Possible UV Completion through Classicalization Mechanism

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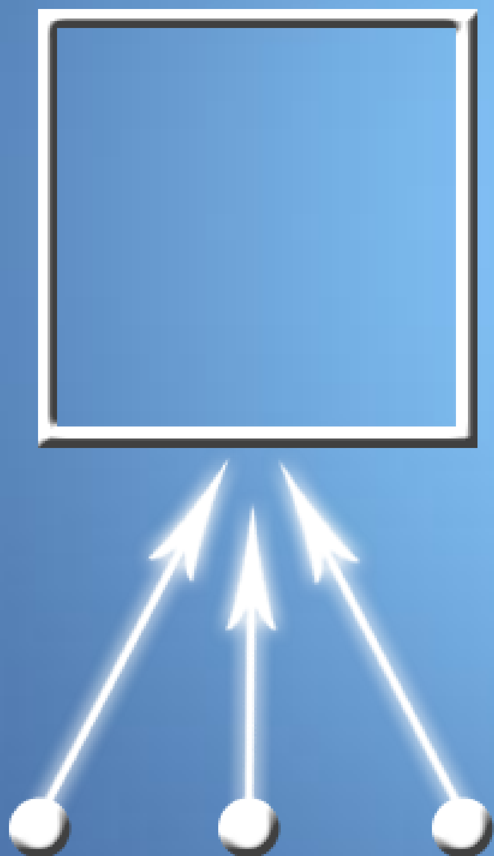
Some things come in

$$\boxed{a^\dagger_{k_1} a^\dagger_{k_2} \dots \mid 0\rangle}$$

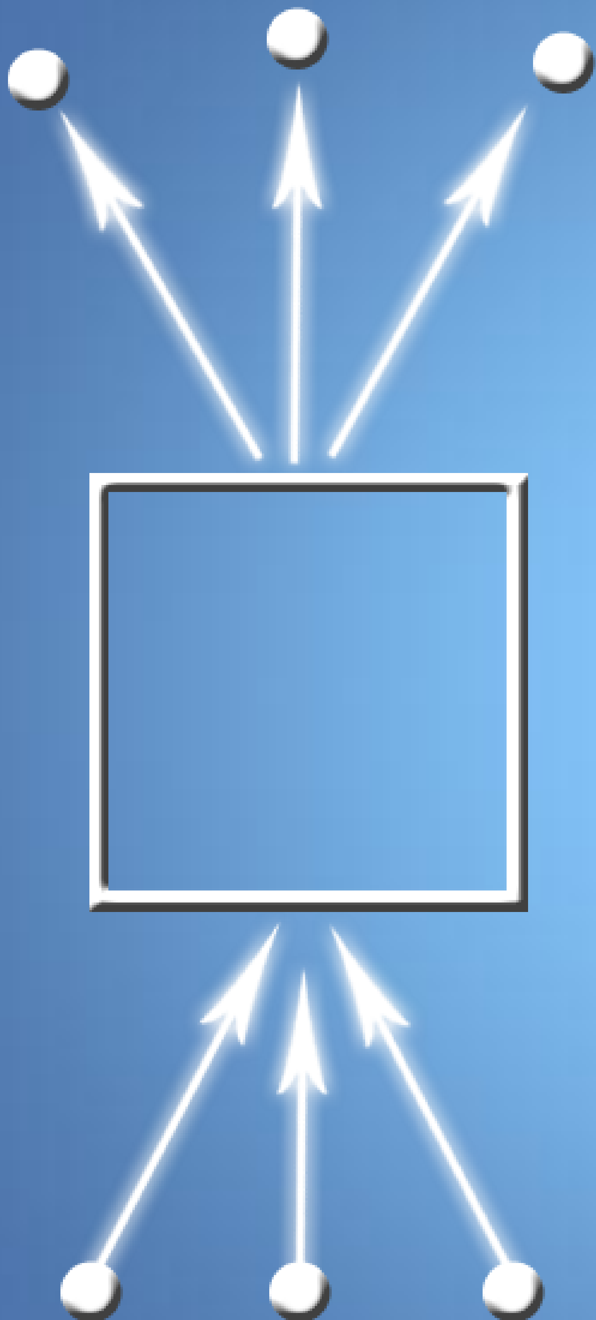


Some things happen

$$T^* e^{i \int d^4x \mathcal{L}_{int}}$$

Some things come in

$$a^\dagger_{k_1} a^\dagger_{k_2} \ldots \mid 0\rangle$$



Some things come out

$$\boxed{a^\dagger_{k_3} a^\dagger_{k_4} \ldots \mid 0\rangle}$$

Some things happen

$$\boxed{T^* e^{i \int d^4x \mathcal{L}_{int}}}$$

Some things come in

$$\boxed{a^\dagger_{k_1} a^\dagger_{k_2} \ldots \mid 0\rangle}$$

$\hat{S}$ Matrix

$$\langle 0 \mid a_{k_4} a_{k_3} \hat{S} a^\dagger_{k_1} a^\dagger_{k_2} \mid 0 \rangle = \langle 0 \mid a_{k_4} a_{k_3} T^* e^{i \int \mathcal{L}_I(x) d^4x} a^\dagger_{k_1} a^\dagger_{k_2} \mid 0 \rangle$$

$$= \langle 0 \mid a_{k_4} a_{k_3} \left[1 + i \int d^4x \mathcal{L}_I(x) + \frac{(i)^2}{2} \int \int d^4x d^4y T^* \mathcal{L}_I(x) \mathcal{L}_I(y) + ... \right] a^\dagger_{k_1} a^\dagger_{k_2} \mid 0 \rangle$$

The $\hat{S}$ matrix is often used for scattering and is usually treated perturbatively.

