Probing Dark Energy with Weak Lensing and Redshift Space Distortions

Molly Swanson

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Outline

Fun with Assumptions

Constructing a metric theory of gravity w/o assuming GR

What GR assumptions are we testing with WL+RSD?

Stacked Weak Lensing

Measuring the potential followed by light

Redshift Space Distortions

Measuring the potential followed by galaxies

Testing GR

A null hypothesis test statistic

Ideas for an estimator with minimal assumptions

Issues

Potential pitfalls and caveats

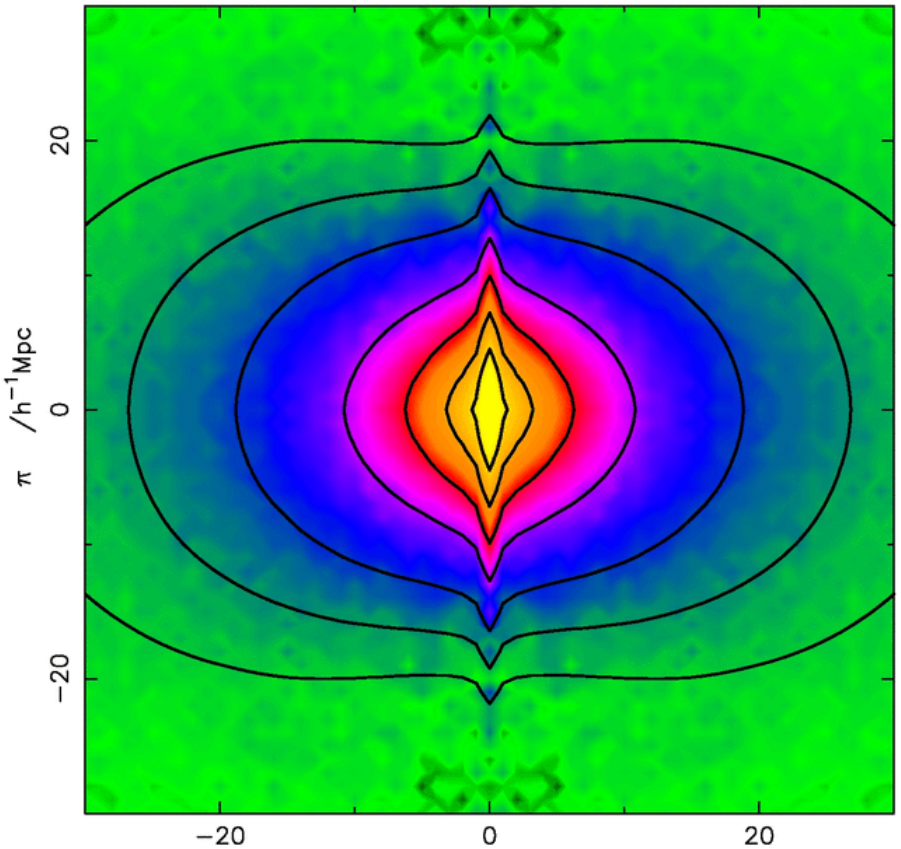
Conclusions

Testing General Relativity

- General metric theory of gravity has two distinct potentials
- Newtonian potential Ψ, curvature potential Φ. GR: $\Phi = \Psi$.

Weak lensing: sensitive to $\Phi + \Psi$ **Peculiar velocities**: sensitive to Ψ

Hawkins et al. (2002), astro−ph/0212375
2dFGRS: β = 0.49 ± 0.09



Goals

- Lay out assumptions of metric theory of gravity
- Understand where GR assumptions enter
- Define observables from stacked weak lensing (SWL) and redshift space distortions (RSD)
- Construct a test comparing SWL and RSD on same sky
- Null hypothesis test statistic E_G for GR assumptions
- Construct an estimator for E_G with minimal assumptions
- Figure out how well DESpec could measure it

Perturbation Theory Equations

Perturbed FLRW metric in Newtonian gauge:

$$ds^2 = -\left[1 + 2\Psi\left(t, \mathbf{x}\right)\right] dt^2 + a\left(t\right)^2 \left[1 - 2\Phi\left(t, \mathbf{x}\right)\right] \left[d\chi^2 + r\left(\chi\right)^2 d\Omega^2\right]$$

Assumptions:

- Vector and tensor modes can be neglected
- Φ and $\Psi \ll 1$
- At zeroth order ($\Phi = \Psi = 0$) the universe is homogenous and isotropic.
- Haven't assumed GR

Conservation of Stress-Energy

$T^{\mu}{}_{\nu;\mu}(t, \mathbf{x}) = 0$ for a fluid characterized by density, pressure, anisotropic stress, and peculiar velocity leads to:

$$\dot{\delta} = -(1+w)\frac{\theta}{a} - 3H\left(\frac{\delta P}{\bar{\rho}} + w\delta\right)$$

$$\dot{\theta} = -H(1-3w)\theta - \frac{\dot{w}}{1+w}\theta - \frac{1}{a}\nabla^2\left(\frac{\delta P/\bar{\rho}}{1+w} - \sigma + \Psi\right)$$

δ is density perturbation, δP is pressure perturbation, σ is anisotropic stress, θ is divergence of peculiar velocity $\mathbf{v}$, $H \equiv \dot{a}/a$, $w \equiv \bar{P}/\bar{\rho}$.

Applies to δ_X and θ_X for uncoupled fluid component X.

