Bias: Gaussian, non-Gaussian, Local, non-Local

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- R.S., L. Hui, M. Manera, and K.C. Chan (arXiv:1108.5512)
- K.C. Chan, R.S. and R. Sheth (arXiv:1201.3614 and in preparation)
- LasDamas: C. McBride et al, M. Manera et al., E. Sefusatti et al. (in preparation)

Primordial Non-Gaussianity from Inflation

Gaussianity is a consequence of:

i) inflaton a single scalar field
ii) slowly rolling
iii) in vacuum state
iv) with canonical kinetic terms

if we relax i) we have for the Bardeen potential,

$$\Phi = \phi + f_{\rm NL}\phi^2$$

which implies for it a bispectrum,

$$B = 2 f_{\rm NL} P_1 P_2 + \text{cyc.} \qquad\qquad -10 < f_{\rm NL}^{\rm local} < 74$$

- For biased tracers (galaxies, halos), this model leads to a scale-dependent bias at large scales (Dalal et al 2008),

$$b_1(k) = b_{10} + \Delta b_1(k, f_{\rm NL})$$

with b~1/k^2 at low-k. Thus the power spectrum of galaxies is sensitive to fnl!!

Beyond Local Primordial Non-Gaussianity

- Within single-field inflationary models, we can break Gaussianity by introducing non-canonical kinetic terms, leading to the so-called equilateral and orthogonal shapes for the primordial bispectrum.

For example, the equilateral model has a Bardeen potential bispectrum,

$$(6f_{\rm NL})^{-1}B_{\rm equil} = -P_1P_2 - 2(P_1P_2P_3)^{2/3} + P_1^{1/3}P_2^{2/3}P_3$$

$$-214 < f_{\rm NL}^{\rm equil} < 266$$

(permutations are understood), whereas the orthogonal model reads

$$(6f_{\rm NL})^{-1}B_{\rm ortho} = -3P_1P_2 - 8(P_1P_2P_3)^{2/3} + 3P_1^{1/3}P_2^{2/3}P_3$$

$$-410 < f_{\rm NL}^{\rm ortho} < 6$$

Generic Predictions in Peak-Background Split

We are interested in establishing as rigorously as possible the validity of the local PNG bias formula

$$\Delta b_1(k, f_{\rm NL}) = \frac{2 f_{\rm NL}}{M(k)}(b_{10} - 1)\delta_c$$

and generalizing it to arbitrary (non-local) PNG. Some issues in derivations,

- proper treatment of filter and transfer function effects

- dependence on primordial bispectrum (cannot be just a number)

- peaks in phi vs peaks in delta approximations

$$\nabla \phi^2 = 2\phi\nabla^2\phi + 2\nabla\phi\cdot\nabla\phi \approx 2\phi\nabla^2\phi ?$$

simulations suggest a somewhat smaller amplitude (depending on halo def)

A full calculation of the PBS change in bias due to arbitrary PNG bispectrum gives,

$$\Delta b(k) = \frac{\partial_{\sigma^2}\Big[I_B(k)\mathcal{F}_0\Big]}{M(k)\,\mathcal{F}_0}$$

$$I_B(k,R) \equiv \frac{1}{P_\phi(k)}\int B_{\delta_R\delta_R\phi}(\boldsymbol{q},\boldsymbol{k}-\boldsymbol{q},-\boldsymbol{k})\,d^3q$$

Note that, unlike the GW86 formula, what matters is the *cross* bispectrum. For local PNG, expanding in powers of k small (with higher-order corrections coming from filter, transfer function, grad-phi terms, etc

$$I_B(k=0,R) \approx 4f_{\rm NL}\;{\sigma_R}^2(m) + \mathcal{O}(k^2)$$

which gives

non-markovian ↓

$$\Delta b(k) = \frac{4f_{\rm NL}}{M(k)}\;\partial_{\ln\sigma^2}\ln(\sigma^2\mathcal{F}_0) < \frac{2f_{\rm NL}}{M(k)}\delta_c\frac{(\partial\mathcal{F}/\partial\delta_\ell)_0}{\mathcal{F}_0} = \frac{2f_{\rm NL}}{M(k)}\delta_c(b_1-1)$$

the precise relationship has to be obtained from the first-crossing prob F0.

In terms of the mass function,

$$\Delta b_1 = \frac{\partial_m \left[I_B(k,m) \left(\frac{d\,n}{d\ln m} \right) \left(\frac{d\sigma_m^2}{dm} \right)^{-1} \right]}{M(k) \left(\frac{d\,n}{d\ln m} \right)}$$

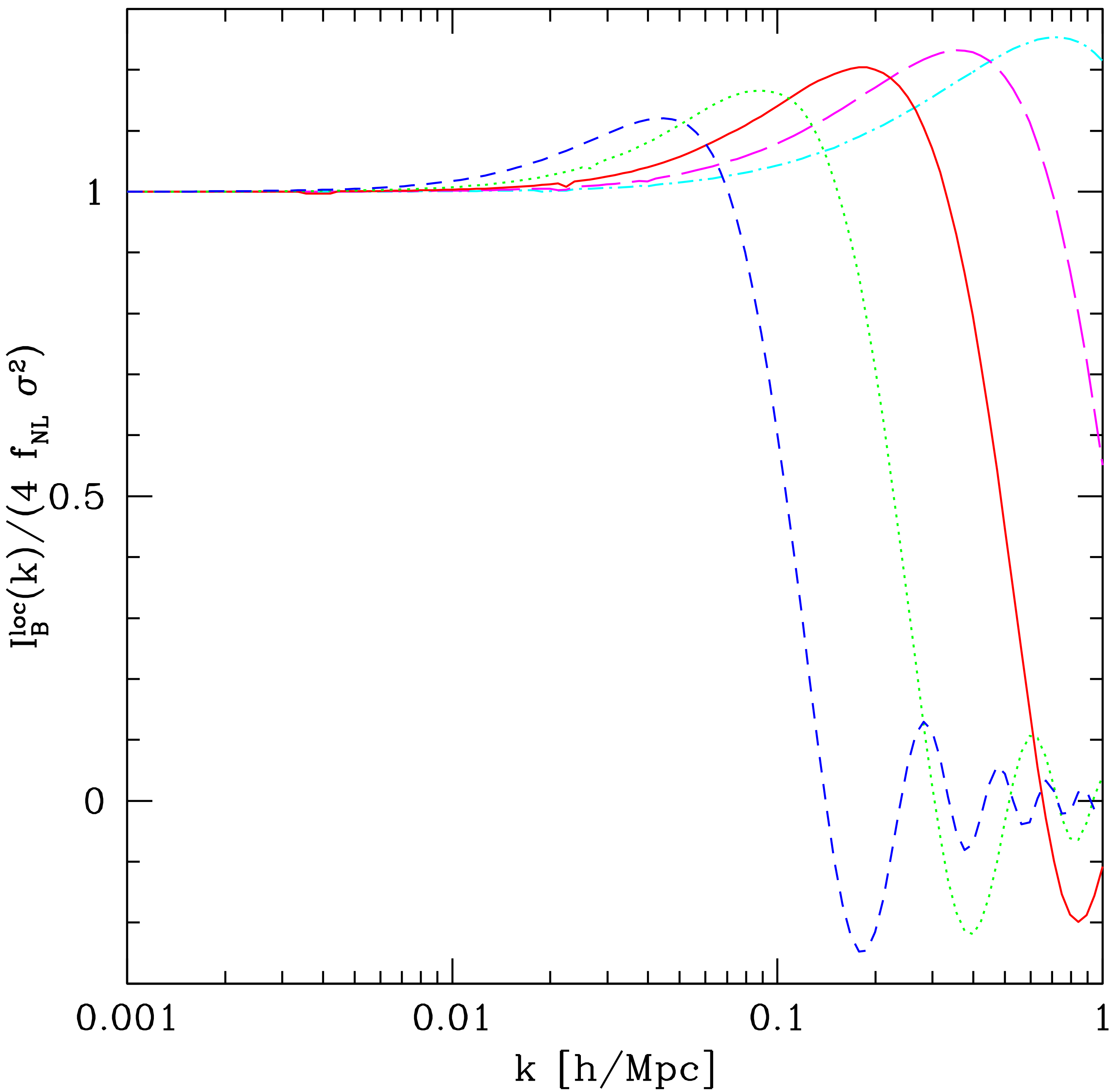
note that, without assuming markovian + universality, this is more general than the usual (b-1) amplitude.

Given a *Gaussian* mass function (not necessarily universal, e.g. measured from simulations), we can compute the scale dependent bias.

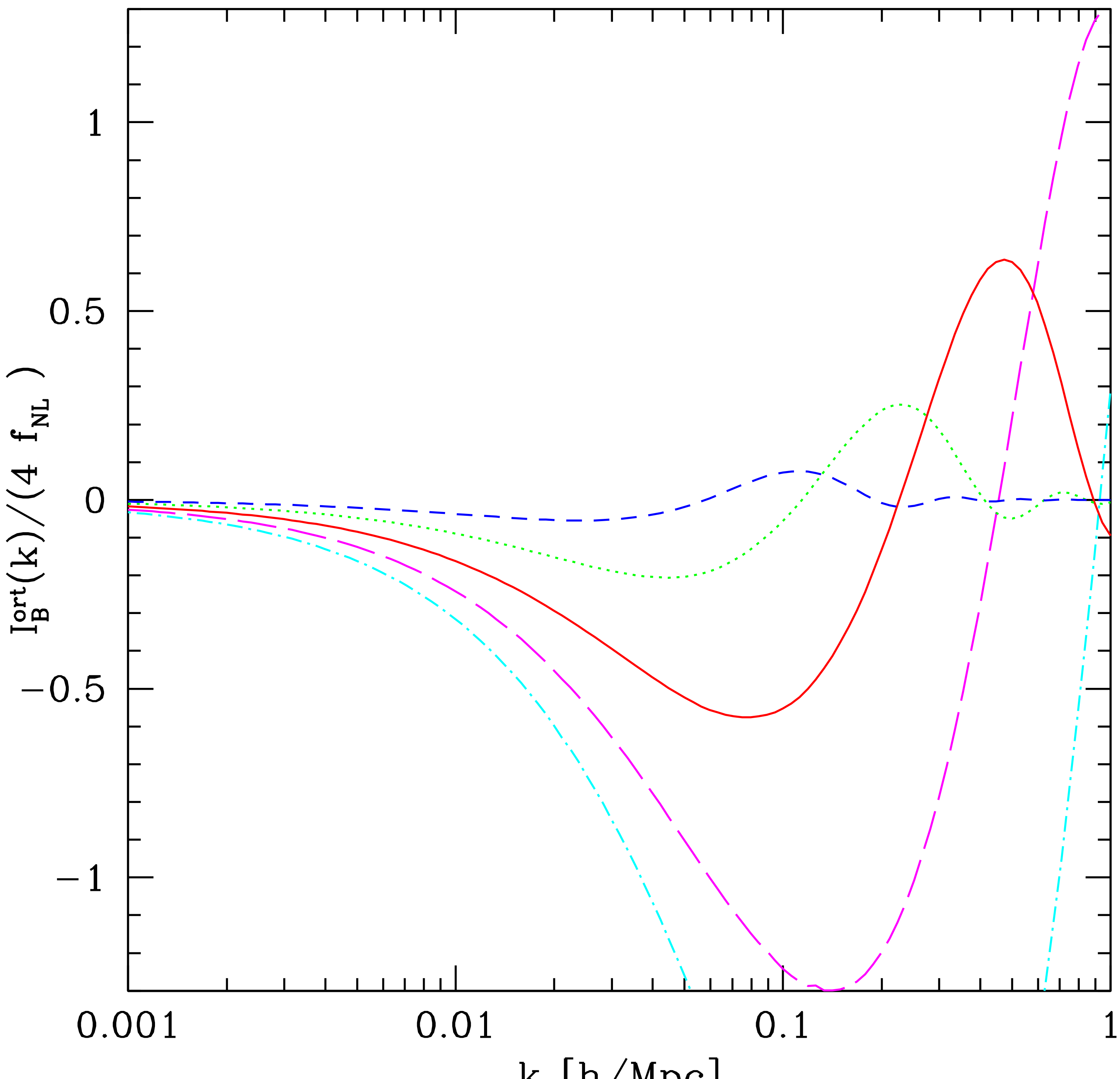
Same for quadratic bias,

$$\Delta b_2 = \frac{\partial_{\sigma_m^2} [I_B(k_1)\, b_{1L}^{(1)}\, \mathcal{F}_0]}{M(k_1)\, \mathcal{F}_0} + k_1 \leftrightarrow k_2$$

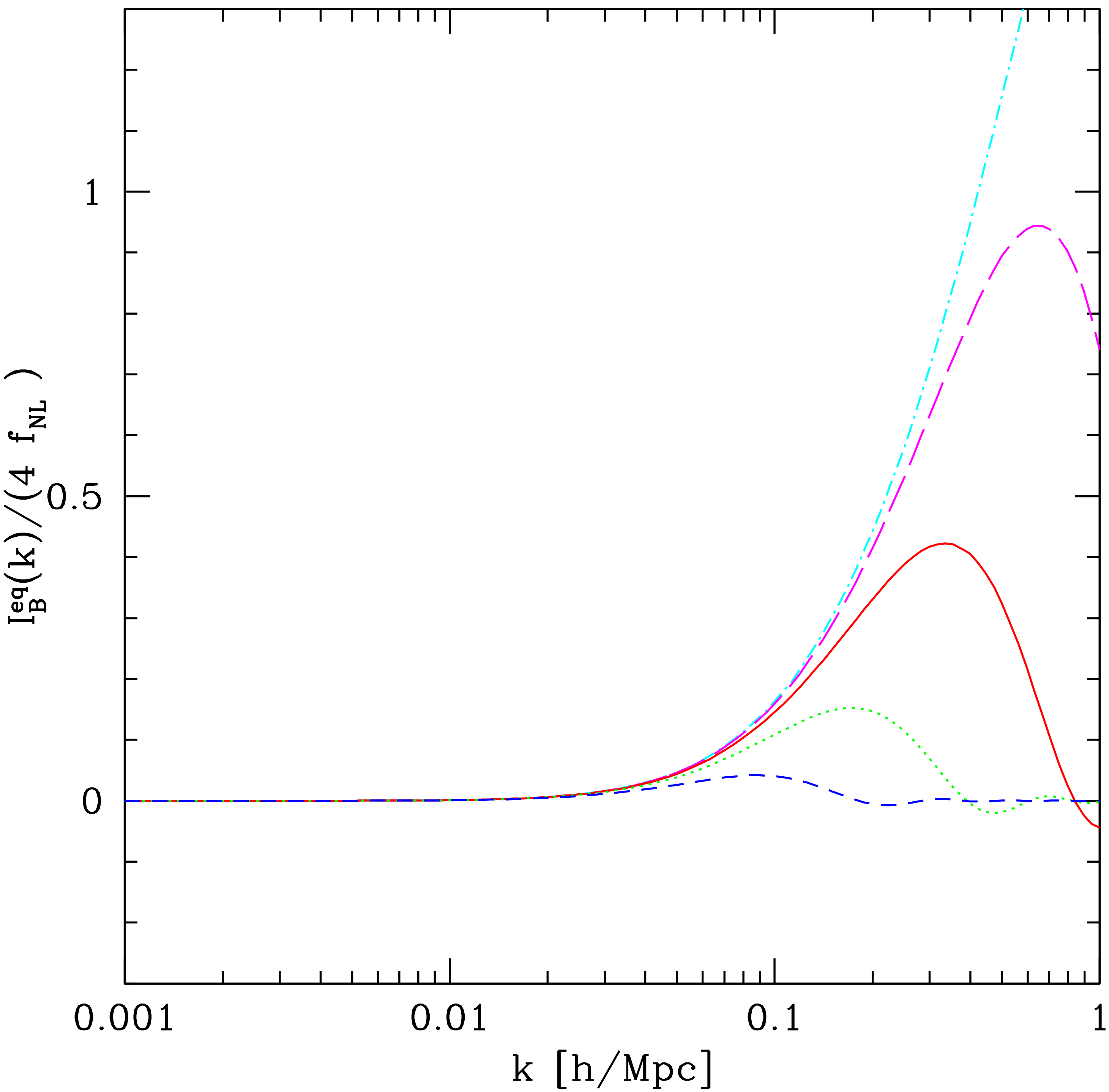
$$\Delta b_2 = \frac{\partial_m \left[I_B(k_1)\, b_{1L}^{(1)} \left(\frac{dn}{d\ln m} \right) \left(\frac{d\sigma_m^2}{dm} \right)^{-1} \right]}{M(k_1) \left(\frac{dn}{d\ln m} \right)} + k_1 \leftrightarrow k_2$$



