Overview of non-Gaussian models

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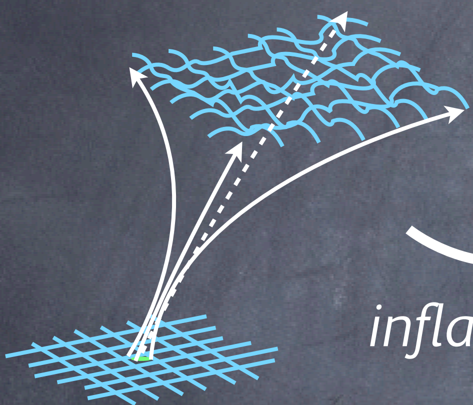
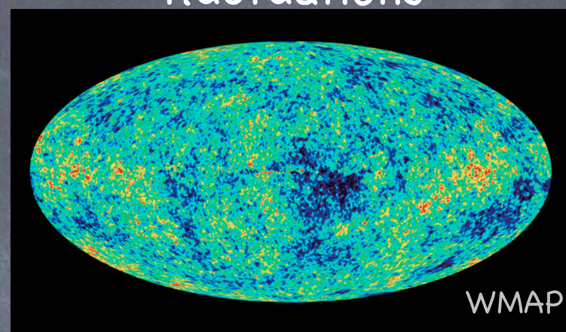
Inflation as the origin of structure

quantum fluctuations

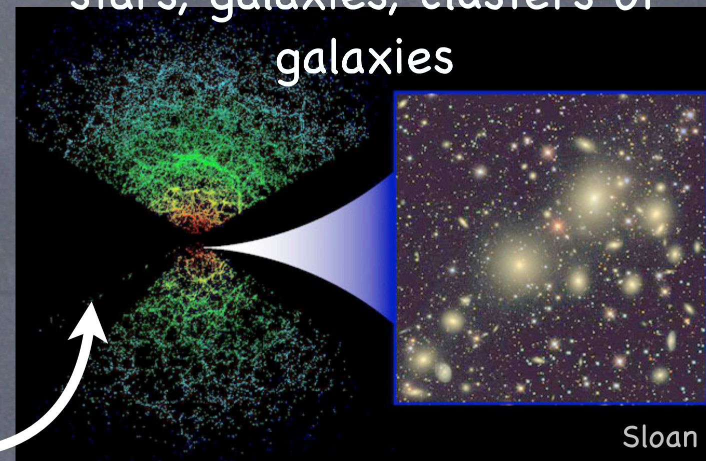


inflation

small matter & energy fluctuations



stars, galaxies, clusters of galaxies

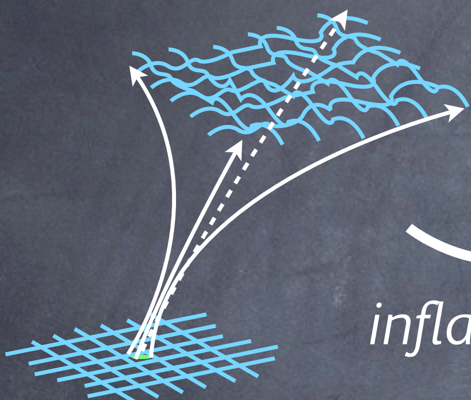


gravitational collapse

$$\delta\varphi_{\text{inflaton}} \longrightarrow \Phi_{\text{curvature}} \rightsquigarrow \begin{matrix} \delta T_{\text{CMB}} \\ \delta\rho_{\text{matter}} \\ \delta n_{\text{galaxies}} \end{matrix}$$

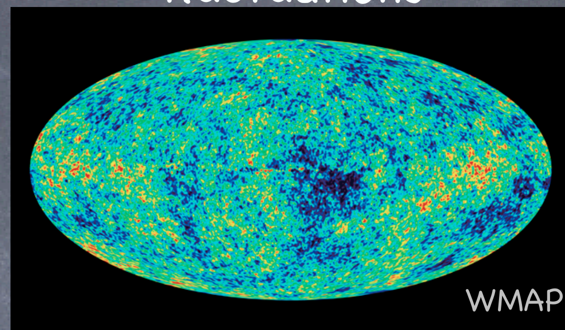
Inflation as the origin of structure

quantum fluctuations



inflation

small matter & energy fluctuations



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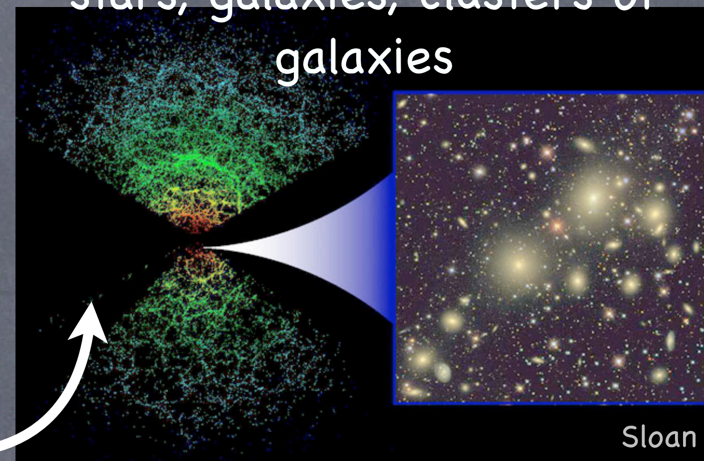


gravitational collapse

statistics of these

probe of this era

$\delta\varphi_{\text{inflaton}} \longrightarrow \Phi_{\text{curvature}} \rightsquigarrow$

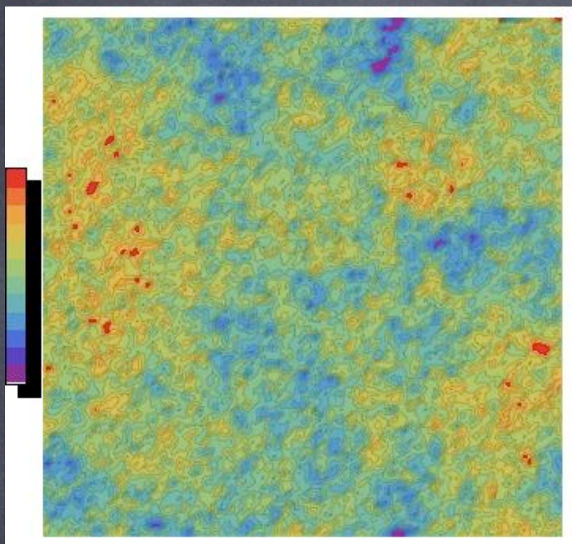
δT_{CMB}

$\delta\rho_{\text{matter}}$

$\delta n_{\text{galaxies}}$

reminder: simplest option, Gaussian

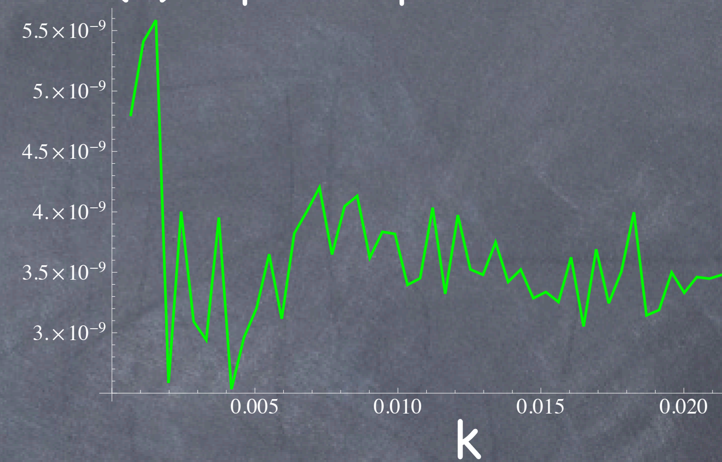
$\Phi_{curvature}$



two-point function:

$\langle \Phi(x)\Phi(y)\rangle \leftrightarrow P_{\Phi}(k)$

$k^2 P_{\Phi}(k)$ power per mode

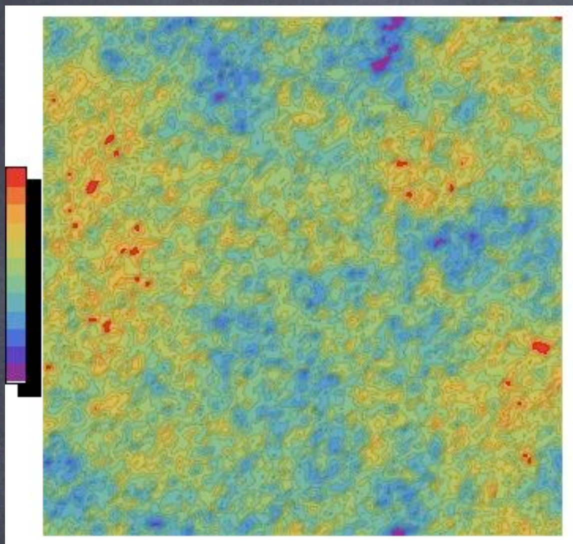


probability distribution:

Φ value

reminder: simplest option, Gaussian

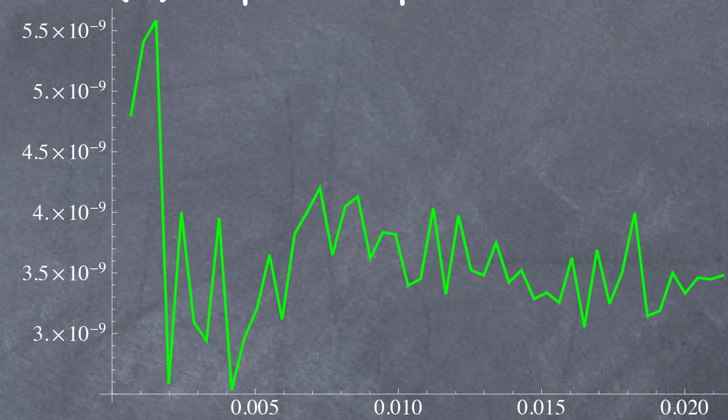
$\Phi_{curvature}$



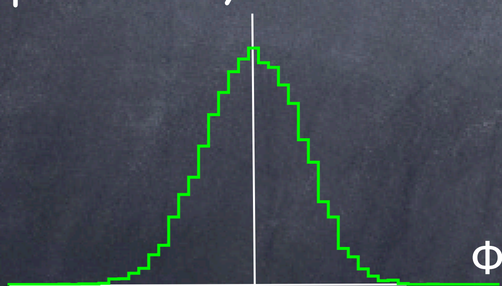
two-point function:

$$\langle \Phi(x)\Phi(y) \rangle \leftrightarrow P_\Phi(k)$$

$k^2 P_\Phi(k)$ power per mode



probability distribution:



$$\langle \Phi(x)\Phi(y)\Phi(z) \rangle = 0$$

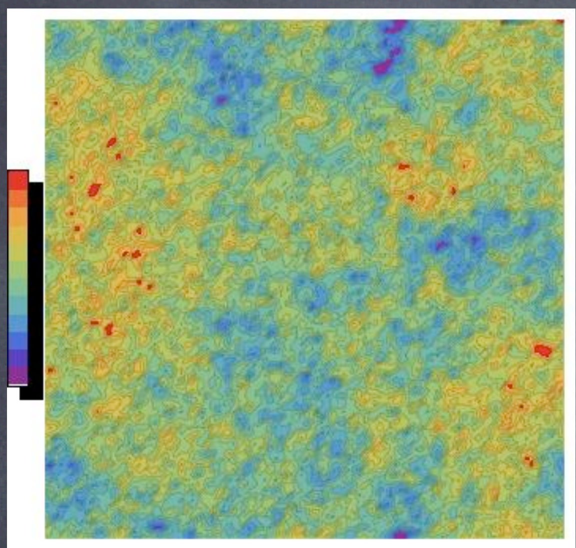
$$\langle \Phi(x)\Phi(y)\Phi(z)\Phi(w) \rangle = \langle \Phi(x)\Phi(y) \rangle\langle \Phi(z)\Phi(w) \rangle + \langle \Phi(x)\Phi(z) \rangle\langle \Phi(y)\Phi(w) \rangle + \langle \Phi(x)\Phi(w) \rangle\langle \Phi(y)\Phi(z) \rangle$$

$$\langle \Phi(x)\Phi(y)\Phi(z)\Phi(w)\Phi(s) \rangle = 0$$

. . .

reminder: simplest option, Gaussian

$\Phi_{\text{curvature}}$



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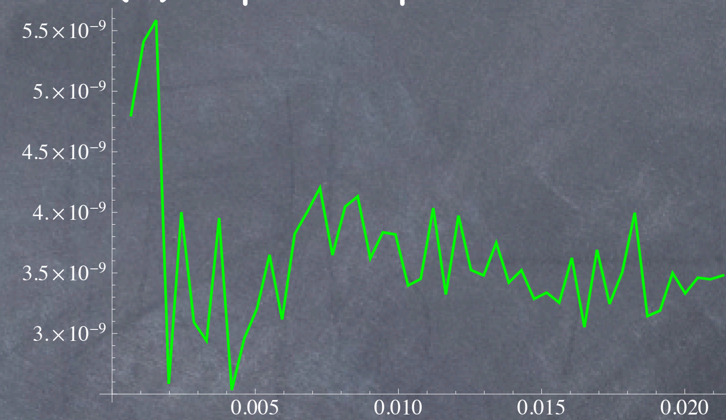
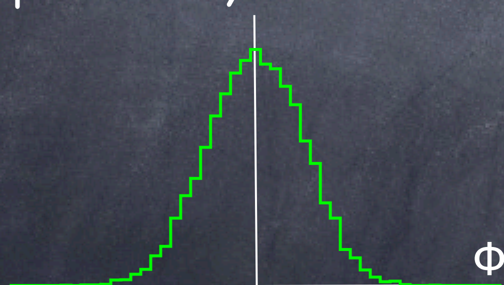


vanishing or trivially related to two-point

probability distribution:



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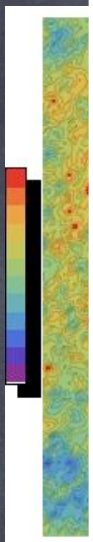


single-field, slow-roll inflation predicts this

observations suggest IC's are nearly Gaussian

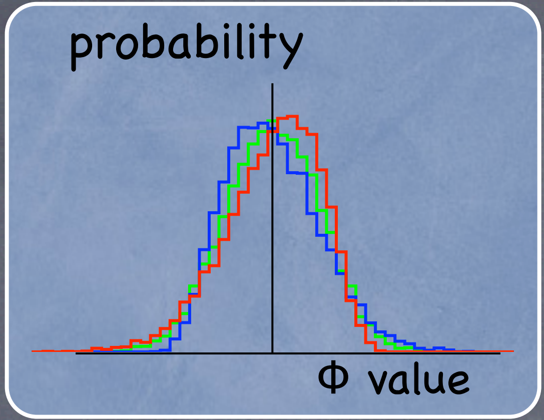
BUT small departures may exist and could provide one of few observational handles on physics of inflation

Example mildly non-Gaussian initial conditions

"f_{NL}"

$$\Phi(x) \sim \delta\sigma(x) + f_{NL}\, \delta\sigma(x)^2$$

Salopek and Bond 1990; Gangui, Lucchin, Matarrese, Mollerach 1994; Komatsu and Spergel 2001



skewness ~ f_{NL}

kurtosis ~ f_{NL}^2

. . .

"f_{NL}"

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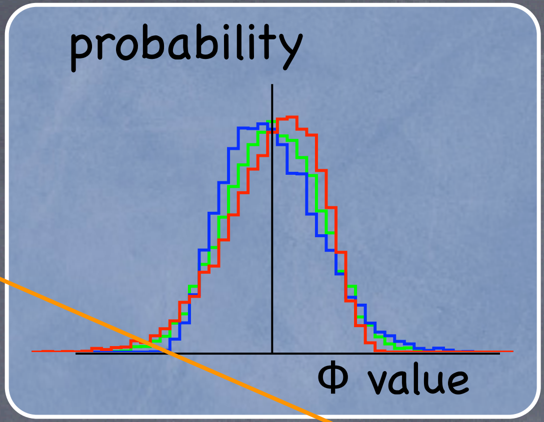

skewness ~ f_{NL}

kurtosis ~ f_{NL}^2

. . .

here, $\delta\sigma$ is a Gaussian field. the non-linear terms in $\delta\sigma$ make Φ non-Gaussian

this map completely specifies Φ statistics

“f_{NL}”

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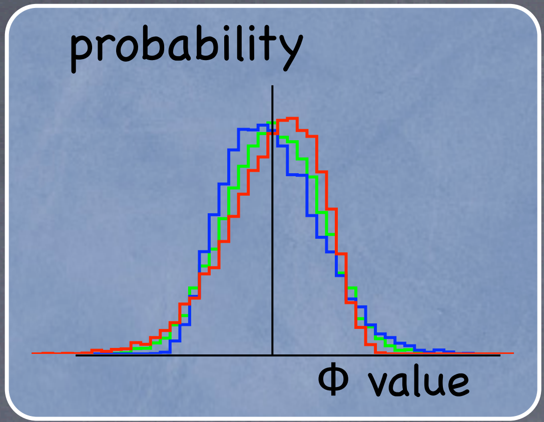


skewness ~ f_{NL}

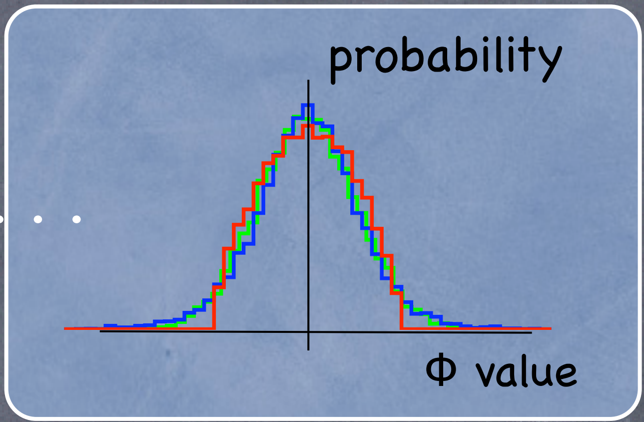
kurtosis ~ f_{NL}^2

. . .

“g_{NL}”

$$\Phi(x) \sim \delta\sigma(x) + g_{NL}\,\delta\sigma(x)^3 + \ldots$$

(Okamoto and Hu 2002; Enqvist and Nurmi 2005)



skewness ~ 0

kurtosis ~ g_{NL}

. . .

"f_{NL}"

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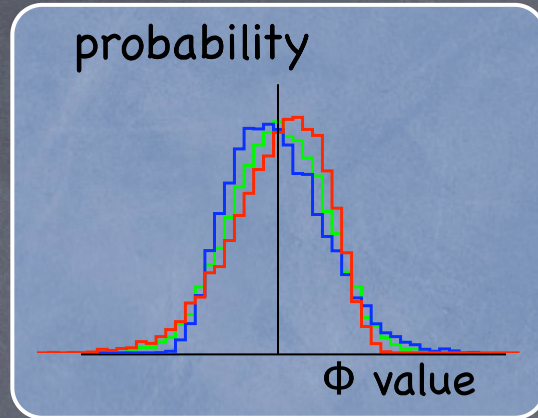

skewness ~ f_{NL}

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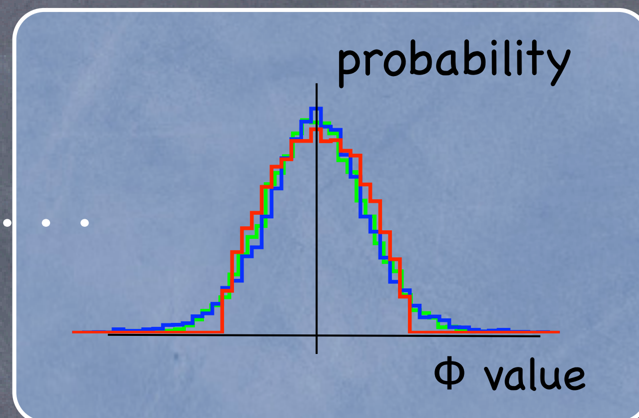


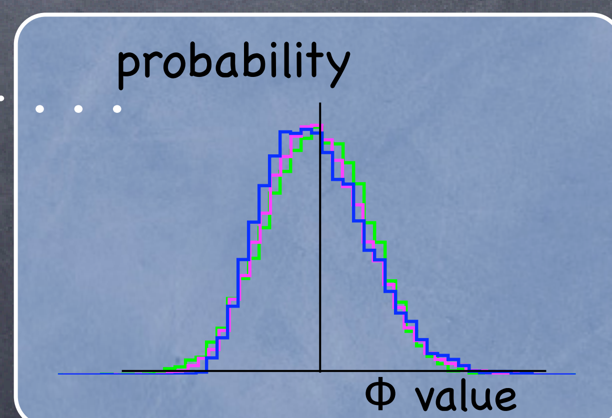
skewness ~ 0

kurtosis ~ g_{NL}

. . .

"τ_{NL}"

$$\Phi(x) \sim \delta\varphi(x) + \delta\sigma(x) + \tilde{f}_{NL}\delta\sigma(x)^2 + \ldots$$



$$\tilde{f}_{NL} = f_{NL}(1 + P_{\varphi\varphi}/P_{\sigma\sigma})^2$$

$$\tau_{NL} = f_{NL}^2(1 + P_{\varphi\varphi}/P_{\sigma\sigma})$$

and $P_{\varphi\sigma} = 0$

skewness ~ f_{NL}

kurtosis ~ τ_{NL}

(Φ=primordial gravitational potential)

(Lyth and Wands 2002; Ichikawa, Suyama, Takahishi, Yamaguchi (2008); Tseliakhovich, Hirata, Slosar 2010)

Those were the non-Gaussian (1-point) probability distributions functions

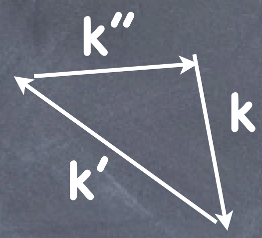
but

more generally, non-Gaussianity introduces non-trivial multi-point correlation functions (or polyspectra)

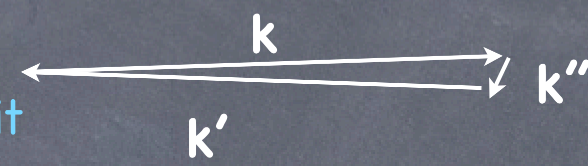
Bispectrum:

$$\langle \Phi(\mathbf{k})\Phi(\mathbf{k}')\Phi(\mathbf{k}'')\rangle = 2f_{NL}\,(P_\Phi(k)\,P_\Phi(k') + \ldots)\,(2\pi)^3\,\delta(\mathbf{k}+\mathbf{k}'+\mathbf{k}'')$$

function of triangle



largest in the "squeezed" limit



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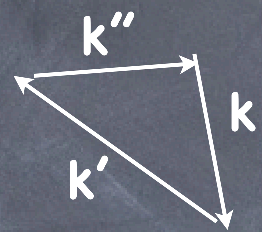

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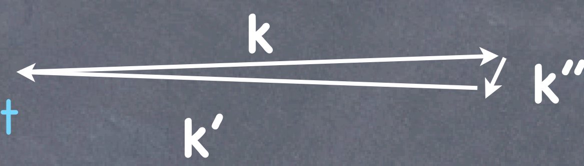
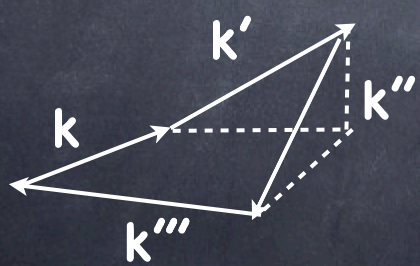


Trispectrum:

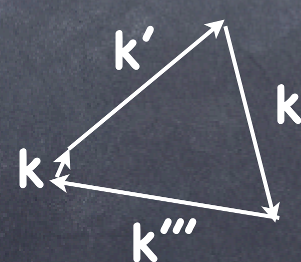
$$\langle \Phi(\mathbf{k})\Phi(\mathbf{k}')\Phi(\mathbf{k}'')\Phi(\mathbf{k}''')\rangle_c = g_{NL}\,(P_\Phi(k)\,P_\Phi(k')\,P_\Phi(k'') + \ldots)\,(2\pi)^3\,\delta(\mathbf{k}+\mathbf{k}'+\mathbf{k}''+\mathbf{k}''')$$

$$+\,2\,\tau_{NL}\,(P_\Phi(k)\,P_\Phi(k')\,P_\Phi(|\mathbf{k}+\mathbf{k}''|) + \ldots)\,(2\pi)^3\,\delta(\mathbf{k}+\mathbf{k}'+\mathbf{k}''+\mathbf{k}''')$$

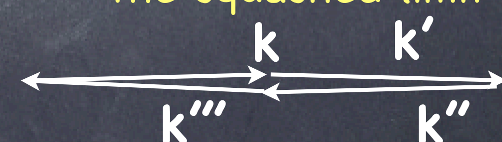
function of a quadrilateral



g_{NL} term peaks in the limit



τ_{NL} term peaks in the squashed limit

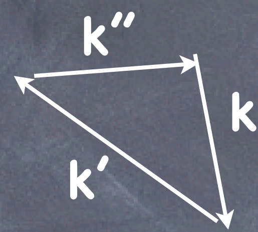


(Φ=primordial gravitational potential)

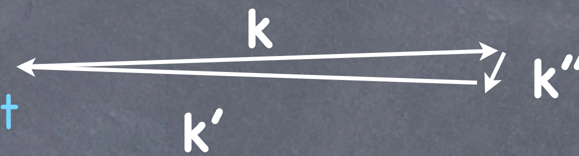
Bispectrum:

$$\langle \Phi(\mathbf{k})\Phi(\mathbf{k}')\Phi(\mathbf{k}'')\rangle = 2f_{NL}\,(P_\Phi(k)\,P_\Phi(k') + \ldots)\,(2\pi)^3\,\delta(\mathbf{k}+\mathbf{k}'+\mathbf{k}'')$$

function of triangle



largest in the "squeezed" limit



Trispectrum:

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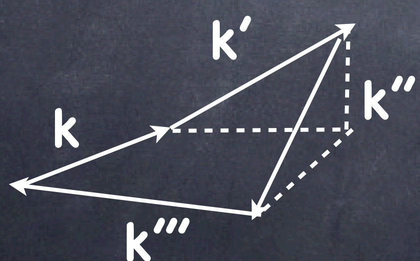
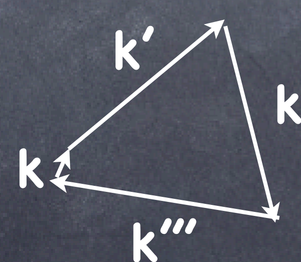
$$+\,2\,\tau_{NL}\,(P_\Phi(k)\,P_\Phi(k')\,P_\Phi(|\mathbf{k}+\mathbf{k}''|) + \ldots)\,(2\pi)^3\,\delta(\mathbf{k}+\mathbf{k}'+\mathbf{k}''+\mathbf{k}''')$$

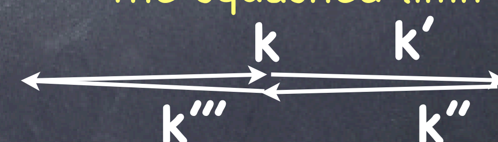
function of a quadrilateral



g_{NL} term peaks in the limit



τ_{NL} term peaks in the squashed limit



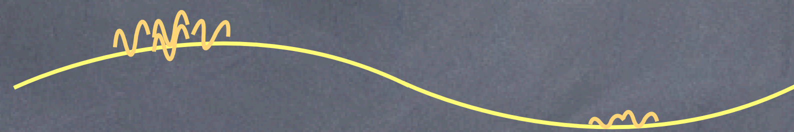
so g_{NL} and τ_{NL} different "shape" trispectra

(Φ=primordial gravitational potential)

Helpful to consider how polyspectra couple different physical scales scales

"f_{NL}" $\Phi \sim \delta\sigma + f_{NL}\, \delta\sigma^2$

$$\langle \Phi_{short}{}^2 \rangle = \langle \sigma_{G,short}{}^2 \rangle \,(1 + 4\, f_{NL}\, \sigma_{G,long}(x))$$



small-scale power depends on large-scale fluctuations!

(Φ=primordial gravitational potential)

Slosar, Hirata, Seljak, Ho, Padmanabhan 2008

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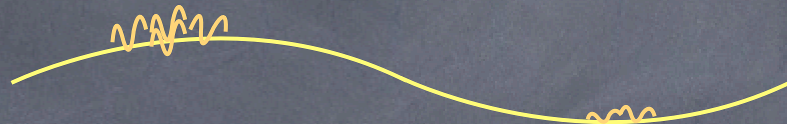

small-scale power depends on large-scale fluctuations!

"g_{NL}" $\Phi \sim \delta\sigma + g_{NL}\delta\sigma^3 + \ldots$

$$\langle \Phi_{short}^3 \rangle = 18\, g_{NL}\, \langle \sigma_{G,short}^2 \rangle^2 \sigma_{G,long}(x) \equiv f_{NL}^{eff}(x) \langle \sigma_{G,short}^2 \rangle^2$$

small-scale skewness depends on large-scale fluctuations!

(Φ=primordial gravitational potential)

"f_{NL}" $\Phi \sim \delta\sigma + f_{NL}\,\delta\sigma^2$

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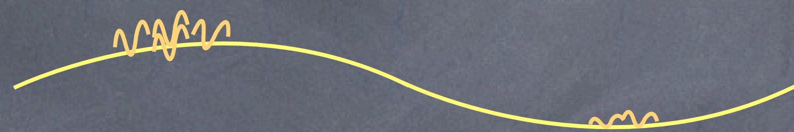

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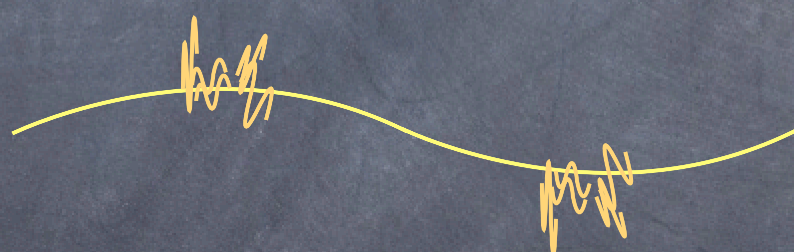
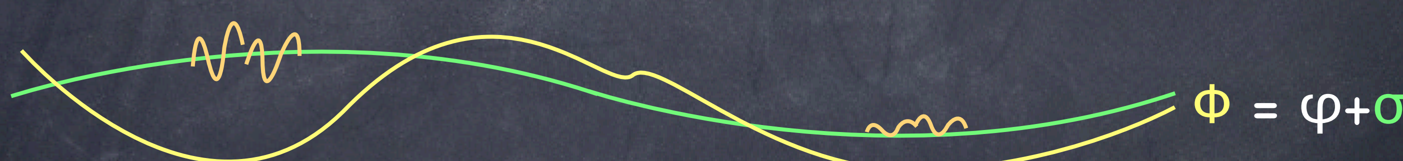


"τ_{NL}" $\Phi \sim \delta\varphi + \delta\sigma + \tilde{f}_{NL}\delta\sigma^2 + \ldots$

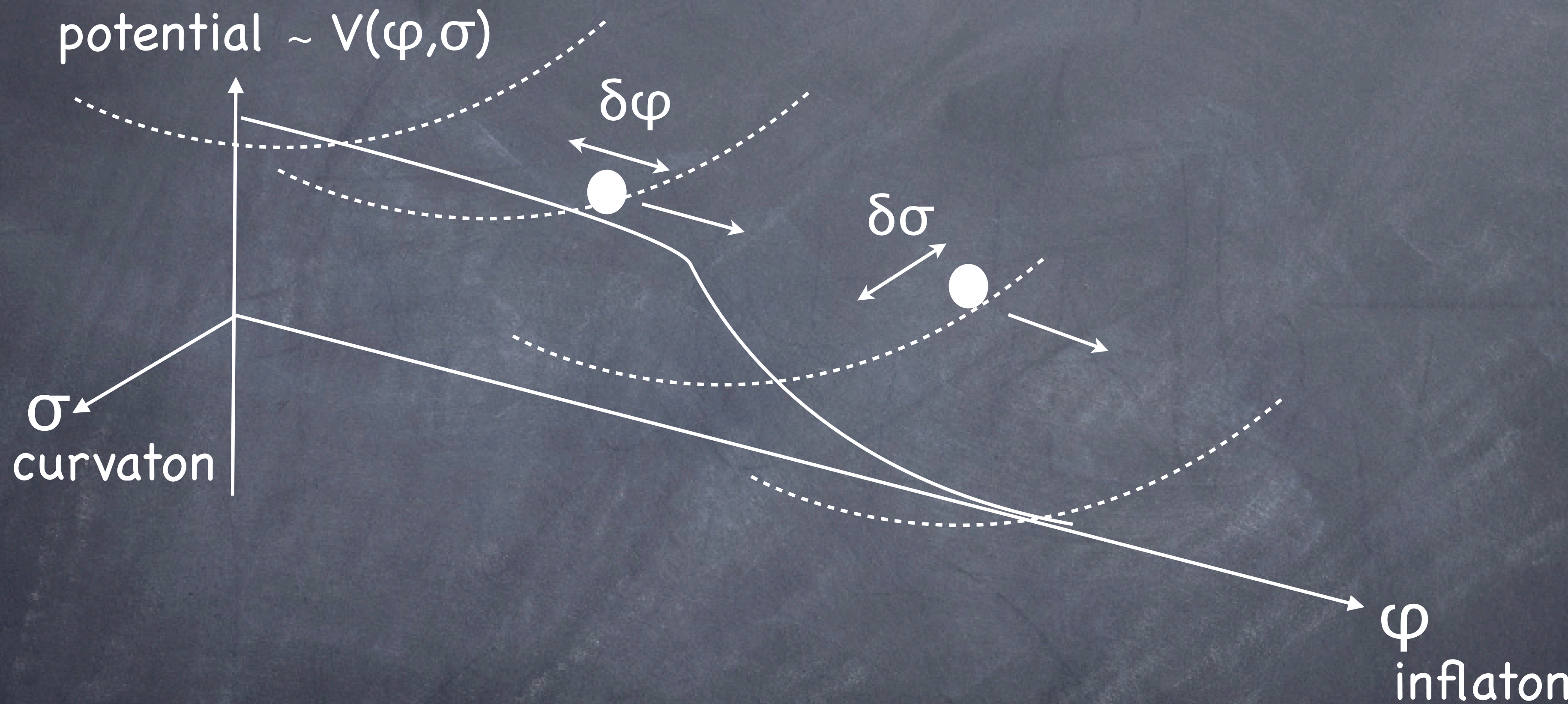
$$\langle \Phi_s^2 \rangle = \langle \Phi_{G,short}^2 \rangle \left(1 + 4\, \tilde{f}_{NL}\sigma_{G,long}(x)\right)$$



(Φ=primordial gravitational potential)

These are cartoon examples
but these types of initial conditions can
arise from real models