Assumptions:

- Energy and momentum are locally conserved.
- $\mathbf{v} \ll c$, δ and $\delta P \ll 1$, no vorticity ($\boldsymbol{\nabla} \times \mathbf{v} = 0$).
- In quasi-static ($\frac{\theta}{a} \gg \dot{\Phi}$), sub-horizon ($k/aH \gg 1$) regime.
- Still haven't assumed GR

Adding in GR

Okay, what happens when we finally add GR?
Apply Einstein Equations $G^{\mu}{}_{\nu} = 8\pi G T^{\mu}{}_{\nu}$
Zeroth order: Friedmann Equation

$$H^2 = \frac{8\pi G}{3}\bar{\rho}_t - \frac{K}{a^2}$$

First order: Poisson and Stress Equations

$$\begin{aligned}\left(\nabla^2 + 3K\right)\Phi &= 4\pi G a^2 \bar{\rho}_t \left(\delta_t + 3\left(1 + w_t\right) H a \nabla^{-2}\theta_t\right) \\ &\approx 4\pi G a^2 \bar{\rho}_t \delta_t \\ -\left(\nabla^2 + 3K\right)\left(\Phi - \Psi\right) &= 12\pi G a^2 \left(1 + w_t\right)\bar{\rho}_t \sigma_t\end{aligned}$$

These apply to total fluid δ_t, not to individual components δ_X.

General Case

Fully generalize:

Zeroth order: Friedmann Equation

$$H^2 = \frac{8\pi G_F(a)}{3}\bar{\rho}_t - \frac{K}{a^2}$$

First order: Poisson and Stress Equations

$$\begin{aligned}\left(\nabla^2 + 3K\right)\Phi &= 4\pi G_P(a,\mathbf{x})\, a^2 \bar{\rho}_t \left(\delta_t + 3(1+w_t)\, Ha\nabla^{-2}\theta_t\right) \\ &\approx 4\pi G_P(a,\mathbf{x})\, a^2 \bar{\rho}_t \delta_t\end{aligned}$$

$$-\left(\nabla^2 + 3K\right)\left(\Phi - \eta(a,\mathbf{x})\,\Psi\right) = 12\pi G_S(a,\mathbf{x})\, a^2 (1+w_t)\, \bar{\rho}_t \sigma_t$$

Simplify General Case

Assume flatness ($K = 0$) and make some simplifications:

Zeroth order: Friedmann Equation

Take H, a to match ΛCDM

First order: Poisson and Stress Equations

$$\nabla^2 \Phi = 4\pi G_{\text{eff}}\left(t, \mathbf{x}\right) a^2 \bar{\rho}_m \delta_m$$

Dark energy clustering has been absorbed into G_{eff}.

$$-\nabla^2 \left(\Phi - \eta_{\text{eff}}\left(a, \mathbf{x}\right) \Psi\right) = 0$$

Anisotropic stress has been absorbed into η_{eff}.

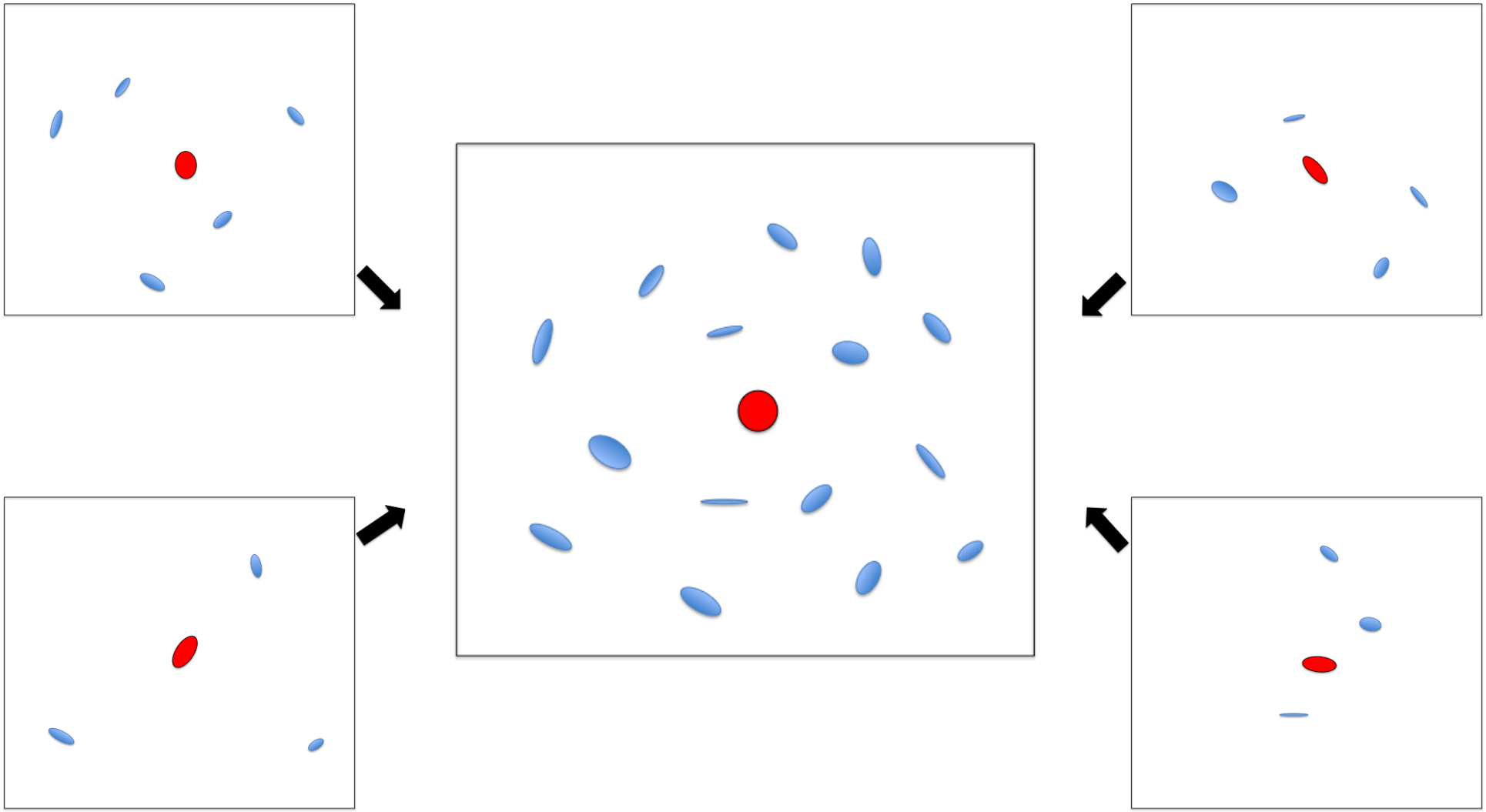
Combine with conservation equations to get growth equation:

$$\ddot{\delta}_m + 2H\delta_m - 4\pi\bar{\rho}_m \frac{G_{\text{eff}}}{\eta_{\text{eff}}}\delta_m = 0$$

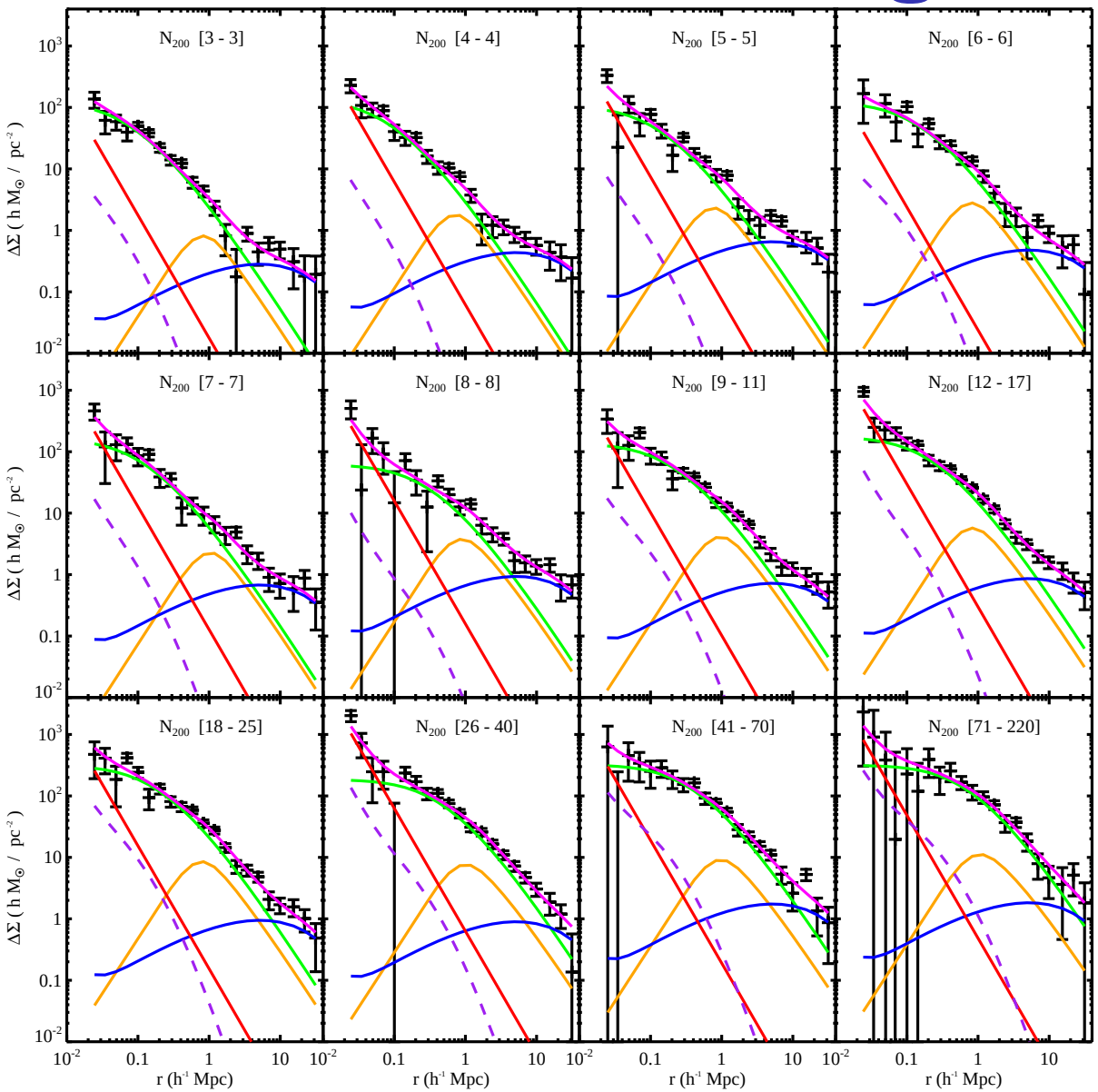
Solution is $\delta_m\left(a, \mathbf{x}\right) = D\left(a\right)\delta_0\left(\mathbf{x}\right)$. Growth factor $f \equiv \frac{d\ln D}{d\ln a}$.

Keep track of G_{eff}, η_{eff}, and f for the rest of the talk!

