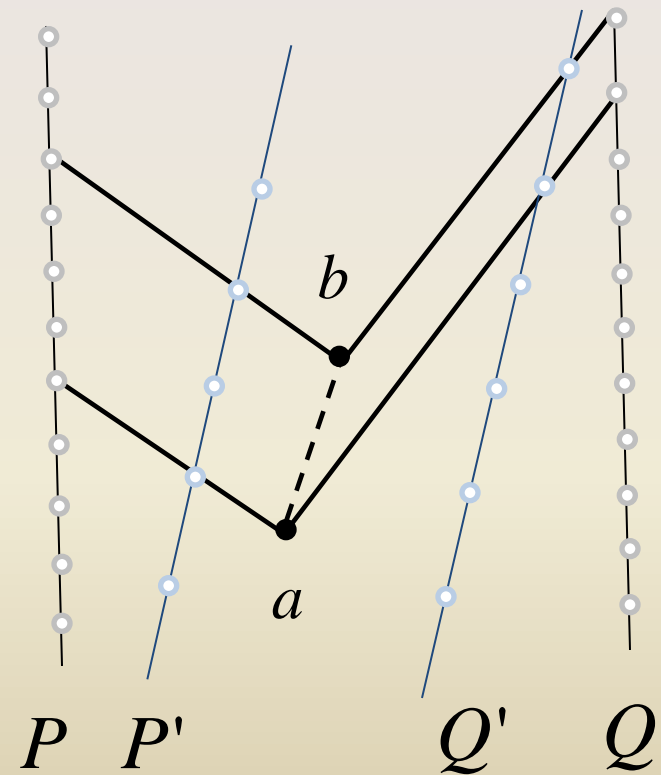


# A Potential Mechanism for Emergent Observer-Based Space-Time

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and  
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Consider the most basic  
properties of interactions and  
see what physics we get

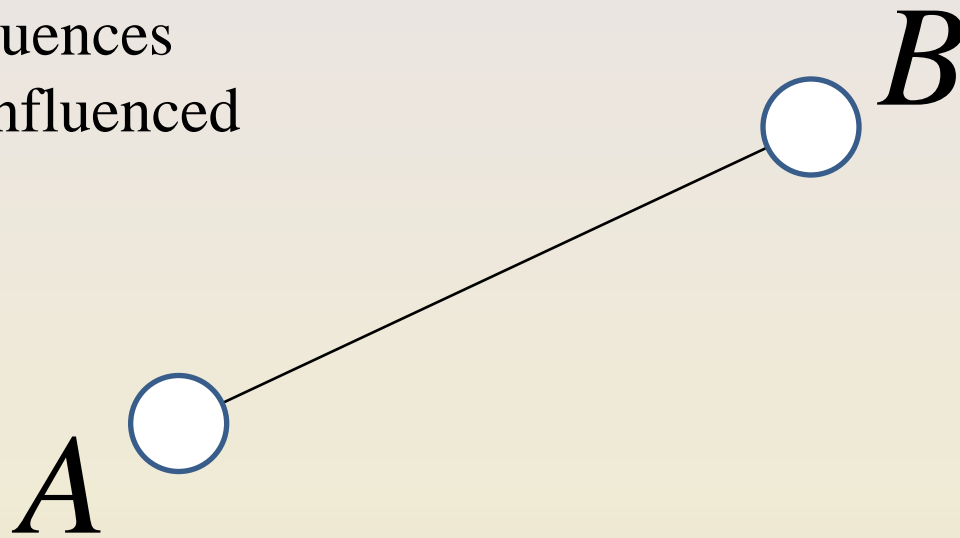
# Interactions and Events

An interaction defines two events and orders them

*Event A: A influences*

*Event B: B is influenced*

$A \rightarrow B$



The direction of the ordering relation is arbitrary.