For instance

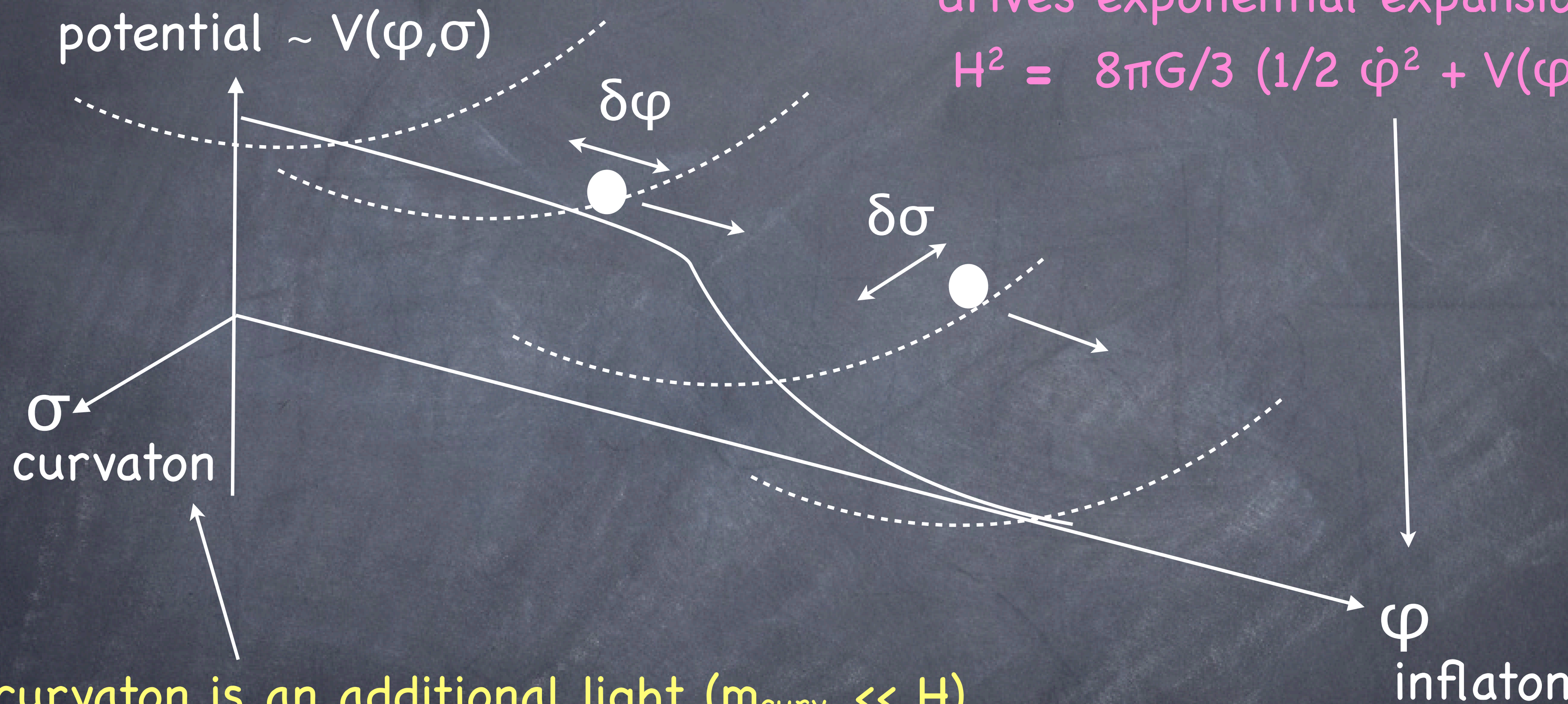


Linde and Mukhanov 1997; Lyth and Wands 2002

For instance

inflaton dominates energy density,
drives exponential expansion

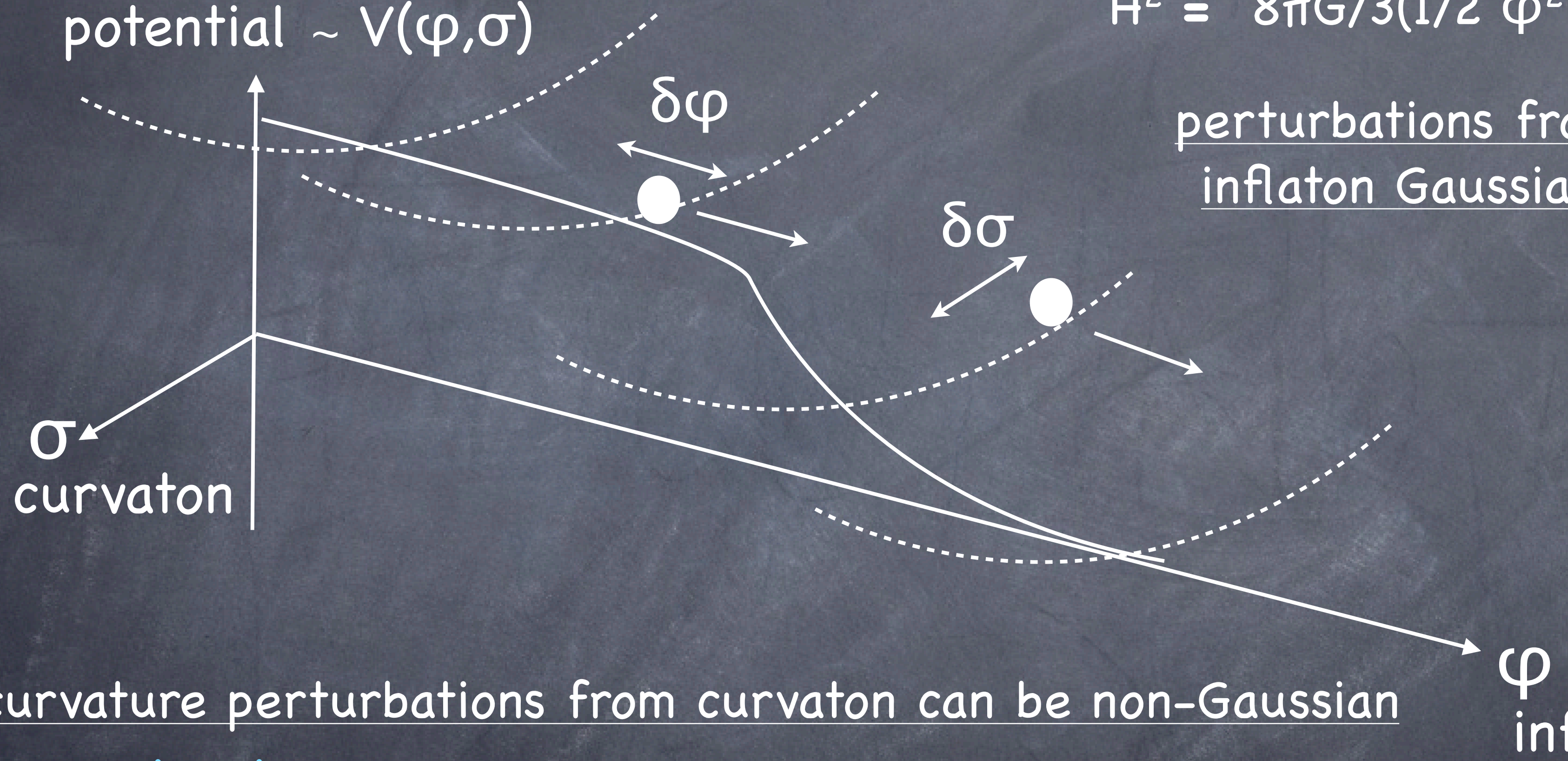
$H^2 = 8\pi G/3 \, (1/2 \, \dot{\varphi}^2 + V(\varphi))$



curvaton is an additional light ($m_{curv.} \ll H$)
field that gets excited and eventually
generates curvature perturbations Φ

Linde and Mukhanov 1997; Lyth and Wands 2002

For instance



total energy dominated by inflaton:

$$H^2 = 8\pi G/3(1/2\ \dot{\varphi}^2 + V(\varphi))$$

perturbations from inflaton Gaussian

curvature perturbations from curvaton can be non-Gaussian

"f_{NL}" $\Phi \sim \delta\sigma + \delta\sigma^2$

"g_{NL}" $\Phi \sim \delta\sigma + \delta\sigma^3 + \ldots$

"τ_{NL}" $\Phi \sim \delta\varphi + \delta\sigma + \delta\sigma^2 + \ldots$

Linde and Mukhanov 1997; Lyth and Wands 2002

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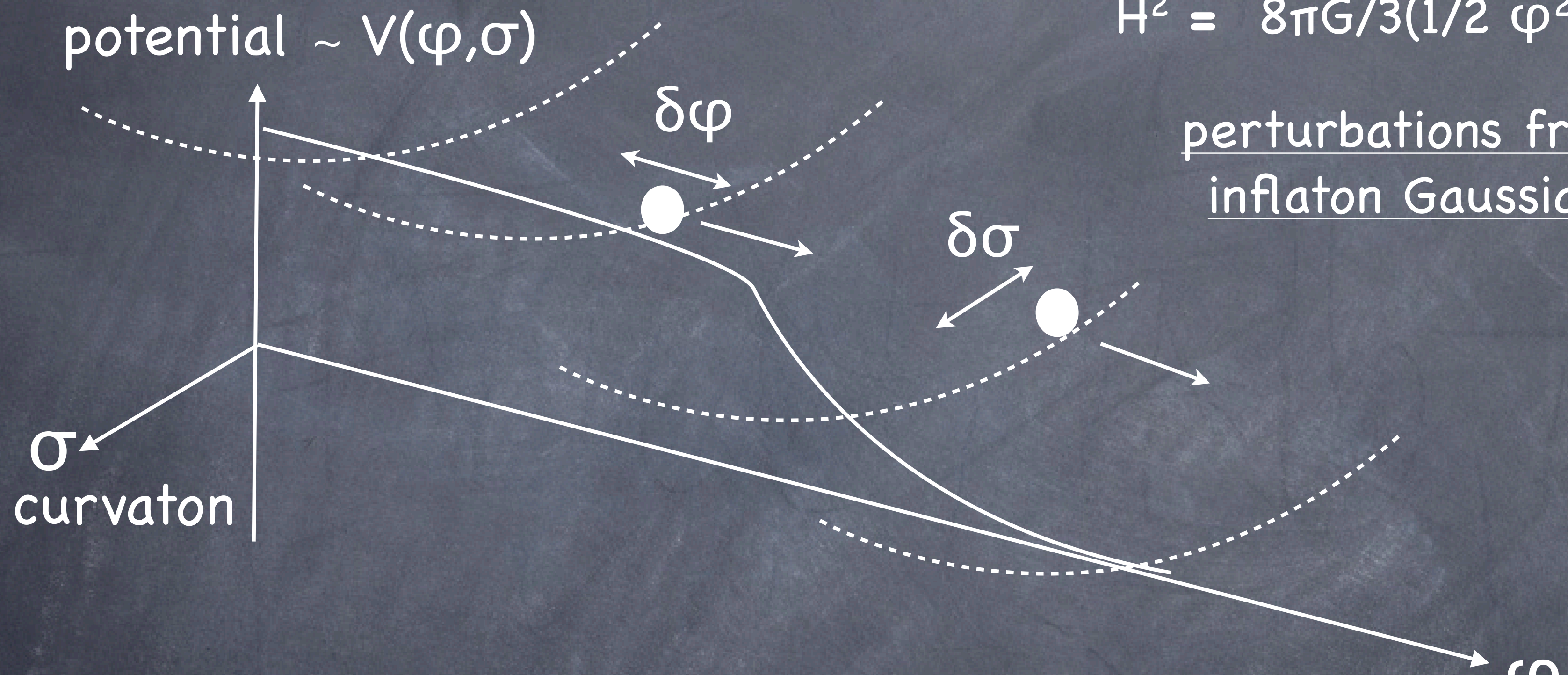

perturbations from inflaton Gaussian

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non-linearities all "local" in position space

Linde and Mukhanov 1997; Lyth and Wands 2002

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non-linearities all "local" in position space

But local models (i.e. $\Phi_{NG}(x)=F(\sigma_G(x))$) of non-Gaussianity is not the only option

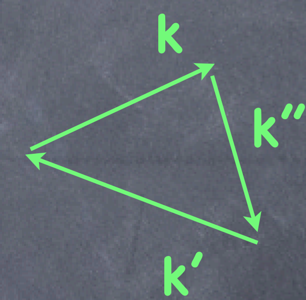
Single-field inflation with strong self-interactions can also generate detectable non-Gaussianity

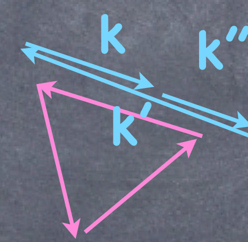
skewness, bispectrum amplitude

$\langle \phi^3 \rangle$ $\quad\quad$ $\langle \Phi(\mathbf{k})\Phi(\mathbf{k}')\Phi(\mathbf{k}'') \rangle \propto 1/c_s^{\,2}$

shape

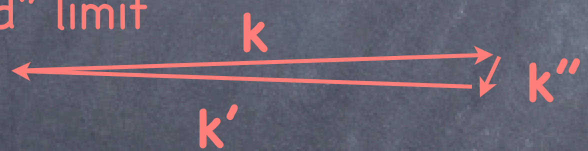
largest in the "equilateral" limit





or "orthogonal" shape

BUT vanish in the "squeezed" limit



see Babich, Creminelli, Zaldarriaga 2004; Chen, Huang, Kachru, Shiu 2006; Senatore, Smith, Zaldarriaga 2011

(e.g. Dirac-Born-Infeld inflation, k-inflation, ghost inflation, inflation w dissipation)

Alishahiha, Silverstein, Tong 2004; Armendariz-Picon, Damour, Mukhanov 1999; Arkani-Hamed, Creminelli, Mukohyama, Zaldarriaga 2004; Nacir, Porto, Senatore, Zaldarriaga 2012

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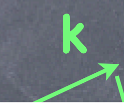

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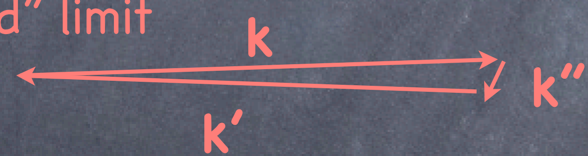

where, vanish means

$0+O(k_L^2/k_s^2)$

see Babich, Creminelli, Zaldarriaga 2004; Chen, Huang,

(e.g. Dirac-Born-Infeld inflation, k-inflation, ghost inflation, inflation w dissipation)

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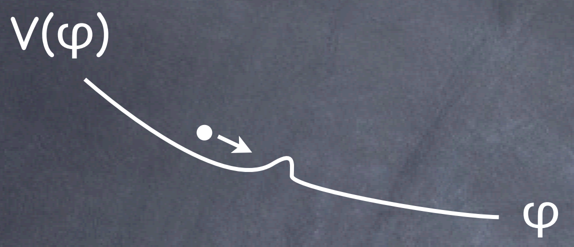
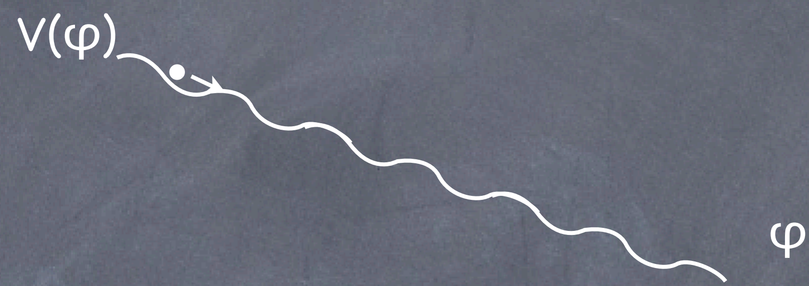
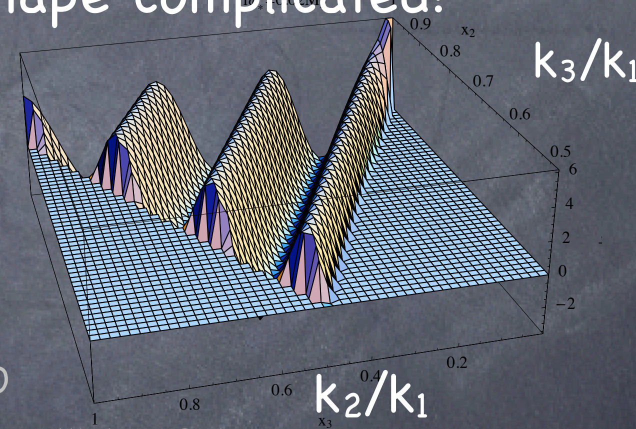
a single-field that violates slow-roll can also generate observable non-Gaussianity





shape complicated!



