

# *Combining photo-z and spec-z surveys: Constraint on linear growth*

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*with* Gary Bernstein (UPenn)

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**1. 'Combining weak-lensing tomography  
and spectroscopic redshift surveys'**

*Yan-Chuan Cai & Gary Bernstein*

*MNRAS, 2012, 422, 1045*

**2. 'Cosmology without cosmic variance'**

*Gary Bernstein & Yan-Chuan Cai*

*MNRAS, 2011, 416, 3009*

**3. 'Optimal linear reconstruction of dark matter from halo catalog'**

*Yan-Chuan Cai, Gary Bernstein & Ravi Sheth*

*MNRAS, 2011, 412, 995*

# Outlines

- Goal –

Measure the growth of structure at high accuracy

- Approach –

Linear redshift-space distortion (RSD)

- Limitations –

Cosmic-variance, galaxy bias, stochasticity

- Improvement –

1. Multi-tracer RSD

2. Combine a lensing survey with galaxy redshift survey

3. Optimal weighting of galaxies

# Measure of Linear Growth

- LPT: growth of density fluctuations is scale independent

$$\delta(\vec{k}, a) = G(a)\delta(\vec{k}, a_0), \quad P(\vec{k}, a) = G^2(a)P(\vec{k}, a_0)$$

- and determined by the expansion history:

$$G(a) = \frac{H(a)}{H_0} \int_0^a \left( \frac{H_0}{a'H(a')} \right)^3 da'$$

In the matter-dominated epoch,

$$G(a) \propto a$$

# Measure of growth

- Measure density fluctuations at two different epochs

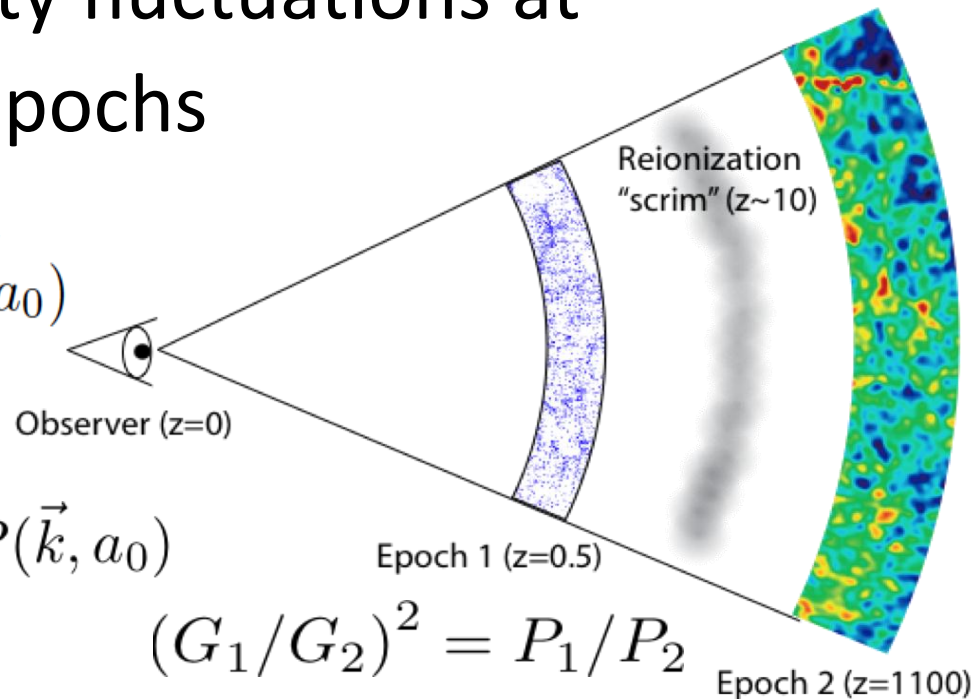
$$P(\vec{k}, a) = G^2(a) P(\vec{k}, a_0)$$

$$P_g(\vec{k}, a) = G^2(a) b^2 P(\vec{k}, a_0)$$

$$(G_1/G_2)^2 = P_1/P_2$$

Note: This 'ratio' is different!