Stacked Weak Lensing



Stacked Weak Lensing in SDSS-II



Johnston et al 2008 0709.1159

Average cluster shear profile in richness bins

Model:
NFW profile
central BCG
neighboring halos
orange: miscentering
dashed: nonlinearity
TOTAL

see Johnston et al, Sheldon et al papers

Stacked Weak Lensing Observable

Observe average tangential shear in annulus:

$$\gamma_T(R) = \bar{\kappa}(< R) - \kappa(R)$$

Convergence κ:

$$\kappa = \frac{1}{2}\int_0^{z_s} \frac{dz}{H(z)} \frac{\chi(z)(\chi(z_s) - \chi(z))}{\chi(z_s)} \nabla_{2D}^2 (\Phi + \Psi)$$

GR assumptions enter when we relate κ to 2D density Σ :

$$\kappa = \Sigma(R)/\Sigma_{\rm crit}$$

$$\Sigma_{\rm crit} \equiv \frac{c^2}{4\pi \tilde{G}_{\rm eff}} \frac{D_s}{D_l D_{ls}}, \text{ where } \tilde{\rm G}_{\rm eff} \equiv \frac{1}{2}\left(1 + \frac{1}{\eta_{\rm eff}}\right) {\rm G}_{\rm eff}$$

Stacked density profile

Mean projected density Σ_{lm} of mass around a lens population l is projected lens-mass cross-correlation function:

$$\Sigma_{lm}(R) = \bar{\rho}_m \int_{-\infty}^{\infty} dx_3 \xi_{lm}(r) \equiv \bar{\rho}_m w_{lm}(R)$$

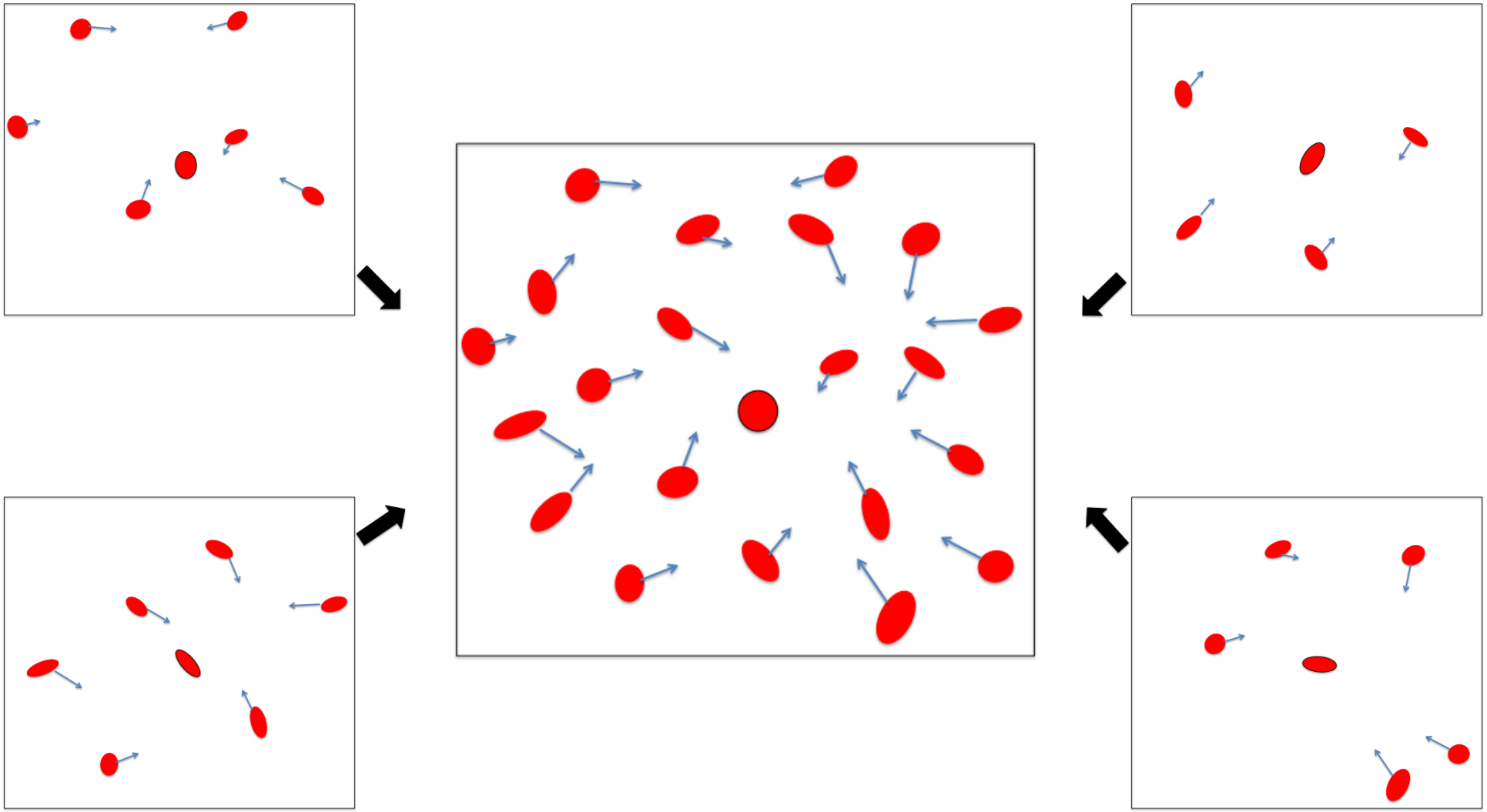
Tangential shear observable is $\Delta\Sigma_{lm}(R)$:

$$\Delta\Sigma_{lm}(R) = \bar{\Sigma}_{lm}(< R) - \Sigma_{lm}(R)$$

Expected value of observable:

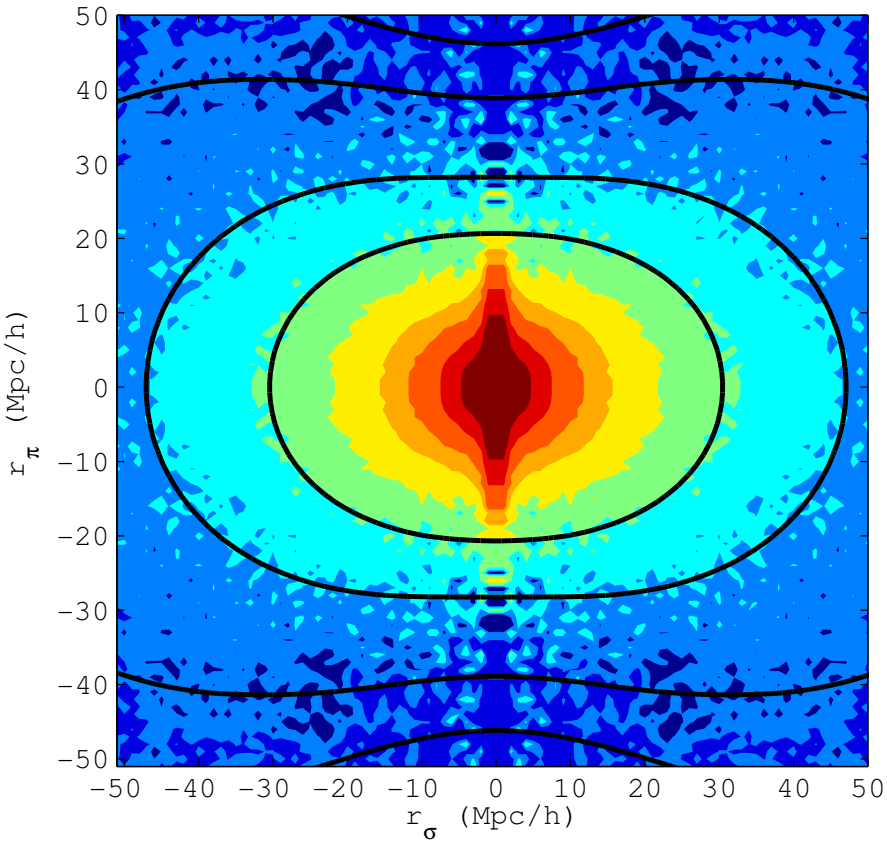
$$\langle \Delta\Sigma_{lm}(R) \rangle = \frac{\tilde{G}_{\mathrm{eff}}}{G} \bar{\rho}_m \left[\bar{w}_{lm}(< R) - w_{lm}(R) \right]$$

Redshift Space Distortions



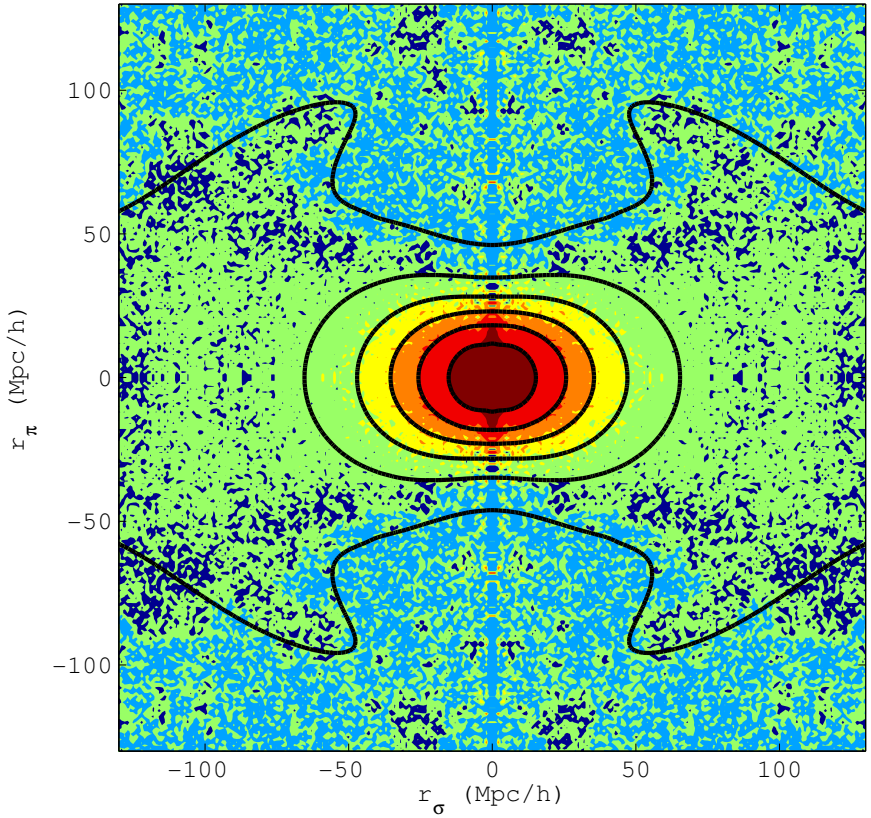
Recent RSD measurements with BOSS

small scales



Reid et al 2012 1203.6641

large scales



Reid et al 2012 1203.6641

RSD observable

2D galaxy power spectrum:

$$P^s_{gg}(k,\,\mu_k) = P_{gg}(k) + 2\mu_k^2 P_{g\Theta}(k) + \mu_k^4 P_{\Theta\Theta}(k)$$

where $\mu_k \equiv \hat{\mathbf{k}} \cdot \hat{\mathbf{n}}$ is cosine of angle between $\mathbf{k}$ and the line of sight, $\Theta \equiv -\theta_g/aH$ is rescaled velocity divergence.