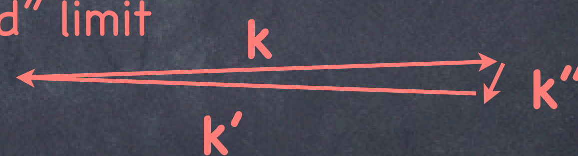
skewness, bispectrum amplitude

$\langle \Phi^3 \rangle$ $\langle \Phi(\mathbf{k})\Phi(\mathbf{k}')\Phi(\mathbf{k}'')\rangle \propto$ local feature in k

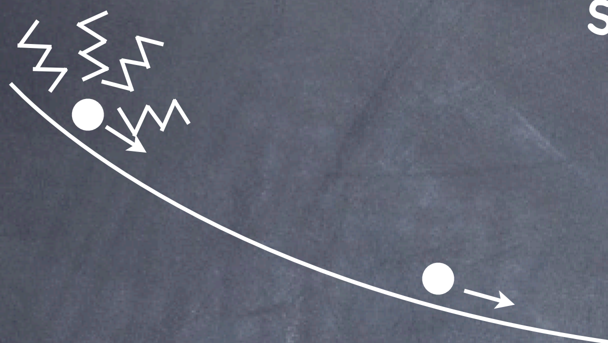
Chen, Easther, Lim 2006; Chen, Easther, Lim 2008; Flauger & Pajer 2010

(e.g. Axion monodromy: McAllister, Silverstein, Westphal 2008; Flauger, McAllister, Pajer, Westphal, Xu 2009)

still vanish in the "squeezed" limit



single-field with modified initial vacuum state generates observable non-Gaussianity

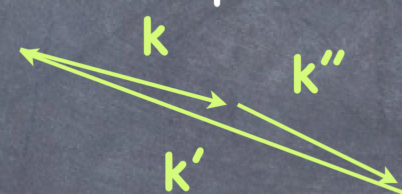


skewness, bispectrum amplitude

$\langle\Phi^3\rangle$ $\quad\quad$ $\langle\Phi(\mathbf{k})\Phi(\mathbf{k}')\Phi(\mathbf{k}'')\rangle \propto \beta_k \sim e^{-k^2/k^2_{\text{cut-off}}}$

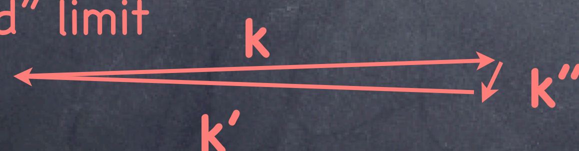
Holman and Tolley 2008

shape



largest in "flattened" configuration

still vanish in the "squeezed" limit



but, may have non-vanishing contributions in a limited, observable k-range

Agullo & Shandera 2012; Ganc & Komatsu 2012

In fact:

single-field inflation predicts

$$\langle \Phi(\mathbf{k})\Phi(\mathbf{k}')\Phi(\mathbf{k}'' \to 0)\rangle \approx \underbrace{(n_s-1)}_{\approx f_{NL}}(2\pi)^3\, \delta(\mathbf{k}+\mathbf{k}')\, P_\Phi(k)\, P_\Phi(k'')$$

where $n_s = d\ln P_\Phi(k)/d\ln k + 4 \approx 1$

the so called "consistency relation"

so $f_{NL} \gtrsim$ few rules it out

Acquaviva, Bartolo, Matarrese, Riotto 2003; Maldacena 2003; Creminelli & Zaldarriaga 2004

(see also Tanaka, Urakawa 2011)

Note:

single-field consistency relation

$$f_{NL} \approx \frac{\partial \ln k^3 P_\Phi}{\partial \ln k} = (n_s - 1)$$

also applies to g_{NL} and τ_{NL}

$$g_{NL} \approx \frac{\partial \ln k^6 B_\Phi}{\partial \ln k} = n_{NG}$$

$$\tau_{NL} \approx (n_s - 1)^2$$

e.g. Chen, Huang, Shiu 2008; Leblond & Pajer 2011
(see also Tanaka, Urakawa 2011)

in terms of physical observables these are strictly zero

also have,

$$\tau_{NL} \gtrsim f_{NL}^{\ 2}$$

Suyama & Yamaguchi 2008; Sugiyama, Komatsu, Futamase 2011; Smith, ML, Zaldarriaga 2011

Single-field models do not generate such extreme couplings of perturbations on short and long length scales

$$\langle\Phi(k_S)\Phi(-k_S-k_L)\Phi(k_L)\rangle \sim \langle P_\Phi(k_S)\Phi(k_L)\rangle \sim f_{NL}\, k_S^{-3}\, k_L^{-3}$$

$m_{\text{other fields}} \gg H$ ----> single-field $\langle \Phi(k_S)\Phi(-k_S-k_L)\Phi(k_L)\rangle \sim 0$

$m_{\text{other fields}} \ll H$ ----> other fields relevant

can get $\langle \Phi(k_S)\Phi(-k_S-k_L)\Phi(k_L)\rangle \sim f_{NL} k_S^{-3} k_L^{-3}$

$m_{\text{other fields}} \sim H$? ----> quasi single-field

Chen & Wang 2010; Baumann & Green 2011

$m_{\text{other fields}} >> H$ ----> single-field $\langle \Phi(k_S)\Phi(-k_S-k_L)\Phi(k_L)\rangle = 0$

$m_{\text{other fields}} << H$ ----> other fields relevant

$\langle \Phi(k_S)\Phi(-k_S-k_L)\Phi(k_L)\rangle = f_{NL} k_s^{-3} k_L^{-3}$

$m_{\text{other fields}} \sim H$? ----> quasi single-field

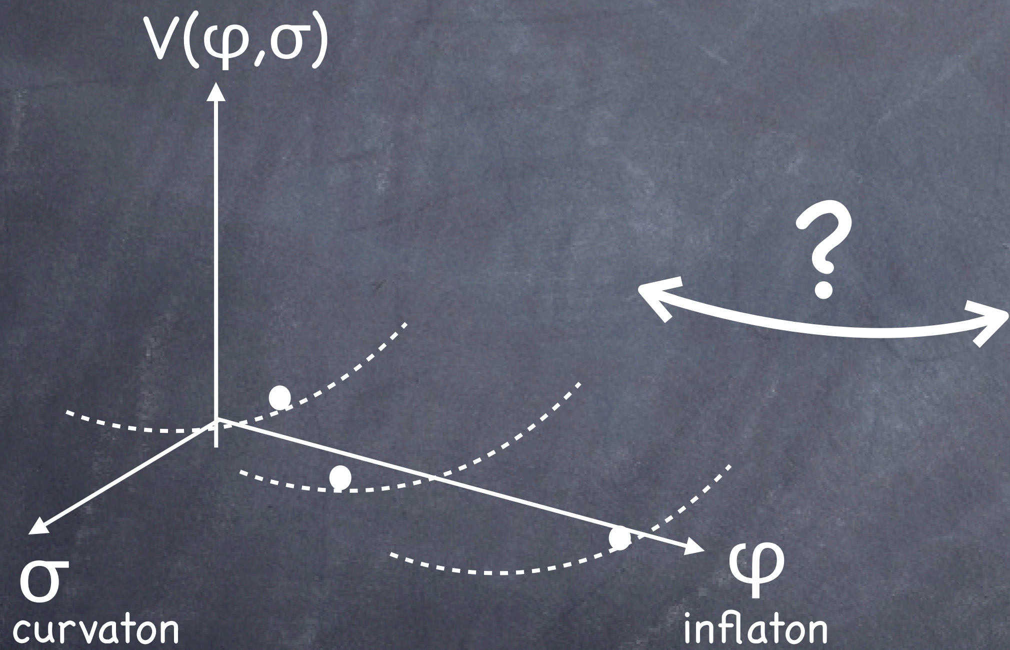
Chen & Wang 2010; Baumann & Green 2011

$$\langle \Phi(k_S)\Phi(-k_S-k_L)\Phi(k_L)\rangle \sim \langle P_\Phi(k_S)\Phi(k_L)\rangle \sim f_{NL}\, k_S^{-3}\, k_L^{-3} \left(\frac{k_L}{k_S}\right)^{3/2-\nu}$$

intermediate scalings possible!

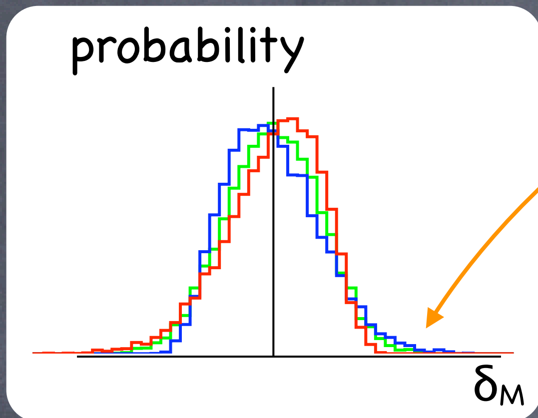
$$\nu \sim \sqrt{9/4 - m^2/H^2}$$

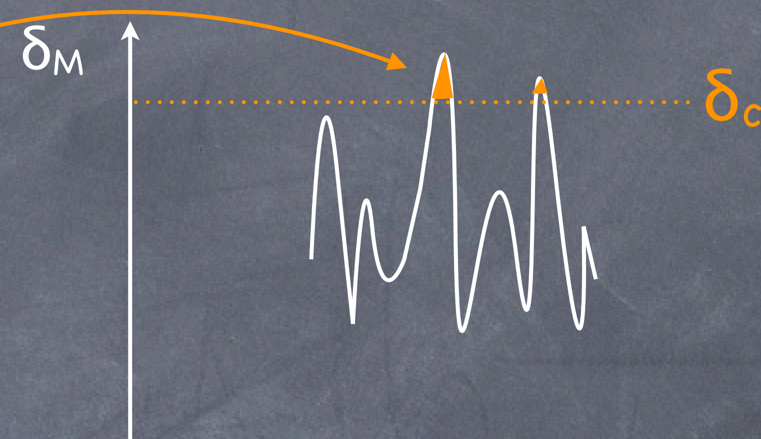
How does primordial non-Gaussianity show up in large-scale structure?



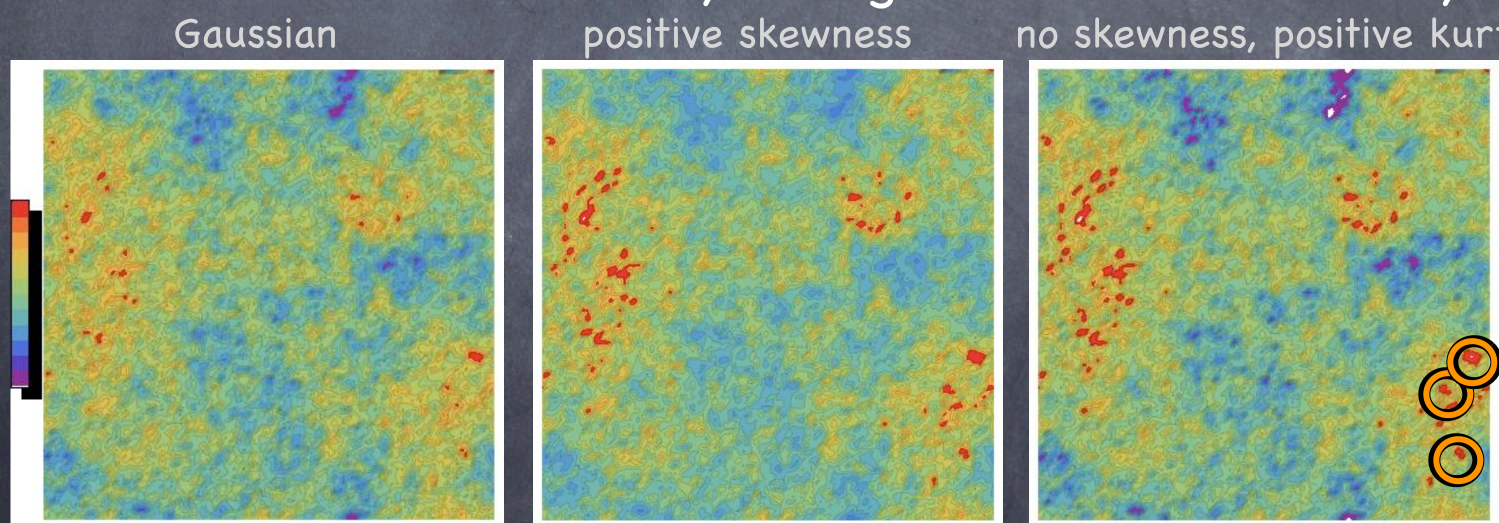
HALO ABUNDANCE

dark matter halos form in peaks of the density field



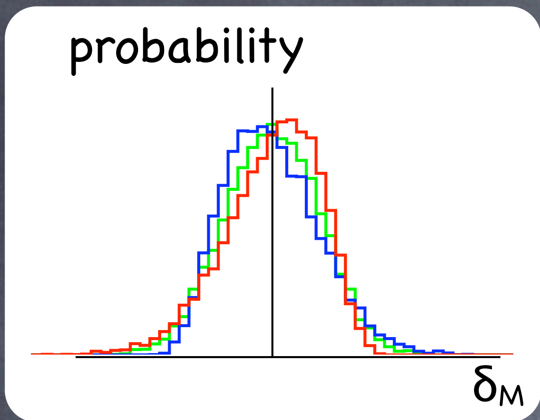


non-Gaussianity changes the number density of peaks



Lucchin & Matarrese 1988; Chiu, Ostriker, Strauss 1998; Robinson, Gawiser, Silk 2000

