On charged black holes in Nonlinear ghost-free Massive Gravity

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September 27, 2012

Outline of the talk

- dRGT action
- On spherically symmetric charged background
- Linearized treatment in weak field limit
- Numerical and analytic analysis
- Conclusion

The dRGT action: de Rham,Gabadadze, Tolley 2010

$$S = \int d^4x\sqrt{-g}\frac{M_p^2}{2}\left[R + m^2\mathcal{U}(g,\phi^a)\right] , \quad (1)$$

where $\mathcal{U}$ is the potential of graviton

$$\mathcal{U}(g,\phi^a) = \mathcal{U}_2 + \alpha_3\mathcal{U}_3 + \alpha_4\mathcal{U}_4 \ . \quad (2)$$

$$\mathcal{U}_2 \equiv [\mathcal{K}]^2 - [\mathcal{K}^2] , \quad (3)$$
$$\mathcal{U}_3 \equiv [\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3] , \quad (4)$$
$$\mathcal{U}_4 \equiv [\mathcal{K}]^4 - 6[\mathcal{K}]^2[\mathcal{K}^2] + 8[\mathcal{K}][\mathcal{K}^3] + 3[\mathcal{K}^2]^2 - 6[\mathcal{K}^4] \ . \quad (5)$$

$$\mathcal{K}^\mu_\nu = \delta^\mu_\nu - \sqrt{g^{\mu\sigma}\eta_{ab}\partial_\sigma\phi^a\partial_\nu\phi^b} \ . \quad (6)$$

where $M_p = 1/\sqrt{8\pi G}$ and ϕ^a are the Stückelberg scalars.
Choosing unitary gauge: $\phi^a = x^\mu\delta^a_\mu$

Spherically symmetric charged background

$$ds^2 = -N^2(r)dt^2 + \frac{dr^2}{F^2(r)} + \frac{r^2 d\Omega_2^2}{H^2(r)} \ , \tag{7}$$

Assume static electric charge in the system:

$$E_r = F_{0r} = E(r) \ , \quad E_\theta = E_\varphi = 0 \ , \quad \vec{B} = 0 \ . \tag{8}$$

Inhomogeneous Maxwell equation:

$$D_\mu F^{\mu\nu} = -J^\nu . \tag{9}$$

Source free $J^\nu = 0$, we obtain

$$E(r) = \frac{QNH}{4\pi F r^2} \tag{10}$$

Weak field limit: Koyama,Niz,Tasinato 2011

$$\begin{aligned} N(r) &= 1+n(r) \\ F(r) &= 1+f(r) \\ H(r) &= 1+h(r) \end{aligned} \tag{11}$$

Rescale the radial component

$$\rho = \frac{r}{H} \Rightarrow 1+\tilde{f} = \frac{1+f}{1+h+\rho h'}$$

Linearized metric

$$ds^2 = -(1+2n(\rho))dt^2 + (1-2\tilde{f}(\rho))d\rho^2 + \rho^2 d\Omega^2 \tag{12}$$

Equation of Motion

$$2\tilde{f} + 2\rho\tilde{f}' + m^2\rho^2\Big[(1 - 2\alpha h + 6\beta h^2)[(1 + \tilde{f})\rho h' + (1 + h)\tilde{f}] + 3h(1 - \alpha h + 2\beta h^2)\Big] + \frac{GQ^2}{4\pi\rho^2} = 0, \quad (13)$$

$$2\tilde{f} + 2\rho n' - m^2\rho^2\Big[n - 2(1 + n + \alpha n)h + (\alpha + \alpha n + 6\beta n)h^2\Big] + \frac{GQ^2(1+n)}{4\pi\rho^2} = 0\ , \quad (14)$$

$$\rho n'[1 - 2\alpha h + 6\beta h^2] + 2\tilde{f}[1 - \alpha h] = 0\ , \quad (15)$$

where $\alpha = 1 + 3\alpha_3$, $2\beta = 3\alpha_3 + 4\alpha_4$.

Solution

Neglecting all high-order terms proportional to m^4, $m^2 GM$, and $m^2 GQ^2$:

$$\tilde{f} \simeq \frac{GQ^2}{8\pi\rho^2} - \frac{GM}{\rho} - \frac{m^2\rho^2}{2}(h - \alpha h^2 + 2\beta h^3) \ , \tag{16}$$

$$n' \simeq -\frac{GQ^2}{4\pi\rho^3} + \frac{GM}{\rho^2} - \frac{m^2\rho}{2}(h - 2\beta h^3) \ , \tag{17}$$

$$\frac{GM}{\rho}(1 - 6\beta h^2) - \frac{GQ^2}{4\pi\rho^2}(\alpha h - 6\beta h^2) = m^2\rho^2 \times [-\frac{3}{2}h + 3\alpha h^2 - (\alpha^2 + 4\beta)h^3 + 6\beta^2 h^5] \ . \tag{18}$$