Aside: Kaiser limit: if $\Theta = f\delta_m$ and $\delta_g = b\delta_m$,

$$P^s_{gg}(k,\,\mu_k) = \left(b + \mu_k^2 f\right)^2 P_{mm}(k).$$

Translating back into position space (Hamilton 1992) gives

$$\xi^s_{gg}(r,\mu) = \xi_0(r)\,\mathcal{P}_0(\mu) + \xi_2(r)\,\mathcal{P}_2(\mu) + \xi_4(r)\,\mathcal{P}_4(\mu)$$

$\mathcal{P}_\ell(\mu)$ are Legendre polynomials and $\xi_\ell(r)$ are moments of $\xi^s_{gg}(r,\mu)$. (monopole, quadrupole, hexadecapole)

Measuring Legendre Polynomial moments

Relate moments to g, Θ correlation functions:

$$\xi_0(r) = \xi_{gg}(r) + \frac{2}{3}\xi_{g\Theta}(r) + \frac{1}{5}\xi_{\Theta\Theta}(r)$$

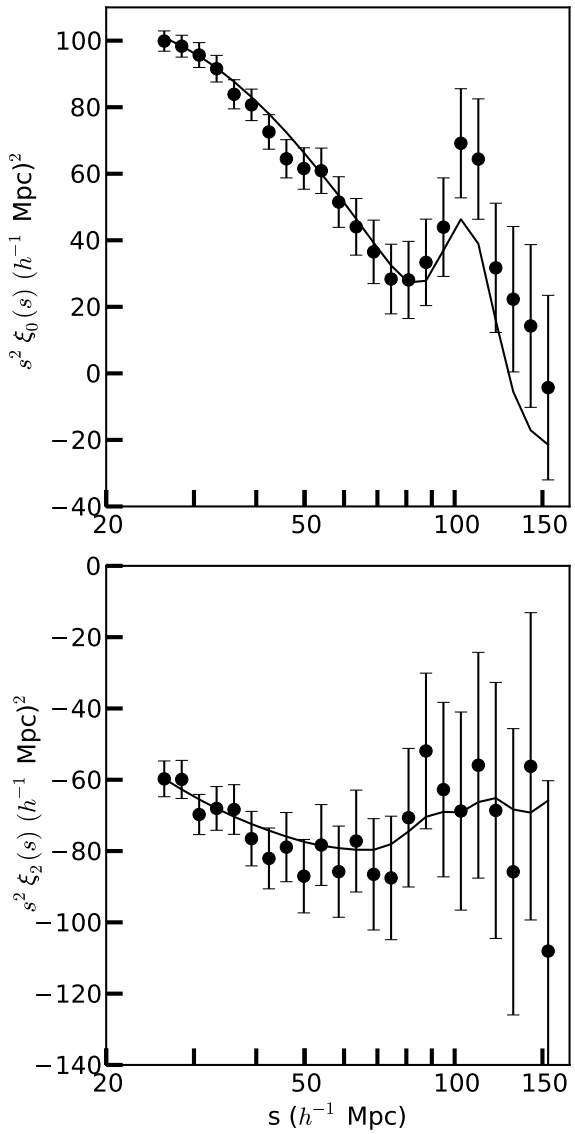
$$\xi_2(r) = \frac{4}{3}\left[\xi_{g\Theta}(r) - \bar{\xi}_{g\Theta}(r)\right] + \frac{4}{7}\left[\xi_{\Theta\Theta}(r) - \bar{\xi}_{\Theta\Theta}(r)\right]$$

$$\xi_4(r) = \frac{8}{35}\left[\xi_{\Theta\Theta}(r) + \frac{5}{2}\bar{\xi}_{\Theta\Theta}(r) - \frac{7}{12}\bar{\bar{\xi}}_{\Theta\Theta}(r)\right]$$

$$\bar{\xi}(r) \equiv 3r^{-3}\int_0^r \xi(r')\, r'^2 dr'$$

$$\bar{\bar{\xi}}(r) \equiv 5r^{-5}\int_0^r \xi(r')\, r'^4 dr'$$

Reid et al 2012 measure monopole and quadrupole directly→



Reid et al 2012 1203.6641

Recast RSD into SWL-type expression

Multipoles can be combined into estimator $\hat{\xi}_{g\Theta}(r)$ for galaxy-velocity cross-correlation. Project into 2D plane:

$$\hat{w}_{g\Theta}(R) = 2\int_R^\infty \hat{\xi}_{g\Theta}(r)\left(r^2 - R^2\right)^{-1/2} rdr$$

and define $\Delta w_{g\Theta}$ in analogy with $\Delta\Sigma_{lm}$:

$$\Delta w_{g\Theta} \equiv \bar{\hat{w}}_{g\Theta}(< R) - \hat{w}_{g\Theta}(R)$$

Haven't assumed GR yet! GR comes in when we relate Θ back to mass via $\Theta = f\delta_m$. f comes from growth equation - can be different under modified gravity.

Plugging this in: [assuming galaxy velocity traces DM velocity]

$$\langle \Delta w_{g\Theta} \rangle = f\left[\bar{w}_{gm}(< R) - w_{gm}(R)\right]$$