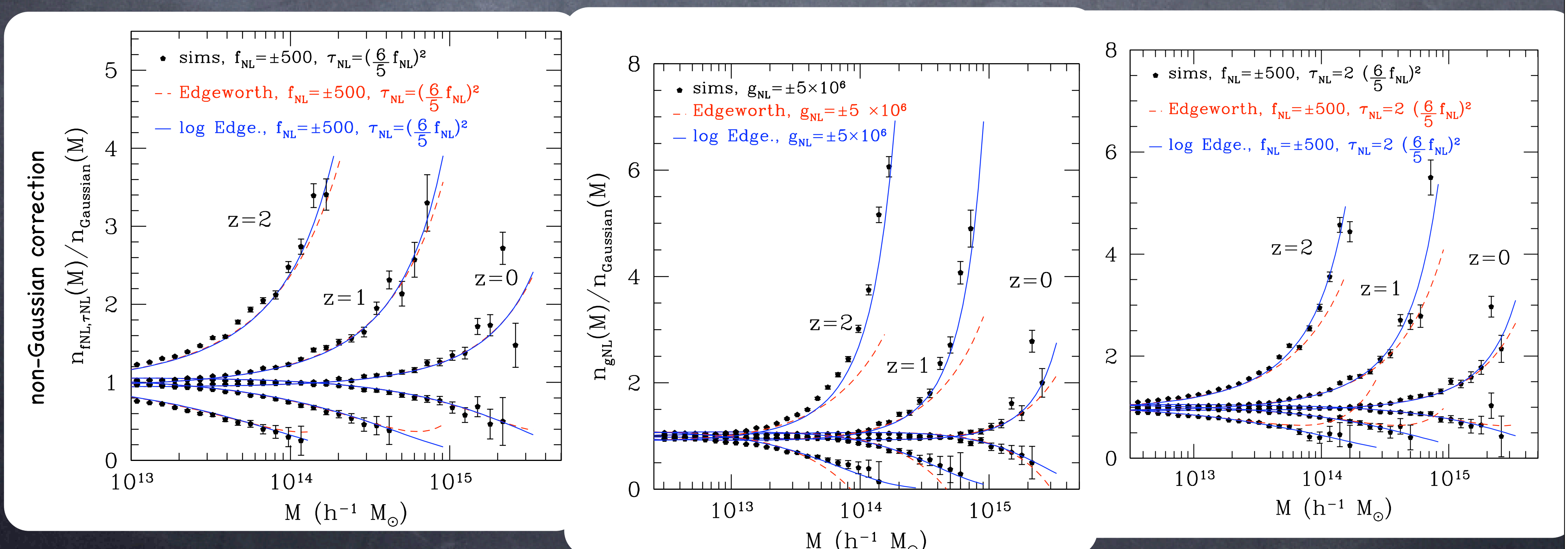
HALO ABUNDANCE



$$\frac{dn_{NG}}{dM} \Big/ \frac{dn_G}{dM}$$

seems to work in comparison to N-body!

(with caveats about how you approximate the PDF)



see also Dalal, Dore, Huterer, Shirokov 2007; Grossi et al 2009; Kang, Norberg, Silk 2009; Pillepich, Porciani, Hahn 2009 ; Desjacques and Seljak 2010; Wagner, Verde, Boubekeur 2010

ML & Smith 2010

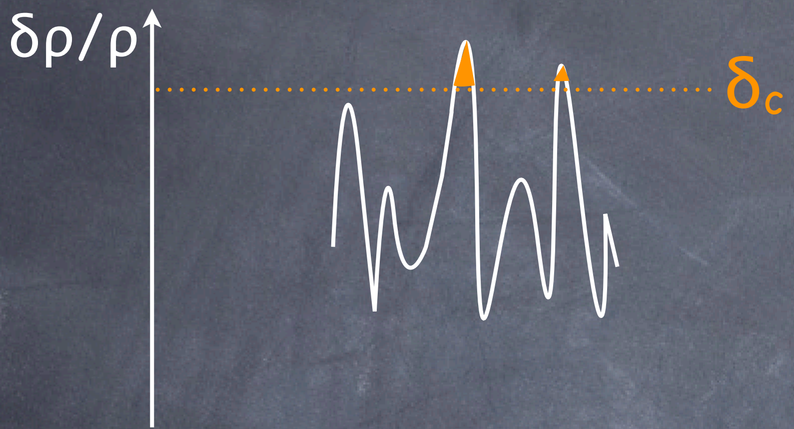
HALO ABUNDANCE

Pros: ingredients just $\langle\delta_M^2\rangle$, $\langle\delta_M^3\rangle$, $\langle\delta_M^4\rangle$ -- insensitive to "shape" of bispectrum trispectrum. in principle $\langle\delta_M^3\rangle$, $\langle\delta_M^4\rangle$ effects not degenerate in dn/dM

Cons: cosmology with cluster abundance is really hard (mass-observable, degeneracy with σ_8 etc)

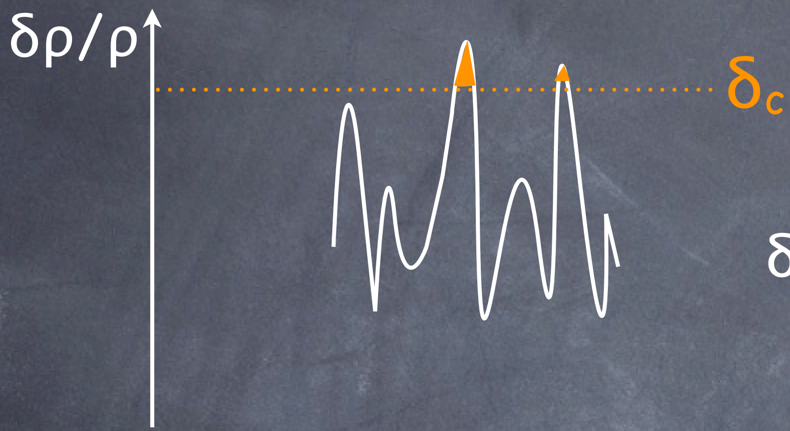
SCALE-DEPENDENT HALO-BIAS

a dark matter halo forms when δρ/ρ is larger than the collapse threshold

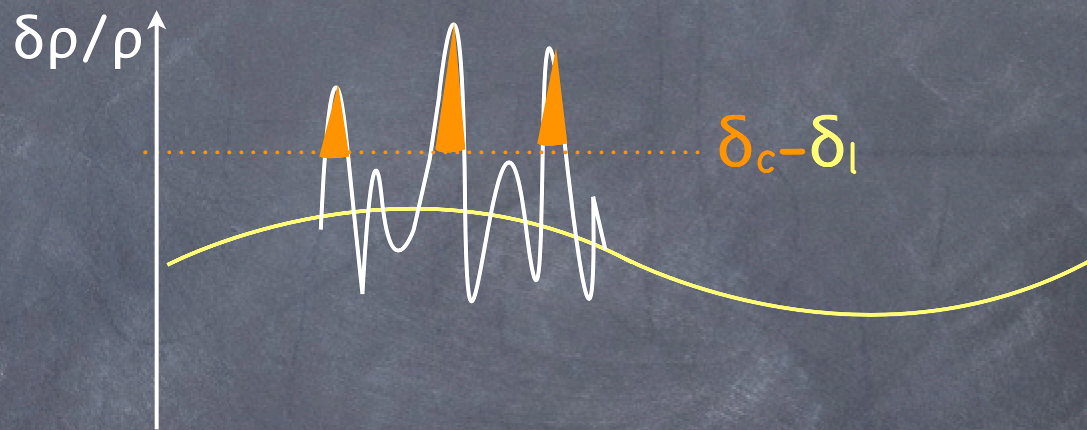


SCALE-DEPENDENT HALO-BIAS

a dark matter halo forms when $\delta\rho/\rho$ is larger than the collapse threshold



which is easier to reach on top of a long wavelength density perturbation



SCALE-DEPENDENT HALO-BIAS

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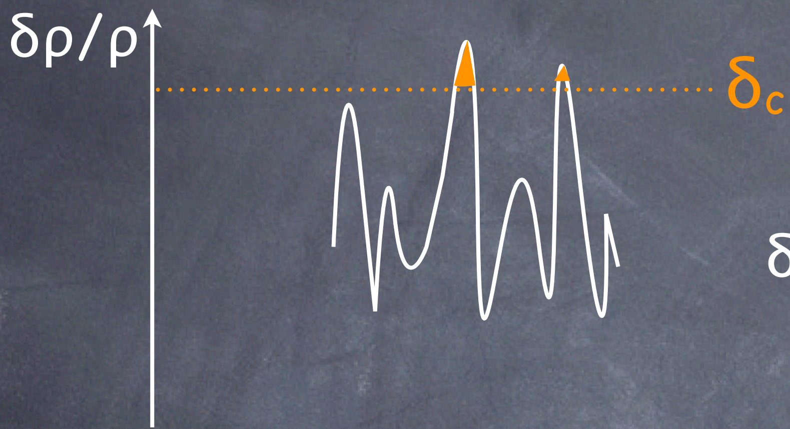

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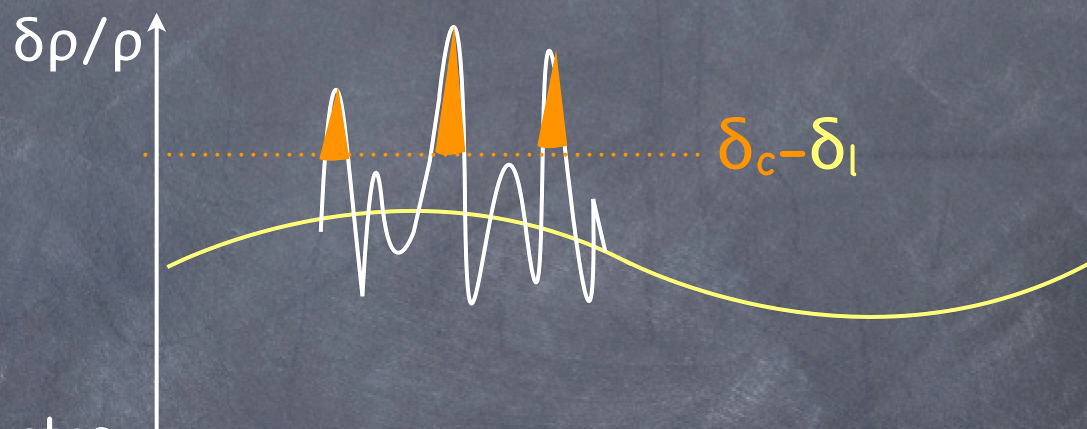

so the number of halos fluctuates depending on δ_l

$$\delta n = \frac{\partial n}{\partial \delta}\delta_l \; . \; . \; .$$

SCALE-DEPENDENT HALO-BIAS

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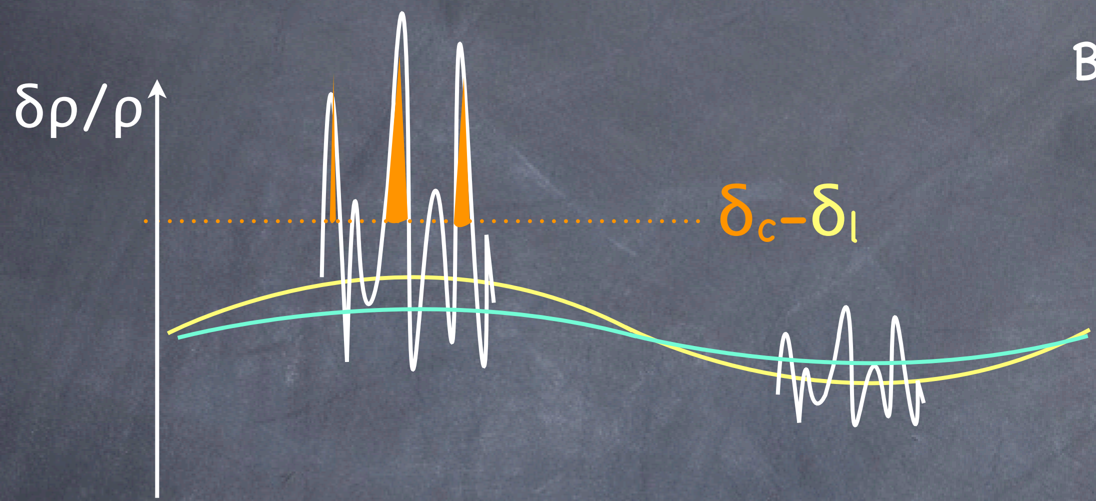

BUT with f_{NL}, the small-scale power fluctuates also depending on Φ_l

Dalal, Doré, Huterer, Shirokov 2007

Matarrese & Verde 2008; Slosar, Hirata, Seljak, Ho, Padmanabhan 2008; Afshordi & Tolley 2008; McDonald 2008