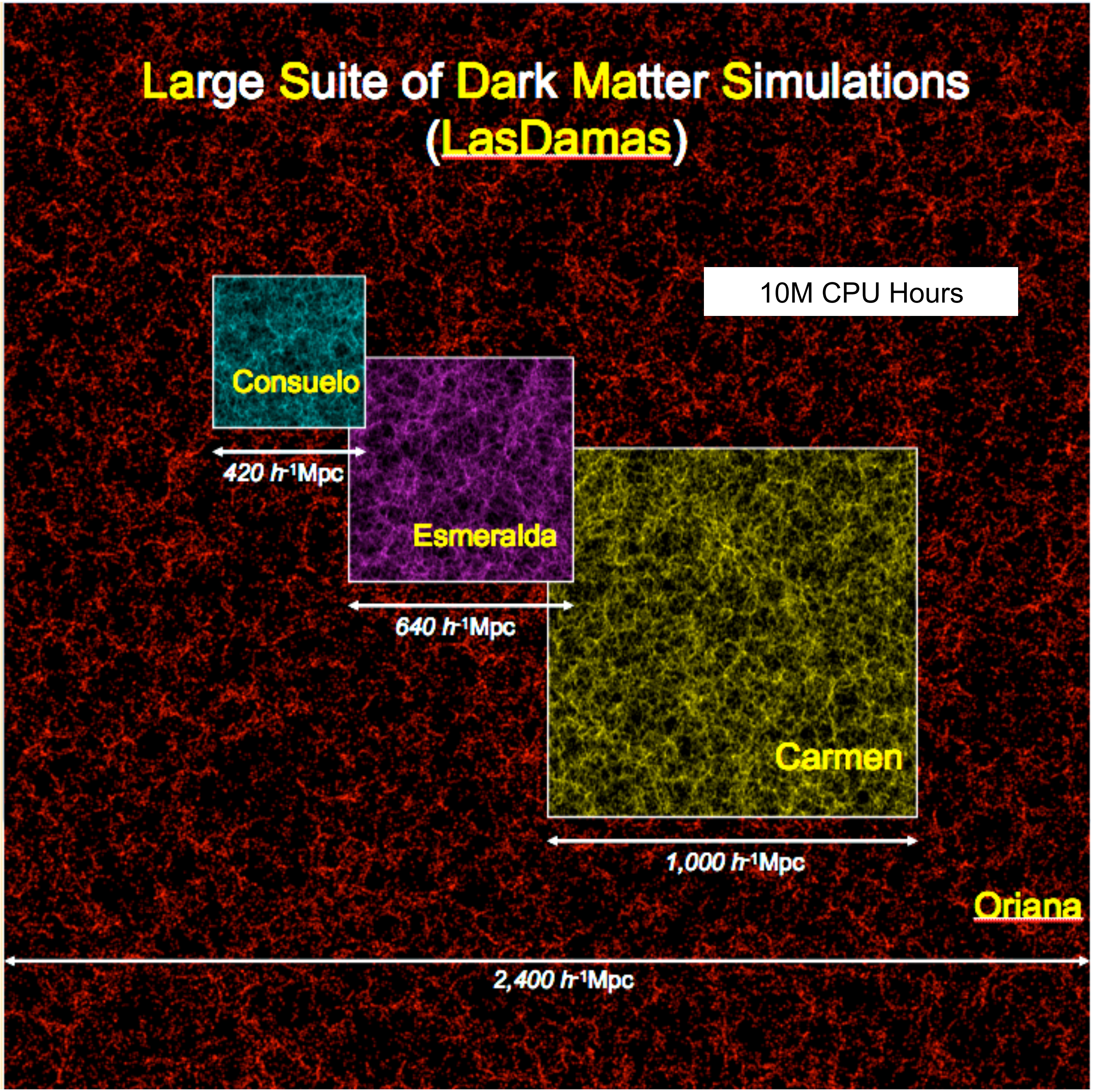
1
0.5
0
0.001
0.01
0.1
1
k [h/Mpc]
$I_B^{loc}(k)/(4\ f_{NL}\ \sigma^2)$



1
0.5
0
−0.5
−1
0.001
0.01
0.1
1
k [h/Mpc]
$I_B^{ort}(k)/(4\ f_{NL})$



1
0.5
0
0.001
0.01
0.1
1
k [h/Mpc]
$I_B^{eq}(k)/(4\ f_{NL})$

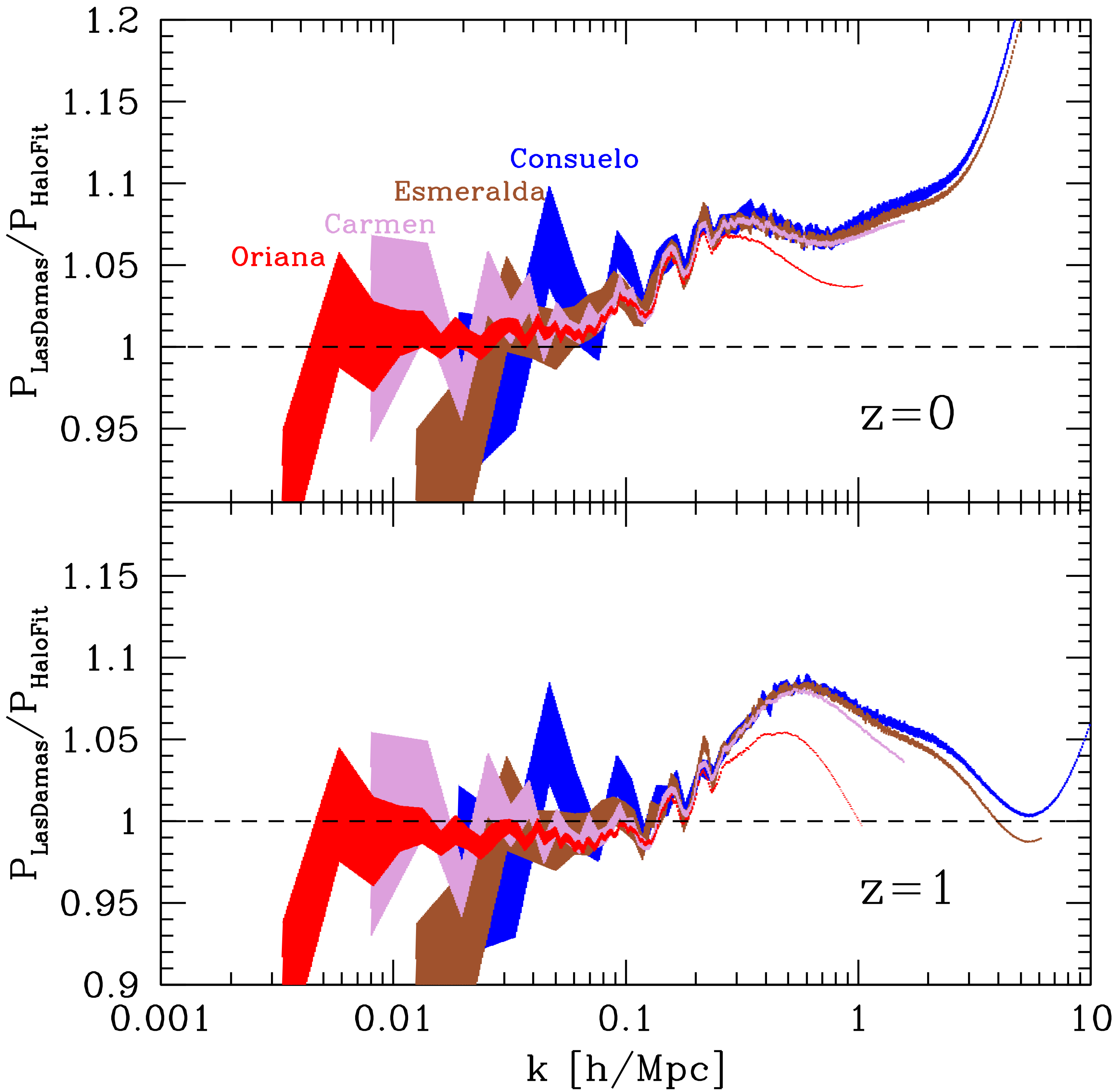


with
A. Berlind,
C. McBride,
M. Manera,
J. Gardner,
M. Busha,
R. Wechsler,
F. van den Bosch

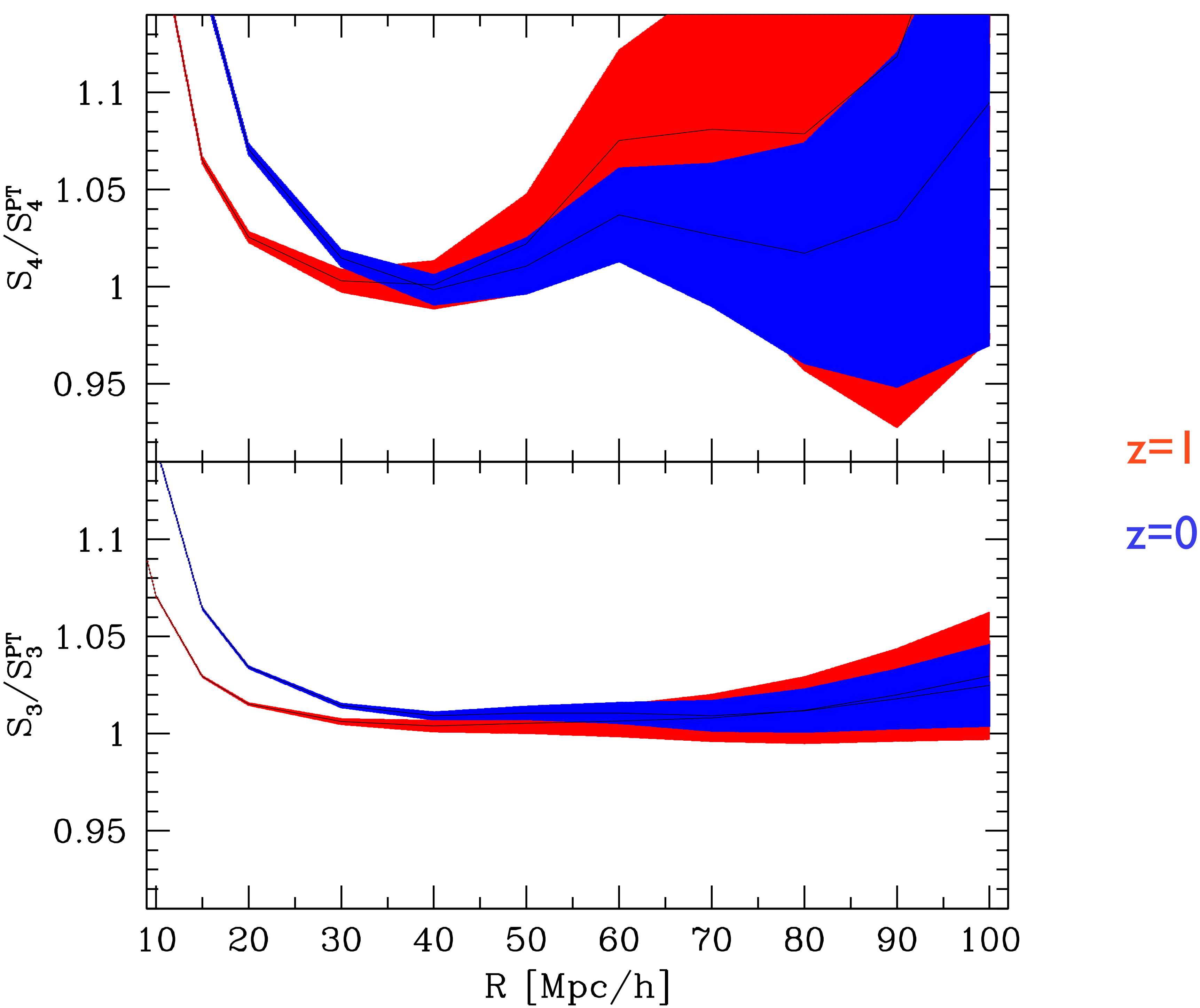
LasDamas Simulations

Name	Sample	Lbox	Npar	mpar	Nrealiz
Oriana (G)	LRG +Main -22	2400	1280^3	4.57E+11	42
Oriana fnl_local=+100	LRG +Main -22	2400	1280^3	4.57E+11	40
Oriana fnl_equi=-400	LRG +Main -22	2400	1280^3	4.57E+11	30
Oriana fnl_orto=-400	LRG +Main -22	2400	1280^3	4.57E+11	37
Carmen	Main -21	1000	1120^3	4.98E+10	42
Esmeralda	Main -20	640	1250^3	9.31E+09	50
Consuelo	Main -19-18	420	1400^3	1.87E+09	50

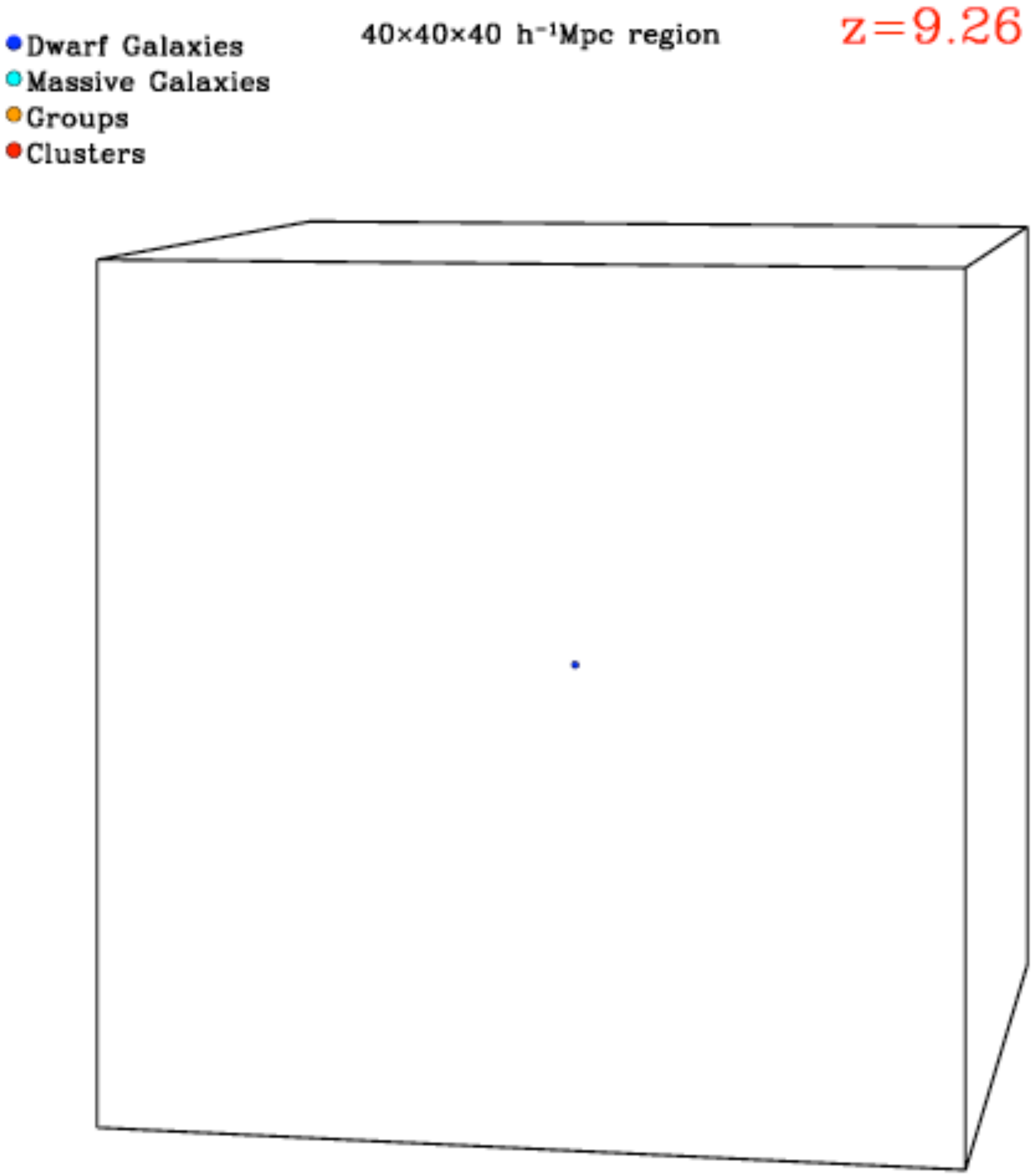
Nmocks=4 x Nrealiz,   2LPT ICs,   Gaussian Mocks available at http://lss.phy.vanderbilt.edu/lasdamas/