The Problem

$$S = \int d^4x \;\; -\frac{1}{2}(\partial\phi)^2 - \frac{1}{4\Lambda^4}(\partial\phi)^4$$

The Problem

Treat perturbatively

$$S = \int d^4x \ \ -\frac{1}{2}(\partial\phi)^2 - \boxed{\frac{1}{4\Lambda^4}(\partial\phi)^4}$$

The Problem

Treat perturbatively

$$S = \int d^4x \quad -\frac{1}{2}(\partial\phi)^2 - \boxed{\frac{1}{4\Lambda^4}(\partial\phi)^4}$$

$$\langle 0 \mid a_{k_4} a_{k_3} \hat{S} a^\dagger_{k_1} a^\dagger_{k_2} \mid 0 \rangle \sim \frac{s^2}{\Lambda^4}$$

$$s \sim E^2_{\mathrm{CM}}$$

Tree level calculation breaks unitarity once $s^2 \sim \Lambda^4$.

Wilsonian Completion

Wilsonian Completion

$$S[l,H] = \int d^4x \quad -\frac{1}{2}(\partial l)^2 - \frac{1}{2}(\partial H)^2 - V(l,H)$$

$$V(l,H) = \frac{1}{2}m^2 l^2 + \frac{1}{2}M^2 H^2 + \frac{1}{4!}g_l l^4 + \frac{1}{4!}g_H H^4 + \frac{1}{4}g_{lH} l^2 H^2 + \frac{1}{2}\tilde{m} l^2 H + \frac{1}{3!}\tilde{g}_H M H^3$$

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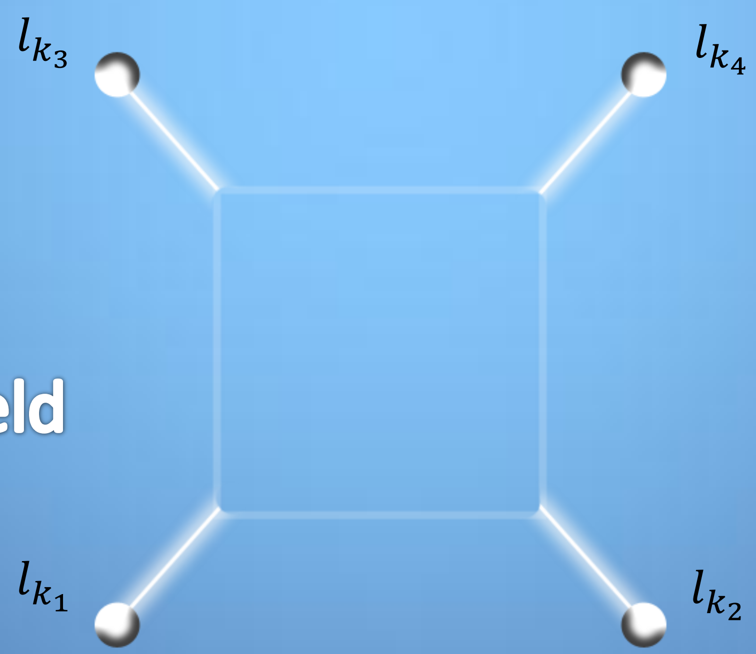


