#### **Reactor Antineutrino Fluxes**

Patrick Huber

Center for Neutrino Physics – Virginia Tech

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#### Motivation

- Nuclear reactors are the brightest available neutrino source ⇒ a large number of past and present experiments
- Recently, reactor neutrino fluxes have been re-evaluated and a 3% upward shift was found Mueller *et al.*, Phys.Rev. C83 (2011) 054615.
- Which in turn implies a reactor neutrino anomaly Phys.Rev. D83 (2011) 073006.
- Double Chooz initially is a single detector experiment





#### **Fission yields of** $\beta$ **emitters**



#### **Neutrinos from fission**

#### $^{235}U + n \to X_1 + X_2 + 2n$

with average masses of  $X_1$  of about A=94 and  $X_2$  of about A=140.  $X_1$  and  $X_2$  have together 142 neutrons. The stable nuclei with A=94 and A=140 are  $\frac{94}{40}Zr$  and  $^{140}_{58}Ce$ , which together have only 136 neutrons. Thus 6  $\beta$ -decays will occur, yielding 6  $\overline{\nu}_e$ . About 2 will be above inverse  $\beta$ -decay threshold. How does one compute the number and spectrum of neutrinos above inverse  $\beta$ -decay threshold?

#### **Neutrinos from fission**

For a single branch energy conservation implies a one-to-one correspondence between  $\beta$  and  $\overline{\nu}$  spectrum.

However, here there are about 500 nuclei and 10 000 individual  $\beta$ -branches involved; many are far away from stability.

Direct  $\beta$  spectroscopy of single nuclei never will be complete, and even then one has to untangle the various branches

 $\gamma$  spectroscopy yields energy levels and branching fractions, but with limitations, *cf.* pandemonium effect

#### $\beta$ -decay – Fermi theory

$$N_{\beta}(W) = K \underbrace{p^2(W - W_0)^2}_{\text{phase space}} F(Z, W) ,$$

where  $W = E/(m_e c^2) + 1$  and  $W_0$  is the value of Wat the endpoint. K is a normalization constant. F(Z, W) is the so called Fermi function and given by

 $F(Z,W) = 2(\gamma+1)(2pR)^{2(\gamma-1)}e^{\pi\alpha ZW/p}\frac{\left|\Gamma(\gamma+i\alpha ZW/p)\right|^2}{\Gamma(2\gamma+1)^2}$ 

 $\gamma = \sqrt{1 - (\alpha Z)^2}$ 

The Fermi function is the modulus square of the electron wave function at the origin.

#### **Corrections to Fermi theory**

 $N_{\beta}(W) = K p^2 (W - W_0)^2 F(Z, W) L_0(Z, W) C(Z, W) S(Z, W)$  $\times G_{\beta}(Z, W) (1 + \delta_{WM} W).$ 

The neutrino spectrum is obtained by the replacements  $W \rightarrow W_0 - W$  and  $G_\beta \rightarrow G_\nu$ . All these correction have been studied 15-30 years ago.

#### Weak magnetism & $\beta$ -spectra

 $g_M$  is call weak magnetism and the question is how it manifests itself in nuclear  $\beta$ -decay. Nuclear structure effects can be summarized by the use of appropriate form factors  $F_X^N$ .

The weak magnetic nuclear,  $F_M^N$  form factor by virtue of CVC is given in terms of the analog EM form factor as

 $F_M^N(0) = \sqrt{2}\mu(0)$ 

The effect on the  $\beta$  decay spectrum is given by

$$1 + \delta_{WM} W \simeq 1 + \frac{4}{3M} \frac{F_M^N(0)}{F_A^N(0)} W$$

# **Impulse approximation**

In the impulse approximation nuclear  $\beta$ -decay is described as the decay of a free nucleon inside the nucleus. The sole effect of the nucleus is to modify the initial and final state densities.

In impulse approximation

 $F_M^N(0) = \mu_p - \mu_n \simeq 4.7$  and  $F_A^N(0) = C_A \simeq 1.27$ , and thus

 $\delta_{WM} \simeq 0.5\% \,\mathrm{MeV}^{-1}$ 

This value, in impulse approximation, is universal for all  $\beta$ -decays since it relies only on free nucleon parameters.