# Partially-Ordered Set of Events

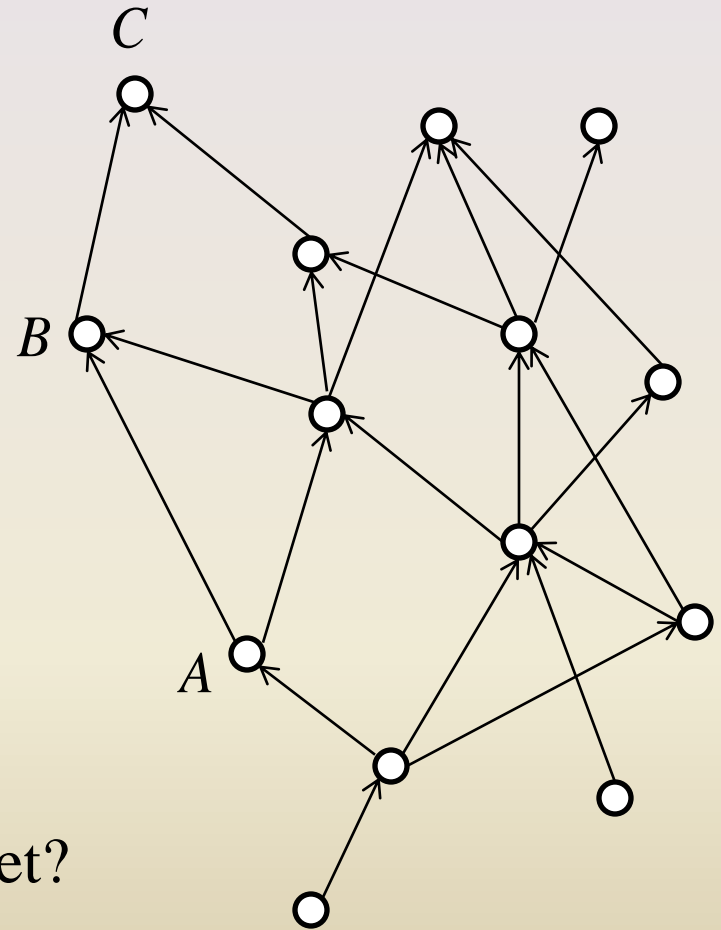
## A Set of Events

along with a

## Binary Ordering Relation

results in a

## Partially-Ordered Set of Events

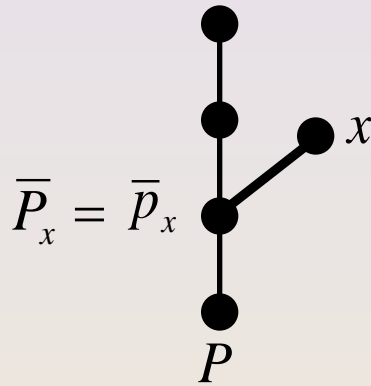


What constraints exist for consistent quantification of this partially-ordered set?

**Chains are Easily Quantified**

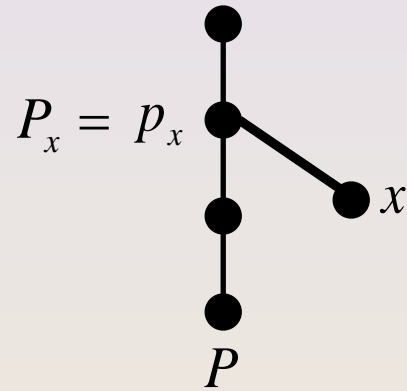
**Other Elements can be Quantified by  
Relating them to a Chain**

# Projection Operator



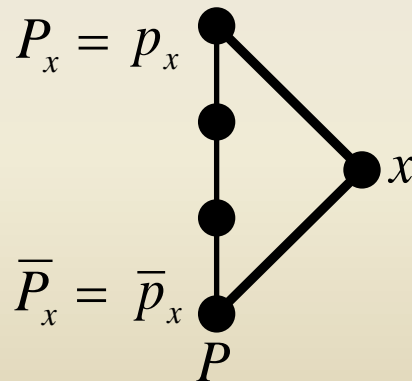
$p_i \leq x$  for all  $p_i \leq \bar{p}_x$

