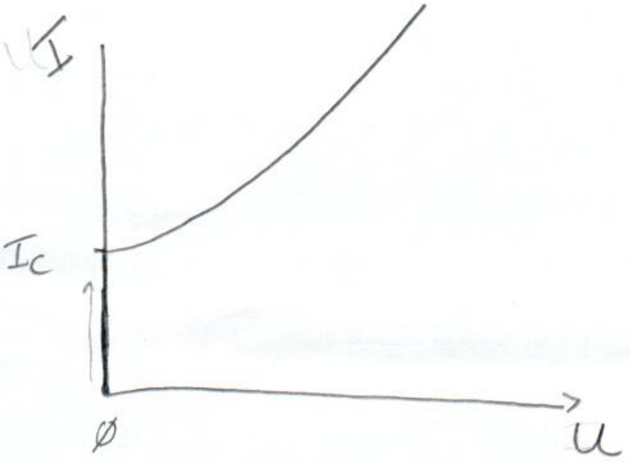


$\langle \delta(t) \rangle \Rightarrow \langle U(t) \rangle = 0$

$I > I_c$



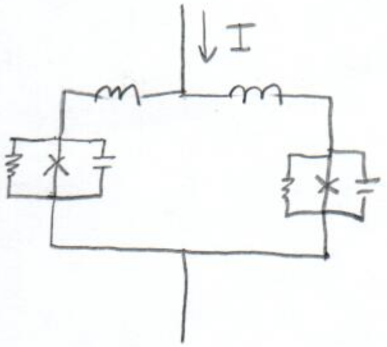
$\langle \delta(t) \rangle \Rightarrow \langle U(t) \rangle \neq 0$



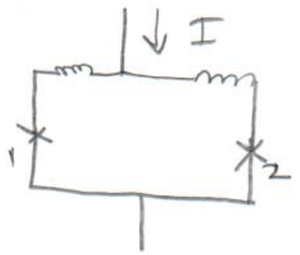
this leads to the following IV curve for a SQUID

The DC SQUID

2 josephson junctions in paralle



⇒ simplify



superconducting ring biased w/ current I

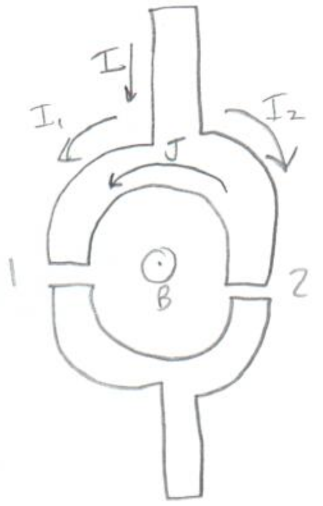
Max super current in the loop = $I_{c1} + I_{c2}$

Bias w/ $I > I_c$

Read V as a function applied Φ

Φ modulates $I_c \Rightarrow$ which changes V when I_c is max V is min

I_c is min V is max



Flux wants to be integer multiple Φ_0

current through each junction is:

$$I_1 = \frac{I}{2} + J$$

$$I_2 = I/2 - J$$

=> Back to the RCSJ model

$$\frac{I}{2} \pm J = I_c \sin\delta + \frac{\hbar}{2eR}\dot{\delta} + \frac{C\hbar}{2e}\ddot{\delta}$$