$l \rightarrow$ light field

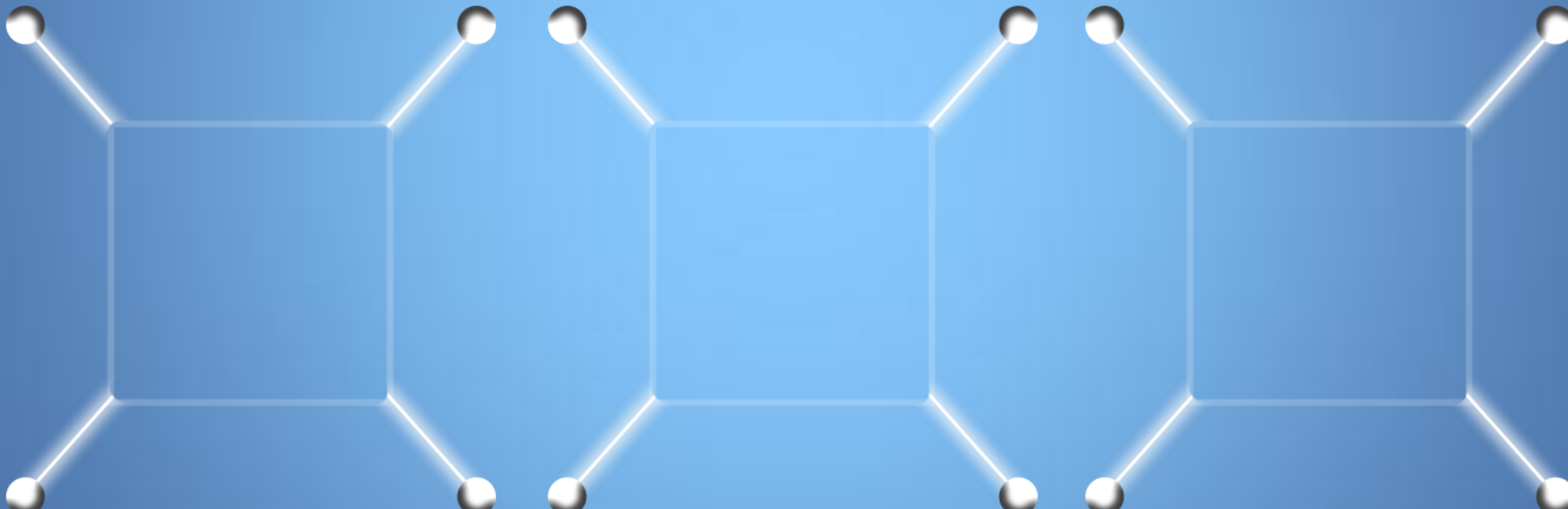
$H \rightarrow$ heavy field

$$\langle 2 \mid \hat{S} \mid 2 \rangle$$

$\langle 2 \mid \hat{S} \mid 2 \rangle$

Wilsonian Completion

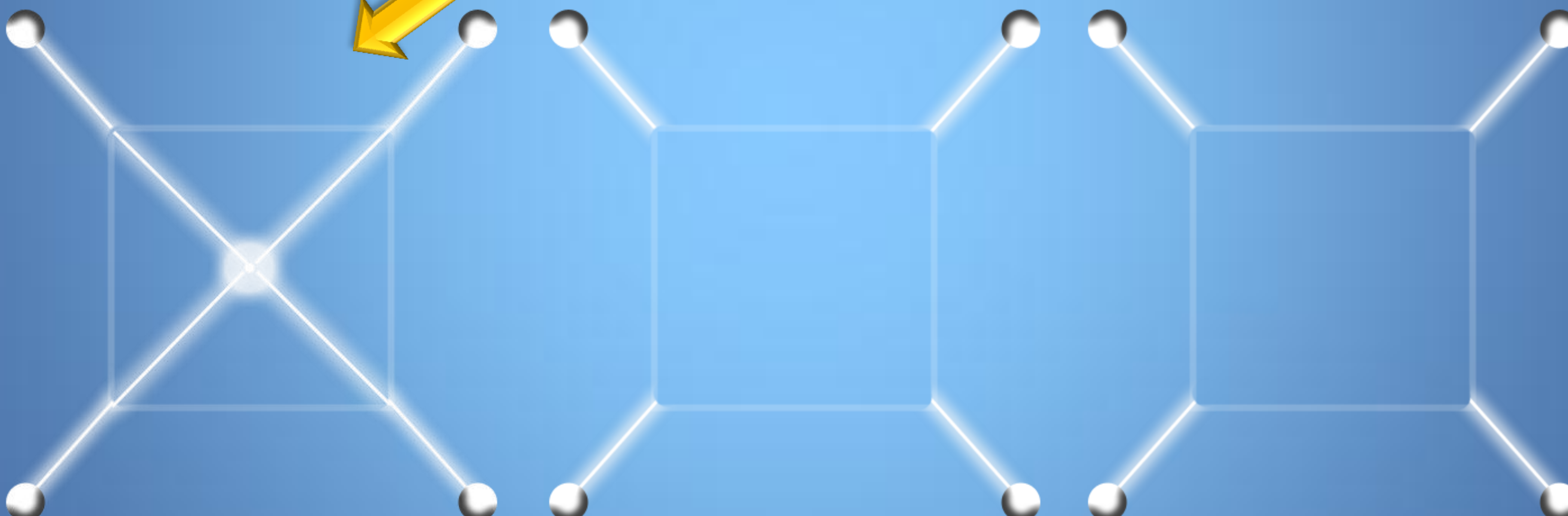
$$V(l,H) = \frac{1}{2}m^2l^2 + \frac{1}{2}M^2H^2 + \frac{1}{4!}g_l l^4 + \frac{1}{4!}g_H H^4 + \frac{1}{4}g_{lH}l^2H^2 + \frac{1}{2}\tilde{m}l^2H + \frac{1}{3!}\tilde{g}_H M H^3$$



$$\langle 2 \mid \hat{S} \mid 2 \rangle$$

Wilsonian Completion

$$V(l,H) = \frac{1}{2}m^2 l^2 + \frac{1}{2}M^2 H^2 + \boxed{\frac{1}{4!}g_l l^4} + \frac{1}{4!}g_H H^4 + \frac{1}{4}g_{lH} l^2 H^2 + \frac{1}{2}\tilde{m} l^2 H + \frac{1}{3!}\tilde{g}_H M H^3$$