$p_i || x$  for all  $p_i > \bar{p}_x$



$p_i \geq x$  for all  $p_i \geq p_x$

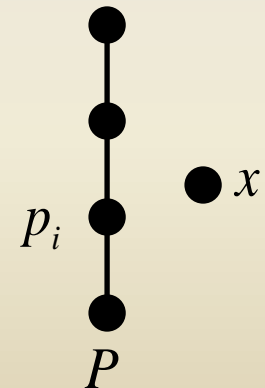
$p_i || x$  for all  $p_i < p_x$



$p_i \leq x$  for all  $p_i \leq \bar{p}_x$

$p_i || x$  for all  $\bar{p}_x < p_i < p_x$

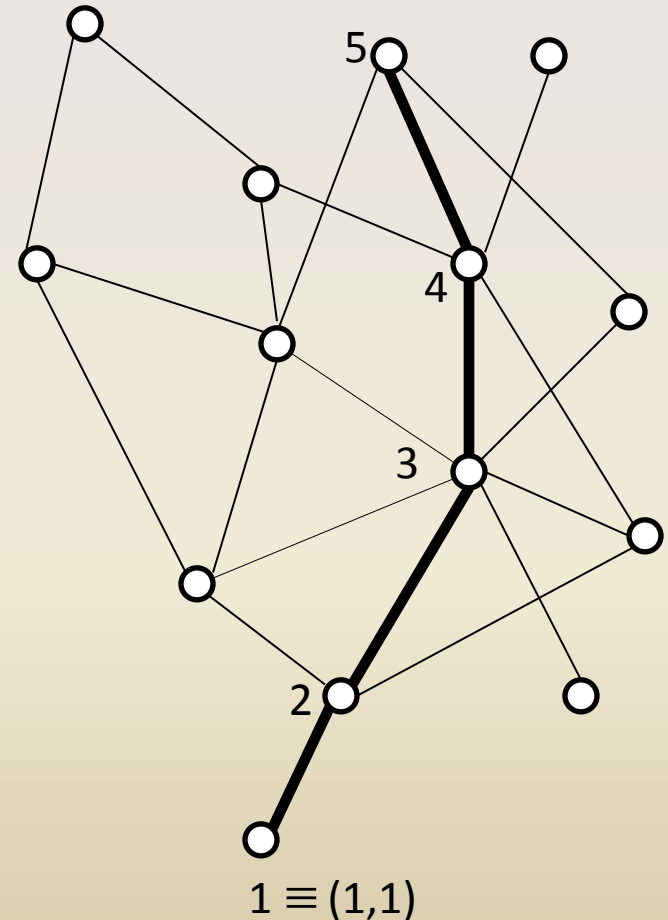
$p_i \geq x$  for all  $p_i \geq p_x$



# Quantification of a Distinguished Chain

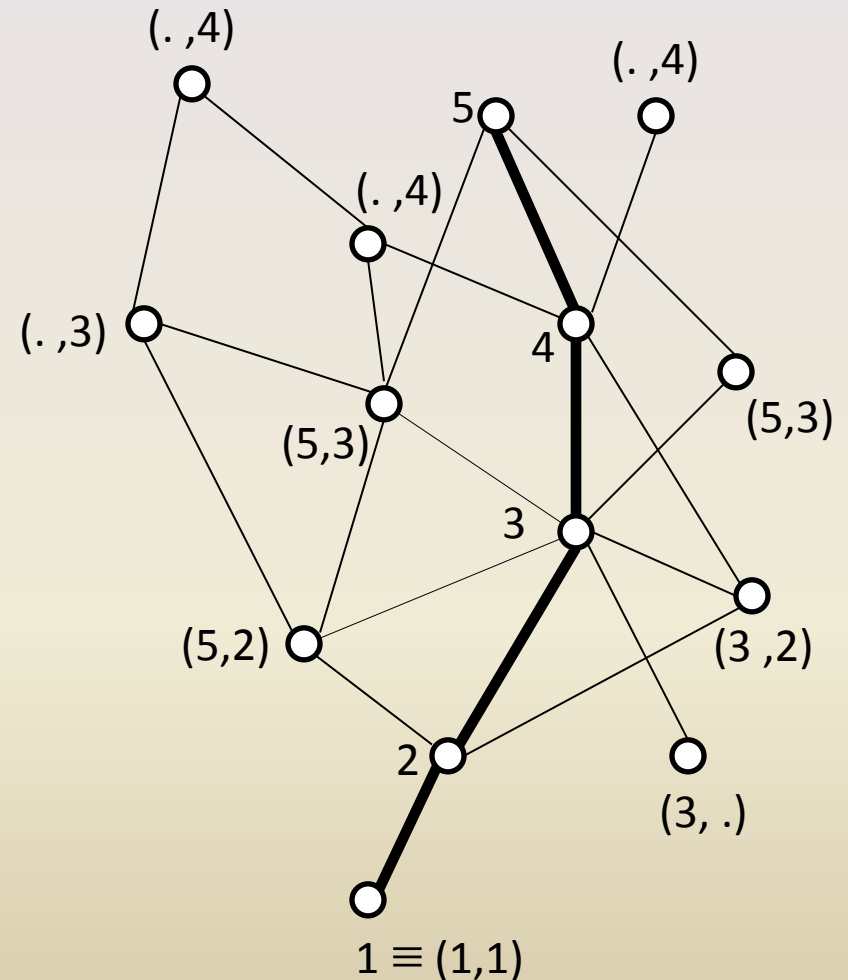
Distinguish a Chain (Observer Chain)

Quantify its elements with a non-decreasing sequence of numbers.



# Quantification via Chain Projection

Quantify additional elements by projection onto the chain.