Phase differences accross the junctions are related to the total flux

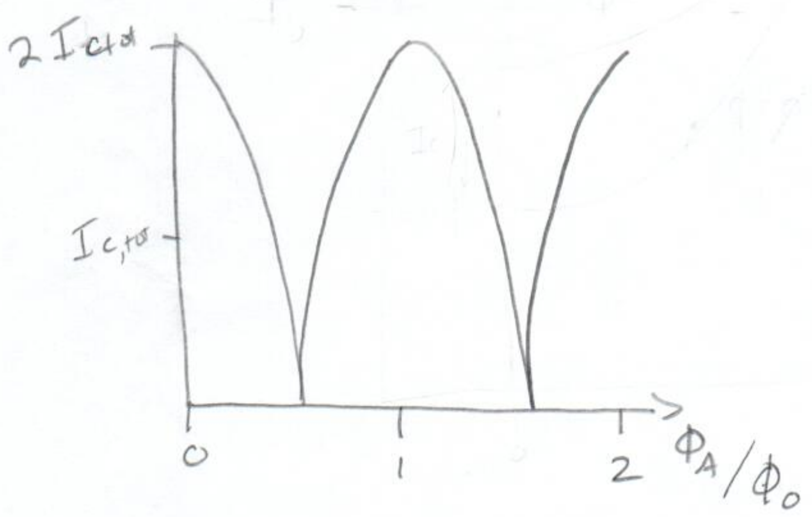
→ 2 components

1) external magnetic field
2) circulating current

N.B. this coupling is dictated by SQUID layout

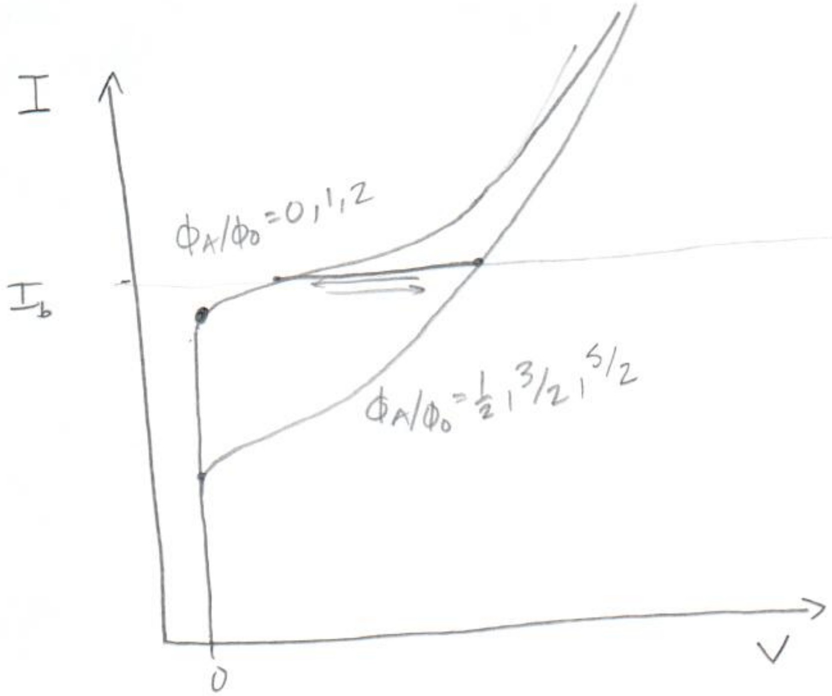
$$\delta_2 - \delta_1 = \frac{2\pi}{\Phi_0}(\Phi_a + LJ)$$

In the static case can solve for $I_c(\Phi_A)$



$$I_{c,tot} = 2I_c\left|\cos\left(\pi\frac{\Phi_A}{\Phi_0}\right)\right|$$

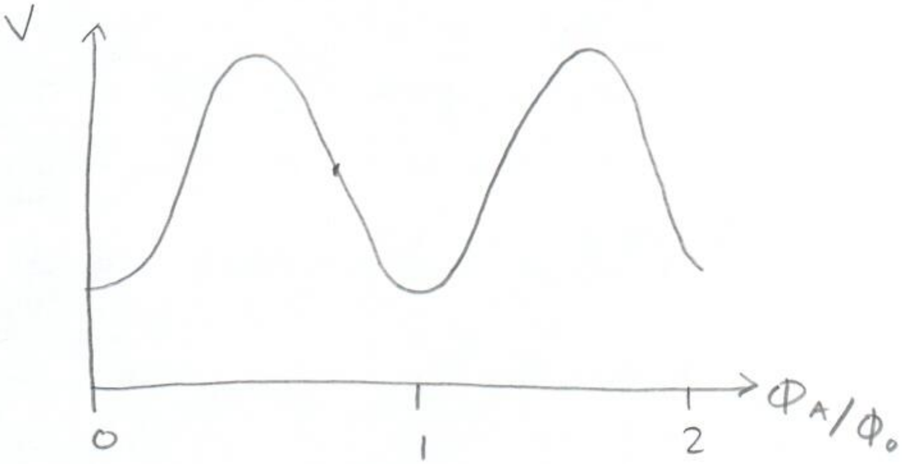
(simplified assumptions



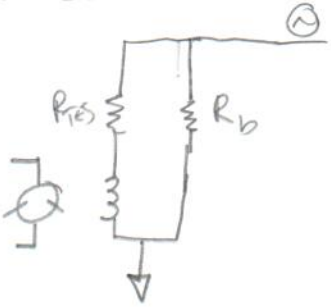
Must solve current equations for non-static case