Case I: $\alpha = \beta = 0$

$$n(\rho) \simeq -\frac{4GMe^{-m\rho}}{3\rho} + \frac{GQ^2}{8\pi\rho^2} + \frac{GmQ^2}{16\pi\rho}\left[e^{m\rho}\mathrm{Ei}(-m\rho) - e^{-m\rho}\mathrm{Ei}(m\rho)\right] , \quad (19)$$

$$\tilde{f}(\rho) \simeq -\frac{2GMe^{-m\rho}(1+m\rho)}{3\rho} + \frac{GQ^2}{8\pi\rho^2} + \frac{GmQ^2}{32\pi\rho} \times \left[(1-m\rho)e^{m\rho}\mathrm{Ei}(-m\rho) - (1+m\rho)e^{-m\rho}\mathrm{Ei}(m\rho)\right] , \quad (20)$$

where $\mathrm{Ei}(x) \equiv \int_{-\infty}^{x} e^t \, d\ln t$. $\gamma \simeq (1+m\rho)/2$ if $Q = 0$.

Case II: $\alpha \neq 0$ and $\beta = 0$

$$\rho_Q \equiv (\frac{GQ^2}{4\pi m^2})^{1/4}\ ,\ \ \rho_V \equiv (\frac{GM}{m^2})^{1/3}\ .$$

When $2GM < \rho < \rho_Q$,

$$n \simeq \frac{GQ^2}{8\pi\rho^2} - \frac{GM}{\rho} + \frac{m^2\rho_Q^2}{2\alpha^{1/2}}\ln(m\rho + c)\ , \tag{21}$$

$$\tilde{f} \simeq \frac{GQ^2}{8\pi\rho^2} - \frac{GM}{\rho} - \frac{m^2\rho_Q^2}{\alpha^{1/2}} + \frac{GM\rho}{2\alpha^{1/2}\rho_Q^2}\ . \tag{22}$$

When $\rho_Q < \rho < \rho_V$,

$$n \simeq \frac{GQ^2}{8\pi\rho^2} - \frac{GM}{\rho} + \frac{GM\rho}{2\alpha^{2/3}\rho_V^2}\ , \tag{23}$$

$$\tilde{f} \simeq \frac{GQ^2}{8\pi\rho^2} - \frac{GM}{\rho} + \frac{GM}{2\alpha^{1/3}\rho_V} + \frac{GM\rho}{2\alpha^{2/3}\rho_V^2}\ . \tag{24}$$

Case II: $\alpha \neq 0$ and $\beta = 0$

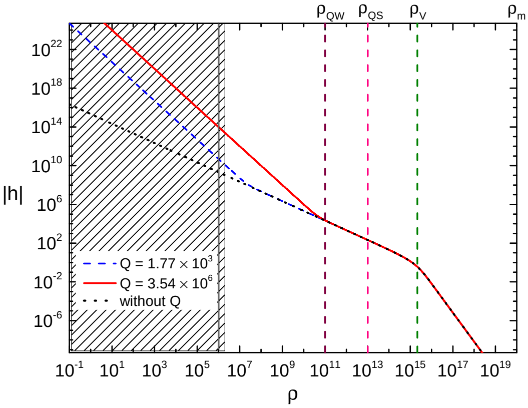

Figure: $\alpha = 1$, $\beta = 0, m = 10^{-20}$ and $M = 10^6$. All dimensional parameters are of Planck units.

Case II: $\alpha \neq 0$ and $\beta = 0$

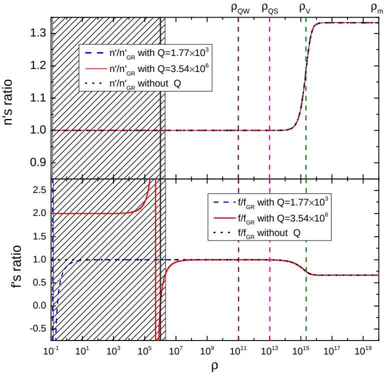

Figure: Plot of the evolutions of the ratios n'/n'_{GR} and $\tilde{f}/\tilde{f}_{GR}$ as functions of radial coordinate ρ. $\alpha = 1$

Case II: $\alpha \neq 0$ and $\beta = 0$

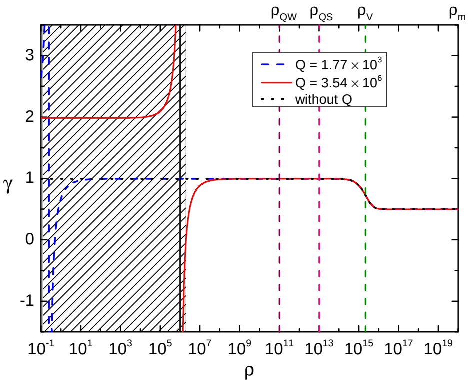

Figure: $\gamma = \frac{\tilde{f}}{n}, \alpha = 1$

Case III: $\alpha \neq 0$ and $\beta < 0$

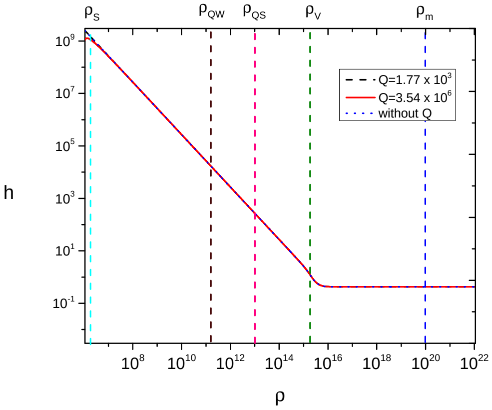

Figure: $\alpha = 1$ and $\beta = -1/2$.