- Limits: A.) sample-variance limited,  
B.) degenerate with galaxy bias



# Growth Rate

$$f(a) \equiv d \ln G(a) / d \ln a$$

- sensitive to gravity

$$f(a) \approx \Omega_m^\gamma(a), \gamma \approx 0.55 \quad (\text{in GR})$$

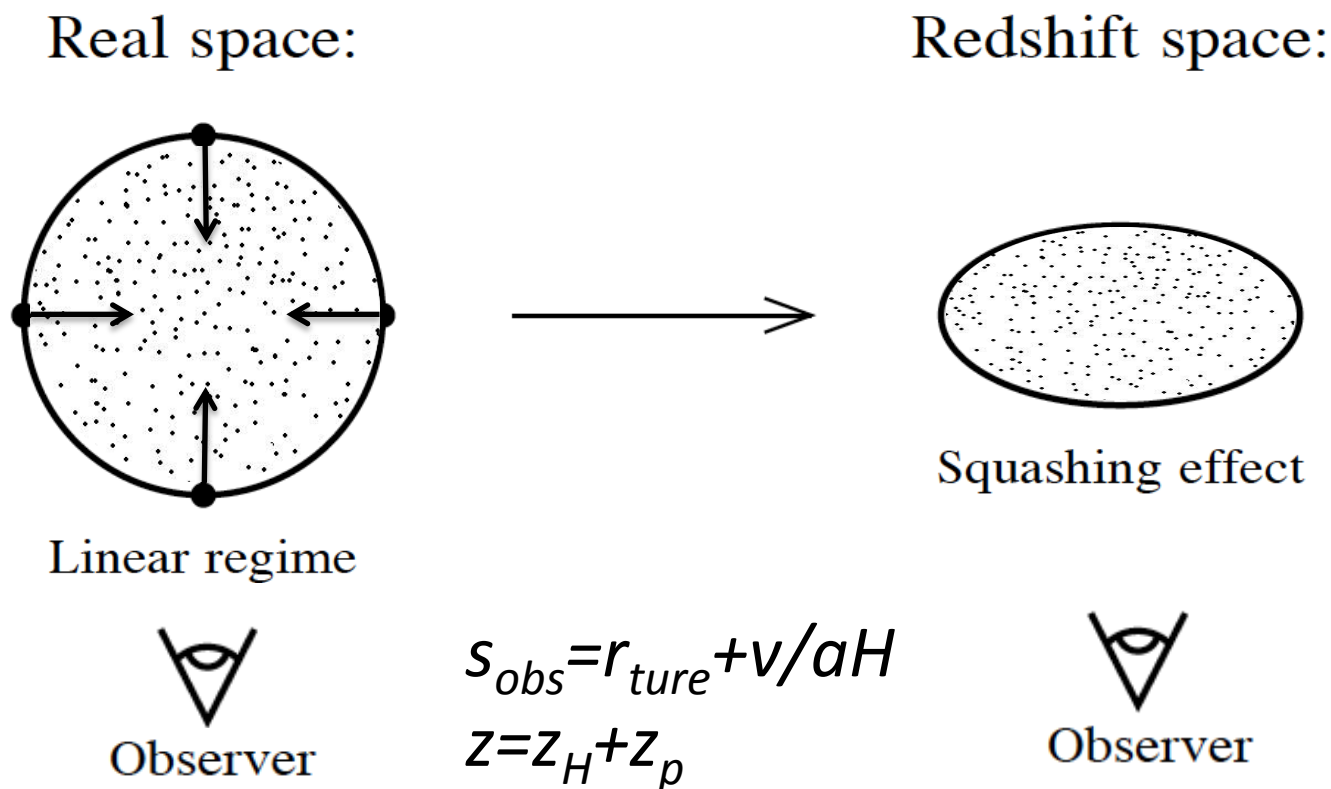
Peebles, 1980; Lahva et al 1991; Linder & Cahn 2007

- sensitive to the dynamics of cosmic potential

$$\dot{\Phi}(\vec{k}, a) \propto [1 - f(a)], \Delta T(\hat{n}) = \frac{2}{c^2} \bar{T}_{CMB} \int_{t_L}^{t_0} \dot{\Phi}(a, \hat{n}) dt \quad (\text{ISW})$$

Sachs & Wolfe 1967

# Redshift Space Distortion (RSD)



*Hamilton 1997*

# Limits of standard RSD:

(Kaiser 1987)  $P_s = (b + f\mu^2)^2 P = (b/f + \mu^2)^2 f^2 P$

- $f$  is degenerate with  $b$ , only constrain  $f^2 P = f^2 G^2 P_0$  &  $b/f$
- Kaiser formula is valid only on very large scales ( $k < 0.03$ ?)
- *Cosmic Variance:*

$$\sigma_{\ln fG} \geq \sqrt{\frac{11}{N_s}}$$



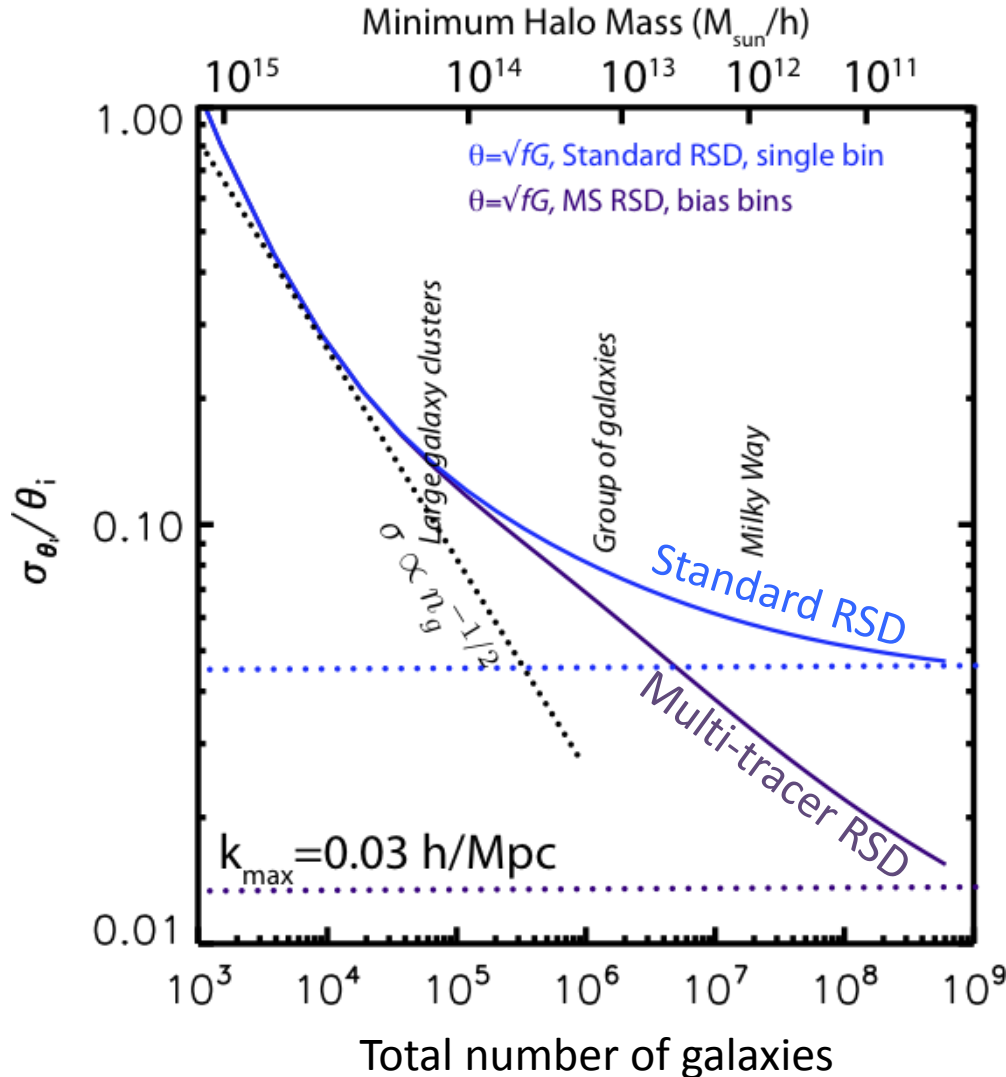
# Multi-tracer RSD

MacDonald & Seljak (2009)