to allow voltage to develop

⇓



→ Magnetic flux coupled via inductor input coil



Can maximize ΔV for $\Delta\Phi_A$

→ find $\frac{\partial V}{\partial \Phi_A}$ is max

Can use V_{PP} as a proxy

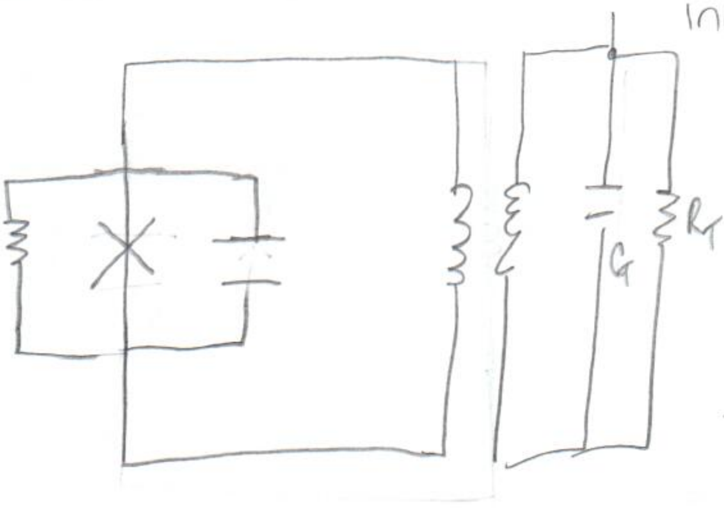
Transimpedance ⇒ quoted in Ω

$$Z_{SQ} = M \frac{\partial V}{\partial \phi}$$

⇒ output voltage given input current

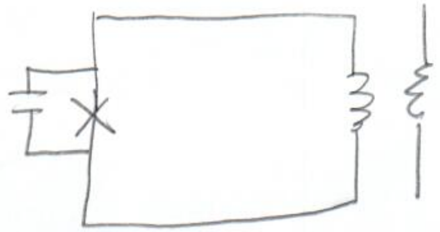
The RF SQUID

Single junction
inductively coupled to
a resonant circuit
which is driven
at its resonant
frequency



Standard

loop has self
inductance, current
that tunnels across junction
also drives flux in the
loop