#### **Isospin analog** $\gamma$ -decays



$$\Gamma(C^{12*} - C^{12})_{M1} = \frac{\alpha E_{\gamma}^3}{3M^2} \left| \sqrt{2\mu(0)} \right|^2$$

$$b:=\sqrt{2}\mu(0)=F_M^N(0)$$

Gamow-Teller matrix element c

$$c = F_A^N(0) = \sqrt{\frac{2ft_{\rm Fermi}}{ft}}$$

and thanks to CVC  $ft_{\text{Fermi}} \simeq 3080 \,\text{s}$  is universal or ot CNP-p. 11

# What is the value of $\delta_{WM}$ ?

Three ways to determine  $\delta_{WM}$ 

- impulse approximation universal value  $0.5\% \,\mathrm{MeV}^{-1}$
- using  $CVC F_M$  from analog M1  $\gamma$ -decay width,  $F_A$  from ft value
- direct measurement in β-spectrum only very few, light nuclei have been studied. In those cases the CVC predictions are confirmed within (sizable) errors.

In the following, we will compare the results from CVC with the ones from the impulse approximation.

#### **CVC** at work

Collect all nuclei for which we

- can identify the isospin analog energy level
- and know  $\Gamma_{M1}$

then, compute the resulting  $\delta_{WM}$ . This exercise has been done in Calaprice, Holstein, Nucl. Phys. A273 (1976) 301. and they find for nuclei with  $ft < 10^6$ 

 $\delta_{WM} = 0.82 \pm 0.4\% \,\mathrm{MeV}^{-1}$ 

which is in reasonable agreement with the impulse approximated value of  $\delta_{WM} = 0.5\% \text{MeV}^{-1}$ . Our result for  $ft < 10^6$  is  $\delta_{WM} = (0.67 \pm 0.26)\% \text{MeV}^{-1}$ .

#### **CVC** at work

| Decay  | $J_i \to J_f$         | $E_{oldsymbol{\gamma}}$ | $\Gamma_{M1}$ | $b_{\gamma}$ ft |                       | c       | $b_{\gamma}/Ac$ | dN/dE                  |
|--|-----------------------|-------------------------|---------------|-----------------|-----------------------|---------|-----------------|------------------------|
|  |                       | (keV)                   | (eV)          |                 | (S)                   |         |                 | $(\%  {\rm MeV^{-1}})$ |
| $^{6}$ He $\rightarrow^{6}$ Li                 | $0^+ \rightarrow 1^+$ | 3563                    | 8.2           | 71.8            | 805.2                 | 2.76    | 4.33            | 0.646                  |
| $^{12}\mathrm{B} \rightarrow ^{12}\mathrm{C}$  | $1^+ \rightarrow 0^+$ | 15110                   | 43.6          | 37.9            | 11640.                | 0.726   | 4.35            | 0.62                   |
| $^{12}\mathrm{N}  ightarrow ^{12}\mathrm{C}$   | $1^+ \rightarrow 0^+$ | 15110                   | 43.6          | 37.9            | 13120.                | 0.684   | 4.62            | 0.6                    |
| $^{18}$ Ne $\rightarrow^{18}$ F                | $0^+ \rightarrow 1^+$ | 1042                    | 0.258         | 242.            | 1233.                 | 2.23    | 6.02            | 0.8                    |
| $^{20}\mathrm{F} \rightarrow^{20} \mathrm{Ne}$ | $2^+ \rightarrow 2^+$ | 8640                    | 4.26          | 45.7            | 93260.                | 0.257   | 8.9             | 1.23                   |
| $^{22}Mg \rightarrow^{22}Na$                   | $0^+ \rightarrow 1^+$ | 74                      | 0.0000233     | 148.            | 4365.                 | 1.19    | 5.67            | 0.757                  |
| $^{24}$ Al $\rightarrow^{24}$ Mg               | $4^+ \rightarrow 4^+$ | 1077                    | 0.046         | 129.            | 8511.                 | 0.85    | 6.35            | 0.85                   |
| $^{26}$ Si $\rightarrow^{26}$ Al               | $0^+ \rightarrow 1^+$ | 829                     | 0.018         | 130.            | 3548.                 | 1.32    | 3.79            | 0.503                  |
| $^{28}$ Al $\rightarrow^{28}$ Si               | $3^+ \rightarrow 2^+$ | 7537                    | 0.3           | 20.8            | 73280.                | 0.29    | 2.57            | 0.362                  |
| $^{28}P \rightarrow ^{28}Si$                   | $3^+ \rightarrow 2^+$ | 7537                    | 0.3           | 20.8            | 70790.                | 0.295   | 2.53            | 0.331                  |
| $^{14}\mathrm{C}  ightarrow ^{14}\mathrm{N}$   | $0^+ \rightarrow 1^+$ | 2313                    | 0.0067        | 9.16            | $1.096 \times 10^9$   | 0.00237 | 276.            | 37.6                   |
| $^{14}$ O $\rightarrow^{14}$ N                 | $0^+ \rightarrow 1^+$ | 2313                    | 0.0067        | 9.16            | $1.901 \times 10^7$   | 0.018   | 36.4            | 4.92                   |
| $^{32}P \rightarrow ^{32}S$                    | $1^+ \rightarrow 0^+$ | 7002                    | 0.3           | 26.6            | $7.943 \times 10^{7}$ | 0.00879 | 94.4            | 12.9                   |