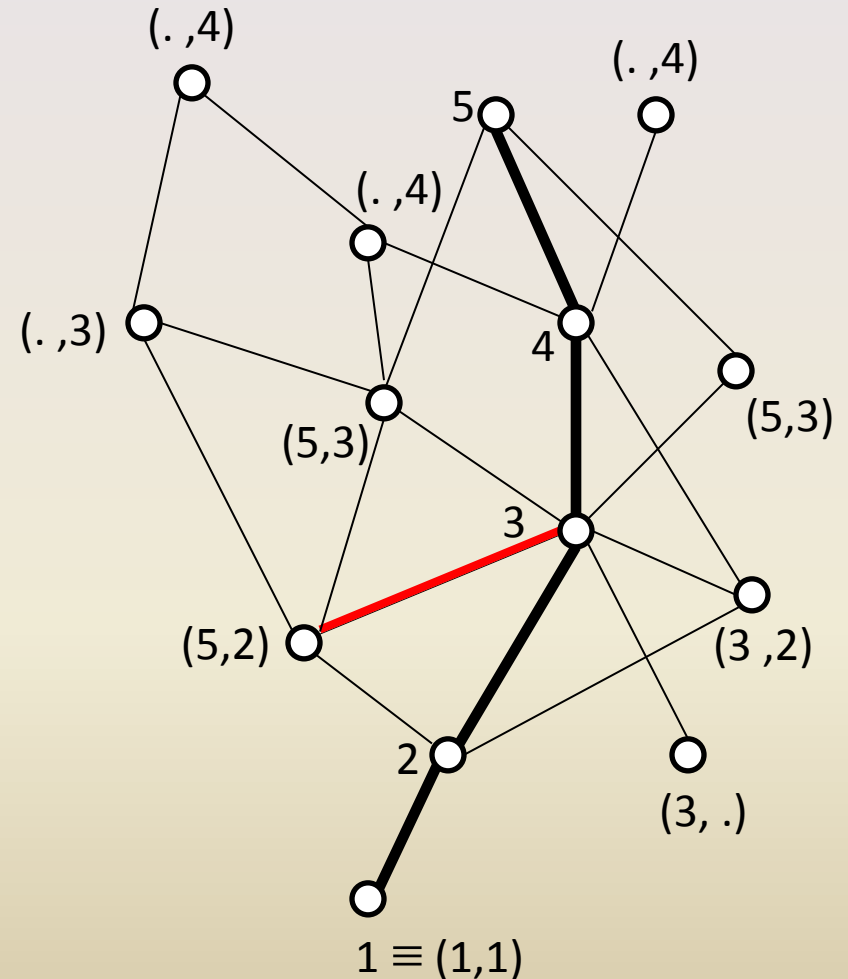




# Intervals

Intervals can be defined by pairs of events

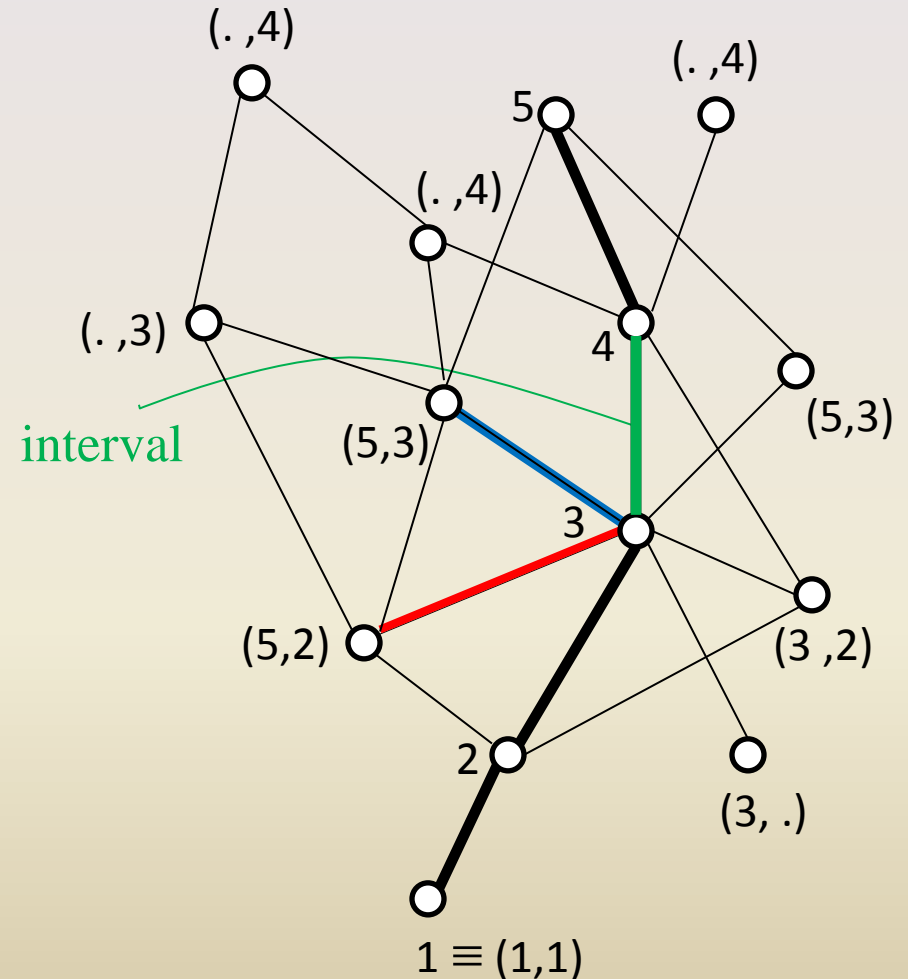
4-numbers  $(5,3;2,3)$   
pair  $(5-3,2-3) = (2,-1)$   
scalar  $(2)(-1) = -2$