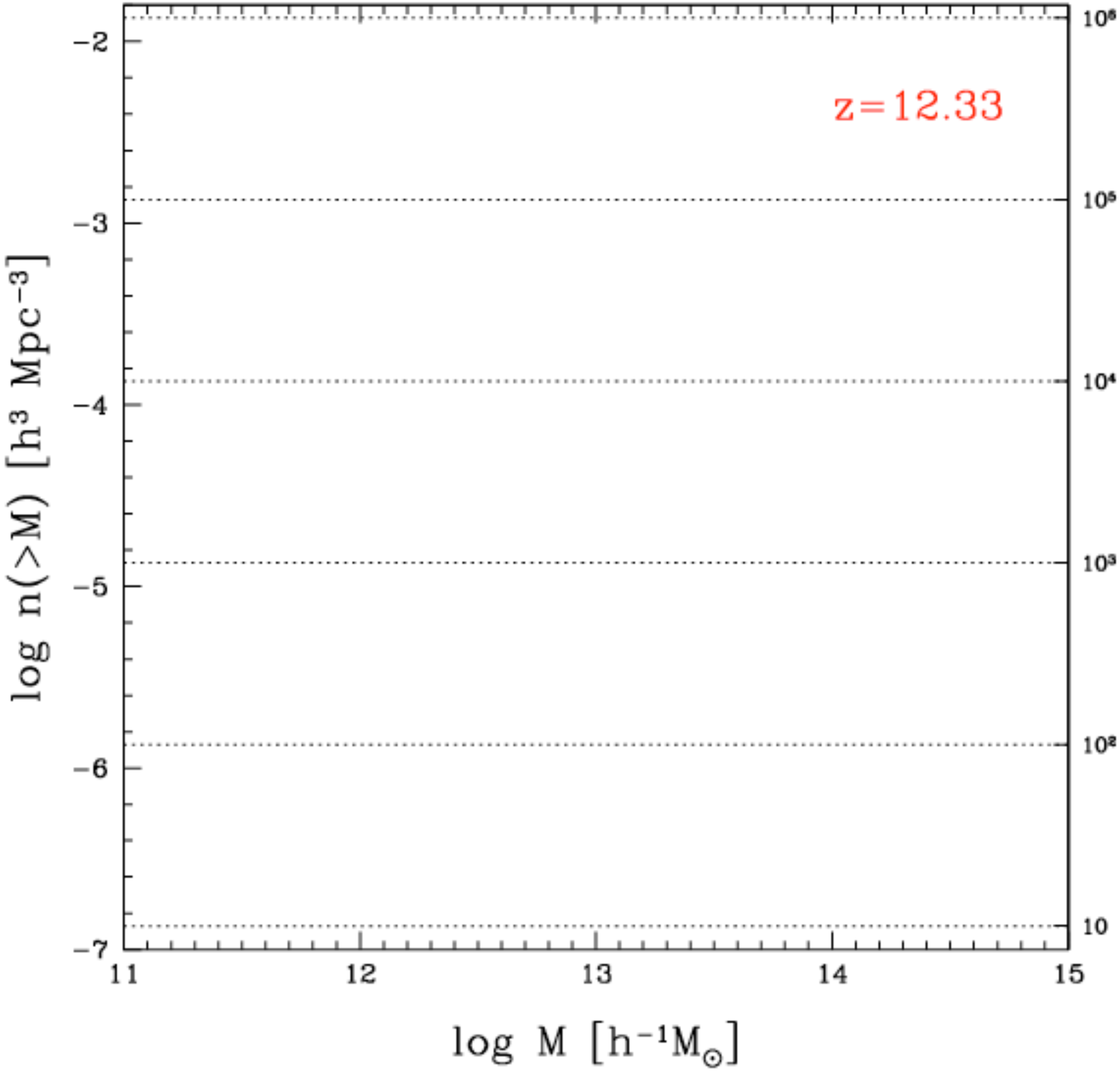


Consuelo
Esmeralda
Carmen
Oriana
z=0
z=1
$P_{LasDamas}/P_{HaloFit}$
k [h/Mpc]
0.001
0.01
0.1
1
10
1.2
1.15
1.1
1.05
1
0.95
0.9



S_4/S_4^{PT}
1.1
1.05
1
0.95
S_3/S_3^{PT}
1.1
1.05
1
0.95
10
20
30
40
50
60
70
80
90
100
R [Mpc/h]
z=1
z=0


Dwarf Galaxies
Massive Galaxies
Groups
Clusters
40×40×40 h⁻¹Mpc region
z=9.26


z=12.33
log n(>M) [h³ Mpc⁻³]
log M [h⁻¹M⊙]
−2
−3
−4
−5
−6
−7
11
12
13
14
15
10⁶
10⁵
10⁴
10³
10²
10

non-local PNG Initial Conditions in Simulations

- In single-field inflationary models, we are instead interested in models that correspond to non-local PNG (due to non-canonical kinetic terms). For example, the equilateral model has a Bardeen potential bispectrum,

$$(6 f_{\rm NL})^{-1} B_{\rm equil} = -P_1 P_2 - 2(P_1 P_2 P_3)^{2/3} + P_1^{1/3} P_2^{2/3} P_3$$

$$-214 < f_{\rm NL}^{\rm equil} < 266$$

(permutations are understood), whereas the orthogonal model template reads

$$(6 f_{\rm NL})^{-1} B_{\rm ortho} = -3 P_1 P_2 - 8(P_1 P_2 P_3)^{2/3} + 3 P_1^{1/3} P_2^{2/3} P_3$$

$$-410 < f_{\rm NL}^{\rm ortho} < 6$$

We are interested in generating such bispectra from quadratic (non-local) models, i.e.

$$\Phi = \phi + f_{\rm NL}\ K[\phi, \phi]$$

where K is the appropriate non-local quadratic kernel that generates the desired bispectrum. For simplicity, here we assume scale-invariance.

- Introduce some handy non-local operators

$$\partial\phi \equiv \sqrt{-\nabla^2}\phi(\mathbf{x}) \equiv \int \mathrm{e}^{-i\mathbf{k}\cdot\mathbf{x}}\; k\,\phi(\mathbf{k})\,d^3k$$

$$\nabla^{-2}A(\mathbf{x}) \equiv -\int \mathrm{e}^{-i\mathbf{k}\cdot\mathbf{x}}\left(\frac{1}{k^2}\right)A(\mathbf{k})\,d^3k$$

$$\partial^{-1}A \equiv \sqrt{-\nabla^{-2}}A \equiv \int \mathrm{e}^{-i\mathbf{k}\cdot\mathbf{x}}\left(\frac{1}{k}\right)A(\mathbf{k})\,d^3k$$

Then the EQ and ORT bispectra templates can be generated by,

$$K[\phi,\phi] = a\phi^2 + b\,\partial^{-1}(\phi\,\partial\phi) + c\,\nabla^{-2}(\phi\;\nabla^2\phi) + d\,\nabla^{-2}(\partial\phi)^2 + e\,\nabla^{-2}\partial^{-1}(\phi\nabla^2\partial\phi) + f\,\nabla^{-2}\partial^{-1}(\nabla^2\phi\;\partial\phi)$$

regularity constraints (one-loop corrections to the power spectrum must preserve scale-invariance in the IR) restrict the free parameters that leave the bispectrum invariant. Note these kernels have correct exchange symmetry.

More precisely,

$$\Phi_{\rm EQ} \;=\; \phi + f_{\rm NL}\Big[-3\phi^2 + 4\,\partial^{-1}(\phi\,\partial\phi) + 2\,\nabla^{-2}(\phi\;\nabla^2\phi) + 2\,\nabla^{-2}(\partial\phi)^2\Big],$$

$$\Phi_{\rm ORT} \;=\; \phi + f_{\rm NL}\Big[-9\phi^2 + 10\,\partial^{-1}(\phi\,\partial\phi) + 8\,\nabla^{-2}(\phi\;\nabla^2\phi) + 8\,\nabla^{-2}(\partial\phi)^2\Big],$$

2LPT Code to generate non-local (and local) PNG publicly available

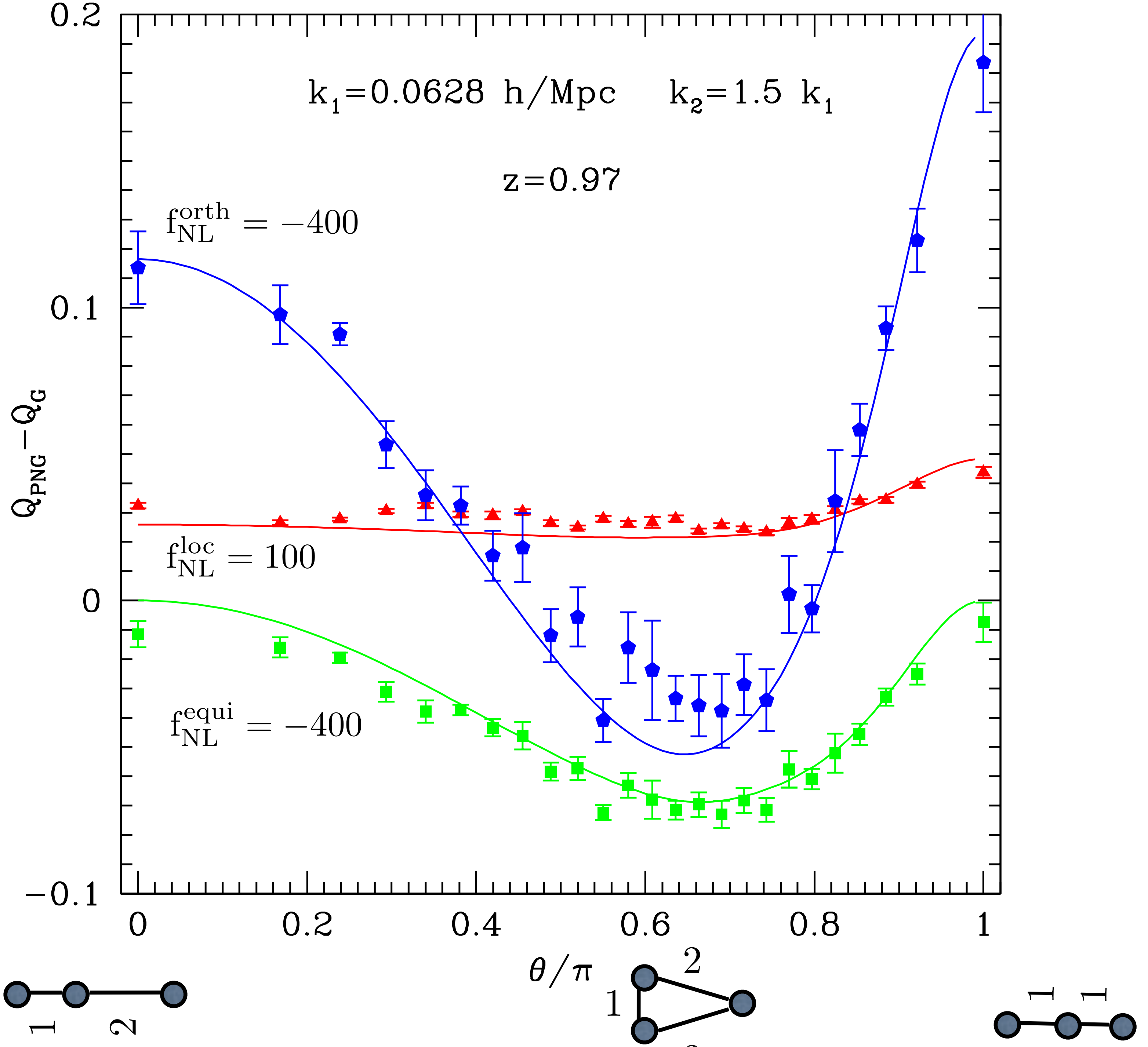
http://cosmo.nyu.edu/roman/2LPT/

Algorithm works for any bispectrum template that is sum of factorizable

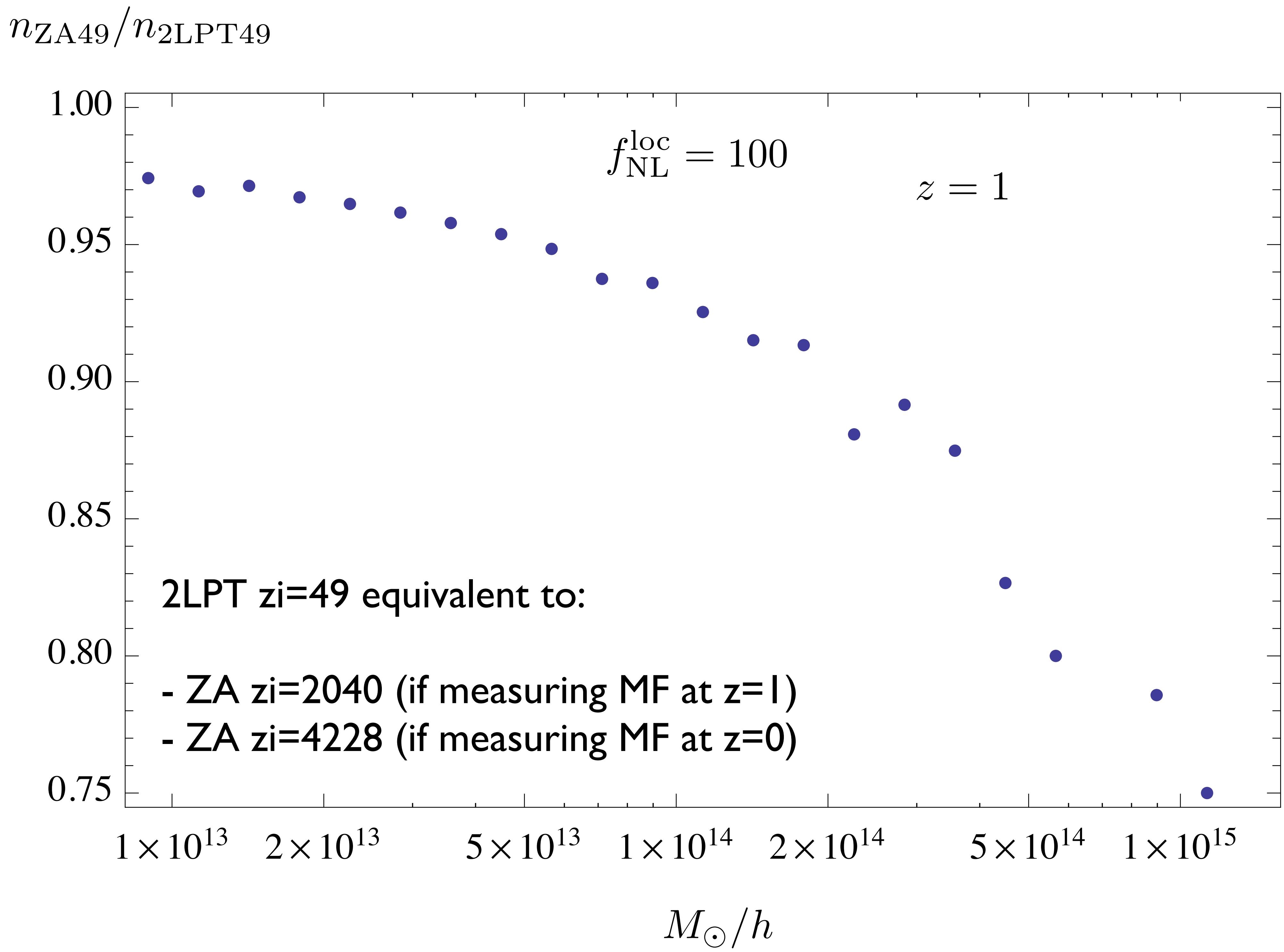
$$B \simeq \sum g_1(k_1) g_1(k_2) g_1(k_3)$$