#### **What happens for large** *ft*?

| Decay   | $J_i \to J_f$         | $E_{\gamma}$ | $\Gamma_{M1}$ | $b_\gamma$ | ft                             | С       | $b_{\gamma}/Ac$ | dN/dE                  |
|---|-----------------------|--------------|---------------|------------|--------------------------------|---------|-----------------|------------------------|
|   |                       | (keV)        | (eV)          |            | (s)                            |         |                 | $(\%  {\rm MeV^{-1}})$ |
| $^{14}\mathrm{C} \rightarrow ^{14}\mathrm{N}$ | $0^+ \rightarrow 1^+$ | 2313         | 0.0067        | 9.16       | $1.096 \times 10^9$            | 0.00237 | 276.            | 37.6                   |
| $^{14}\mathrm{O} \rightarrow^{14}\mathrm{N}$  | $0^+ \rightarrow 1^+$ | 2313         | 0.0067        | 9.16       | $1.901 \times 10^7$            | 0.018   | 36.4            | 4.92                   |
| $^{32}P \rightarrow ^{32}S$                   | $1^+ \rightarrow 0^+$ | 7002         | 0.3           | 26.6       | $\overline{7.943} \times 10^7$ | 0.00879 | 94.4            | 12.9                   |

Including these large ft nuclei, we have

 $\delta_{WM} = (4.78 \pm 10.5) \% \,\mathrm{MeV}^{-1}$ 

which is about 10 times the impulse approximated value and this are about 3 nuclei out of 10-20...

NB, a shift of  $\delta_{WM}$  by  $1\% \text{MeV}^{-1}$  shifts the total neutrino flux above inverse  $\beta$ -decay threshold by  $\sim 2\%$ .





E. Christensen, PH, P. Jaffke, in preparation

Shown is the distribution of  $\log ft$  and  $Q_\beta$  throughout the ENSDF data base. Indeed, this confirms that there should be very few allowed decays with  $\log ft > 6$ .





E. Christensen, PH, P. Jaffke, in preparation

Here we weight each  $\beta$ -emitter by its fission yield, which emphasizes both large values of log ft as well as forbidden decays. For forbidden decays the previous dicussions do generally not apply!

# **Large** *ft* **and forbiddness!!**



#### E. Christensen, PH, P. Jaffke, in preparation

Conversion to neutrinos and the IBD cross section enhance the contributions from large  $\log ft$  and forbidden decays even more – room for significant theory uncertainties

#### **Complete** $\beta$ **-shape**



# Computation of Neutrino Spectrum

#### **Extraction of** $\nu$ **-spectrum**

We can measure the total  $\beta$ -spectrum

$$\mathcal{N}_{\beta}(E_e) = \int dE_0 N_{\beta}(E_e, E_0; \bar{Z}) \eta(E_0) \,. \tag{1}$$

with  $\overline{Z}$  effective nuclear charge and try to "fit" the underlying distribution of endpoints,  $\eta(E_0)$ .