# Intervals

Intervals can be defined by pairs of events

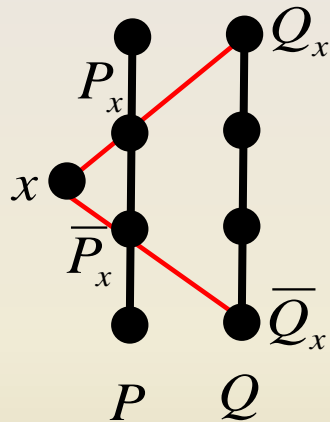
4-numbers	$(5,3;2,3)$
pair	$(5-3,2-3) = (2,-1)$
scalar	$(2)(-1) = -2$
pair	$(5-3,3-3) = (2,0)$
scalar	$(2)(0) = 0$
pair	$(4-3,4-3) = (1,1)$
scalar	$(1)(1) = 1$



# Chains Induce Structure

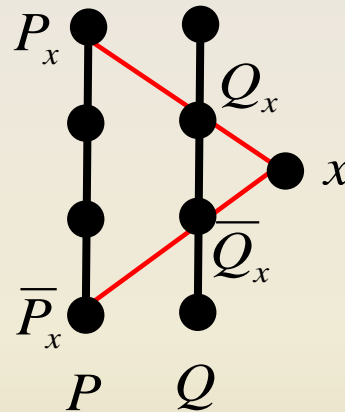
# Collinearity

An element  $x$  is said to be collinear with a finite chain  $P$  and a finite chain  $Q$ , iff the projections of  $x$  onto  $P$ , can be found by first projecting  $x$  onto  $Q$  and then onto  $P$ , and vice versa by interchanging the roles of  $P$  and  $Q$ .



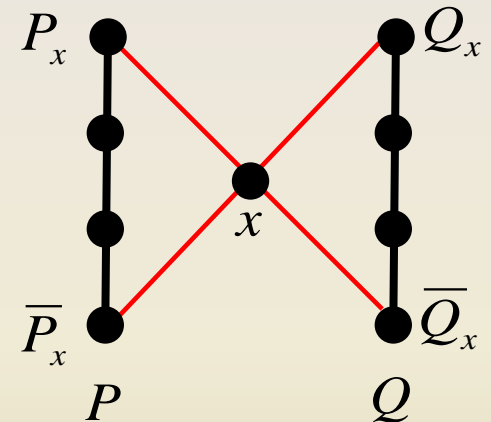
$$QP_x = Q_x$$

$$\bar{Q}\bar{P}_x = \bar{Q}_x$$



$$PQ_x = P_x$$

$$\bar{P}\bar{Q}_x = \bar{P}_x$$



$$Q\bar{P}_x = Q_x$$

$$\bar{Q}P_x = \bar{Q}_x$$

**Collinearity allows some chains to be *ordered***

# **Coordinated Chains Measure the Same Length for One Interval**

# Coordinated Chains

A set of chains are coordinated if an interval on one projects onto an interval of the same length on the others.

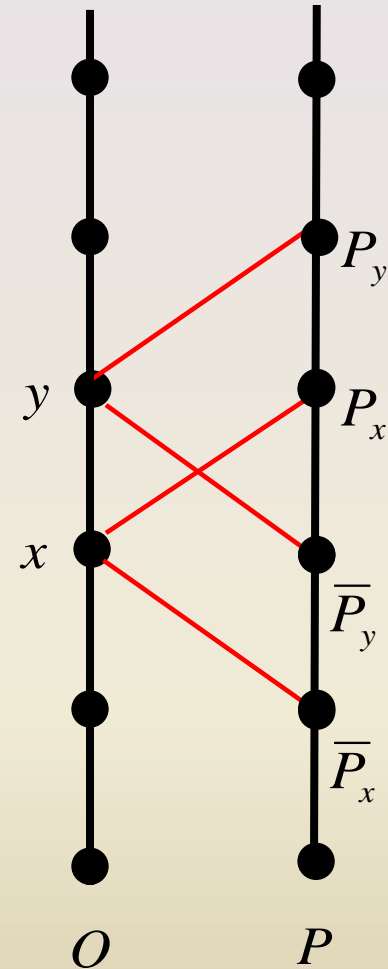
$$v(y) - v(x) = v(P_y) - v(P_x)$$

$$v(y) - v(x) = v(\bar{P}_y) - v(\bar{P}_x)$$

$$[x, y] |_P : (v(P_y) - v(P_x), v(\bar{P}_y), v(\bar{P}_x))$$

Equivalently, intervals can be quantified by forward projections alone using two coordinated chains.