- Advantages:
  - 1.) Sample-variance-free measure of bias ratios  $b_i/b_j$
  - 2.) Measure  $fG$  for individual modes, get  $\sigma_{\ln fG} \geq \sqrt{1/N_m}$ , 11x better than standard RSD!
- Limitation:

No constraint on  $f$ , as  $f$  is still degenerate with the mean  $b$

# Single or multi-tracer RSD



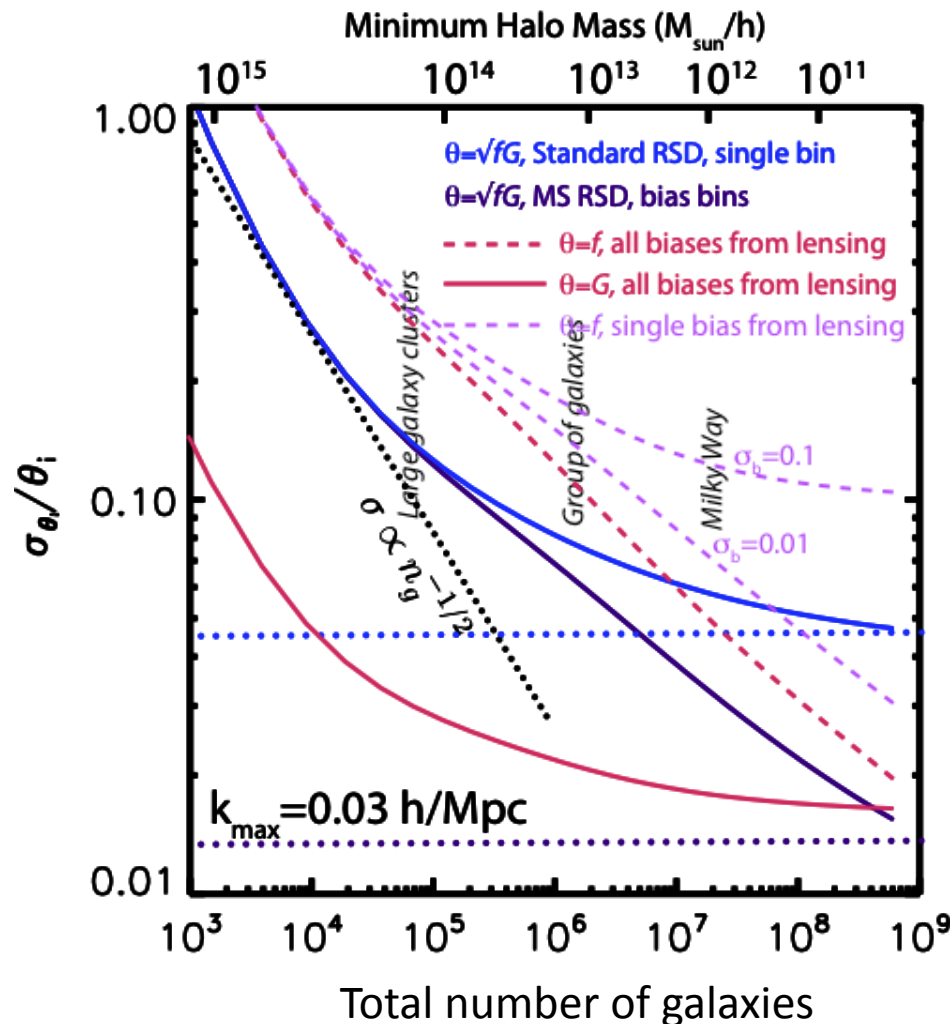
Redshift survey of half the sky at  $z=0.5$ ,  $\Delta z=0.1$ :  $V=2.5h^{-3} \text{ Gpc}^3$ .

Assume:

each main halo host one galaxy

- At  $n < 10^5$ , going for a deeper survey is the same as going wider
- At  $n > 10^5$ , going deeper gains slowly in standard RSD, but more quickly in Multi-tracer RSD