This is a so called Fredholm integral equation of the first kind – mathematically ill-posed, *i.e.* solutions tend to oscillate, needs regulator (typically energy average), however that will introduce a bias.

This approach is know as "virtual branches"

#### **Virtual branches**



1 – fit an allowed  $\beta$ -spectrum with free normalization  $\eta$  and endpoint energy  $E_0$  the last *s* data points

- 2 delete the last s data points
- 3 -subtract the fitted spectrum from the data
- 4 goto 1

Invert each virtual branch using energy conservation into a neutrino spectrum and add them all.

# $\beta$ spectrum from fission



<sup>235</sup>U foil inside theHigh Flux Reactor atILL

Electron spectroscopy with a magnetic spectrometer

Schreckenbach, et al. PLB 160, 325 (1985).

#### **Effective nuclear charge**

In order to compute all the QED corrections we need to know the nuclear charge Z of the decaying nucleus.

Using virtual branches, the fit itself cannot determine Z since many choices for Z will produce an excellent fit of the  $\beta$ -spectrum

 $\Rightarrow$  use nuclear database to find how the average nuclear charge changes as a function of  $E_0$ , this is what is called effective nuclear charge  $\overline{Z}(E_0)$ .

Weigh each nucleus by its fission yield and bin the resulting distribution in  $E_0$  and fit a second order polynomial to it.

#### **Effective nuclear charge**

The nuclear databases have two fundamental shortcomings

- they are incomplete for the most neutron-rich nuclei we only know the  $Q_{gs \rightarrow gs}$ , *i.e.* the mass differences
- they are incorrect for many of the neutron-rich nuclei,  $\gamma$ -spectroscopy tends to overlook faint lines and thus too much weight is given to branches with large values of  $E_0$ , aka pandemonium effect

Simulation using our synthetic data set: by removing a fraction of the most neutron-rich nuclei and/or by randomly distributing the decays of a given branch onto several branches with  $0 < E_0 < Q_{gs \rightarrow gs}$ .

#### **Effective nuclear charge**



Spread between lines – effect of incompleteness and incorrectness of nuclear database (ENSDF). Only place in this analysis, where database enters directly.

#### From first principles?



In Mueller *et al.*, Phys.Rev. C83 (2011) 054615 an attempt was made to compute the neutrino spectrum from fission yields and information on individual  $\beta$  decay branches from databases.

The resulting cumulative  $\beta$  spectrum should match the ILL measurement.

About 10-15% of electrons are missing, Mueller *et al.* use virtual branches for that small remainder.



#### **Bias**

Use synthetic data sets derived from cumulative fission yields and ENSDF, which represent the real data within 10-20% and compute bias



#### Approximately 500 nuclei and 8000 $\beta$ -branches.

#### **Statistical Error**

Use synthetic data sets and fluctuate  $\beta$ -spectrum within the variance of the actual data.



Amplification of stat. errors of input data by factor 7.

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# **Result for** <sup>235</sup>U



Shift with respect to ILL results, due toa) different effective nuclear charge distributionb) branch-by-branch application of shape corrections

# Summary

- Independent, complimentary analysis of ILL data
- Confirms overall, energy averaged upward shift

Differences with respect to Mueller *et al.*, Phys.Rev. **C83** (2011) 054615.

- More accurate  $\beta$ -shape
- Small electron residuals
- Quantified errors
- Significant shape differences origin is understood
- Weak magnetism important open theory issues

Backup Slides

#### **Finite size corrections – I**

Finite size of charge distribution affects outgoing electron wave function

$$L_0(Z, W) = 1 + 13 \frac{(\alpha Z)^2}{60} - WR\alpha Z \frac{41 - 26\gamma}{15(2\gamma - 1)} - \alpha ZR\gamma \frac{17 - 2\gamma}{30W(2\gamma - 1)} \dots$$

Parametrization of numerical solutions, only small associated error. This expression is effectively very close to the Mueller *et al.* one.