Null Hypothesis Test Statistic

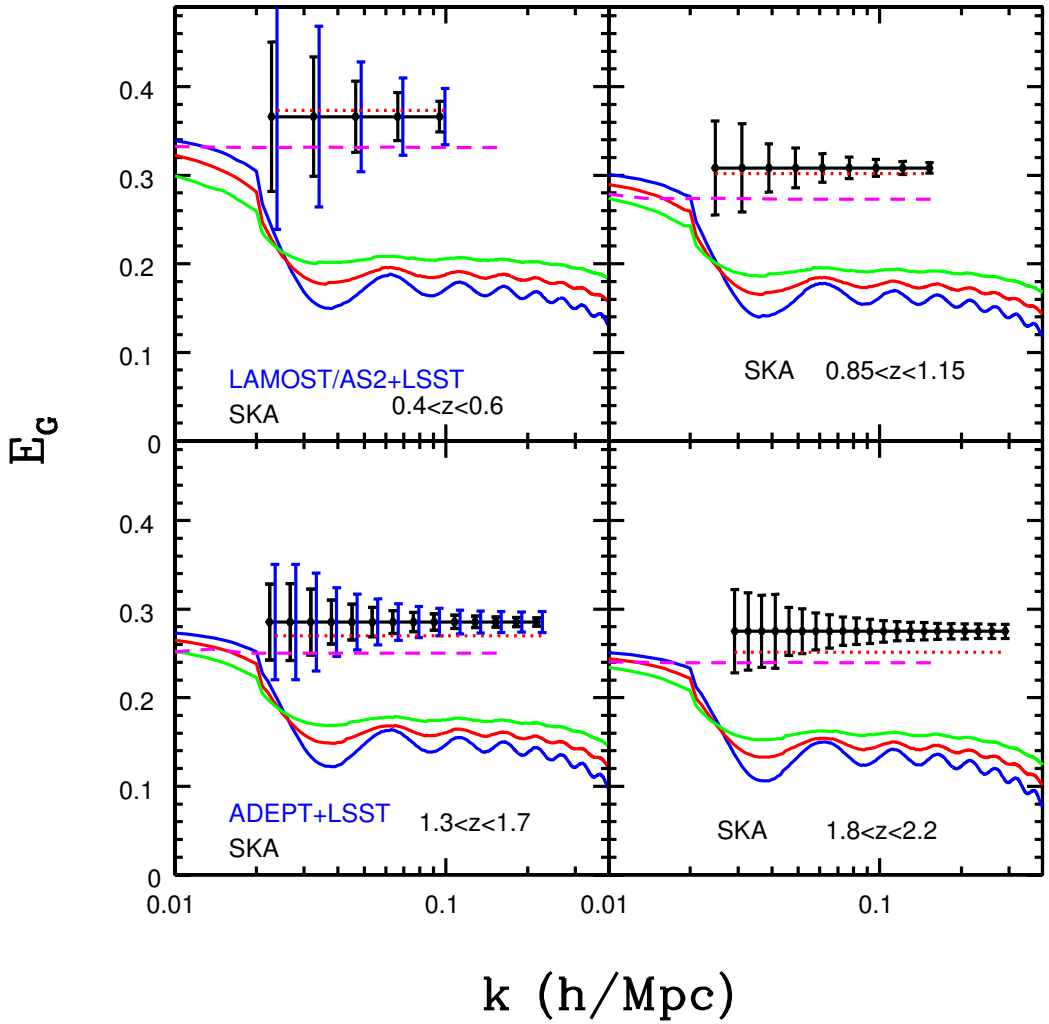
Zhang et al 2007 (0704.1932) define a test of GR by comparing these 2 methods:

- Estimate $P_{\nabla^2(\Phi+\Psi)g} \equiv \langle \nabla^2 (\Phi + \Psi) \delta_g \rangle$ from lensing
  - $\nabla^2 (\Phi + \Psi) = \left(\tilde{G}_{\rm eff}/G \right) 3H_0^2 a^{-1} \Omega_{m0} \delta_m$
- Estimate $P_{g\Theta} \equiv \langle \delta_g \Theta \rangle$ from redshift space distortions
  - $\Theta = f \delta_m$
- Define ratio E_G :

$$E_G \equiv \frac{P_{\nabla^2(\Phi+\Psi)g}}{3H_0^2 a^{-1} P_{g\Theta}} \sim \frac{\tilde{G}_{\rm eff} \Omega_{m0}}{Gf}$$

- δ_g - and thus all galaxy bias ugliness - cancels out

Forecasts



Zhang et al 2007 0704.1932

Null hypothesis:
$E_G = \Omega_{m0\Lambda\mathrm{CDM}}/f(z)_{\Lambda\mathrm{CDM}}$
If not, then GR is wrong!
OR...

- DE is clustered or has anisotropic stress
- DE and DM are coupled
- There is velocity bias
- ...

Black line: ΛCDM
Dotted: flat DGP
Dashed: $f(r)$
Colored: TEVES

To do: DESpec forecast

Application to SDSS-II

Reyes et al 2010 (1003.2185) applied this to SDSS-II data with an annulus method:

$$\hat{E}_G^{\mathrm{Reyes}}(R) \equiv \frac{1}{\beta}\frac{\Upsilon_{gm}(R)}{\Upsilon_{gg}(R)}$$

$\Upsilon_{gm}(R)$ is $\Delta\Sigma_{gm}(R)$ from SWL with scales $R < R_0$ excised:

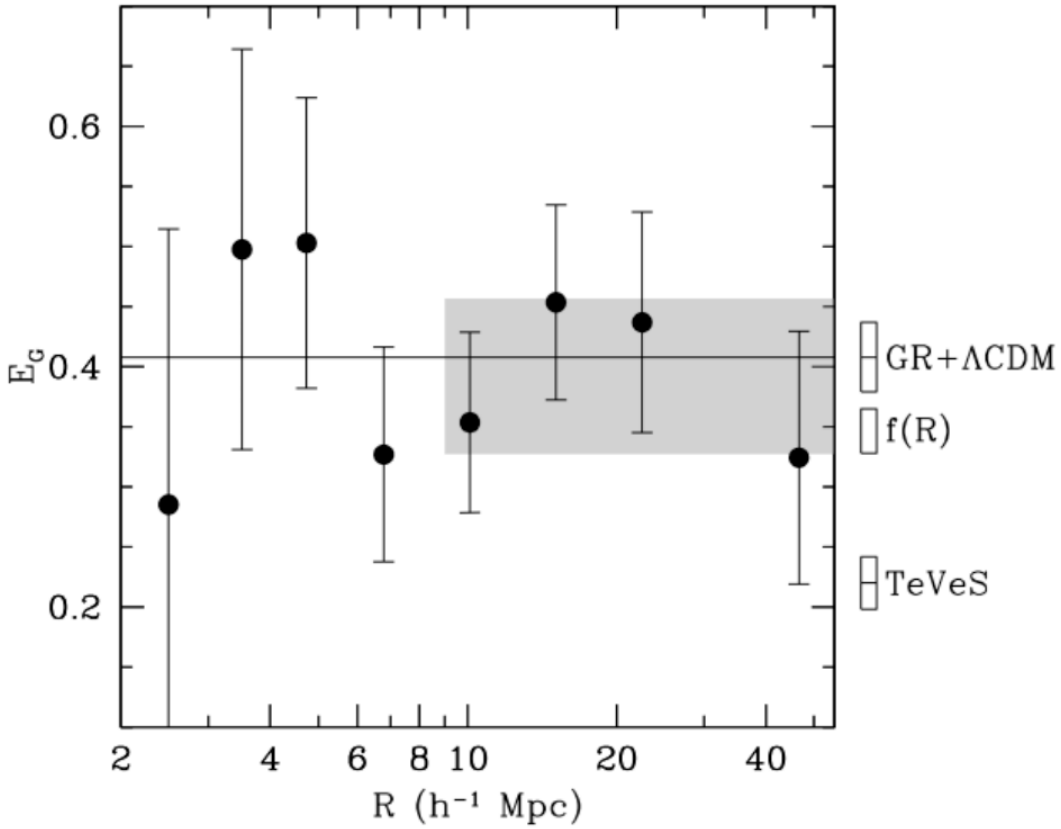
$$\Upsilon_{gm}(R) \equiv \Delta\Sigma_{gm}(R) - \left(\frac{R_0}{R}\right)^2 \Delta\Sigma_{gm}(R_0)$$

$\Upsilon_{gg}(R)$ is defined similarly using gg projected corrfunc:

$$\Upsilon_{gg}(R) \equiv \Delta w_{gg}(R) - \left(\frac{R_0}{R}\right)^2 \Delta w_{gg}(R_0)$$

β is RSD parameter f/b from LRG $P(k)$ (Tegmark et al 2006)

Results for E_G from SDSS-II



Reyes et al 2010 1003.2185

$E_G^{\Lambda\mathrm{CDM}} = 0.408 \pm 0.029$
at $z = 0.32$
(mean survey redshift)

$\hat{E}_G^{\mathrm{Reyes}}(R) = 0.392 \pm 0.065$
on scales of $10 - 50\, h^{-1}$ Mpc

Consistent with GR!

Building from Reyes et al method

How can we improve upon the Reyes et al measurement for applying this test to DESpec data?
$\hat{E}_G^{\mathrm{Reyes}}(R)$ bundles several assumptions into $\beta = f/b$.
$\beta w_{gg}(R) = w_{g\Theta}(R)$ if:

- $w_{gg}(R)$ and β are measured for the same galaxies. (Reyes et al use similarly selected LRGs for both.)
- galaxy bias b is not stochastic or scale-dependent.
- galaxy bias b measured from $P(k)$ analysis cancels perfectly with b from $w_{gg}(R)$ (not true if b is scale-dependent).
- all of the assumptions in the $P(k)$ analysis are valid.

These are $< 5 - 10\%$ effects vs. Reyes et al 15% error bars.

Minimal assumption statistic

Can we do better? Try to build an $\hat{E}_G(R)$ without making these assumptions. (Work in progress)
Combine multipoles of $\xi^s_{gg}(r,\mu)$ to get estimator for $\xi_{g\Theta}(r)$:

$$\hat{\xi}_{g\Theta}(r) \equiv \frac{3}{4}\xi_2(r) - \frac{15}{8}\xi_4(r) - \frac{3}{16}\int_r^\infty \left[12\xi_2(r') - 175\left(\frac{r}{r'}\right)^2 \xi_4(r') + 75\xi_4(r')\right]\frac{dr'}{r'}$$

Project along line of sight to get $\hat{w}_{g\Theta}(R)$ and integrate within radius to match SWL:

$$\Delta w_{g\Theta} \equiv \bar{\hat{w}}_{g\Theta}(<R) - \hat{w}_{g\Theta}(R)$$

Now we can construct $\hat{E}_G(R) \equiv \frac{\Upsilon_{gm}(R)}{\Upsilon_{g\Theta}(R)}$ without β.

Potential pitfalls and caveats

- How feasible would this be with DESpec data?
  - Can we measure $\xi_4(r)$ well enough to integrate over it 4 times?
  - Can we get around this using cleverly-weighted sums of pairs? (e.g. Reid et al 2012's technique for $\xi_1(r)$ and $\xi_2(r)$)
  - Can SWL lensing measurements go to large enough scales to be in linear regime?
- Can we use clusters as the lens population?
  - $\left\langle \hat{\xi}_{g\Theta} \right\rangle = \frac{1}{2}\left(\xi_{g\Theta} + \xi_{c\Theta}\right)$ doesn't cancel elegantly with $\Delta\Sigma_{cm}$.
- Degeneracies with Alcock-Paczynski or magnification bias?
- Velocity bias! Can't get rid of it. Test independently?

Conclusions

- Combining stacked weak lensing and redshift space distortions provides powerful test of GR
- Nice results from SDSS-II already
- DESpec would be an excellent dataset to do such a test
- Promising ideas for generating observables with minimal assumptions about galaxy bias
- ... but can't get rid of velocity bias assumption
- Still a lot of work to to!