Overhead over local fnl is only about 35% (same Npar ln Npar scaling)

Bispectrum

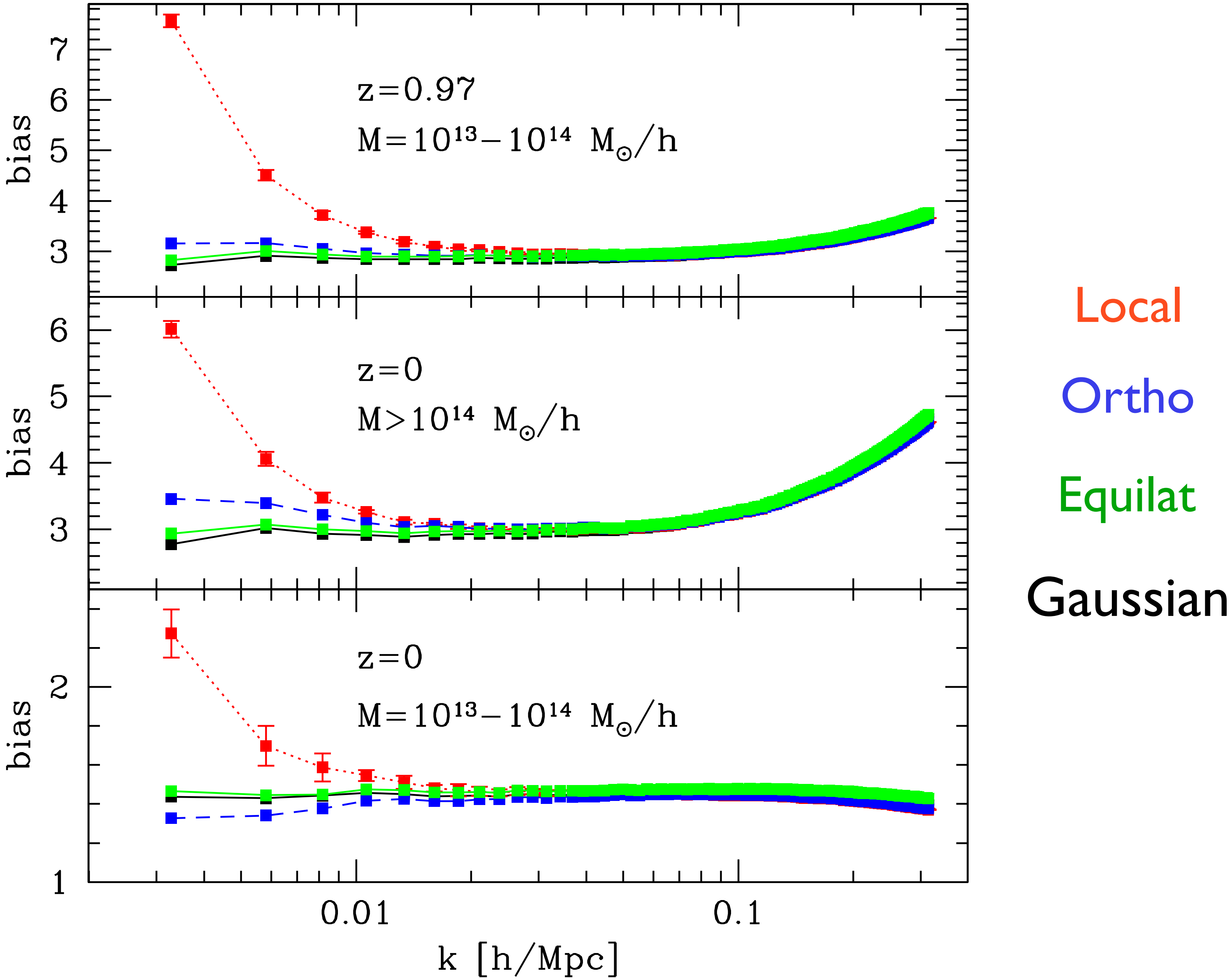


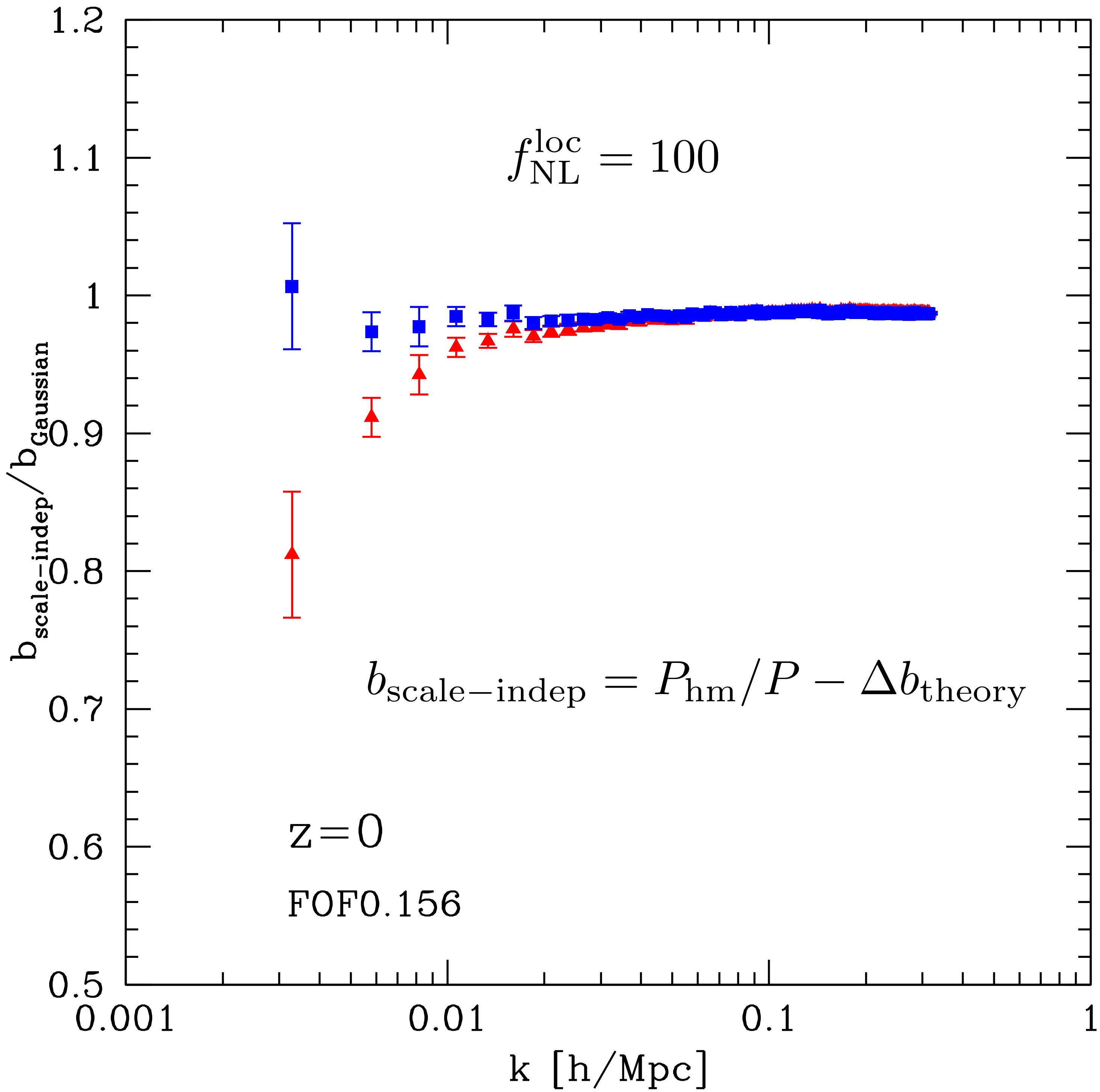
k_1=0.0628 h/Mpc k_2=1.5 k_1
z=0.97
$f_{NL}^{orth} = -400$
$f_{NL}^{loc} = 100$
$f_{NL}^{equi} = -400$
$Q_{PNG} - Q_G$
θ/π
0.2
0.1
0
−0.1
0
0.2
0.4
0.6
0.8
1


n_{ZA49}/n_{2LPT49}
1.00
0.95
0.90
0.85
0.80
0.75
$f_{NL}^{loc} = 100$
$z = 1$
2LPT zi=49 equivalent to:
- ZA zi=2040 (if measuring MF at z=1)
- ZA zi=4228 (if measuring MF at z=0)
1×10^{13}
2×10^{13}
5×10^{13}
1×10^{14}
2×10^{14}
5×10^{14}
1×10^{15}
$M_\odot/h$

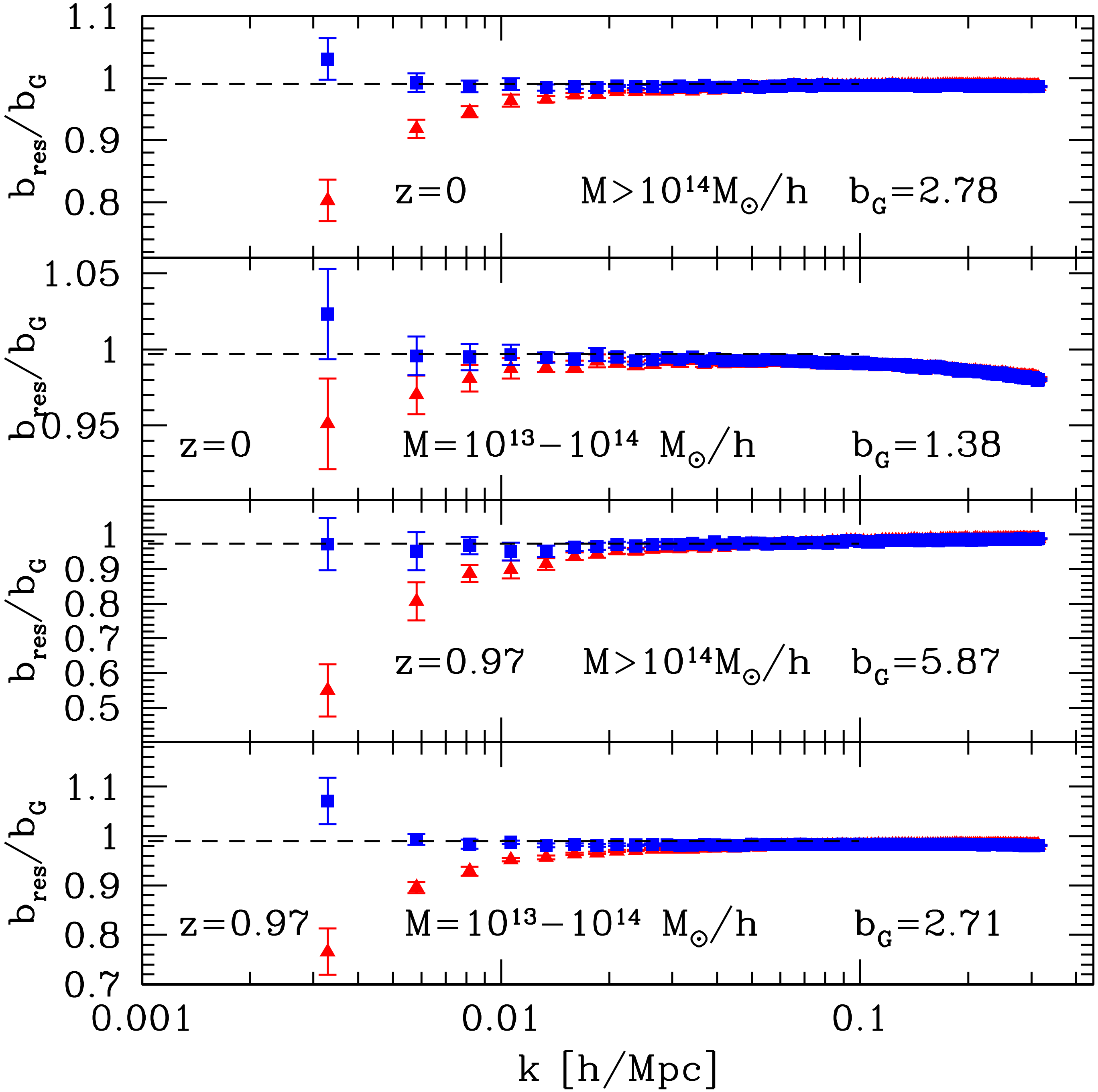
Scale-dependent Bias from Power Spectrum

$$b(k) = \frac{P_{gm}}{P_{mm}}$$




1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
0.001
0.01
0.1
1
$b_{scale-indep}/b_{Gaussian}$
k [h/Mpc]
$f_{NL}^{loc} = 100$
$b_{scale-indep} = P_{hm}/P - \Delta b_{theory}$
z=0
FOF0.156