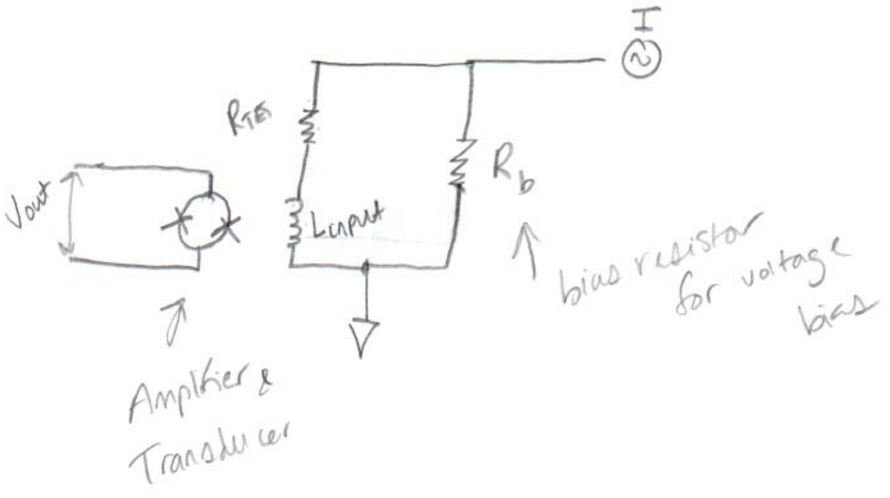
Dissipationless

-> higher noise than DC SQUID
-> require AC readout
-> simpler fab

Flux coupling into the SQUID
=> self inductance depends on
mean flux

-> Change resonance of the tank

Simple Readout w/ DC SQUID



But really, we use arrays of SQUIDs

$\Rightarrow$ can yield few mV of signal

series squid array

$\rightarrow$ voltages increase w N

$\rightarrow$ noise adds quadrature

$$\sigma_{SSA}^2 = N \sigma_S^2 \Rightarrow \sigma_{SSA} = \sqrt{N}\sigma_S$$

$\rightarrow$ win in S/N by $\sqrt{N}$

Caveat for SSA

$\Rightarrow$ increased sensitivity to trapped flux & external magnetic fields

$\rightarrow$ don't want gradients across the SSA so that they add coherently

For CMB experiments

Multiplexing

Multiplexing required

1) Time division multiplexing } DC squids
2) Frequency division multiplexing }
3) μ-multiplexing — RF squids
4) MKIDs

First 3 all use SQUIDs, but in different ways.
Will talk about #4 later (also Erik Shirokoff)

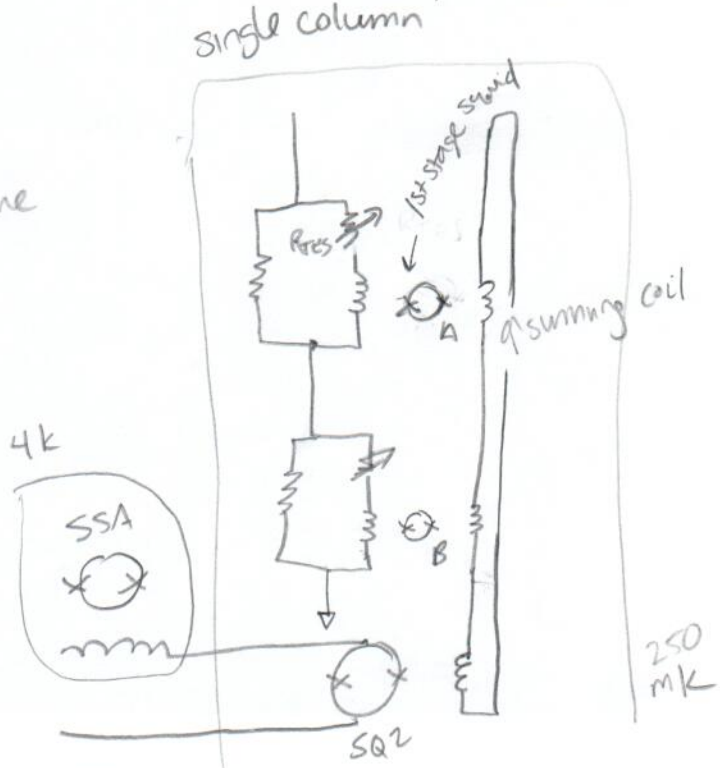
TESs require subkelvin operation ⇒ limited power dissipation of a cryocooler
⇒ 2 wires per detector not possible
as focal plane sizes increase
(& we are there now)

TDM

⇒ TES is DC biased

First stage SQUID inductively coupled to each TES

Uses a common bias line



SQUIDs are biased sequentially
→ A, then B

the summing coils couples into 2nd stage squid

→ couples into an SSA for amplification

Uses flux lock loop in multiple stages

→ feeds back on output of SSA
to keep it constant

Dsc ⇒ FB

Updated TDM architecture for Adv. Act (eliminates RS SA2)
⇒ uses Flux Activated switches for row select
(Array of josephson junctions)

Example of switching
rate ⇒ row rate = 560kHz for ACT

full array = 15.2kHz
(every 65μs)

limitation ⇒ limited sampling rate
→ high-f noise is aliased into signal band

⇒ results in increased noise

→ can be mitigated by bandwidth limiting signals from TES above Nyquist
↓
Nyquist inductors

