Cosmological constant from Massive Gravity

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Wayne Hu, Mark Wyman, PG (1205.4241): Given a Lagrangian containing a massive graviton, a cosmological constant arises from any isotropic matter distribution.

- In 2011 D'Amico et al. showed that in an FRW metric the massive gravity stress-tensor is a cosmological constant.
- Our result is based on, and generalizes, D'Amico's and includes, among others, FRW, Schwarzschild-de Sitter, and perturbations thereof.

Massive Gravity Lagrangian

Fierz and Pauli in 1939: First massive gravity Lagrangian,

$$\mathcal{L}_m = M_{Pl}^2 \sqrt{-g} \left(R + \frac{m^2}{4} \left[h_{\mu\nu}^2 - (h_\mu^\mu)^2 \right] \right) \tag{1}$$

- This Lagrangian has the vDVZ discontinuity at tree level: predictions differ from ordinary GR in the $m \to 0$ limit.
- Vainshtein mechanism (1972): tree level approximation not valid for distances to source smaller than $R_V = (m^{-4} R_S)^{1/5}$: nonlinear interactions comparable to linear ones even for weak fields.

Massive Gravity Lagrangian

- Add nonlinear interactions
- But these will naively have a 6th ghost degree of freedom, the Boulware-Deser mode.
- Possible to find a Lagrangian that is nonlinear, but no BD ghost?

Hassan, Rosen 2012: YES!

Massive Gravity Lagrangian

$$\mathcal{L}_G = \frac{M_{\rm pl}^2}{2}\sqrt{-g}\left[R - \frac{m^2}{4}\mathcal{U}(g_{\mu\nu}, \mathcal{K}_{\mu\nu})\right].$$

The potential is expressed in terms of the tensor

$$\mathcal{K}^{\mu}{}_{\nu} = \delta^{\mu}{}_{\nu} - \sqrt{\mathbf{\Sigma}}^{\mu}{}_{\nu}.$$

where

$$\Sigma^{\mu}_{\nu} \equiv g^{\mu\alpha}\partial_{\alpha}\phi^{a}\partial_{\nu}\phi^{b}\eta_{ab}$$

The ϕ^a's are the Stückelberg fields introduced to restore diffeomorphism invariance.

Massive Gravity Potential

$$
\begin{aligned}
-\mathcal{U}/4 = [\mathcal{K}]^2 - [\mathcal{K}^2] + \alpha_3\Big([\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3]\Big) \\
+ \alpha_4\Big([\mathcal{K}]^4 - 6[\mathcal{K}]^2[\mathcal{K}^2] + 8[\mathcal{K}][\mathcal{K}^3] + 3[\mathcal{K}^2]^2 \\
- 6[\mathcal{K}^4]\Big),
\end{aligned}
$$

$$
\mathcal{K}^{\mu}{}_{\nu} = \delta^{\mu}{}_{\nu} - \sqrt{\mathbf{\Sigma}}^{\mu}{}_{\nu}.
$$

Ansatz for the metric and Stückelberg fields

- An isotropic metric that is fully general:

$$ds^2 = -b^2(r,t)dt^2 + a^2(r,t)(dr^2 + r^2 d\Omega^2).$$

- Spherically symmetric Stückelberg fields:

$$\phi^0 = f(t,r),$$
$$\phi^i = g(t,r)\frac{x^i}{r},$$

$$\Sigma^\mu_\nu \equiv g^{\mu\alpha}\partial_\alpha\phi^a\partial_\nu\phi^b\eta_{ab}$$

and get a block-diagonal Σ matrix.

Equations of motion

The resulting potential-generating matrix Σ then looks as follows:

$$\mathbf{\Sigma} = \begin{pmatrix} \frac{\dot{f}^2 - \dot{g}^2}{b^2} & \frac{\dot{f} f' - \dot{g} g'}{b^2} & 0 & 0 \\ \frac{\dot{g} g' - \dot{f} f'}{a^2} & \frac{-f'^2 + g'^2}{a^2} & 0 & 0 \\ 0 & 0 & \frac{g^2}{a^2 r^2} & 0 \\ 0 & 0 & 0 & \frac{g^2}{a^2 r^2} \end{pmatrix},$$

We want to calculate the contribution of the massive graviton to the stress tensor, i.e. we need f and g.

Equations of motion

The f equation of motion reads (here $\alpha_3 = \alpha_4 = 0$):

$$\partial_t \left[\frac{a^3 r^2}{\sqrt{X}} \left(\frac{\dot{f}}{b} + \mu \frac{g'}{a} \right) \left(3 - \frac{2g}{ar} \right) - \mu a^2 r^2 g' \right]$$
$$- \partial_r \left[\frac{a^2 b r^2}{\sqrt{X}} \left(\mu \frac{\dot{g}}{b} + \frac{f'}{a} \right) \left(3 - \frac{2g}{ar} \right) - \mu a^2 r^2 \dot{g} \right] = 0,$$

The g equation of motion is similar with a nonzero right hand side.
One simple solution to the f equation of motion is

$$g = \frac{3}{2} a(r,t) r$$

Solution and Stress-energy Tensor

We can now compute all components of the stress-energy tensor. It is the exact equivalent of a cosmological constant:

$$(m^2 M_{\rm pl}^2)\, T^{\mu(\mathcal{K})}_{\nu} = \begin{pmatrix} -\rho_{\mathcal{K}} & 0 & 0 & 0 \\ 0 & p_{\mathcal{K}} & 0 & 0 \\ 0 & 0 & p_{\mathcal{K}} & 0 \\ 0 & 0 & 0 & p_{\mathcal{K}} \end{pmatrix},$$

where

$$\rho_{\mathcal{K}} = -p_{\mathcal{K}} = \frac{3}{4} m^2 M_{\rm pl}^2.$$

This result is an exact solution for any isotropic distribution of matter, and for both small and large perturbations. Thus it includes previously obtained solutions, e.g.:

- FRW for $b(t,r) = 1$ and $a(t,r) = a(t)$ found by D'Amico et al.
- Isotropic perturbations around FRW with $a^2(t,r) = a^2(t)[1 + 2\Phi(t,r)]$ and $b^2(t,r) = [1 + 2\Psi(t,r)]$.
- In the vacuum, Schwarzschild-de Sitter solution found by Koyama et al.

Open questions

- Cosmological constant for even more general inhomogeneous matter fields? Will their stress-energy remain homogeneous?
- What about perturbations around our solution? Conflicting results in the literature: Scalar fluctuations: kinetic term $= 0$ (Gumrukcuoglu et al.), $\neq 0$ (Koyama et al.)