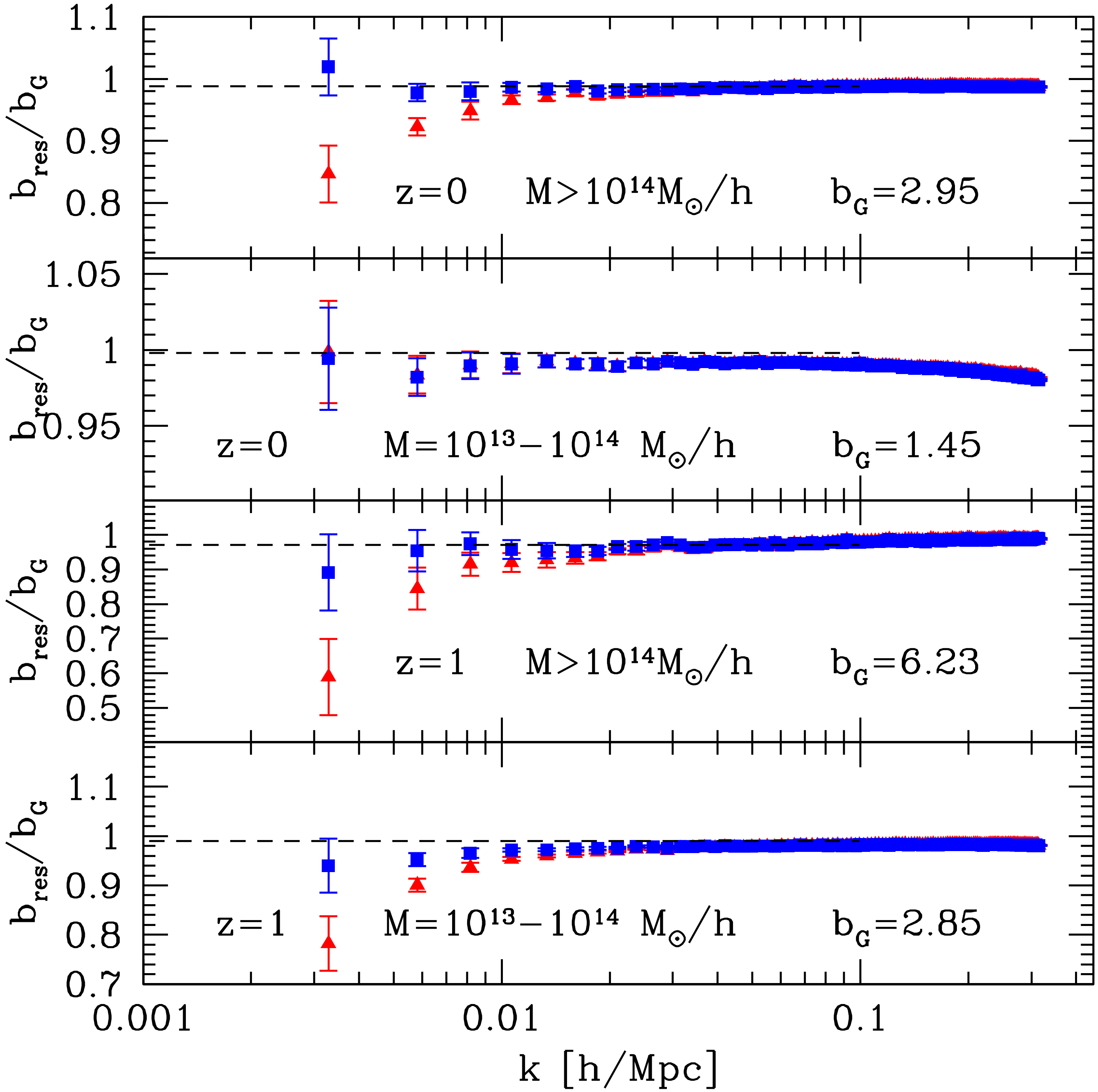
$$b_{\rm scale-indep} = P_{\rm hm}/P - \Delta b_{\rm theory}$$



$f_{\rm NL}^{\rm loc} = 100$

FOF0.2

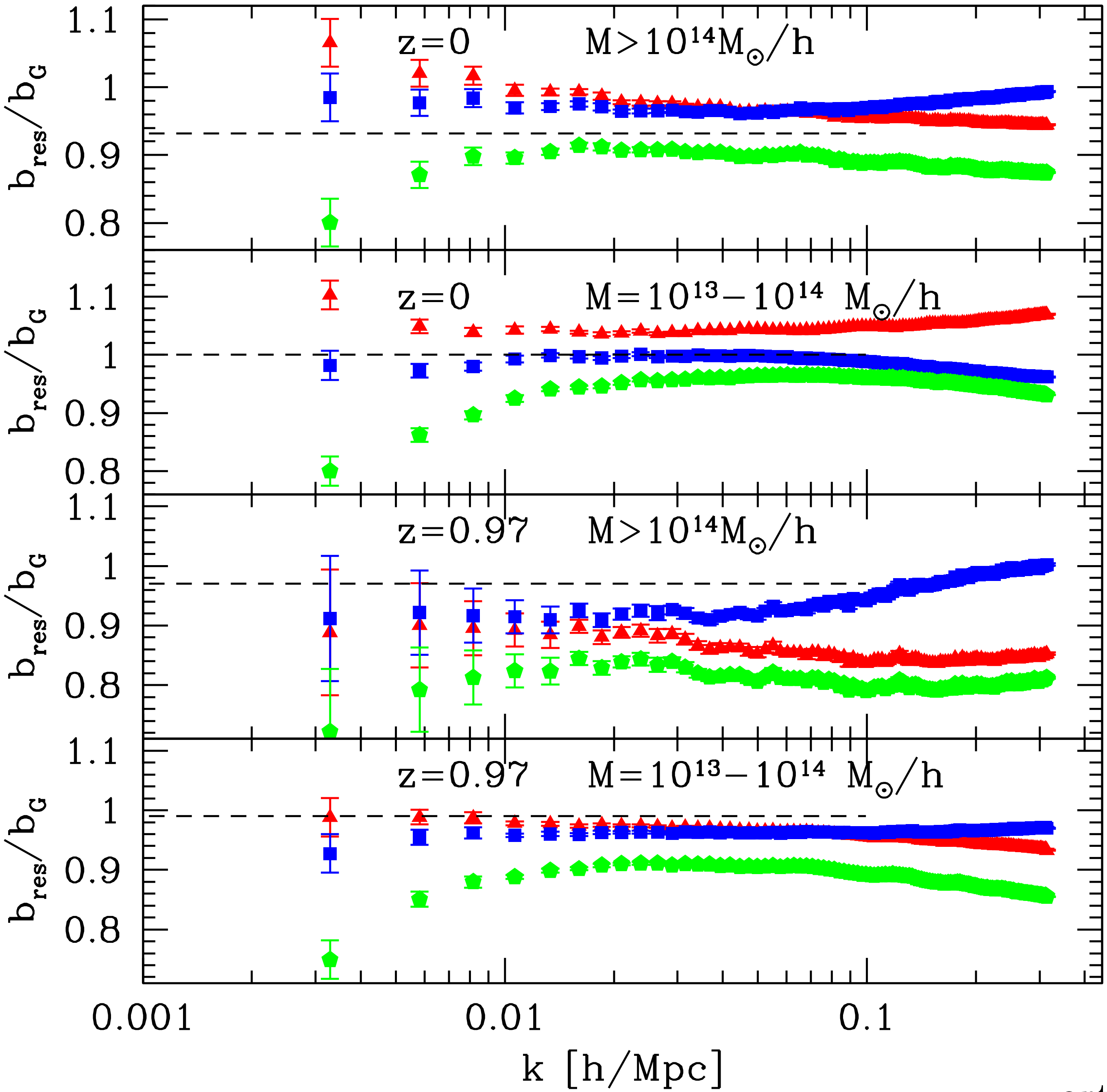
$$b_{\text{scale-indep}} = P_{\text{hm}}/P - \Delta b_{\text{theory}}$$



$f_{\text{NL}}^{\text{loc}} = 100$

FOF0.156

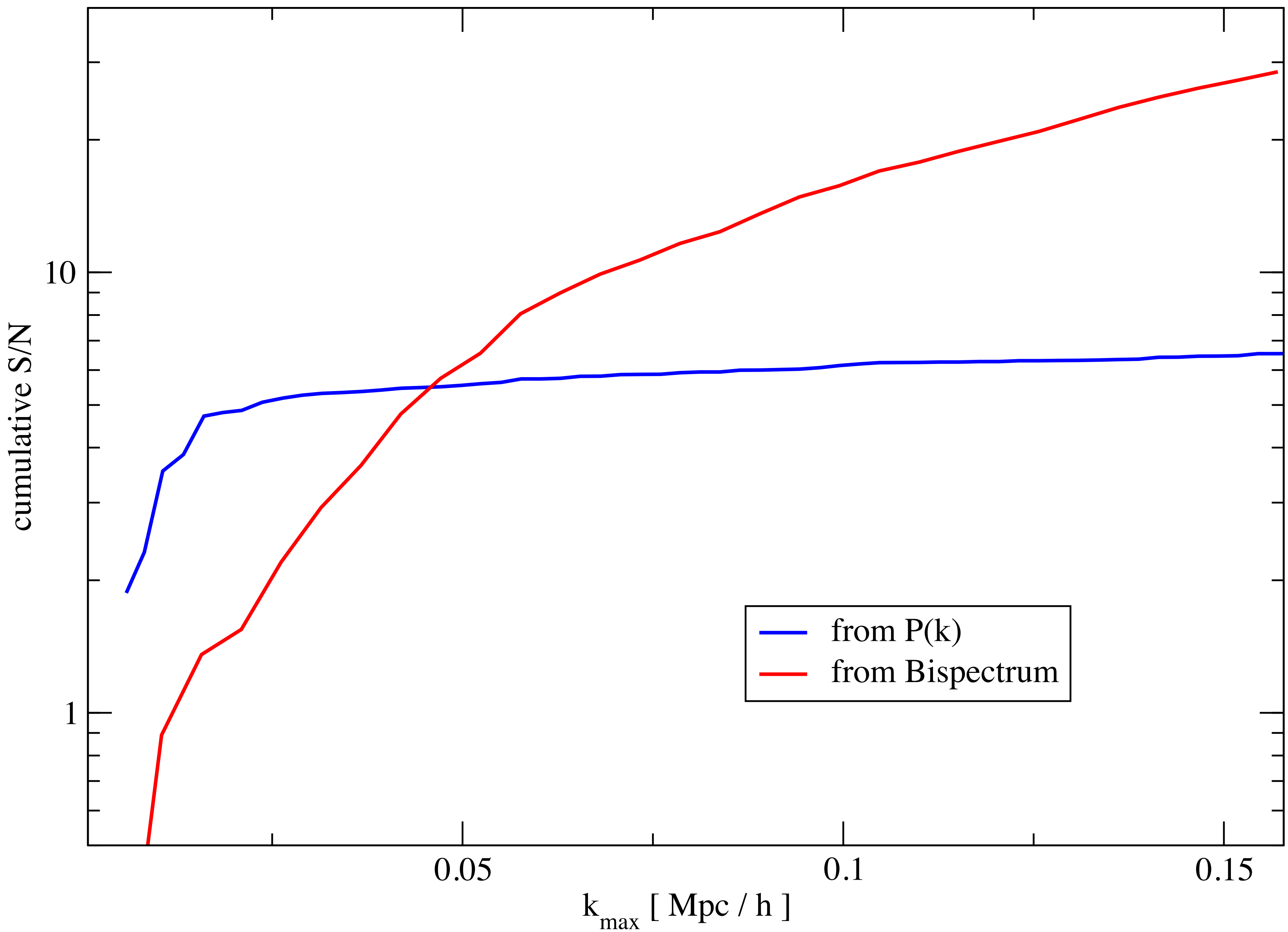
$$b_{\rm scale-indep} = P_{\rm hm}/P - \Delta b_{\rm theory}$$



FOF0.156

$$f_{\rm NL}^{\rm orthog} = -400$$

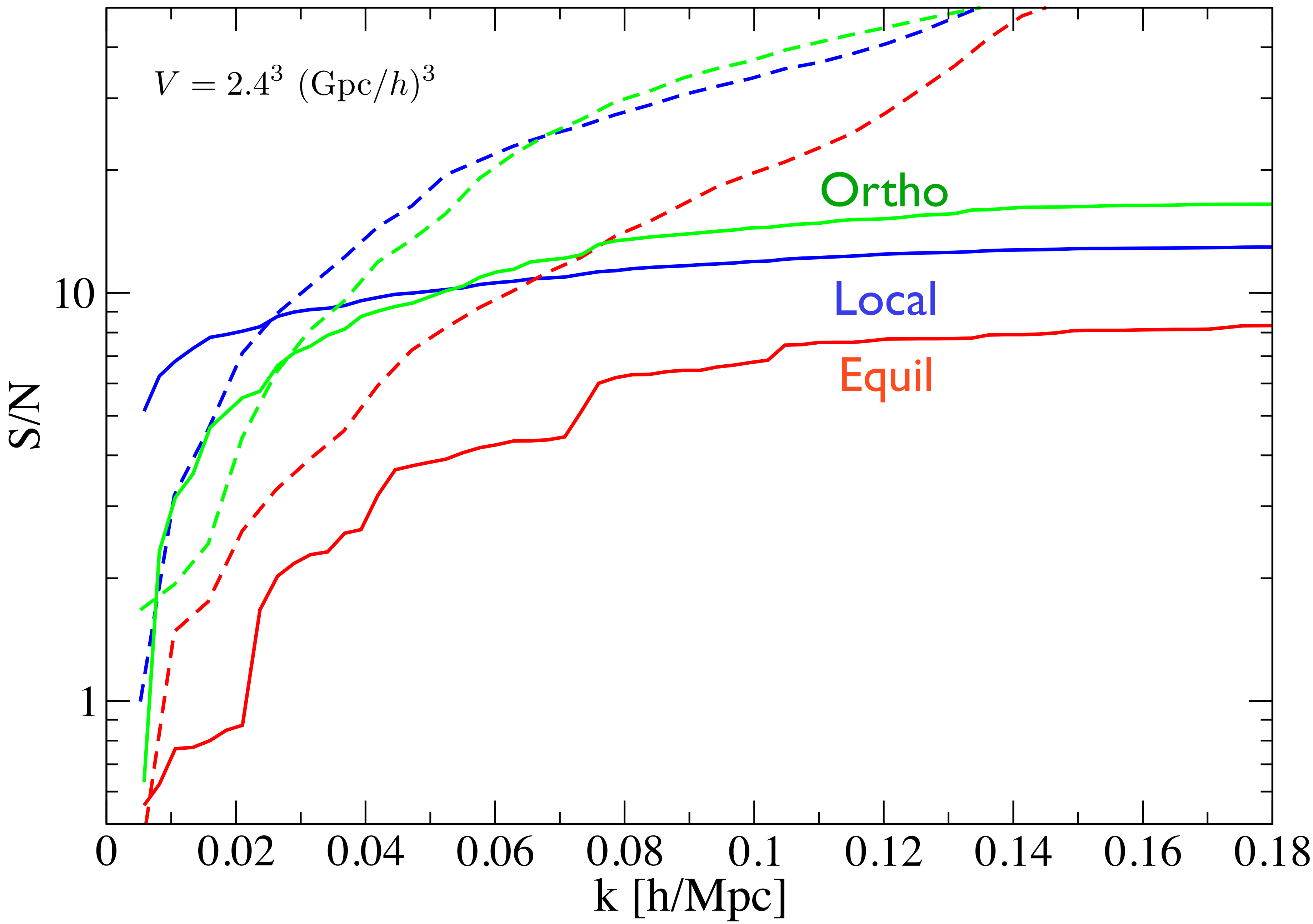
naive BOSS signal to noise for local fnl=100



in pple enough (in pow + bisp) to detect fnl(loc)~few (competitive with CMB)