$\langle 2 \mid \hat{S} \mid 2 \rangle$

Wilsonian Completion

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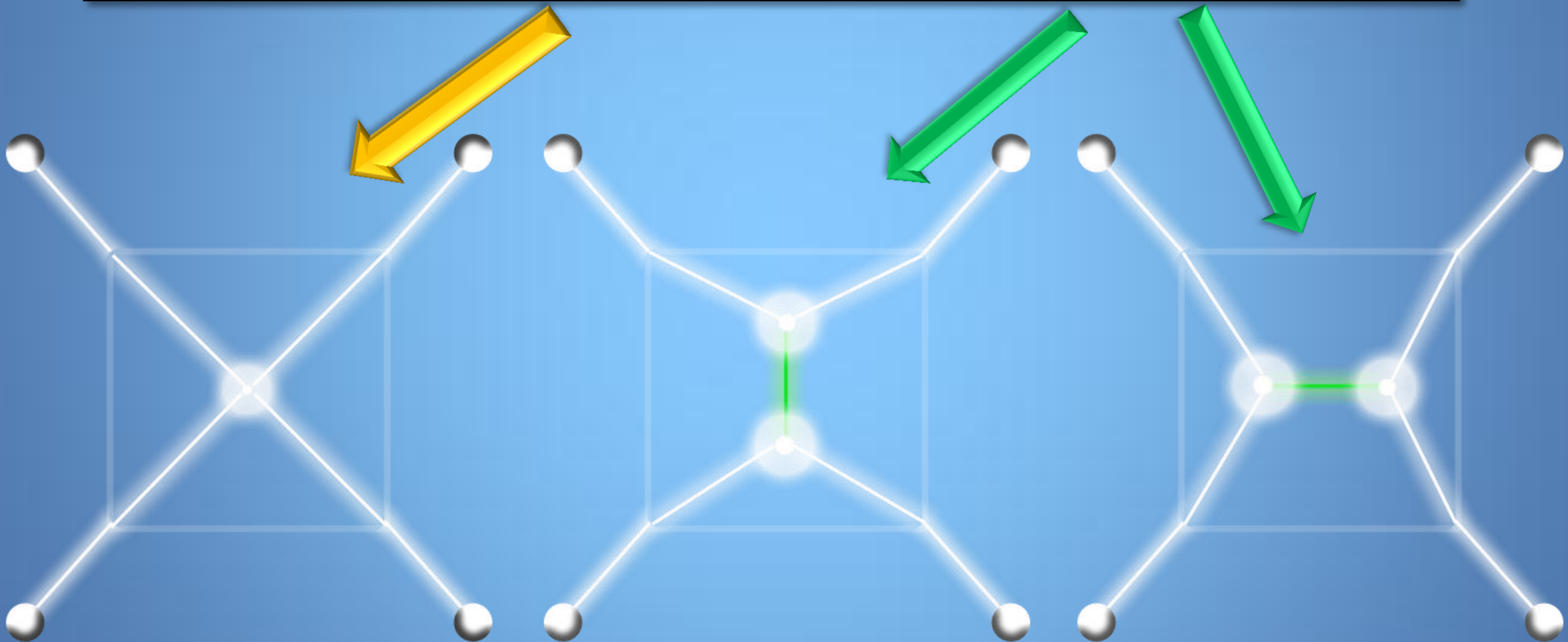

$$\langle 2 \mid \hat{S} \mid 2 \rangle$$

$$\langle 0 \mid a_{k_4} a_{k_3} \hat{S} a^\dagger_{k_1} a^\dagger_{k_2} \mid 0 \rangle = i(2\pi)^4 \delta^{(4)}(k_1 + k_2 - k_3 - k_4) \mathcal{A}(k_1, k_2, k_3, k_4)$$

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$$\mathcal{A}(k_1, k_2, k_3, k_4) = -g_l + \tilde{m}^2 \left[\frac{1}{s + M^2} + \frac{1}{t + M^2} + \frac{1}{u + M^2} \right]$$

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$$\mathcal{A}(k_1, k_2, k_3, k_4) \sim -g_l + \frac{3\tilde{m}^2}{M^2} + \frac{4\tilde{m}^2 m^2}{M^4} + \frac{\tilde{m}^2 s^2}{M^6} + \ldots$$

$$\langle 2 \mid \hat{S} \mid 2 \rangle$$

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Breaks unitarity when $s^2 \sim M^4$

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Breaks unitarity when $s^2 \sim M^4$

$S[l, H] \rightarrow S_{\text{eff,NR}}[l]$

Below energy scale M, $S_{\text{eff,NR}}[l]$ is *effectively* correct.