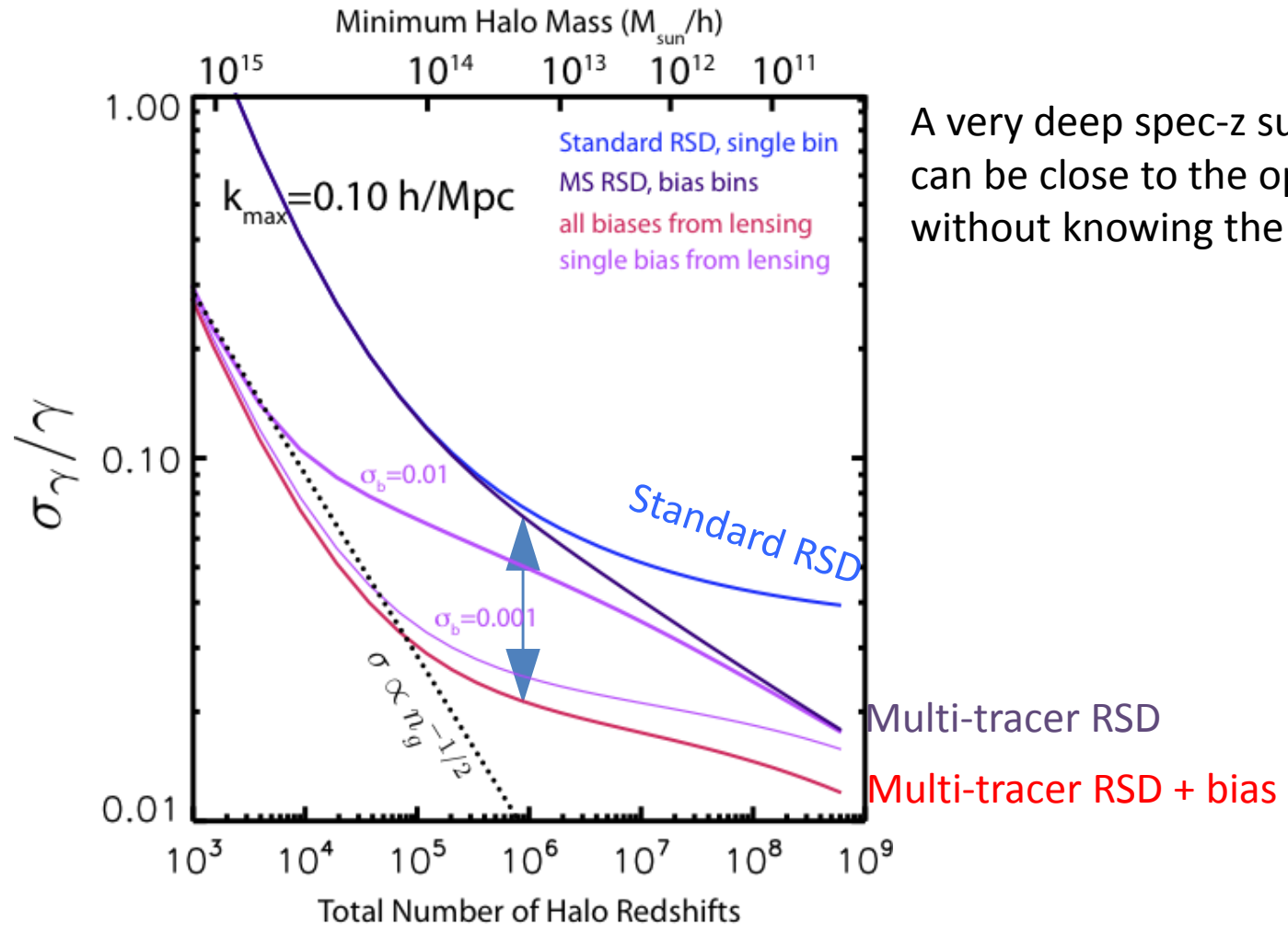
# Constraints with known bias



- Constraint on  $f$  keep raising as a survey goes deeper, can be 5% at  $M_{\text{min}} < 10^{12} M_{\text{Sun}}$
- Constraint on  $G$  is better than  $f$ , but not gaining much for going deeper
- Need high accuracy in measuring  $b$

# Constraints on gravity

$$f(a) \approx \Omega_m^\gamma(a)$$



A very deep spec-z survey can be close to the optimal, without knowing the bias

Knowing bias+RSD is like having 10 Universes to measure!

# Combining lensing with RSD

- A deep lensing (photo-z) survey ( $z \sim 2$ )
- A spec-z survey over the same volume
- Split galaxies into z-bins for both surveys
- Measure  $b$  and  $P$  at each  $z$  from shear-galaxy tomography
- Perform Multi-tracer RSD in each z-bin, with the  $b$  &  $P$  measurement from lensing

# Covariance

$$\mathbf{C} = \begin{bmatrix} \mathbf{C}^{KK} & \mathbf{C}^{gK} \\ (\mathbf{C}^{gK})^T & \mathbf{C}^{gg} \end{bmatrix}$$

(Shape noise)

$$C_{ij}^{KK}(l) = \sum_{k=1}^{\min\{i,j\}-1} A_{ik} A_{jk} \Delta \chi_k P_k(l) F_k^2 + \sigma_\epsilon^2 \delta_{ij} / n_i,$$

$$C_{ij}^{gg}(l) = D_i^{-2} \Delta \chi_i^{-1} P_i(l/D_i) \bar{b}_i^2 \delta_{ij} + \mathcal{N}_i(l), \quad (\text{Stochasticity})$$

$$C_{ij}^{gK}(l) = A_{ji} D_i^{-1} P_i(l/D_i) \bar{b}_i F_i \quad | i < j,$$

Parameters: f, G, b, E

Limits: shape noise & stochasticity

# Optimal weighting of halos

Stochasticity:  $E^2 = \frac{\langle (\delta_m - \hat{\delta}_m)^2 \rangle}{\langle \delta_m^2 \rangle} = 1 - r^2 \sim \left(\frac{\sigma_b}{b}\right)^2$

Mass estimator:  $\hat{\delta}_m \equiv \sum w_i \delta_i = \mathbf{w} \cdot \boldsymbol{\delta}$

$i^{\text{th}}$  halo mass bin:  $\delta_i$ , weight function:  $w_i$

$$\Rightarrow E^2 = 1 - 2\mathbf{b}^T \mathbf{w} + \mathbf{w}^T \mathbf{C} \mathbf{w} / P.$$