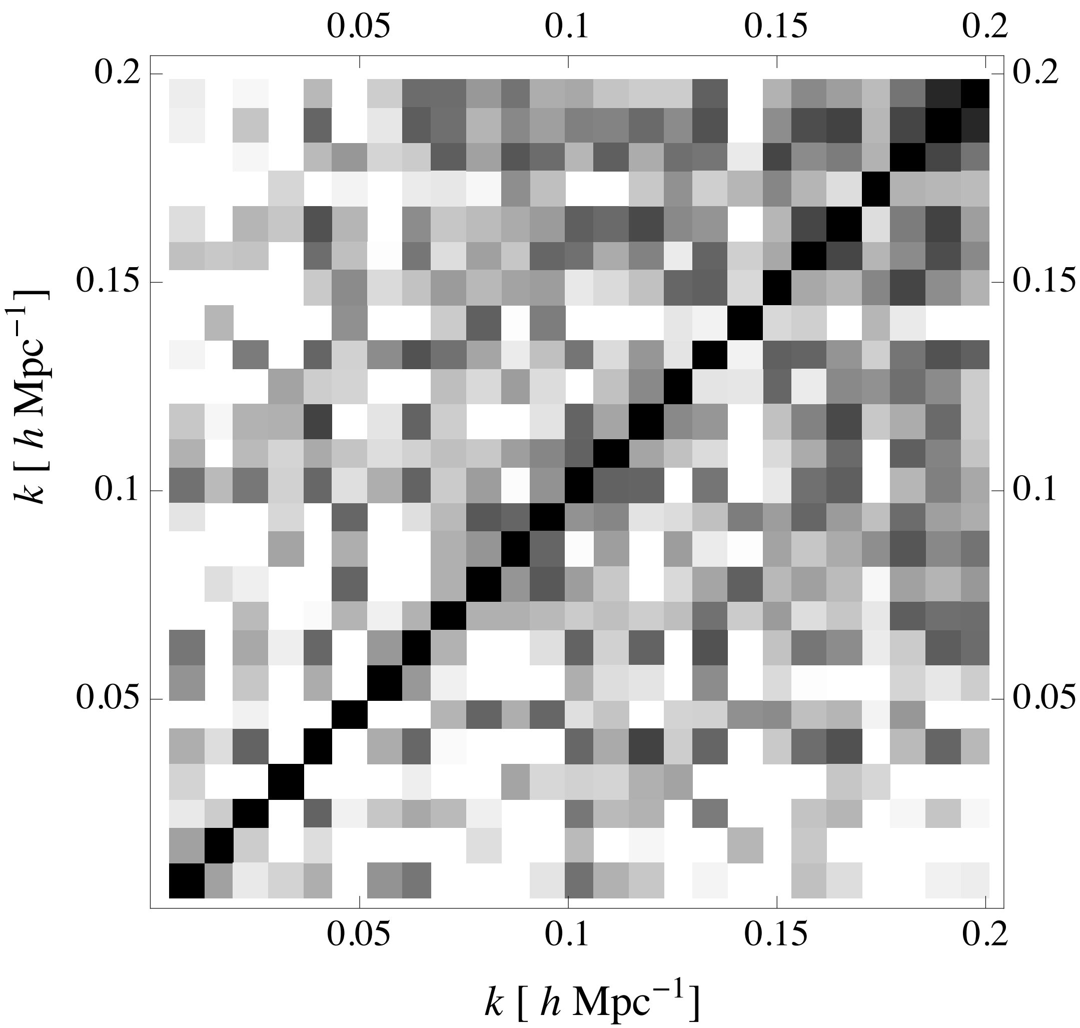
Halo S/N for Non-Gaussian Models, z=1.0, M>10^14 Mo

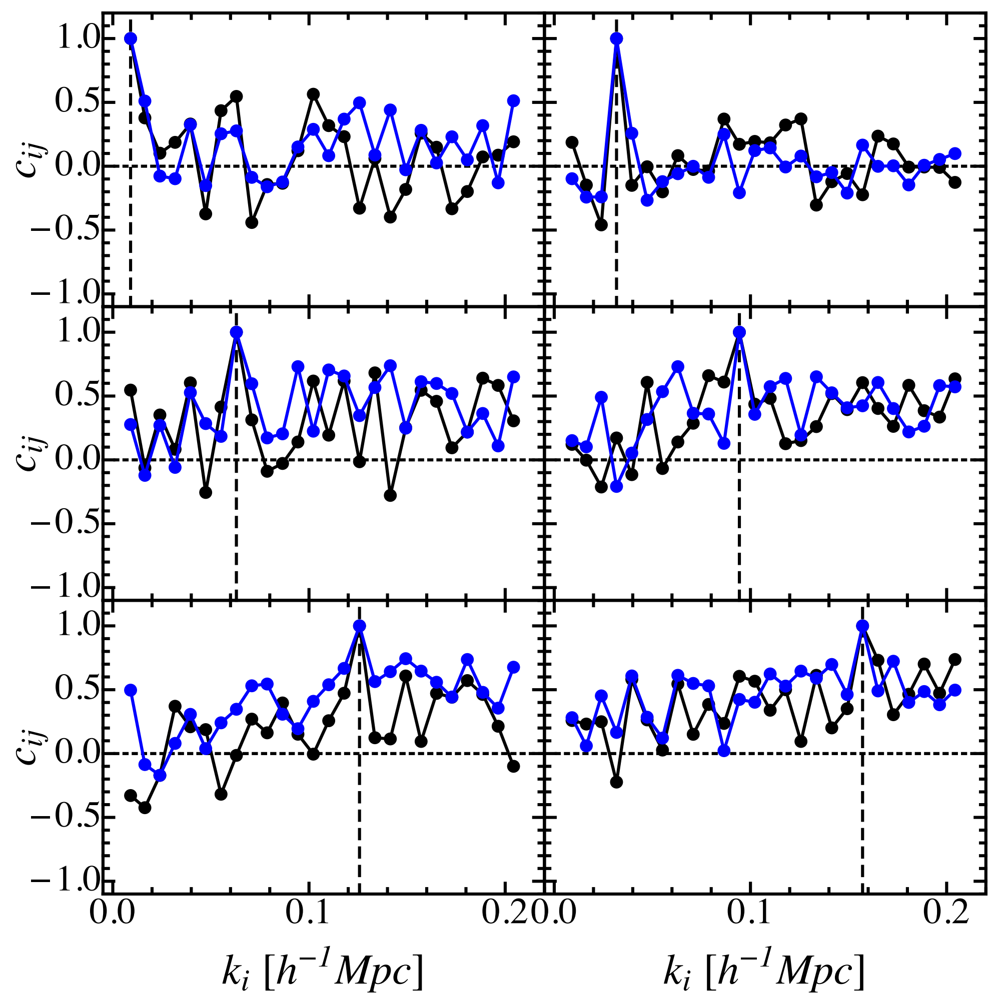
dashed= from bispectrum, solid=from power



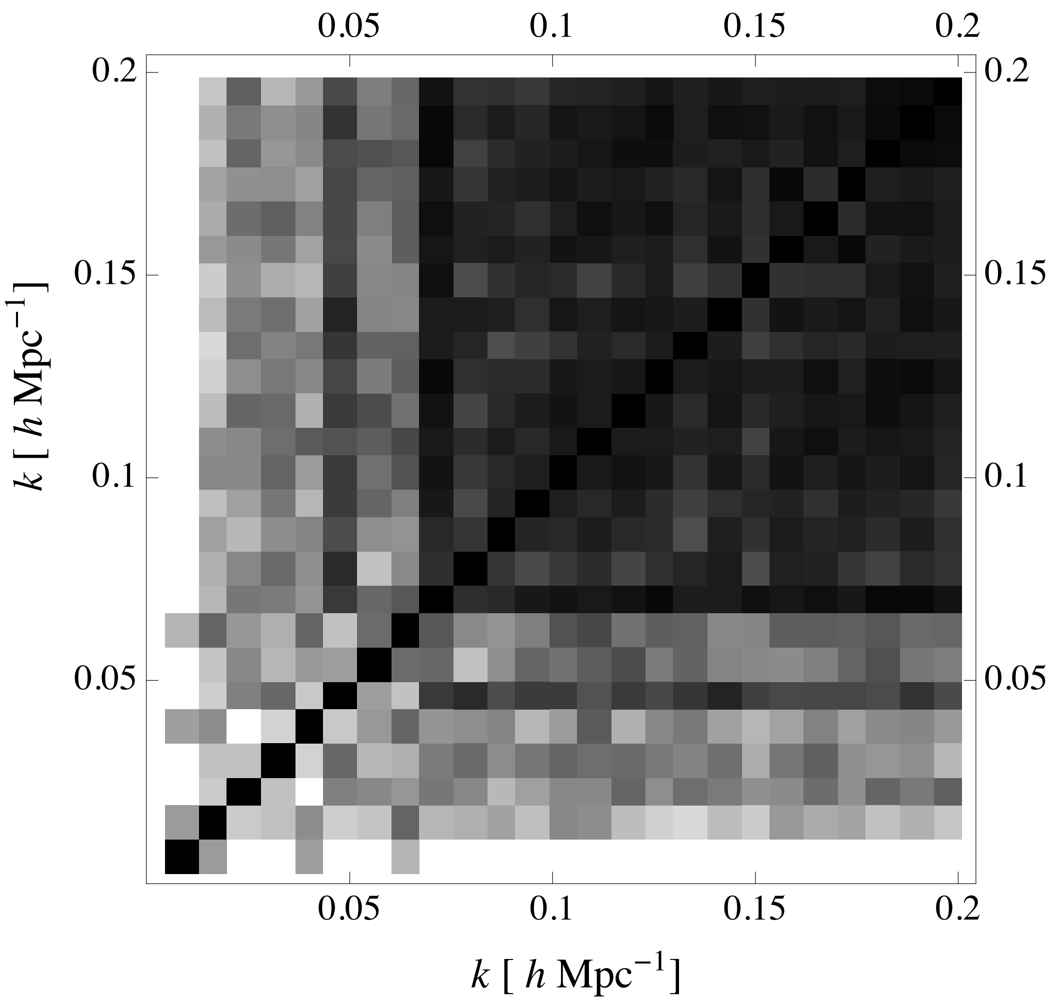
COVARIANCE !!!

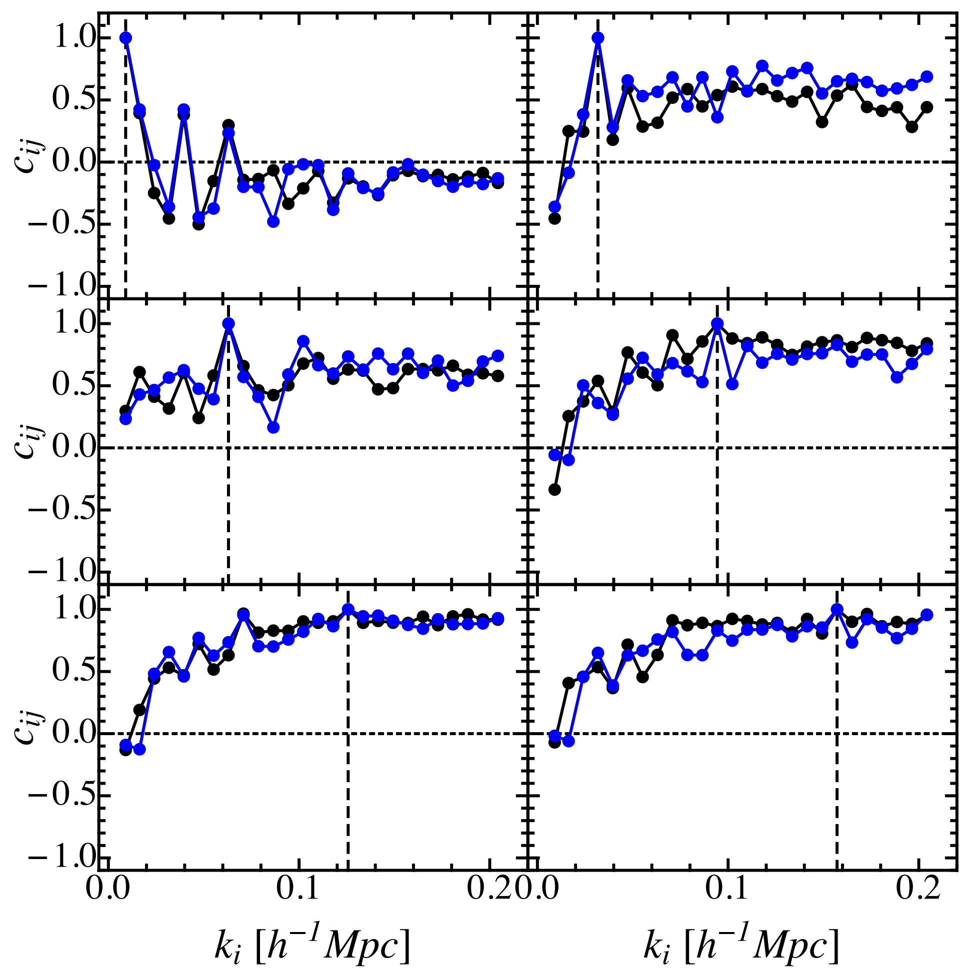
(Squeezed) Bispectrum Covariance: Periodic Box