Energy

Renormalizable

$\boxed{S[l, H]}$ ✓

M

Non-renormalizable

$\boxed{S_{\text{eff,NR}}[l]}$ ✓

Solution

Solution

Wilsonian Completion

Solution

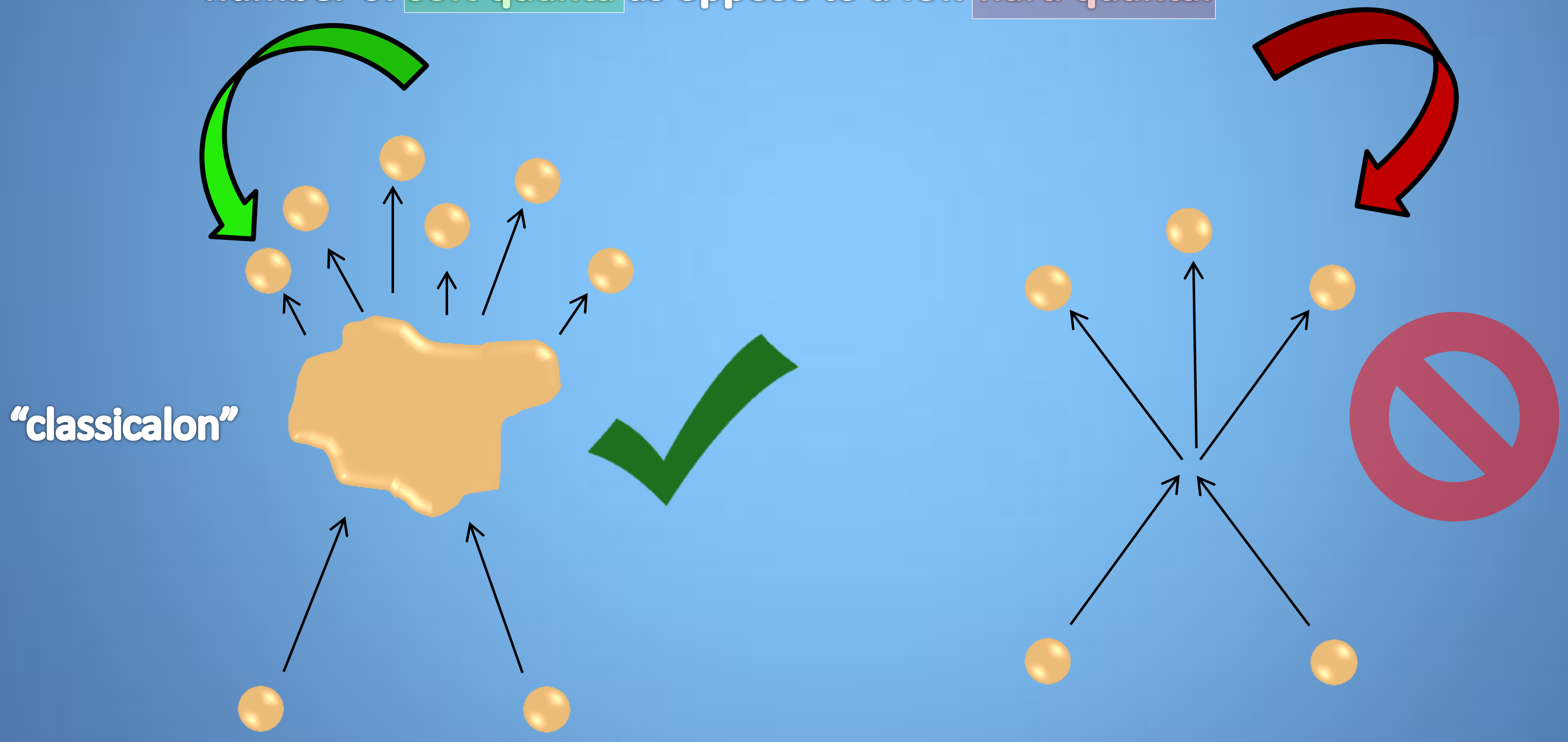
~~Wilsonian Completion~~

Hypothesis (Dvali et al.): arXiv:1010.1415v2 [hep-ph]

Classicalization

Classicalization

At large energies, scattering prefers processes involving a large number of soft quanta as oppose to a few hard quanta.



$2 \rightarrow N$

$2 \rightarrow$ few

Why Classical?

Why Classical?

$$S = \int d^4x \quad -\frac{1}{2}(\partial\phi)^2 - \frac{1}{4\Lambda^4}(\partial\phi)^4$$

Goldstone

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Goldstone

$$[S] = \hbar$$

$$[\phi] = \frac{\sqrt{\hbar}}{L}$$

$$[\Lambda^4] = \frac{\hbar}{L^4}$$

$$[E] = \frac{\hbar}{L}$$

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$$[E] = \frac{\hbar}{L}$$

$$\lim_{\hbar \to 0}$$

Classical length **increases** with energy

$$r_* \sim \frac{1}{\Lambda}\left(\frac{E}{\Lambda}\right)^{\frac{1}{3}}$$

Why Classical?

$$S = \int d^4x \quad -\frac{1}{2}(\partial\phi)^2 - \frac{1}{\Lambda^3}\Box\phi(\partial\phi)^2$$

Galileon

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(another)

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Galileon

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What is r_*?