Adv. Act 64 rows × 32 colums

Frequency Division Multiplexing

→ This is the one you'll use in the lab this afternoon

ACBIAS

TES is AC biased

$I(t) = A\cos\omega_c t$ current through TES from voltage bias

optical signal → $I_\gamma = a_\gamma \cos\omega_\gamma t$

Define the modulated waveform $I_m = A\cos\omega_c t\,(1 + a_\gamma \cos\omega_\gamma t)$

$$= A\cos\omega_c t + a_\gamma A\cos\omega_c t\cos\omega_\gamma t$$

⇒ trig identity

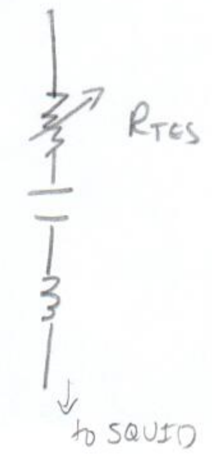
$$= \underbrace{A\cos\omega_c t}_{\text{original carrier}} + \frac{a_\gamma A}{2}\Big[\underbrace{\cos(\omega_c-\omega_\gamma)t + \cos(\omega_c+\omega_\gamma)t}_{\text{sky signal @ } \omega_c \pm \omega_\gamma}\Big]$$

→ the sidebands

Amplitude Modulation

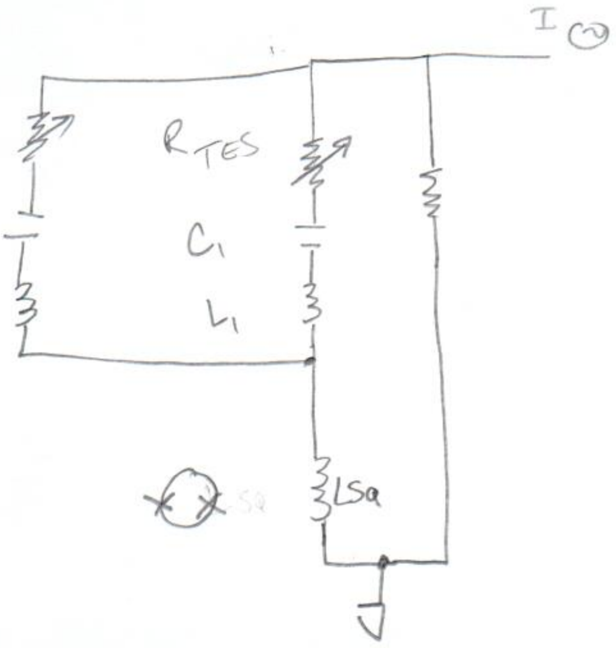
Can multiply by unmodulated reference @ original bias frequency to demodulate down to base band & measure $\int a_\gamma \cos\omega_\gamma t\, dt$

In FDM → each TES in a module is assigned a unique resonant frequency



$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$\Delta f = \frac{R}{2\pi L}$$



Make a parallel network that is summed together an then inductively coupled into a DC SQUID

AC Biased TES
DC SQUID → for amplification

Feedback

old generation → combo of FLL & injected nuller

→New generation
→ digital waveform w/ active feedback (subtract carriers & sidebands)
→ nuller becomes the signal you readout

Limitations

How many channels can pack into a given bandwidth w/ acceptable crosstalk

Macroscopic components

$\Delta f_{elec} > 5.8\, \Delta f_{thermal}$

Electrical circuit bandwidth must be 5.8x > TES bandwidth
→or TES could oscillate at frequencies & circuit couldn't respond

μ-mux

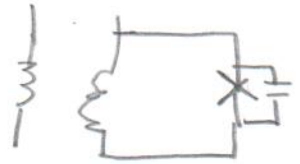
dissipationless rf SQUID used to provide gain between TES & HEMT

→ self inductance depends on the flux

→ couple to microwave resonater (GHz frequencies)

→ each w/ unique resonant frequency

=>



TES is DC biased w/ parallel shunt resistor

→ multiple TESs share single pair of bias wires

Modulations in TES resistance modulate flux through SQUID

→ couples to the resonator & modulates its resonant frequency

Measure the shifting resonance w/ a fixed tone → single coax

SQUID noise

HEMT noise

TLS noise

Mustang2 → 215 elements

SO

NB: HEMT = high electron mobility transistor

→ cryogenic amplifier