#### **Finite size corrections – II**

Convolution of electron wave function with nucleon wave function over the volume of the nucleus

$$(Z,W) = 1 + C_0 + C_1 W + C_2 W^2 \text{ with}$$

$$C_0 = -\frac{233}{630} (\alpha Z)^2 - \frac{(W_0 R)^2}{5} + \frac{2}{35} W_0 R \alpha Z,$$

$$C_1 = -\frac{21}{35} R \alpha Z + \frac{4}{9} W_0 R^2,$$

$$C_2 = -\frac{4}{9} R^2.$$

Small associated theory error. This expression is not taken into account by Mueller *et al.*, quantitatively largest  $\beta$ -shape difference.

#### **Screening correction**

All of the atomic bound state electrons screen the charge of the nucleus – correction to Fermi function

$$\bar{W} = W - V_0$$
,  $\bar{p} = \sqrt{\bar{W}^2 - 1}$ ,  $y = \frac{\alpha Z W}{p}$   $\bar{y} = \frac{\alpha Z \bar{W}}{\bar{p}}$   $\tilde{Z} = Z - 1$ .

 $V_0$  is the so called screening potential

$$V_0 = \alpha^2 \tilde{Z}^{4/3} N(\tilde{Z}) \,,$$

and  $N(\tilde{Z})$  is taken from numerics.

$$S(Z,W) = \frac{\overline{W}}{W} \left(\frac{\overline{p}}{p}\right)^{(2\gamma-1)} e^{\pi(\overline{y}-y)} \frac{|\Gamma(\gamma+i\overline{y})|^2}{\Gamma(2\gamma+1)^2} \quad \text{for} \quad W > V_0 \,,$$

Small associated theory error. This expression is not taken into account by Mueller *et al*..

#### **Radiative correction - I**

Order  $\alpha$  QED correction to electron spectrum, by Sirlin, 1967

$$g_{\beta} = 3\log M_N - \frac{3}{4} + 4\left(\frac{\tanh^{-1}\beta}{\beta}\right)\left(\frac{W_0 - W}{3W} - \frac{3}{2} + \log\left[2(W_0 - W)\right]\right) + \frac{4}{\beta}L\left(\frac{2\beta}{1+\beta}\right) + \frac{1}{\beta}\tanh^{-1}\beta\left(2(1+\beta^2) + \frac{(W_0 - W)^2}{6W^2} - 4\tanh^{-1}\beta\right)$$

where L(x) is the Spence function, The complete correction is then given by

$$G_{\beta}(Z,W) = 1 + \frac{\alpha}{2\pi}g_{\beta}.$$

Small associated theory error.

#### **Radiative correction - II**

Order  $\alpha$  QED correction to neutrino spectrum, recent calculation by Sirlin, Phys. Rev. **D84**, 014021 (2011).

$$h_{\nu} = 3\ln M_N + \frac{23}{4} - \frac{8}{\hat{\beta}}L\left(\frac{2\hat{\beta}}{1+\hat{\beta}}\right) + 8\left(\frac{\tanh^{-1}\hat{\beta}}{\hat{\beta}} - 1\right)\ln(2\hat{W}\hat{\beta}) + 4\frac{\tanh^{-1}\hat{\beta}}{\hat{\beta}}\left(\frac{7+3\hat{\beta}^2}{8} - 2\tanh^{-1}\hat{\beta}\right)$$

$$G_{\nu}(Z,W) = 1 + \frac{\alpha}{2\pi}h_{\nu}.$$

Very small correction.

#### Weak currents

In the following we assume  $q^2 \ll M_W$  and hence charged current weak interactions can be described by a current-current interaction.

$$-rac{G_F}{\sqrt{2}}V_{ud}J^h_\mu J^l_\mu$$

where

$$J^{h}_{\mu} = \bar{\psi}_{u} \gamma_{\mu} (1 + \gamma_{5}) \psi_{d} = V^{h}_{\mu} + A^{h}_{\mu}$$

However, we are not dealing with free quarks ...