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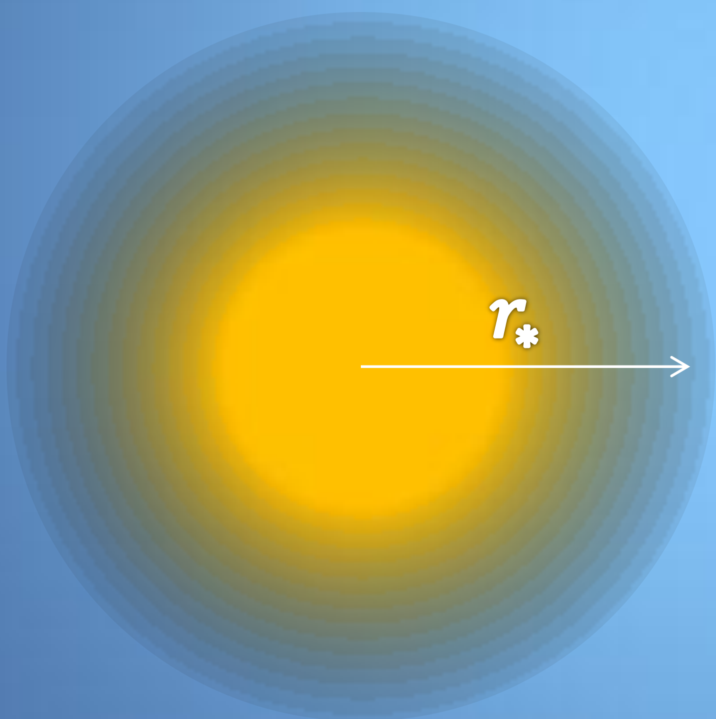
Goldstone



$$E \sim \int_{r<r_*} dr \;\; r^2 (\partial_r \phi)^2 \sim r_* \phi^2$$

$$(\partial\phi)^2 \sim \frac{1}{\Lambda^4} (\partial\phi)^4$$

$$r_* \sim \frac{1}{\Lambda} \left(\frac{E}{\Lambda} \right)^{\frac{1}{3}}$$

What is r_*?

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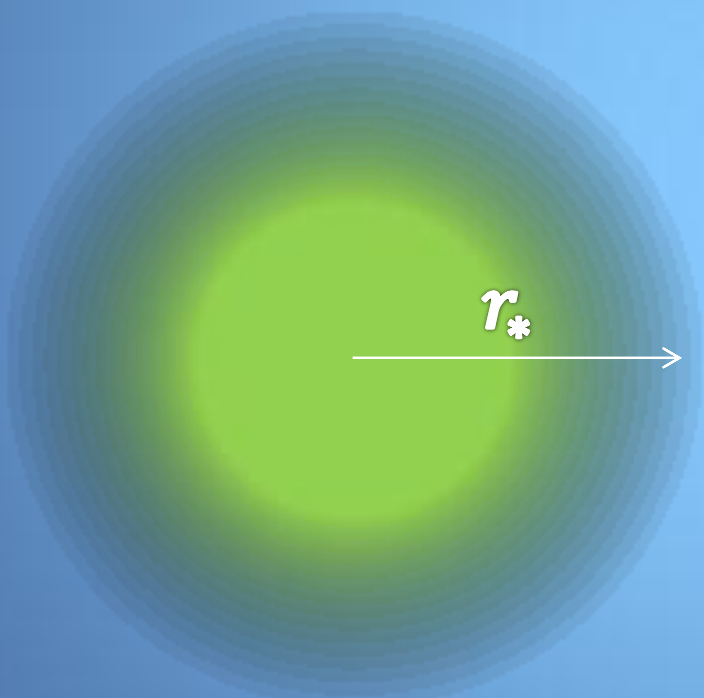

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$$(\partial\phi)^2 \sim \frac{1}{\Lambda^3} \Box\phi (\partial\phi)^2$$

$$r_* \sim \frac{1}{\Lambda} \left(\frac{E}{\Lambda} \right)^{\frac{1}{5}}$$

Region I

S_{weak}

Perturbation theory works

Region II

r_*

S_{strong}

Perturbation theory doesn't work

Self Energy

- Divergent classical and quantum self energies arise with the notion of a point particle.
- Regularization and renormalization help to artificially make the quantum self energy finite.