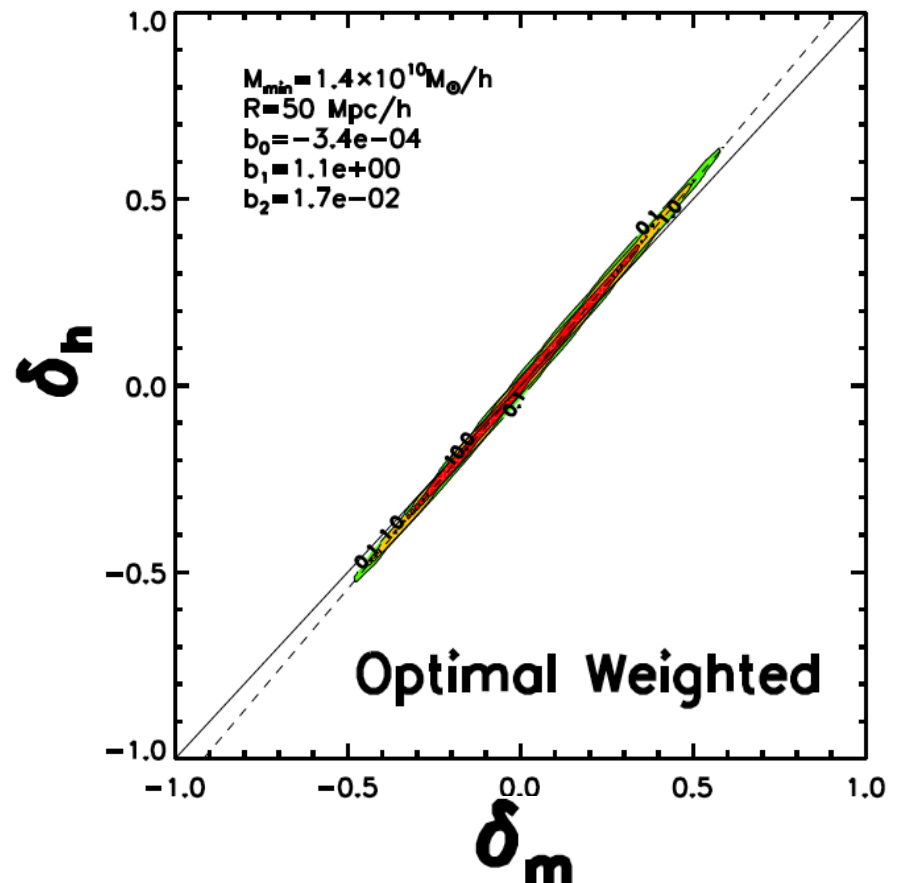
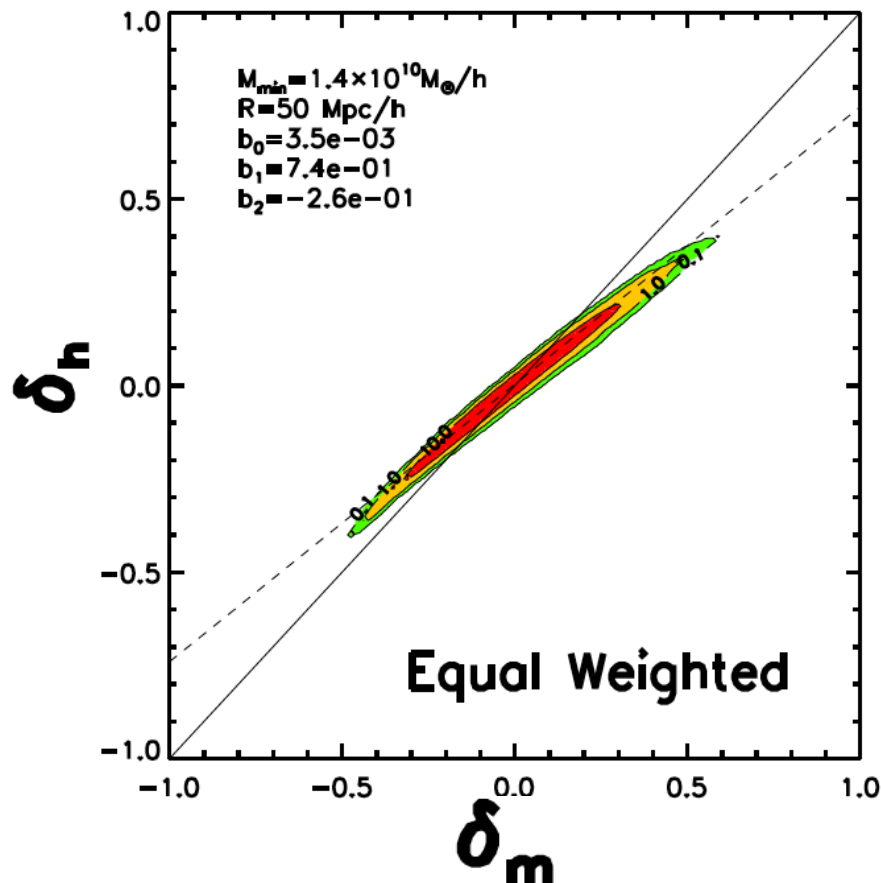
Minimizing  $E^2 \Rightarrow \begin{cases} \mathbf{w}_{\text{opt}} &= (\mathbf{C}/P)^{-1} \mathbf{b} \\ E_{\text{opt}}^2 &= 1 - \mathbf{b}^T (\mathbf{C}/P)^{-1} \mathbf{b}. \end{cases}$

Halo bias:  $b_i = \langle \delta_m \delta_i \rangle / P$ ,  $\langle \delta_i \delta_j \rangle = C_{ij}$ .