$$[x, y] |_{OP} : (v(y) - v(x), v(P_y) - v(P_x))$$

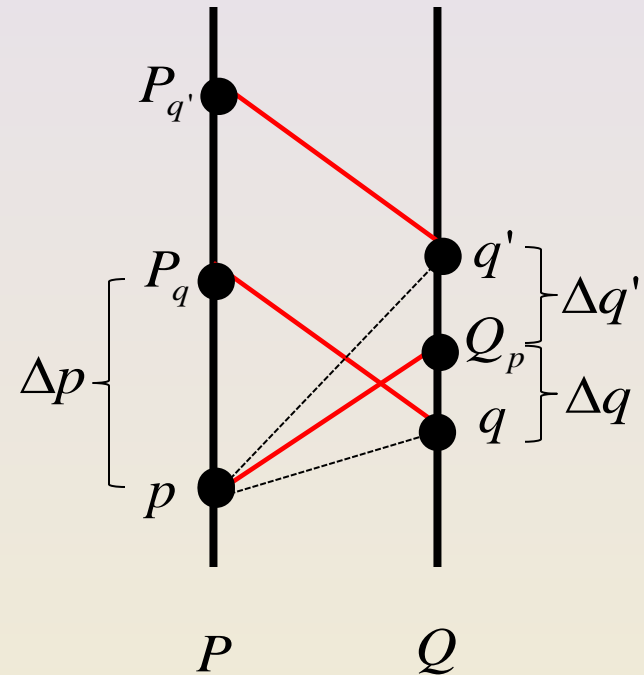


# Distance Between Coordinated Chains

# Distance Between Coordinated Chains

The distance between each pair of chains is found through chain projection using:

- Associativity of joining distances across chains
- arbitrary choice of events on the chains



$$[p, q]_{PQ} = (v(P_q) - v(p), v(q) - v(Q_p)) = (\Delta p, \Delta q)$$

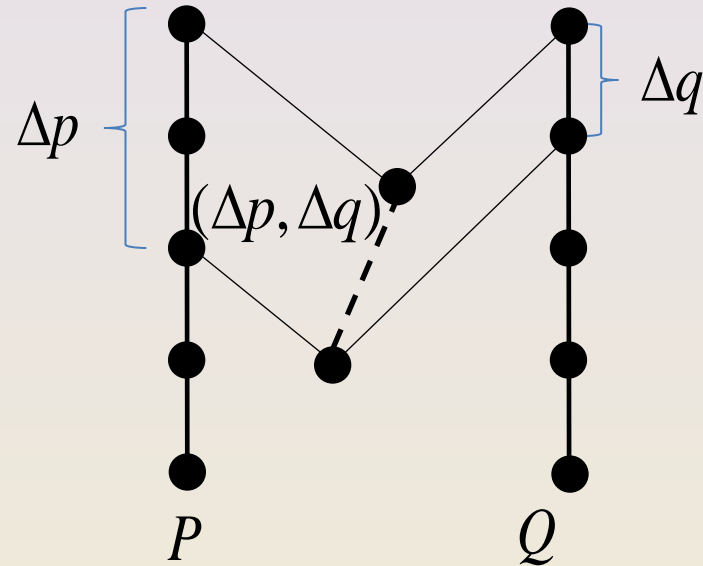
$$D([p, q]) \propto |\Delta p - \Delta q|$$

$$D([p, q]) \propto |\Delta p - \Delta q'|$$



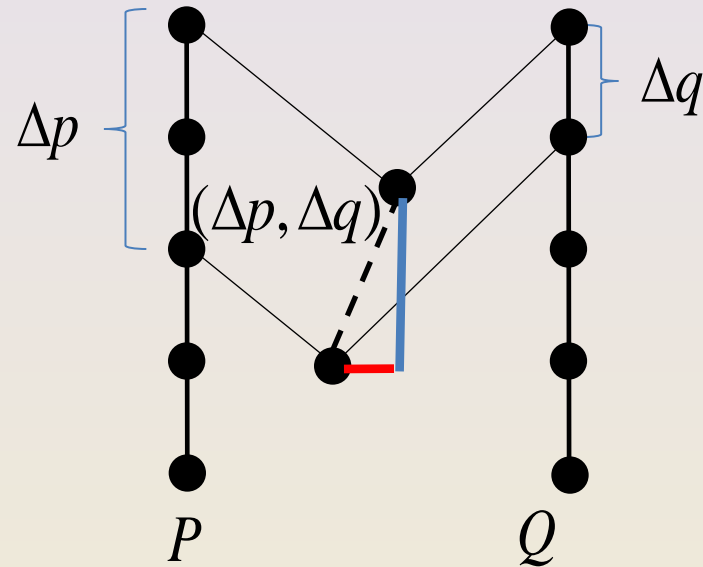
# The Minkowskian Form Emerges from the Pair and Scalar Quantifications