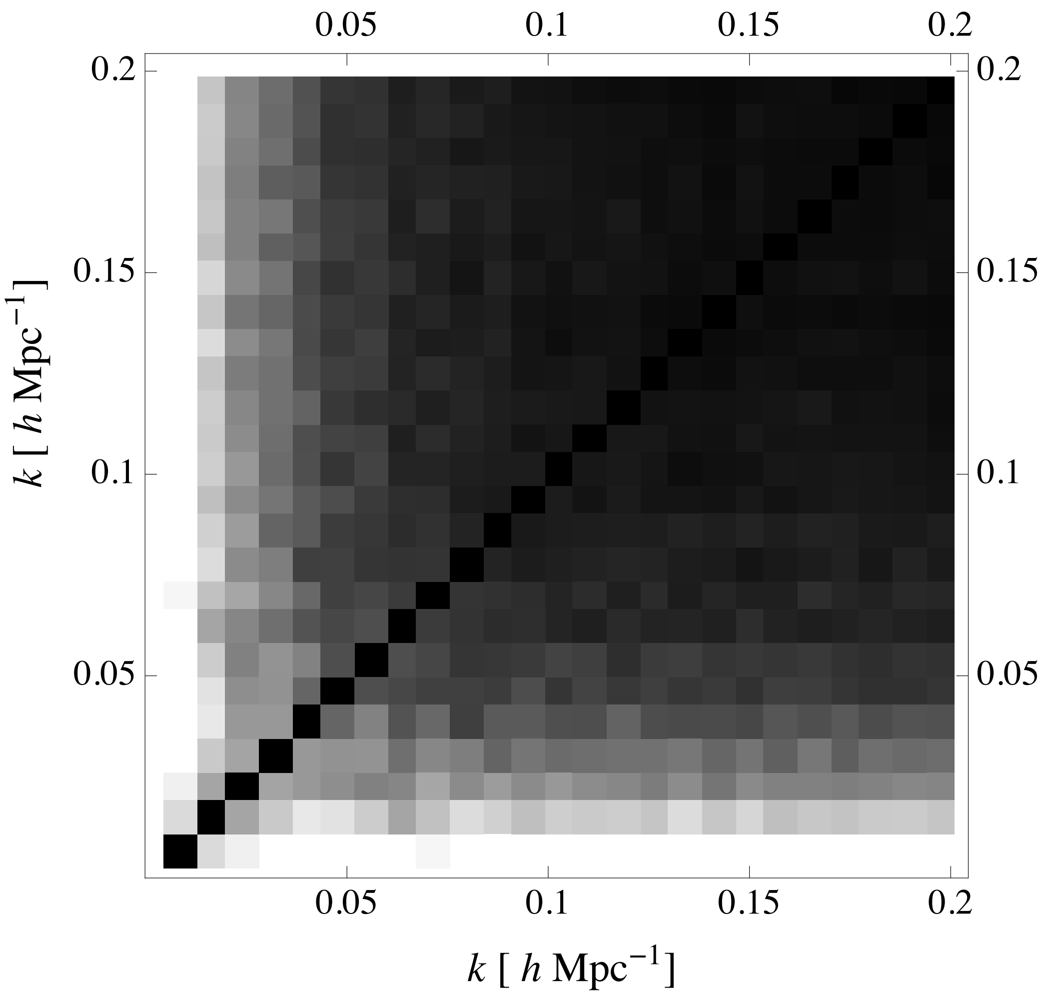


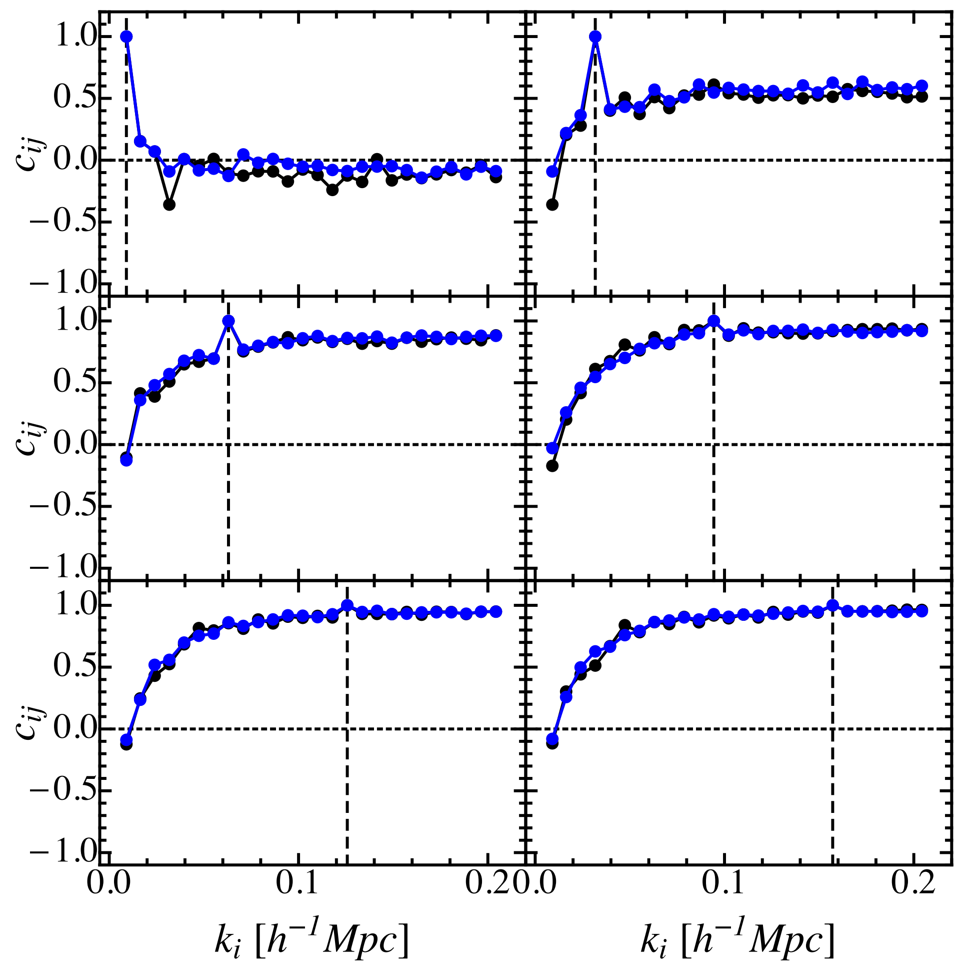
(Squeezed) Bispectrum Covariance: Sphere





(Squeezed) Bispectrum Covariance: Survey Geometry





Beyond Local Bias (Gaussian)

Suppose at some time t*, objects form with local bias,

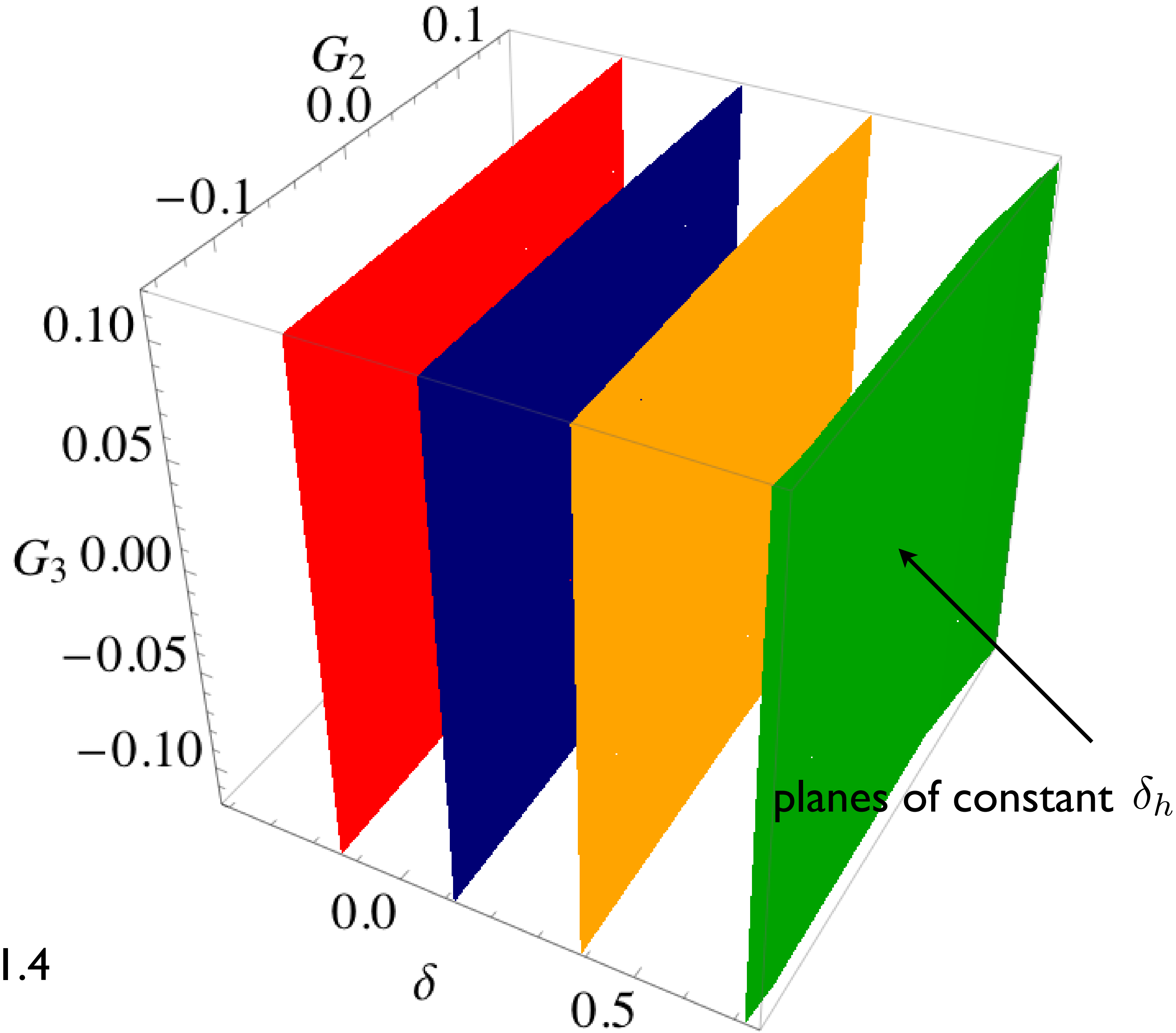
$$\delta_{\rm g}^* = b_1^*\, \delta_* + \frac{b_2^*}{2!}\, \delta_*^2 + \frac{b_3^*}{3!}\, \delta_*^3 + \dots$$

As time goes on, does bias stay local?

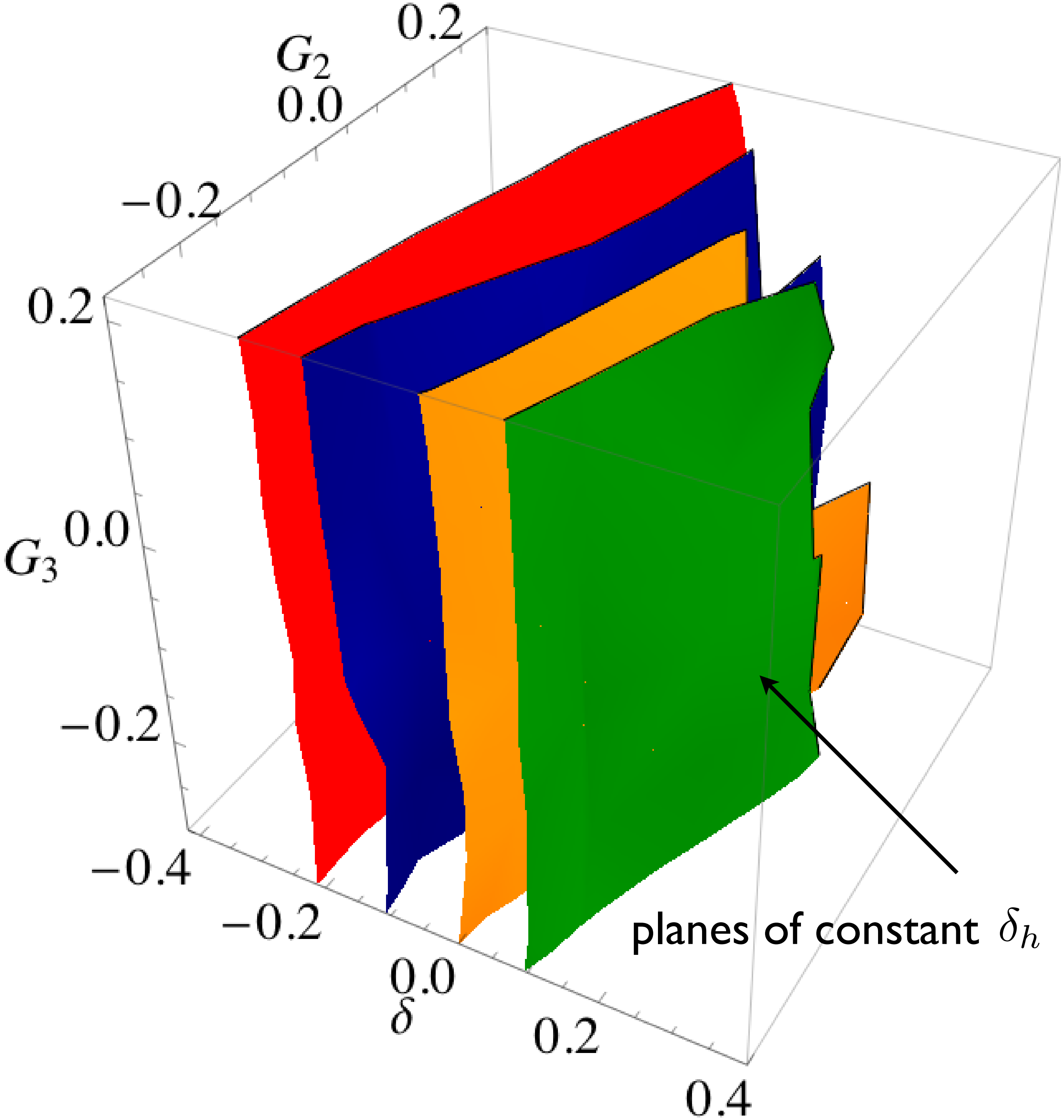
The answer is (a resounding) no!

$$\begin{aligned}
\delta_{\rm g}^{\rm Nloc} &= \gamma_2\, \mathcal{G}_2\,(\Phi_{\rm v})(1+\beta\,\delta) \\
&\quad + \gamma_3 \left(\mathcal{G}_3(\Phi_{\rm v}) + \frac{6}{7}\, \mathcal{G}_2(\Phi_{\rm v}^{(1)}, \Phi_{\rm 2LPT}) \right) + \dots
\end{aligned}$$

$$\begin{aligned}
\mathcal{G}_2(\Phi_{\rm v}) &= (\nabla_{ij}\Phi_{\rm v})^2 - (\nabla^2\Phi_{\rm v})^2, \\
\mathcal{G}_3(\Phi_{\rm v}) &= (\nabla^2\Phi_{\rm v})^3 + 2\nabla_{ij}\Phi_{\rm v}\nabla_{jk}\Phi_{\rm v}\nabla_{ki}\Phi_{\rm v} - 3(\nabla_{ij}\Phi_{\rm v})^2\nabla^2\Phi_{\rm v}.
\end{aligned}$$



linear bias ~ 1.4



linear bias ~ 4

TABLE II. Local Eulerian bias parameters b_1 and b_2 obtained from halo-matter-matter bispectrum fits for all triangles with $k < 0.1\,h\,\mathrm{Mpc}^{-1}$. We also include the large-scale bias $b_\times$ obtained from the halo-matter power spectrum, to be compared with b_1. The last column indicates the goodness of the fit assuming a diagonal covariance matrix ($N_{\mathrm{dof}} = 148$).

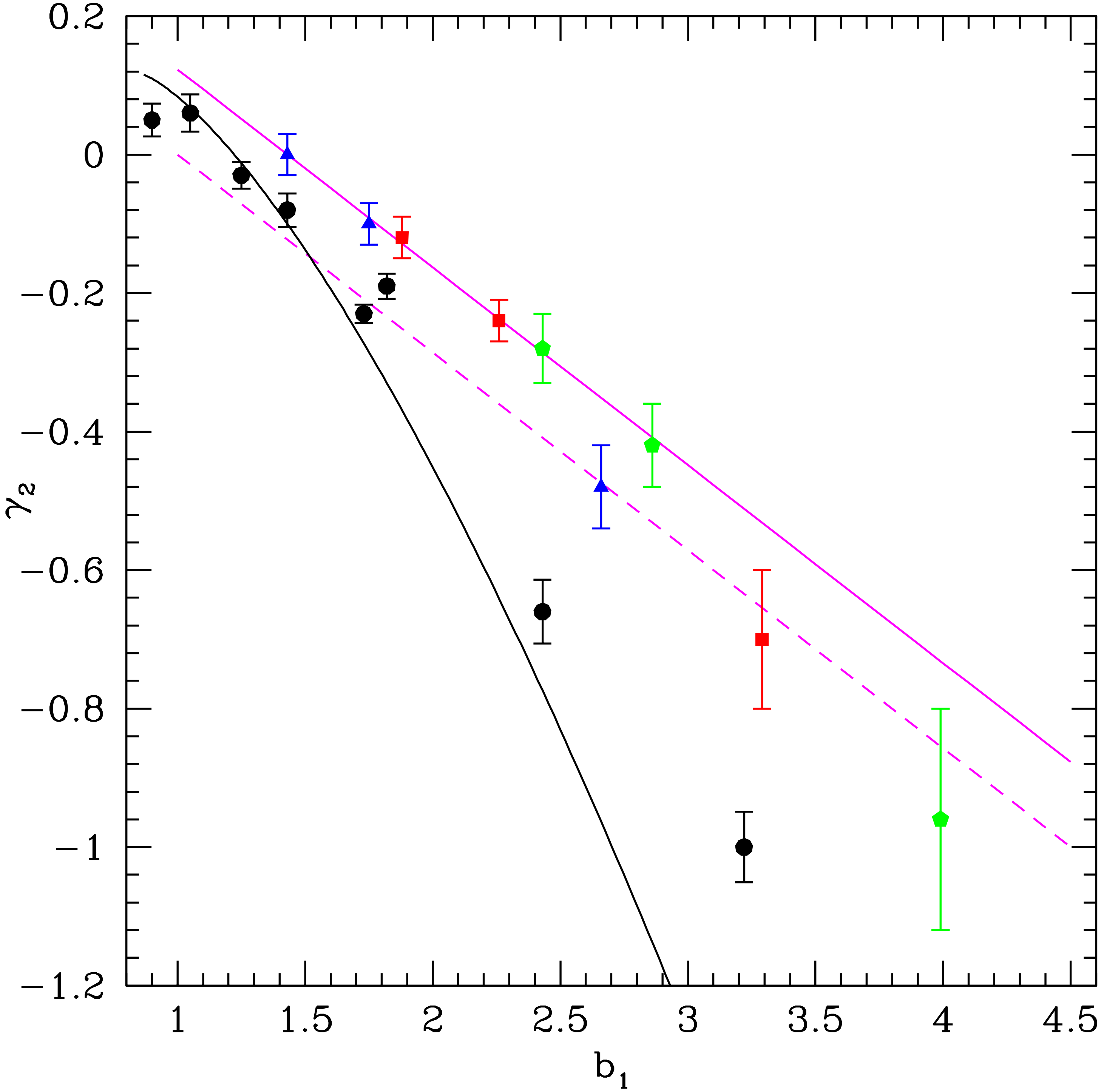
Sample	$b_\times$	b_1	b_2	χ^2/dof
LMz0	1.43	1.42 ± 0.01	-0.91 ± 0.03	1.86
MMz0	1.75	1.71 ± 0.01	-0.55 ± 0.03	1.29
HMz0	2.66	2.37 ± 0.02	2.98 ± 0.07	3.74
LMz0.5	1.88	1.77 ± 0.01	-0.15 ± 0.03	0.91
MMz0.5	2.26	2.13 ± 0.01	0.67 ± 0.03	0.87
HMz0.5	3.29	2.84 ± 0.03	5.89 ± 0.10	3.77
LMz1	2.43	2.22 ± 0.01	1.27 ± 0.04	0.89
MMz1	2.86	2.62 ± 0.02	2.77 ± 0.06	1.07
HMz1	3.99	3.41 ± 0.05	9.98 ± 0.14	3.42

TABLE III. Eulerian bias parameters b_1 and b_2 obtained from doing a *Lagrangian* local bias model fit to the bispectrum.