Mass power spectrum:  $P = \langle \delta_m^2 \rangle$

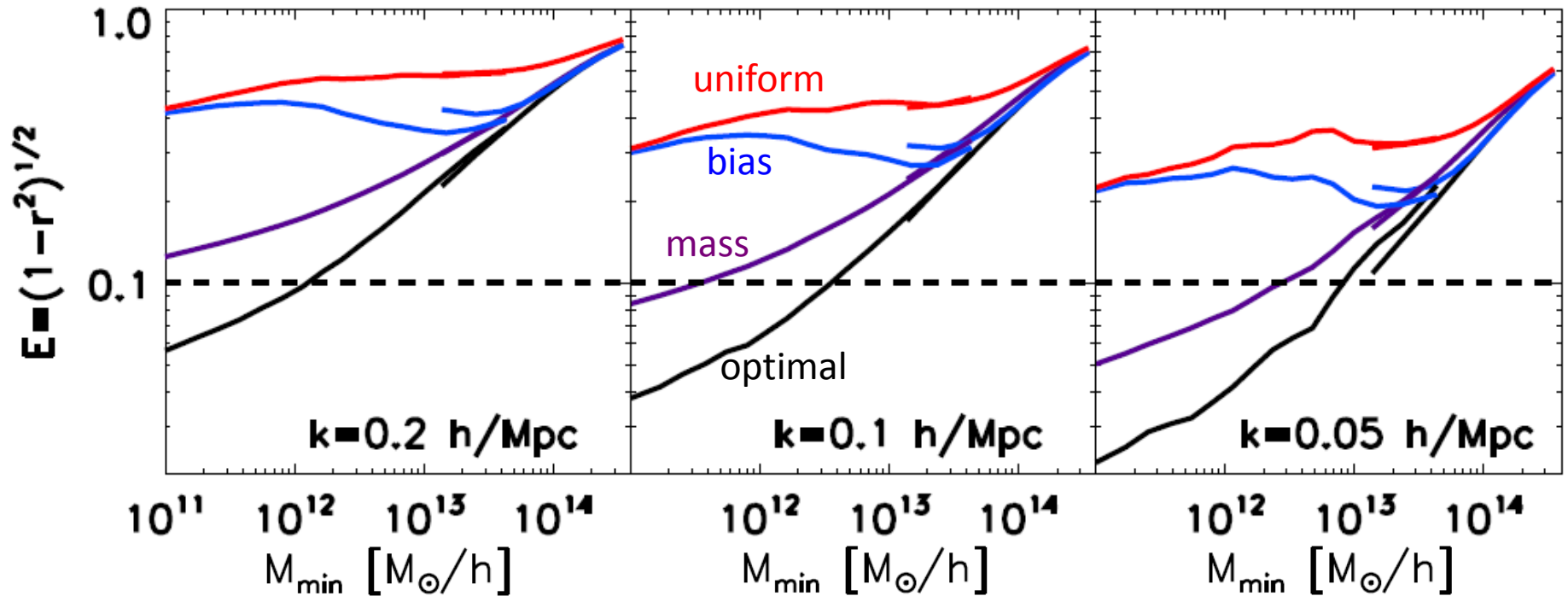
# no weighting VS optimal weighting

$M_{\min} = 1.4 \times 10^{10} M_{\odot}/h$  , smoothed at  $R = 50 \text{ Mpc}/h$



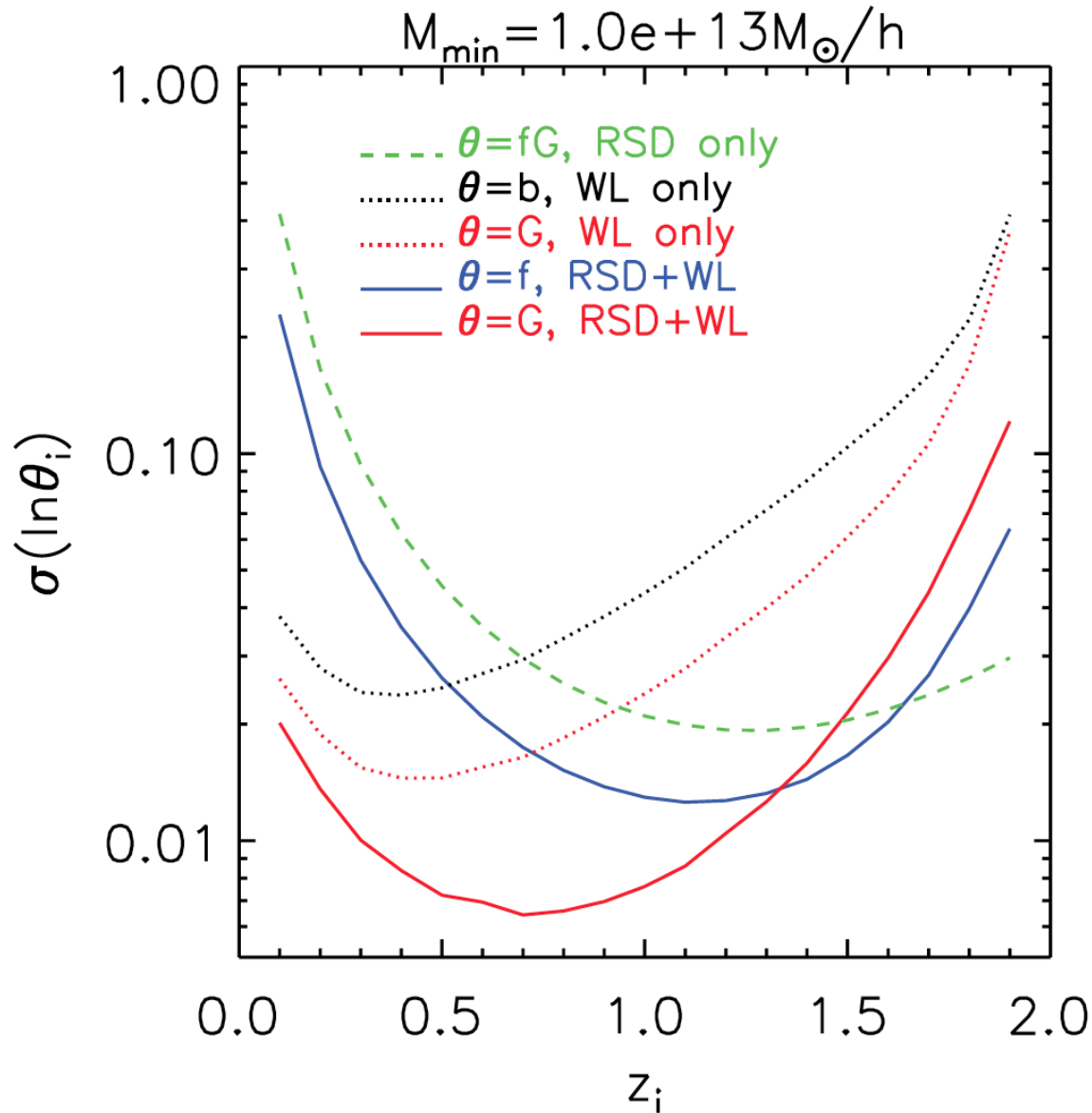


# Minimal $E$ from weighting halos



- Bias, mass or equal weighting is not the optimal
- $E_{\text{opt}}$  can be significantly lower than 10% ( $r=0.995$ )