SCALE-DEPENDENT HALO-BIAS

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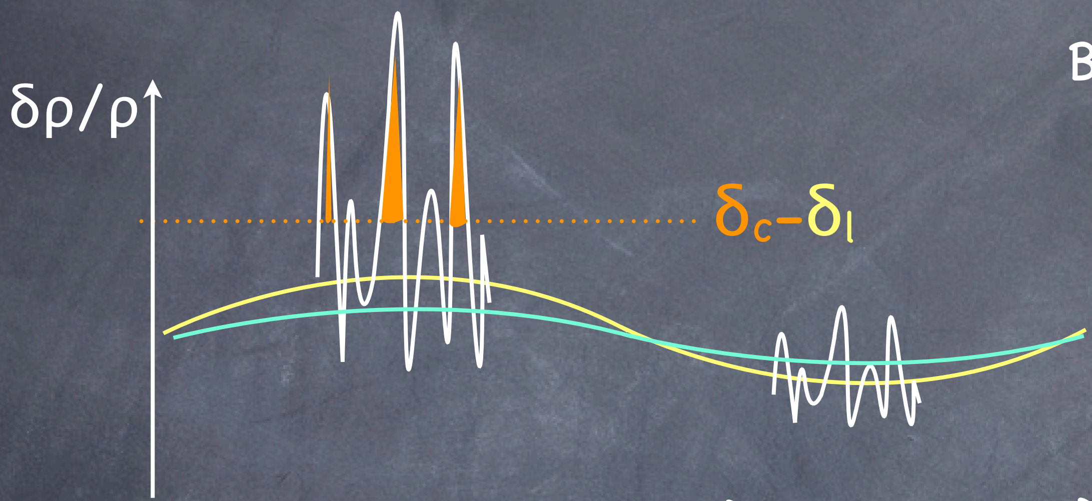

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$$\delta n = \frac{\partial n}{\partial \delta}\delta_l + 4 f_{NL}\frac{\partial n}{\partial P_s}\Phi_l \ldots$$

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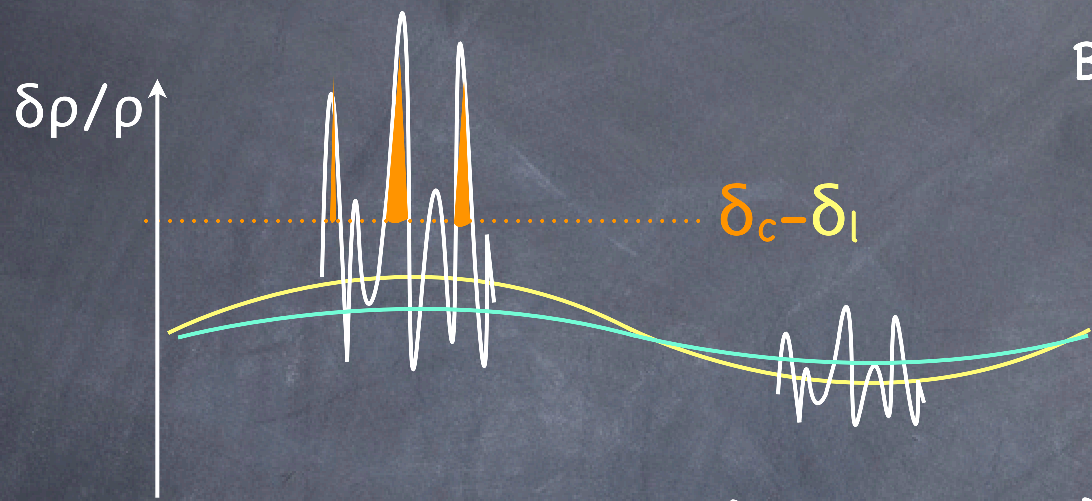

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Poisson's $\nabla^2 \Phi_l \sim 4\pi G\, \delta_l$

$$\delta n \sim \left(\frac{\partial n}{\partial \delta} + \frac{4 f_{NL}}{k^2} \frac{\partial n}{\partial P_s} \right) \delta_l$$

Dalal, Doré, Huterer, Shirokov 2007

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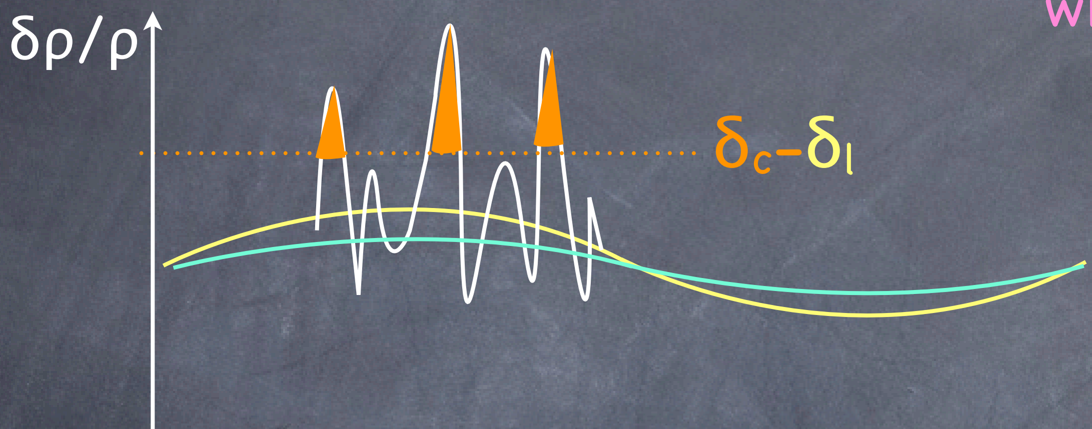
SCALE-DEPENDENT HALO-BIAS

a dark matter halo forms when $\delta\rho/\rho$ is larger than the collapse threshold



with g_{NL} non-Gaussianity, the small-scale skewness fluctuates with Φ_l

so the number of halos fluctuates depending on δ_l and Φ

$$\delta n = \frac{\partial n}{\partial \delta}\,\delta_l + 18 g_{NL}\,\frac{\partial n}{\partial S_3}\,\Phi_l \ldots$$

Desjacques & Seljak 2009; Smith, Ferraro, ML 2011

SCALE-DEPENDENT HALO-BIAS

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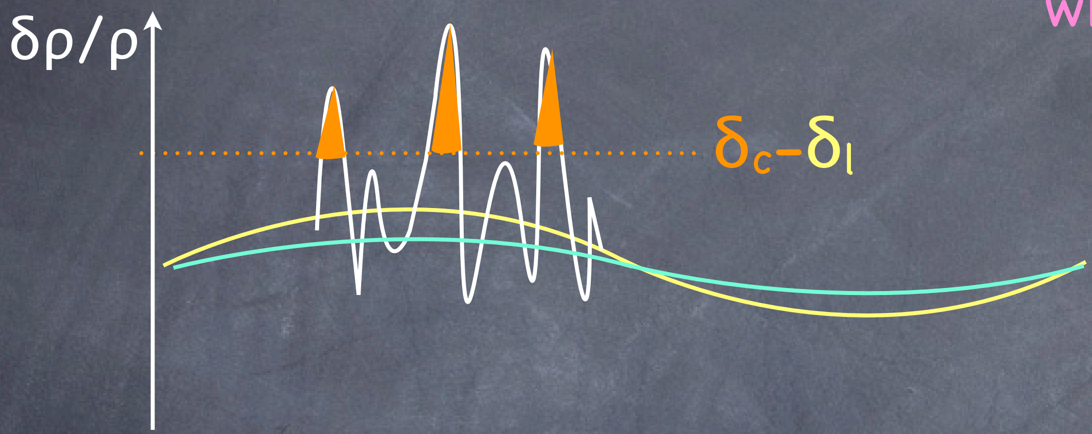

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$$\delta n = \frac{\partial n}{\partial \delta}\,\delta_l + 18 g_{NL}\,\frac{\partial n}{\partial S_3}\,\Phi_l \ldots$$

$$\approx \left(\frac{\partial n}{\partial \delta} + 18 g_{NL} \frac{\partial n}{\partial S_3} / k^2 \right) \delta_l(k) \ldots$$

$\nabla^2 \Phi_l \sim 4\pi G\, \delta_l$

bias depends on Fourier scale k

Desjacques & Seljak 2009; Smith, Ferraro, ML 2011

SCALE-DEPENDENT HALO-BIAS

local non-Gaussianity

$$\Phi(x)=\Phi_G(x)+ f_{NL}\,(\Phi_G(x)^2-\langle\Phi_G{}^2\rangle) + g_{NL}(\Phi_G(x)^3-\Phi_G\langle\Phi_G{}^2\rangle)$$

→ scale dependent halo bias

$$b_{fNL,gNL}\,(k) \sim b + \frac{f_{NL},g_{NL}\ \text{x constant}}{k^2}$$

impossible to generate with single field inflation!

e.g. Creminell, D'Amico, Musso, Noreña 2011

Smith, Ferraro, ML 2011

(Desjacques and Seljak 2010; Desjacques, Jeong, Schmidt 2011; Scoccimarro et al 2012

SCALE-DEPENDENT HALO-BIAS

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impossible to generate with single field inflation!

e.g. Creminell, D'Amico, Musso, Noreña 2011

observational systematics may be hard!

precise values of f_{NL}, g_{NL} will require care -- but seeing $1/k^2$ is the most exciting part

Smith, Ferraro, ML 2011

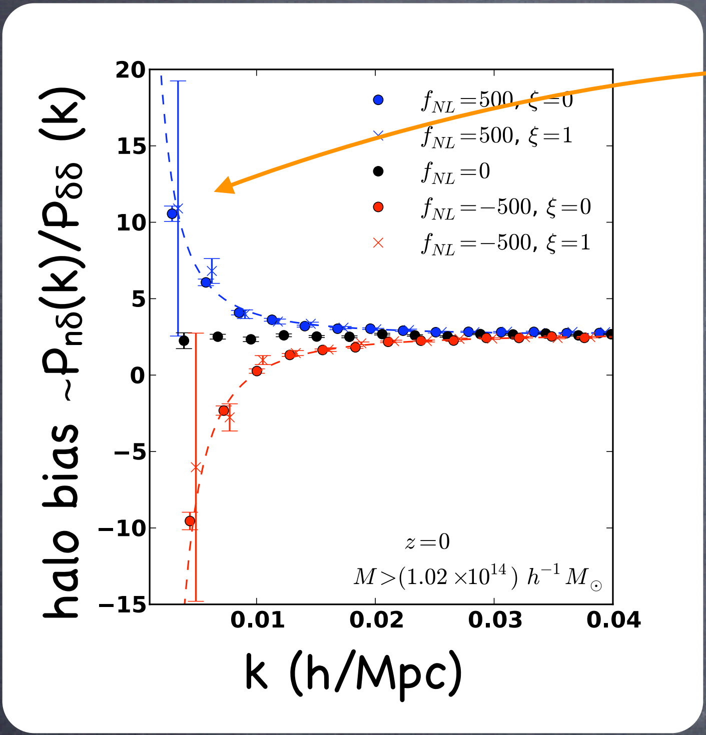
(Desjacques and Seljak 2010; Desjacques, Jeong, Schmidt 2011; Scoccimarro et al 2012

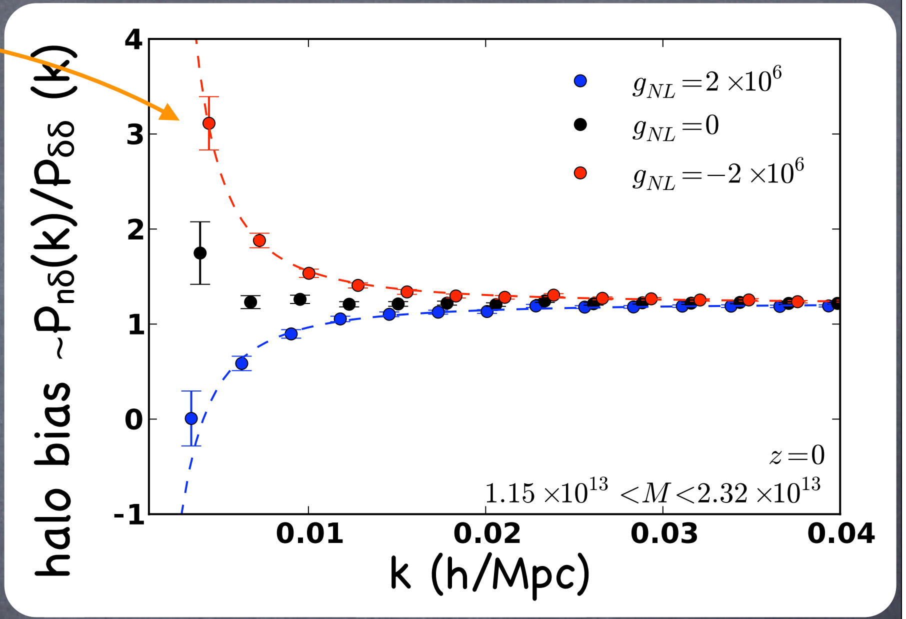
SCALE-DEPENDENT HALO-BIAS

f_{NL}

scales as $1/k^2$

g_{NL}





Dalal, Doré, Huterer, Shirokov 2007
Smith, ML 2010 Smith, Ferraro, ML 2011
Pillepich, Porciani, Hahn 2008; Desjacques, Seljak, Iliev 2008; Grossi et al 2009
Shandera, Dalal, Huterer 2010
(Desjacques and Seljak 2010; Desjacques, Jeong, Schmidt 2011; Scoccimarro et al 2012)