Self Energy

Goldstone

Region I

$$\boxed{S_I = \int d^4x \quad -\frac{1}{2}(\partial\phi)^2 - \phi J}$$

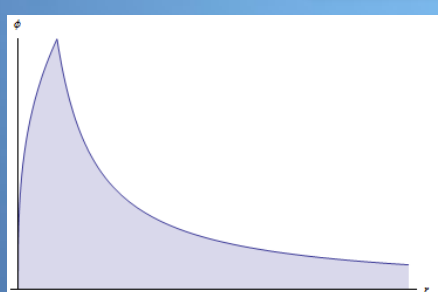
$$\boxed{\phi_I \sim \frac{M}{r}}$$

Region II

$$\boxed{S_{II} = \int d^4x \quad -\frac{1}{\Lambda^4}(\partial\phi)^4 - \phi J}$$

$$\boxed{\phi_{II} \sim (M\Lambda^4)^{\frac{1}{3}} r^{\frac{1}{3}}}$$

$$\boxed{E_{tot} \sim \frac{M^2}{r_*} + (M\Lambda)^{(4/3)} r_*^{1/3}}$$



Finite self energy

Self Energy

Galileon

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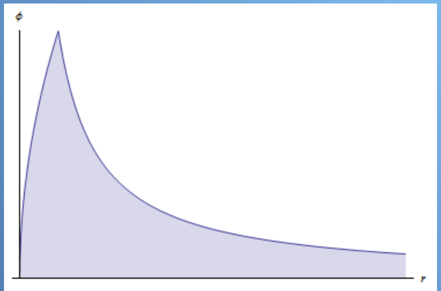
$$E_{tot} \sim \frac{M^2}{r_*} + (M\Lambda)^{(3/2)} r_*^{1/2}$$



Finite self energy

What is a “Classicalon”?

What is a “Classicalon”?

Claim:

Coherent State

$$\boxed{|\,\alpha\rangle \sim e^{\alpha a^\dagger}\,|\,0\rangle}$$

$$\boxed{\Delta x_\alpha \Delta p_\alpha = \frac{\hbar}{2}}$$

Black Holes: coherent state of gravitons

Electric Field: coherent state of photons

Semiclassical Calculation

Lasma, Bezrukov arXiv:1206.5311v1 [hep-th]

$$\boxed{\sigma(E,N)=\sum_f |\langle f \mid P_E P_N \hat{S}\hat{A} \mid 0\rangle|^2}$$

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$$\sigma(E,N)=\sum_f |\langle f \mid P_E P_N \hat{S}\hat{A} \mid 0\rangle|^2$$

$$\sigma(E,N)=\int db^*_{\mathbf{k}}db_{\mathbf{k}}d\xi d\eta \mathcal{D}\phi\mathcal{D}\phi' \exp\Big(-\int d\mathbf{k} b^*_{\mathbf{k}}b_{\mathbf{k}}e^{i\omega_{\mathbf{k}}\xi+i\eta}+iE\xi+iN\eta+B_i(0,\phi_i)+B_f(b^*_{\mathbf{k}},\phi_f)$$
$$+B^*_i(0,\phi'_i)+B^*_f(b_{\mathbf{k}},\phi'_f)+iS[\phi]-iS[\phi']+J\phi(0)+J\phi(0)'\Big)$$

Semiclassical Calculation

Lasma, Bezrukov arXiv:1206.5311v1 [hep-th]

$$\boxed{\sigma(E,N) = \sum_f |\langle f \mid P_E P_N \hat{S}\hat{A} \mid 0\rangle|^2}$$

$$\sigma(E,N) = \int db^*_{\mathbf{k}} db_{\mathbf{k}} d\xi d\eta \mathcal{D}\phi\mathcal{D}\phi' \exp\Big(-\int d\mathbf{k} b^*_{\mathbf{k}} b_{\mathbf{k}} e^{i\omega_{\mathbf{k}}\xi + i\eta} + iE\xi + iN\eta + B_i(0,\phi_i) + B_f(b^*_{\mathbf{k}},\phi_f)$$
$$+B^*_i(0,\phi'_i) + B^*_f(b_{\mathbf{k}},\phi'_f) + iS[\phi] - iS[\phi'] + J\phi(0) + J\phi(0)'\Big)$$

$$\boxed{B_i(0,\phi) = -\frac{1}{2}\int d\mathbf{k}\omega_{\mathbf{k}}\phi_i(\mathbf{k})\phi_i(-\mathbf{k})}$$

$$B_f(b^*_{\mathbf{k}},\phi_f) = -\frac{1}{2}\int d\mathbf{k} b^*_{\mathbf{k}} b^*_{-\mathbf{k}} e^{2i\omega_{\mathbf{k}}T_f} + \int d\mathbf{k}\sqrt{2\omega_k} b^*_{\mathbf{k}}\phi_f(\mathbf{k})e^{i\omega_{\mathbf{k}}T_f} - \frac{1}{2}\int d\mathbf{k}\omega_{\mathbf{k}}\phi_f(\mathbf{k})\phi_f(-\mathbf{k})$$