# Minkowski



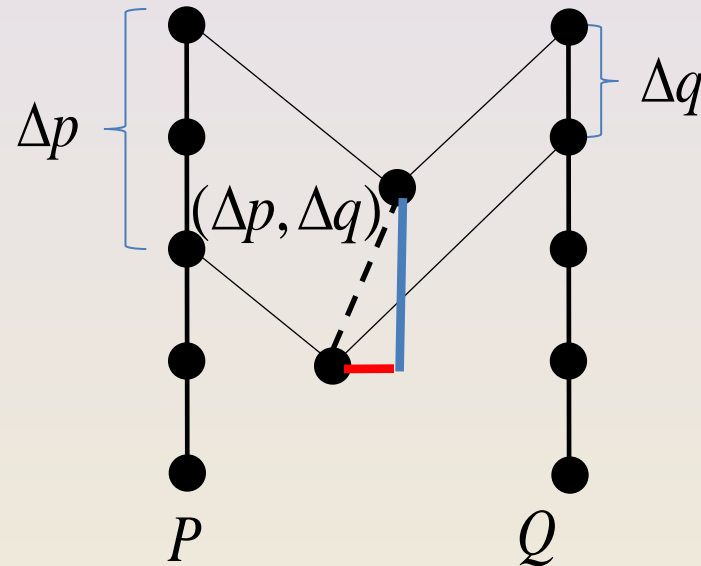
$$(\Delta p, \Delta q) = \left( \frac{\Delta p + \Delta q}{2}, \frac{\Delta p + \Delta q}{2} \right) + \left( \frac{\Delta p - \Delta q}{2}, \frac{-(\Delta p - \Delta q)}{2} \right)$$

# Minkowski



$$(\Delta p, \Delta q) = \underbrace{\left( \frac{\Delta p + \Delta q}{2}, \frac{\Delta p + \Delta q}{2} \right)}_{\substack{\text{Symmetric} \\ (\Delta t, \Delta t)}} + \underbrace{\left( \frac{\Delta p - \Delta q}{2}, \frac{-(\Delta p - \Delta q)}{2} \right)}_{\substack{\text{Antisymmetric} \\ (\Delta x, -\Delta x)}}$$

# Minkowski



$$(\Delta p, \Delta q) = \left( \frac{\Delta p + \Delta q}{2}, \frac{\Delta p + \Delta q}{2} \right) + \left( \frac{\Delta p - \Delta q}{2}, \frac{-(\Delta p - \Delta q)}{2} \right)$$

Interval Scalar

$$\Delta p \Delta q = \left( \frac{\Delta p + \Delta q}{2} \right)^2 - \left( \frac{\Delta p - \Delta q}{2} \right)^2$$

$$\Delta p \Delta q = \Delta t^2 - \Delta x^2 \quad \textit{The Minkowskian Form}$$