SCALE-DEPENDENT HALO-BIAS

bias coefficient for g_{NL} in terms of mass

$$b_{g_{NL}}(k) = b + \frac{3 g_{NL}}{k^2} \frac{\partial \ln n(M)}{\partial f_{NL}}$$

contrast w/f_{NL} where coefficient in terms of bias

$$b_{f_{NL}}(k) = b + \frac{2\,\delta_c\, f_{NL}\,(b-1)}{k^2}$$

SCALE-DEPENDENT HALO-BIAS

bias coefficient for g_{NL} in terms of mass

$$b_{g_{NL}}(k) = b + \frac{3 g_{NL}}{k^2} \frac{\partial \ln n(M)}{\partial f_{NL}}$$

contrast w/f_{NL} where coefficient in terms of bias

$$b_{f_{NL}}(k) = b + \frac{2\, \delta_c\, f_{NL}\, (b-1)}{k^2}$$

we have a fit for g_{NL} in terms of bias:

$$b_{g_{NL}}(k) \sim b + g_{NL} \frac{\text{non-linear function}(b)}{k^2}$$

Smith, Ferraro, ML 2011

form will depend on selection of population in M, z

SCALE-DEPENDENT HALO-BIAS

bias coefficient for g_{NL} in terms of mass

but! exact $1/k^2$ not necessarily expected!

contrast ... bias

we have a fit for g_{NL} in terms of bias:

$$b_{g_{NL}}(k) \sim b + g_{NL} \frac{\text{non-linear function}(b)}{k^2}$$

Smith, Ferraro, ML 2011

form will depend on selection of population in M, z

SCALE-DEPENDENT HALO-BIAS

generalized local ansatz

$$\langle \Phi(k_S)\Phi(-k_S-k_L)\Phi(k_L)\rangle \sim \xi_\sigma(k_S) k_S^{-3}\, k_L^{-3}$$

$$\langle \Phi(k_S)\Phi(-k_S-k_L)\Phi(k_L)\rangle \sim \xi_{\sigma\varphi}(k_S)\, \xi_{\sigma\varphi}(k_L) k_S^{-3}\, k_L^{-3}$$

Shandera, Dalal, Huterer 2010

SCALE-DEPENDENT HALO-BIAS

generalized local ansatz

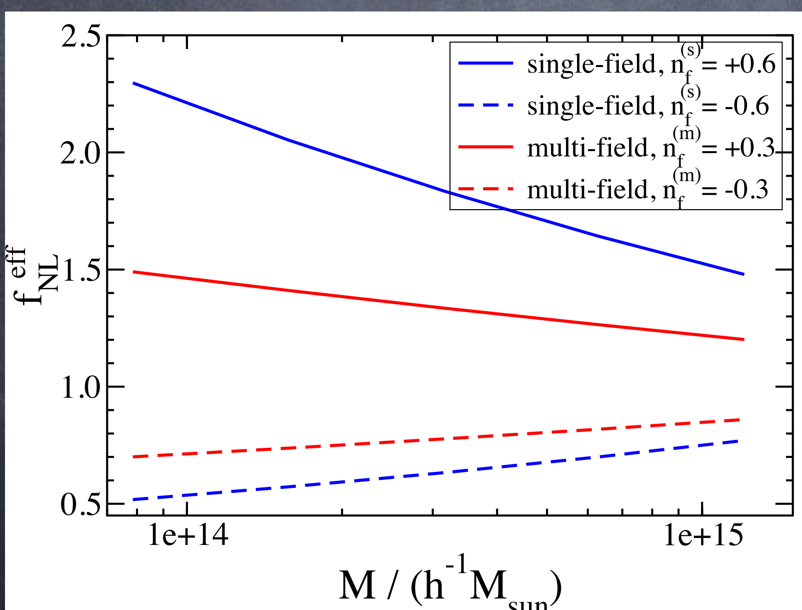
$$\langle \Phi(k_S)\Phi(-k_S-k_L)\Phi(k_L)\rangle \sim \xi_\sigma(k_S)\, k_S^{-3}\, k_L^{-3}$$

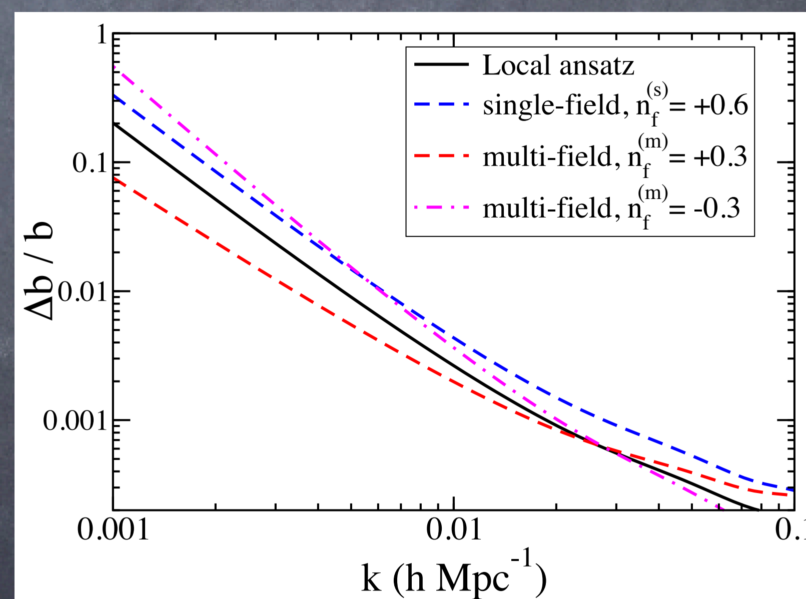
$$\langle \Phi(k_S)\Phi(-k_S-k_L)\Phi(k_L)\rangle \sim \xi_{\sigma\varphi}(k_S)\, \xi_{\sigma\varphi}(k_L)\, k_S^{-3}\, k_L^{-3}$$

$$\xi(k) \sim k^{\text{slow-roll}}$$

$$f_{NL}(M) \sim \xi_{\sigma\varphi}(k_S)\, \xi_\sigma(k_S)$$

$$b(k) \sim \xi_{\sigma\varphi}(k)\, k^{-2} \sim k^{-2+\text{slow-roll}}$$





$$k_S \sim (\rho/M)^{1/3}$$

Shandera, Dalal, Huterer 2010

SCALE-DEPENDENT HALO-BIAS

quasi-single field models?

$$\langle \Phi(k_S)\Phi(-k_S-k_L)\Phi(k_L)\rangle \sim \langle P_\Phi(k_S)\Phi(k_L)\rangle \sim f_{NL}\; k_S^{-3}\; k_L^{-3} \left(\frac{k_L}{k_S}\right)^{3/2-\nu}$$

$$f_{NL}(M) \sim f_{NL}\; k_s^{-3/2+\nu}$$

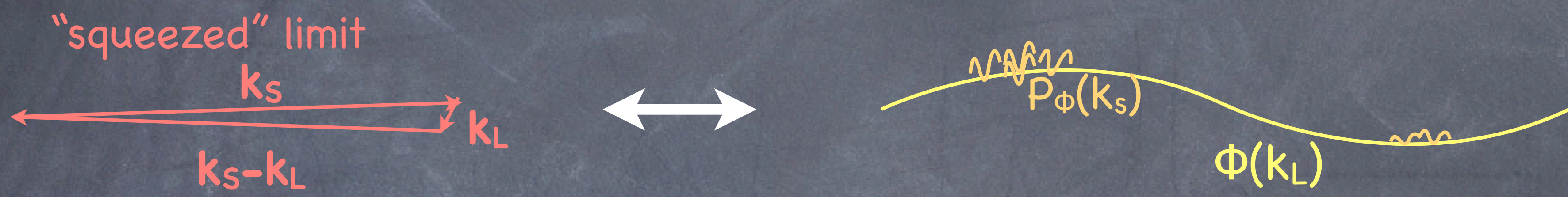
$$k_S \sim (\rho/M)^{1/3}$$

$$b(k) \sim k^{-2+3/2-\nu}$$

$$0 \leq \nu \leq 3/2$$

MORE:

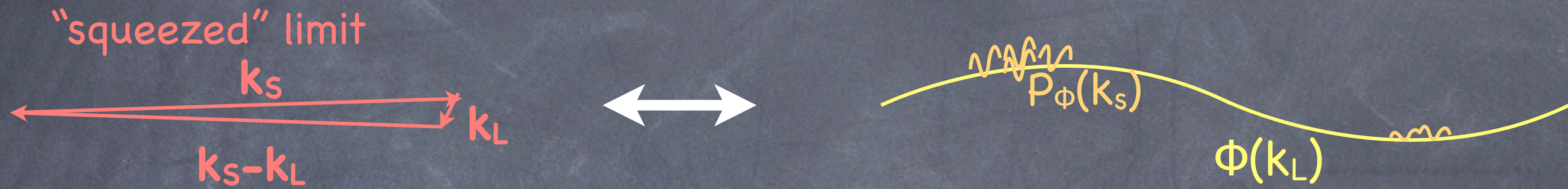
scale-dep bias only probes a particular configuration of bispectrum (or higher)



and, it's one that vanishes in single-field models

MORE:

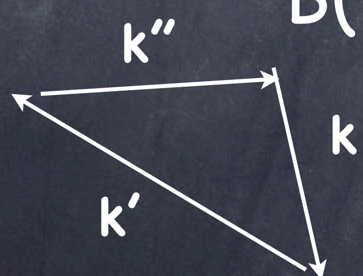
scale-dep bias only probes a particular configuration of bispectrum (or higher)



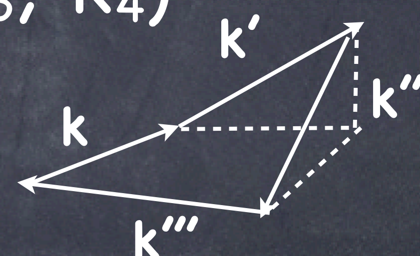
and, it's one that vanishes in single-field models

FULL bispectrum, trispectrum sensitive to more general models, contains more information

$B(k_1, k_2, k_3)$



$T(k_1, k_2, k_3, k_4)$

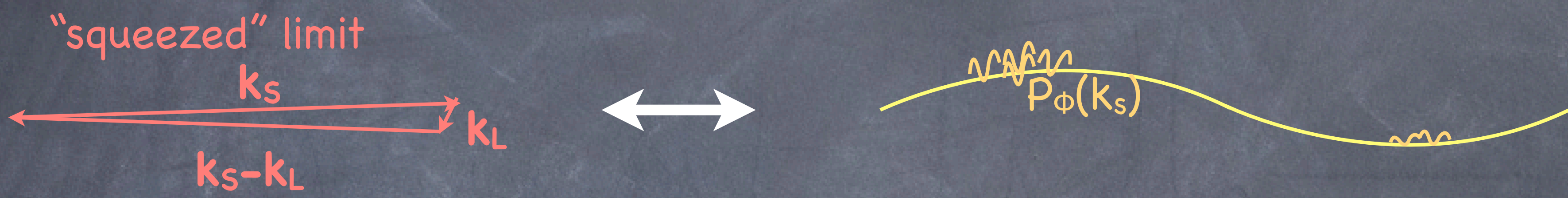


Summary

- Lots of different kinds of non-Gaussian initial conditions
- qualitatively different shapes & scalings of non-Gaussianity from qualitatively different models
- halo abundance sensitive to local statistics of δ_M
- halo clustering (scale-dep bias) probes squeezed limits of bispectrum, trispectrum -- power to rule out single-field inflation
- analytic description for the halo mass function looks good compared with N-body so far
- Analytic descriptions of halo bias agree well with sims

First theory breakout session summary:

scale-dep bias only probes a particular configuration of bispectrum (or higher)



Every bispectrum has a squeezed limit
It just might be very small.....

Seeing anything in scale-dep. bias/squeezed limit is indicative of new physics incredibly exciting

the current limits are already interesting

First theory breakout session summary:

Since every bispectrum (i.e. models other than f_{NL} local) has a squeezed limit, scale dependent bias constrains a broad space of theories.

<u>However</u>, scale-dependent bias in other theories will not have the usual form: $b_0+2\delta_c\, f_{NL}(b_0-1)/k^2$

more powerful to fit:

$$b(k)=b_0+f(M)/k^\alpha$$

where f(M) is a function of mass (that can be calculated from a non-Gaussian model) that is proportional to the amplitude of non-Gaussianity (e.g. $\propto f_{NL}, g_{NL}$) and it's probably safe to assume $0 \leq \alpha \leq 3$

special values of α:

$0 \leq \alpha \leq 2$: quasi-single-field
$\alpha = 2$: exact local model (f_{NL}, g_{NL})
$\alpha = 2 \pm \varepsilon$: two fields contributing to primordial perturbations
$\alpha = 3$: modified initial state

First theory breakout session summary:

AGAIN, seeing anything in scale-dep. bias/squeezed limit is indicative of new physics incredibly exciting

A detection would mean there are other signatures to go after and help distinguish between models

the current limits are already interesting

There are non-Gaussian models that have vanishingly small squeezed limits (and therefore vanishingly small scale-dep bias) BUT detectably large signals in other, non-squeezed configurations. SO we should continue to explore other observables (e.g. galaxy bispectrum in non-squeezed configurations)