Sample	$b_\times$	b_1	b_2	χ^2/dof
LMz0	1.43	1.48 ± 0.01	-1.26 ± 0.04	2.12
MMz0	1.75	1.81 ± 0.01	-1.15 ± 0.03	1.36
HMz0	2.66	2.59 ± 0.02	1.78 ± 0.07	2.73
LMz0.5	1.88	1.87 ± 0.01	-0.79 ± 0.04	0.94
MMz0.5	2.26	2.30 ± 0.01	-0.26 ± 0.04	0.72
HMz0.5	3.29	3.12 ± 0.03	4.34 ± 0.11	2.91
LMz1	2.43	2.40 ± 0.02	0.27 ± 0.05	0.77
MMz1	2.86	2.85 ± 0.02	1.45 ± 0.06	0.82
HMz1	3.99	3.77 ± 0.05	7.97 ± 0.16	2.74

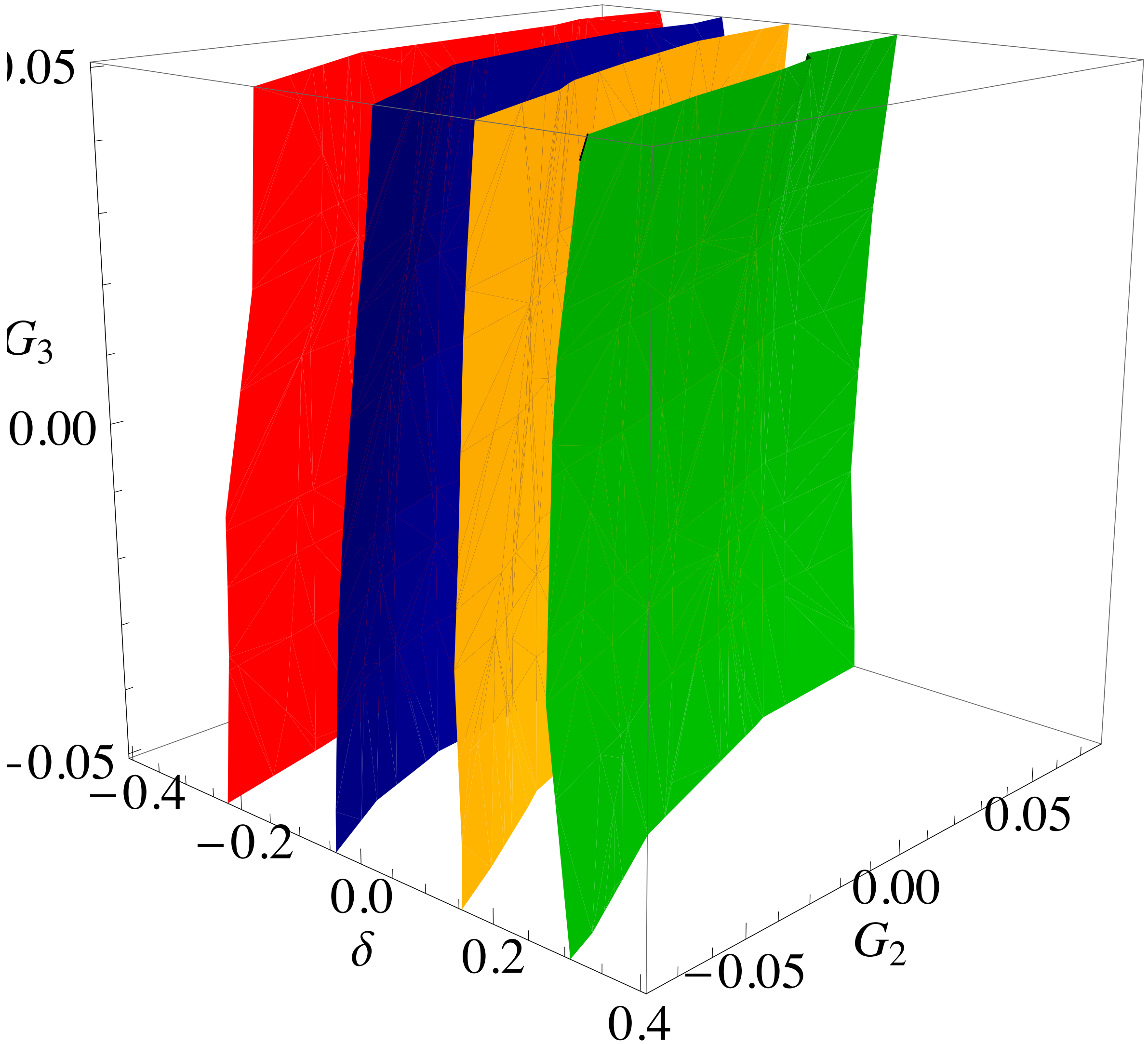
TABLE IV. Eulerian bias parameters b_1 and b_2 and non-local γ_2 parameter obtained from doing a quadratic non-local bias model fit to the bispectrum. For comparison purposes, note that a non-zero γ_2 gives an effective $-(4/3)\gamma_2$ contribution to b_2 (see top panel in Fig. 8). Here $N_{\rm dof} = 147$.

Sample	$b_\times$	b_1	b_2	γ_2	χ^2/dof
LMz0	1.43	1.42 ± 0.02	-0.92 ± 0.08	-0.01 ± 0.03	1.87
MMz0	1.75	1.76 ± 0.02	-0.81 ± 0.08	-0.10 ± 0.03	1.19
HMz0	2.66	2.61 ± 0.04	1.71 ± 0.18	-0.48 ± 0.06	2.74
LMz0.5	1.88	1.83 ± 0.02	-0.46 ± 0.09	-0.12 ± 0.03	0.84
MMz0.5	2.26	2.24 ± 0.02	0.05 ± 0.09	-0.24 ± 0.03	0.67
HMz0.5	3.29	3.16 ± 0.06	4.10 ± 0.28	-0.70 ± 0.10	2.91
LMz1	2.43	2.35 ± 0.03	0.57 ± 0.13	-0.28 ± 0.05	0.74
MMz1	2.86	2.80 ± 0.03	1.70 ± 0.16	-0.42 ± 0.06	0.80
HMz1	3.99	3.84 ± 0.08	7.55 ± 0.41	-0.96 ± 0.16	2.73


0.2
0
−0.2
−0.4
−0.6
−0.8
−1
−1.2
γ_2
1
1.5
2
2.5
3
3.5
4
4.5
b_1

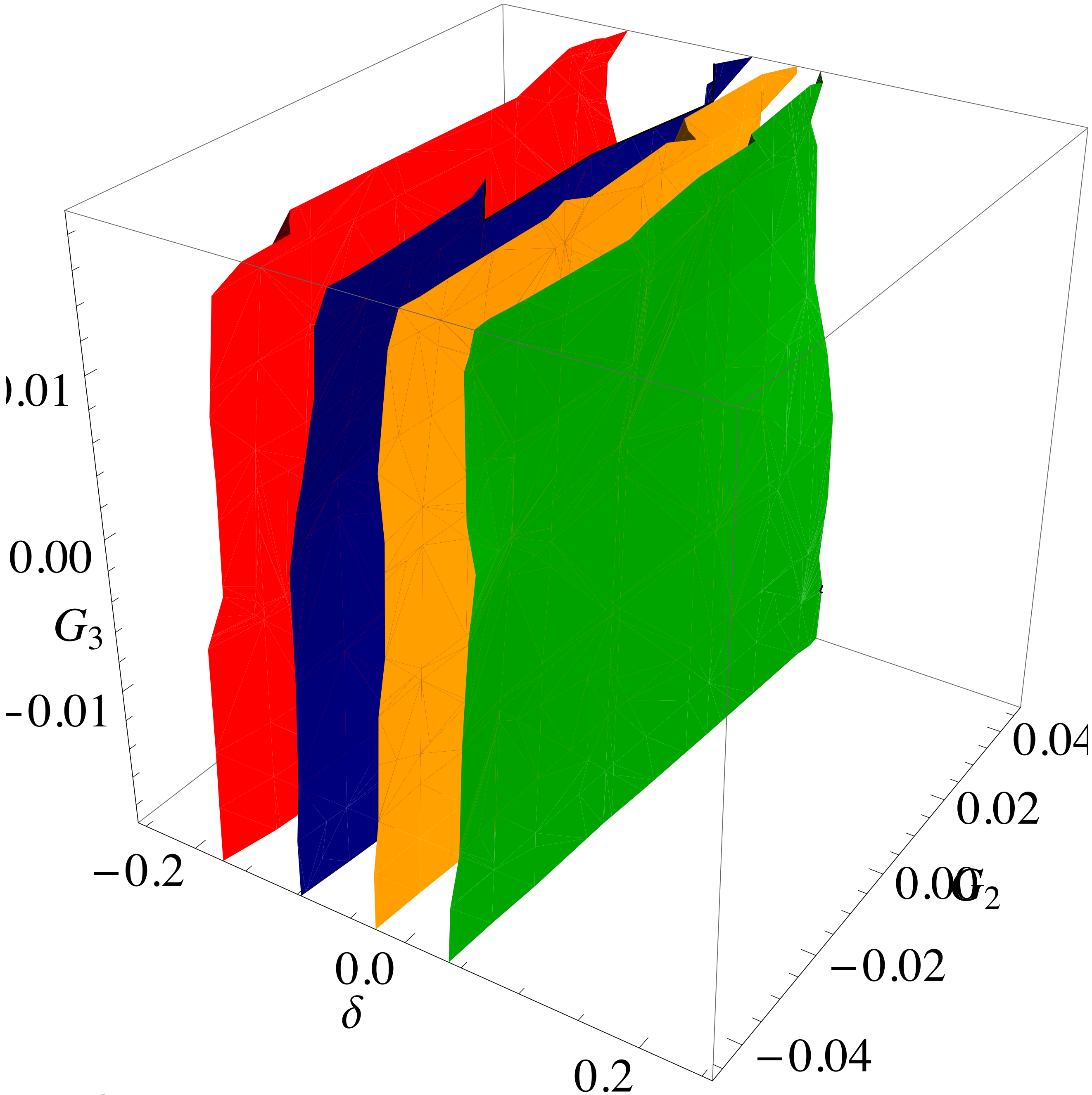
Halos in Lagrangian space: Lag bias is non-local too.

Out[1059]=



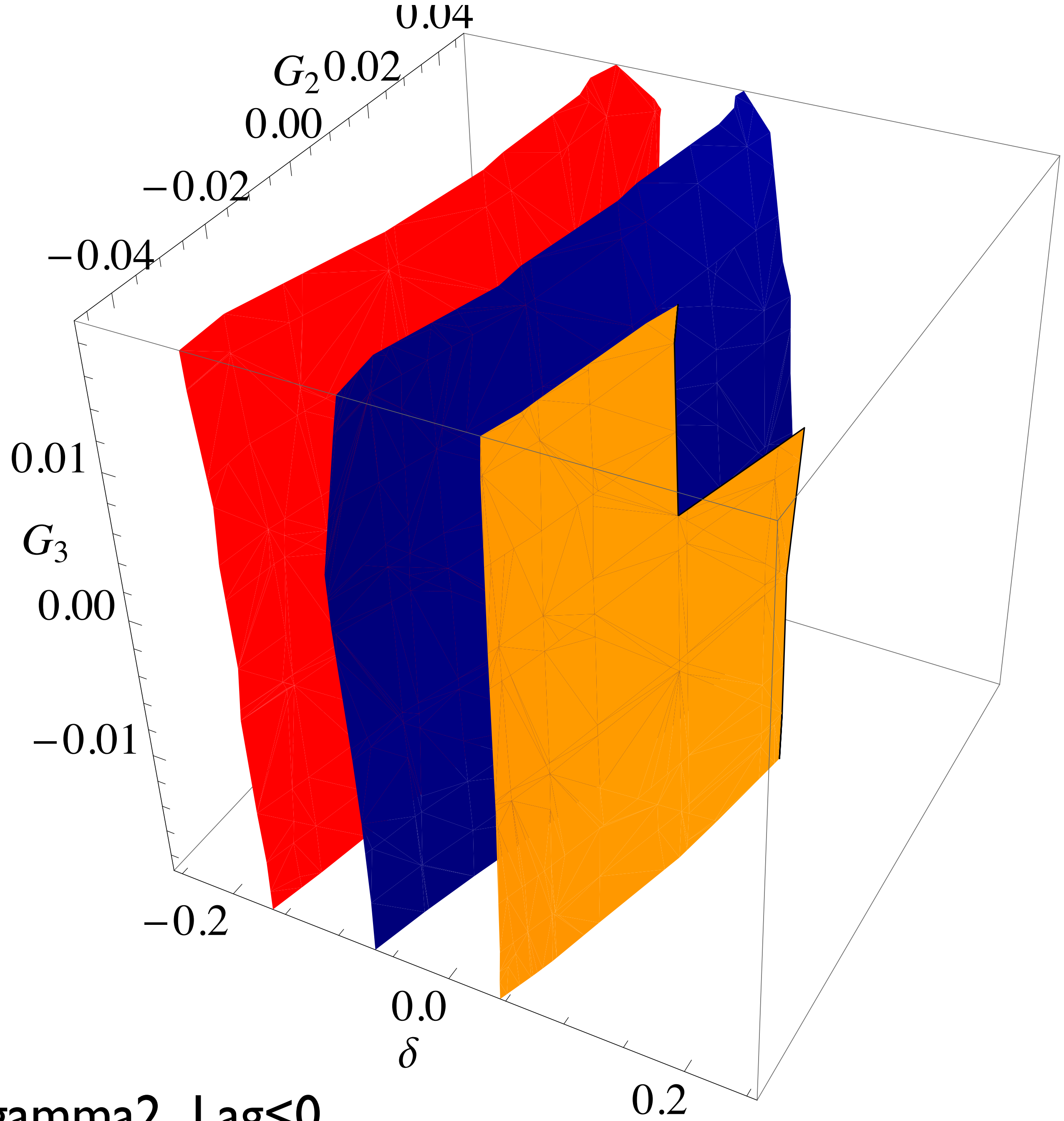
At low-mass gamma2_Lag>0

Out[1154]=



Then gamma2_Lag~0

Out[1201]=



At high-mass gamma2_Lag<0

Conclusions

- More precise modeling of the scale-dependent bias is possible :)

- Non-local PNG initial conditions very doable for most common templates :)

- If somebody tells you wonderful things about the bispectrum, ask for their covariance matrix :(

- Local bias (even for Gaussian ICs) not enough, not even in Lagrangian space :(