**Changes from one Observer Chain to  
another is given by  
Lorentz Transformations**

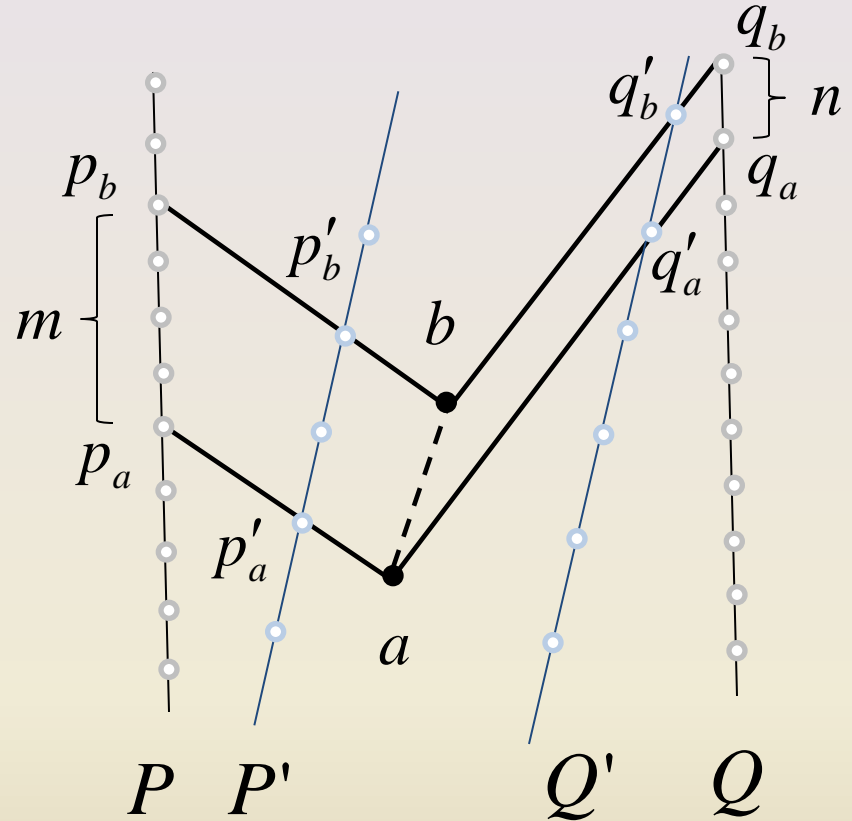
# Transformations

$$(\Delta p', \Delta q') = \left( \Delta p \sqrt{\frac{n}{m}}, \Delta q \sqrt{\frac{m}{n}} \right)$$

$$\Delta p' \Delta q' = \Delta p \Delta q$$

Scalar is preserved

Equivalent to Bondi's K-calculus



# Lorentz Transformation

$$(\Delta p', \Delta q') = \left( \Delta p \sqrt{\frac{n}{m}}, \Delta q \sqrt{\frac{m}{n}} \right)$$

$$\left( \frac{\Delta t' + \Delta x'}{2}, \frac{\Delta t' - \Delta x'}{2} \right) = \left( \left( \frac{\Delta t + \Delta x}{2} \right) \sqrt{\frac{n}{m}}, \left( \frac{\Delta t - \Delta x}{2} \right) \sqrt{\frac{m}{n}} \right)$$

$$\left\{ \begin{array}{l} \Delta t' = \frac{1}{\sqrt{1-\beta^2}} \Delta t + \frac{-\beta}{\sqrt{1-\beta^2}} \Delta x \\ \Delta x' = \frac{-\beta}{\sqrt{1-\beta^2}} \Delta t + \frac{1}{\sqrt{1-\beta^2}} \Delta x \end{array} \right.$$

Lorentz Transformation with

$$\beta = \frac{m-n}{m+n}$$

$\beta \in [0,1]$  has a maximum invariant value of 1.

# CONCLUSIONS

Treating the universe as a network of events and quantifying the network results in the Minkowski metric and Lorentz transformations in the case where observers are coordinated.

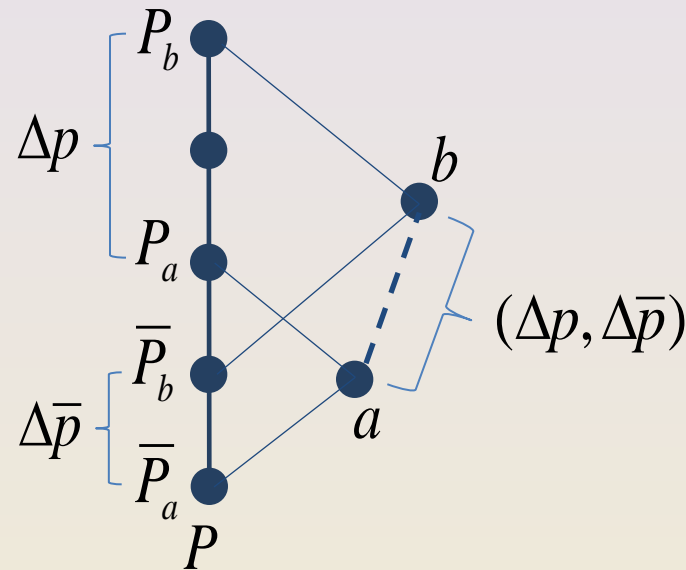
This suggests that the mathematics of space and time emerges as the unique means for an embedded observer to quantify the network.

Knuth K.H., Bahreyni N. 2012.  
**arXiv:1209.0881v1 [math-ph]**



# Intervals Can Be Quantified with 4, 2 and 1 Numbers

# Quantification of a Generalized Interval



The generalized interval  $[a, b]$  can be represented by

Four numbers

$$(P_a, P_b ; \bar{P}_a, \bar{P}_b)$$

Two numbers (**pair**)

$$(P_a - P_b ; \bar{P}_a - \bar{P}_b) \equiv (\Delta p, \Delta \bar{p})$$

One number (**scalar**)

$$(P_a - P_b)(\bar{P}_a - \bar{P}_b) = (\Delta p)(\Delta \bar{p})$$