#### **Induced currents**

Describe protons and neutrons as spinors which are solutions to the free Dirac equation, but which are **not** point-like, we obtain for the hadronic current

$$V_{\mu}^{h} = i\bar{\psi}_{p} \left[ g_{V}(q^{2})\gamma_{\mu} + \frac{g_{M}(q^{2})}{8M}\sigma_{\mu\nu}q_{\nu} + ig_{S}(q^{2})q_{\mu} \right]\psi_{n}$$

$$A_{\mu}^{h} = i\bar{\psi}_{p} \left[ g_{A}(q^{2})\gamma_{\mu}\gamma_{5} + \frac{g_{T}(q^{2})}{8M}\sigma_{\mu\nu}q_{\nu}\gamma_{5} + ig_{P}(q^{2})q_{\mu}\gamma_{5} \right]\psi_{n}$$

In the limit  $q^2 \rightarrow 0$  the form factors  $g_X(q^2) \rightarrow g_X$ , *i.e.* new induced couplings, which are not present in the SM Lagrangian, but are induced by the bound state QCD dynamics.

#### Isospin

Proton and neutron can be regarded as a two state system in the same way a spin 1/2 system has two states  $\Rightarrow$  isospin.

In complete analogy we chose the Pauli matrices as basis, but call them  $\tau$  to avoid confusion with regular spin  $\vec{\tau} = (\tau_1, \tau_2, \tau_3)$ , we define the new 8-component spinor

$$\Psi = \left(\begin{array}{c} \psi_p \\ \psi_n \end{array}\right)$$

and we define the isospin ladder operators as  $\tau^a = \tau^{\pm} = \tau_1 \pm i\tau_2$ , with  $\tau^+$  corresponding to  $\beta^-$ -decay and  $\tau^-$  to  $\beta^+$ -decay.

#### Weak isovector current

Using isospin notation we can write the Lorentz vector part of the weak charged current as

and see that it transform as a vector in isospin space, therefore this together with the corresponding Lorentz axial vector  $A^h_{\mu}$  part, which has the same isospin structure, is also called the weak isovector current.

#### **EM isovector current**

The fundamental EM current is given by

$$V^{EM}_{\mu} = i\frac{2}{3}\bar{\psi}_{u}\gamma_{\mu}\psi_{u} - i\frac{1}{3}\bar{\psi}\gamma_{\mu}\psi_{d}$$

which transforms as Lorentz vector. How does it transform under isospin?

$$V_{\mu}^{EM} = \underbrace{iQ_{+}\bar{\Psi}_{q}\gamma_{\mu}\Psi_{q}\mathbb{1}}_{\text{isoscalar}} + \underbrace{iQ_{-}\bar{\Psi}_{q}\gamma_{\mu}\Psi_{q}\tau^{3}}_{\text{isovector}}$$

with  $Q_{\pm} = \frac{1}{2} \left( \frac{2}{3} \mp \frac{1}{3} \right)$ .

A triplet of isovector currents

Next, we can dress up the isovector part of  $V_{\mu}^{EM}$ ,  $v_{\mu}^{EM}$  to account for nucleon structure

$$= i\bar{\Psi} \left[ F_1^V(q^2)\gamma_\mu + \frac{F_2^V(q^2)}{2M}\sigma_{\mu\nu}q_\nu + iF_3^V(q^2)q_\mu \right] Q_-\tau_3\Psi$$

EN

Compare with the Lorentz vector part of the weak isovector current

$$T^{h}_{\mu} = i\bar{\Psi} \left[ g_{V}(q^{2})\gamma_{\mu} + \frac{g_{M}(q^{2})}{8M}\sigma_{\mu\nu}q_{\nu} + ig_{S}(q^{2})q_{\mu} \right] \frac{1}{2}\tau^{a}\Psi$$

These three currents form a triplet of isovector currents and this observation was made by Feynman and Gell-Mann in 1958.

#### **Conserved vector currents**

We know that  $V_{\mu}^{EM}$  is a conserved quantity which is a direct consequence of U(1) gauge invariance in the SM.

This implies that all components of the triplet are conserved.

This is termed the Conserved Vector Current (CVC), which in the SM is a result not an input.

$$g_V(q^2) = F_1^V(q^2) \xrightarrow{q^2 \to 0} 1$$
  

$$g_M(q^2) = F_2^V(q^2)$$
  

$$g_S(q^2) = F_3^V(q^2) = 0$$