# Tomographic constraint on $f$ , $G$ & $b$

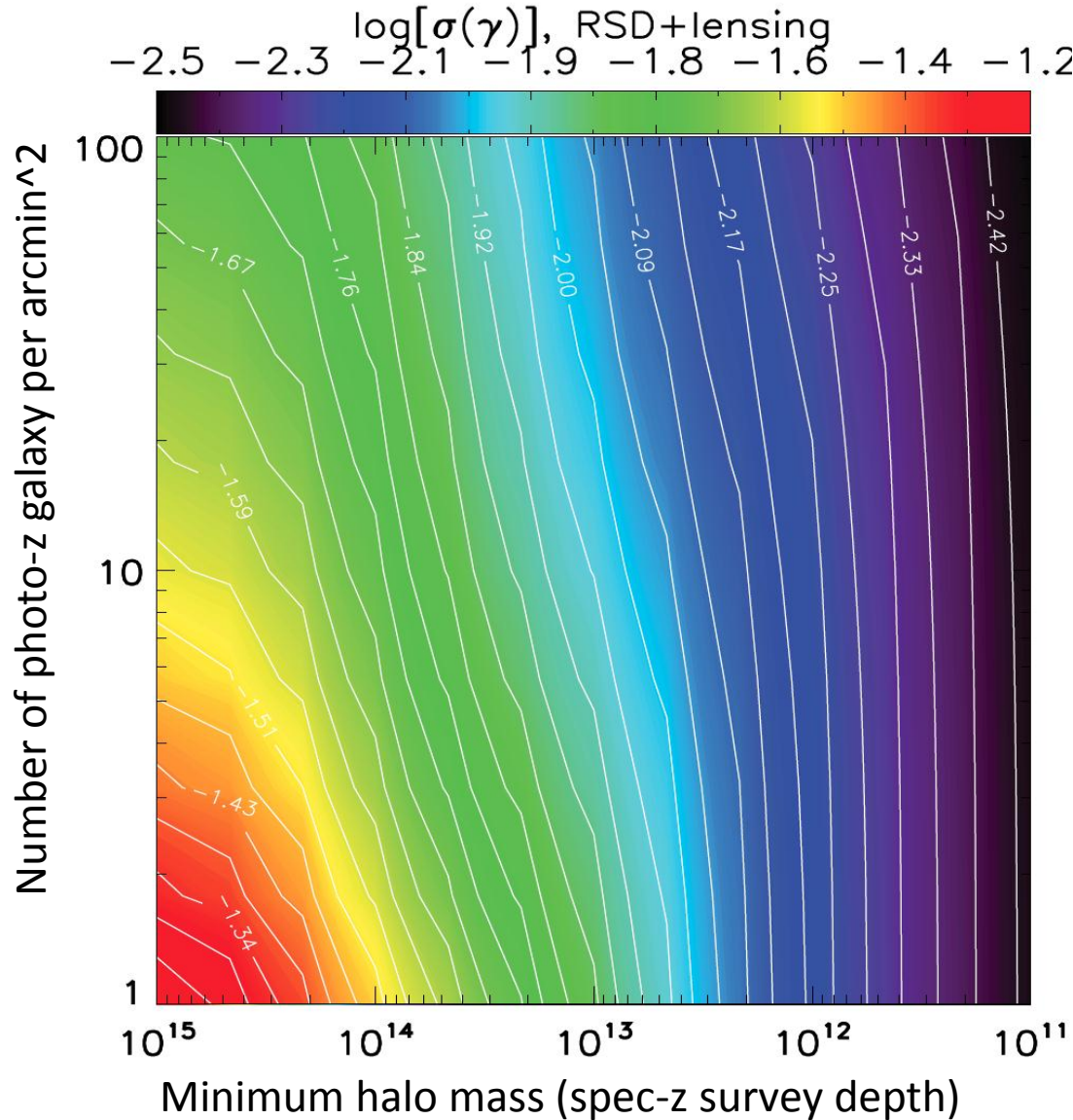


20  $z$ -bins,  $z_{\max} = 2.0$   
 $k_{\max} = 0.1 h/\text{Mpc}$

Percent-level  
Constraint is  
possible when  
LRGs are resolved

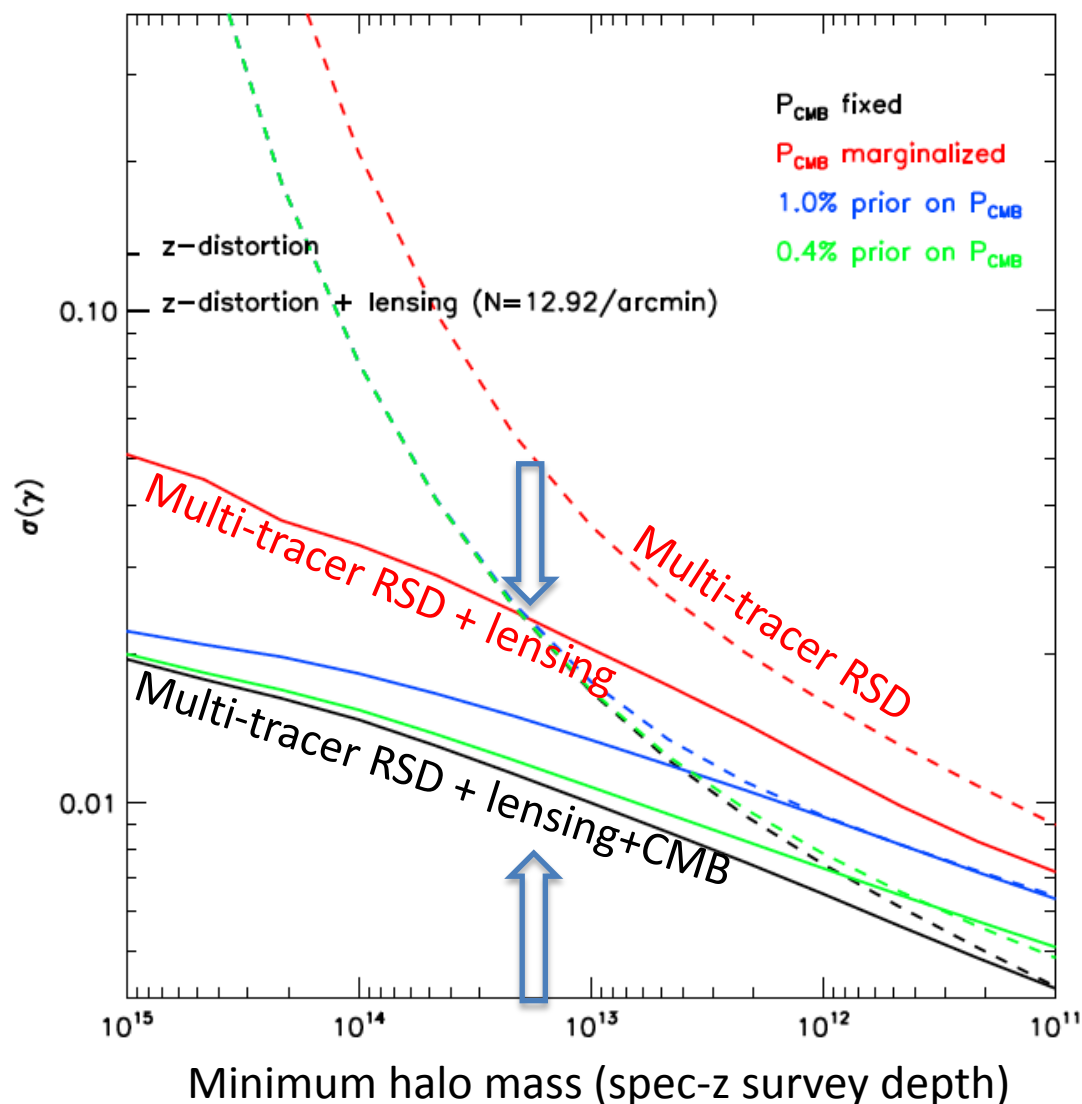
# Constraints on gravity (Lensing + RSD)