Case III: $\alpha \neq 0$ and $\beta < 0$, Analytic analysis

$$h \simeq \frac{1}{\beta^{1/3}}\left(\frac{{\rho_Q}^4}{\rho^4} - \frac{{\rho_V}^3}{\rho^3}\right)^{1/3}, \quad (25)$$

$$n' \simeq -\frac{m^2}{2\beta^{1/3}}\left(\frac{{\rho_Q}^4}{\rho} - {\rho_V}^3\right)^{1/3}, \quad (26)$$

$$\tilde{f} \simeq -\frac{GQ^2}{8\pi\rho^2} + \frac{\alpha m^2}{2\beta^{2/3}}\left(\frac{{\rho_Q}^4}{\rho} - {\rho_V}^3\right)^{2/3} - \frac{m^2}{2\beta^{1/3}}\left({\rho_Q}^4 - \rho{\rho_V}^3\right)^{1/3}\rho^{2/3}. \quad (27)$$

Case III: $\alpha \neq 0$ and $\beta > 0$

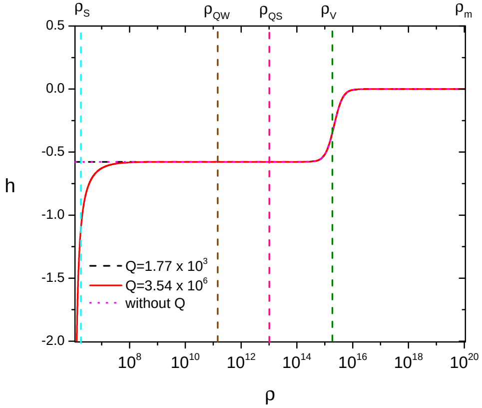

Figure: $\alpha = 1$ and $\beta = 3$

Case III: $\alpha \neq 0$ and $\beta > 0$

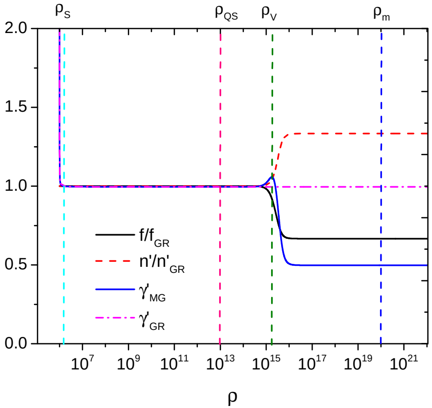

Figure: $\alpha = 1,\ \beta = 3.$

Case III: $\alpha \neq 0$ and $\beta > 0$, Analytic analysis

Neglecting terms proportional to h^3 and h^5, approximate solution

$$h = \frac{-\sqrt{(3\rho^4 - 2\alpha\rho_Q{}^4)^2 + 48\rho\rho_V{}^3(\alpha\rho^4 - 2\beta\rho_Q{}^4 + 2\beta\rho\rho_V{}^3)}}{12(\alpha\rho^4 - 2\beta\rho_Q{}^4 + 2\beta\rho\rho_V{}^3)} + \frac{3\rho^4 - 2\alpha\rho_Q{}^4}{12(\alpha\rho^4 - 2\beta\rho_Q{}^4 + 2\beta\rho\rho_V{}^3)} \qquad (28)$$

Special solution: $\beta = \alpha^2/6$

$$h = 1/\alpha, \qquad H = \frac{1+\alpha}{\alpha}$$

Apply coordinate transformation $t \to \frac{\alpha}{1+\alpha} t$, $r \to \frac{1+\alpha}{\alpha} r$,

$$ds^2 = -A(r)dt^2 + \frac{dr^2}{A(r)} + r^2 d\Omega^2 , \qquad (29)$$

where

$$A(r) = 1 + \frac{r_Q^2}{r^2} - \frac{\tilde{r}_S}{r} - \frac{r^2}{r_\Lambda^2} , \qquad (30)$$

$$\tilde{r}_S \equiv \frac{\alpha^3 r_M}{(1+\alpha)^3} , \quad r_\Lambda \equiv \frac{\sqrt{3\alpha}}{m} . \qquad (31)$$

Conclusion

- We checked the black hole solution with static electric charge in the frame of nonlinear massive gravity
- We find some constraints on the parameter space
- $\alpha = 0$, $\beta = 0$ ruled out because of the vDVZ discontinuity
- $\alpha \neq 0$, $\beta = 0$ shows the Vainshtein mechanism
- $\alpha \neq 0$, $\beta < 0$ ruled out due to dramatic change to GR
- $\alpha \neq 0$, $\beta > 0$ shows the Vainshtein mechanism
- Exactly analytic solution: $\beta = \frac{\alpha^2}{6}$