$$f(a) \approx \Omega_m^\gamma(a)$$



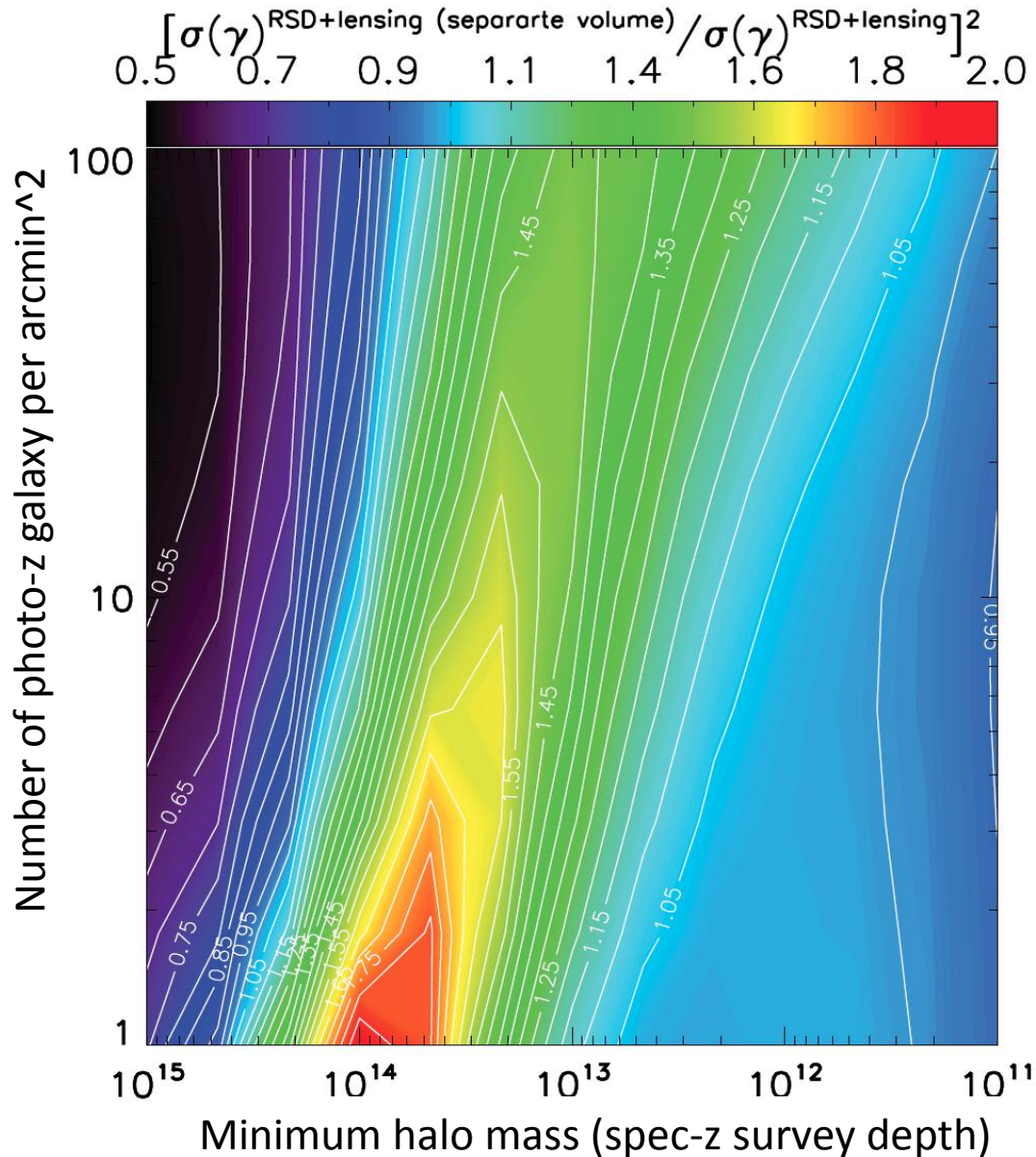
When spec-z is deep, additional lensing survey is not useful

# Joint constraint on gravity



- Lensing tomography helps most for sparse spec-z survey, if multi-tracers are available
- When LRGs are resolved, a factor of 2 – 3 improvement can be achieved
- Having CMB measurement can help to improve the constraint by a factor of 2

# Overlap VS separate sky



A factor of 1.5 increase in the survey volume when two surveys overlaps

# Summary

- Measurements on the growth are severely limited by the finite volume of the observable Universe
- Multi-tracer of the same field help to improve for deep survey
- Combining redshift surveys with gravitational lensing provides, in principle, limitless precision in a finite volume.
- High accuracy measurements of  $b$  is crucial for tightening the constraint on growth, bias stochasticity need to be concerned
- Combined lensing/RSD survey is much better than RSD alone
- Overlapping sky is in general more powerful than separate sky in constraining growth, but the difference is small/moderate. The level of improvement depends on the design of surveys.