Saddle Point Approximation

$$\begin{array}{ccc} \phi & \to & \Phi/g \\ \phi' & \to & \Phi'/g \\ J & \to & j/g \\ b & \to & \beta/g \\ b^* & \to & \beta^*/g \\ E & \to & \epsilon/g^2 \\ N & \to & n/g^2 \end{array}$$

$$S(\phi, m, g) \to S(\Phi/g, m, g) = \frac{1}{g^2} s(\Phi, m)$$

g^2 " = " $\hbar$

Saddle Point Approximation

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$$S(\phi, m, g) \to S(\Phi/g, m, g) = \frac{1}{g^2} s(\Phi, m)$$

g^2 "=" $\hbar$

$$\phi_i(\mathbf{k}) = \frac{a^*_{-\mathbf{k}}}{\sqrt{2\omega_\mathbf{k}}} e^{i\omega_\mathbf{k} t}$$

$$\phi_f(\mathbf{k}) = \frac{1}{\sqrt{2\omega_\mathbf{k}}} (b_\mathbf{k} e^{\omega_\mathbf{k} T - \theta - i\omega_\mathbf{k} t} + b^*_{-\mathbf{k}} e^{i\omega_\mathbf{k} t})$$

Saddle Point Approximation

g^2 " = " $\hbar$

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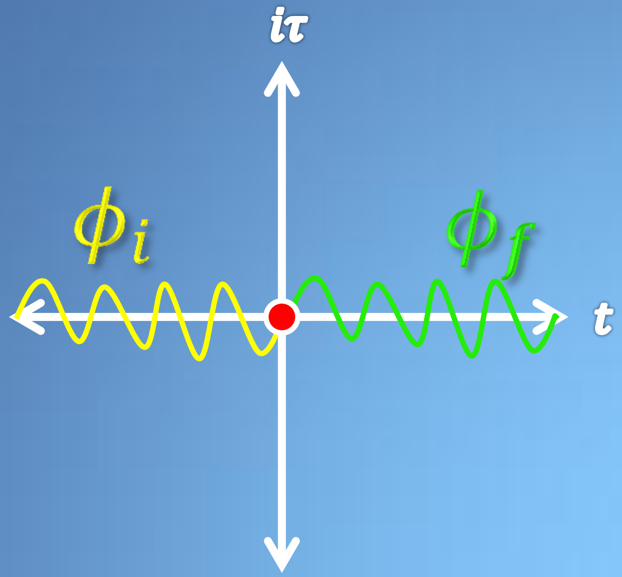
$$\sigma(E, N) \sim e^{-2\Im(S[\phi])/g^2}$$

Son, Nucl.Phys. B477 (1996) 378-406



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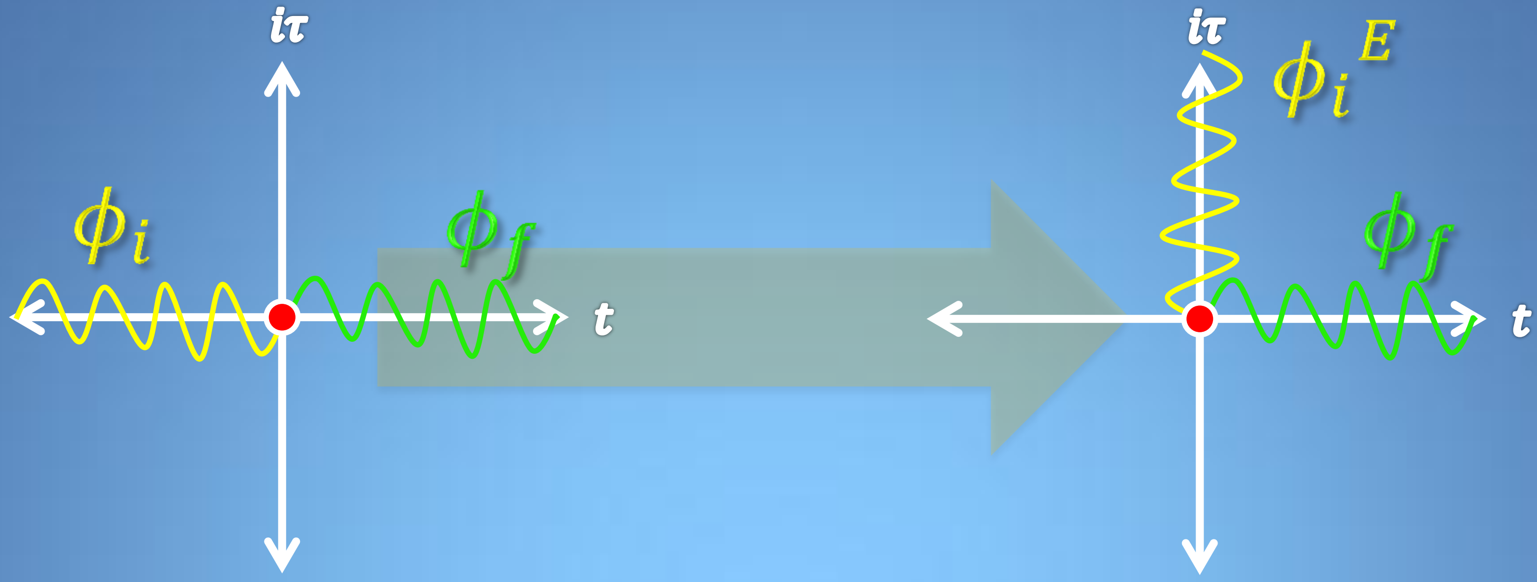

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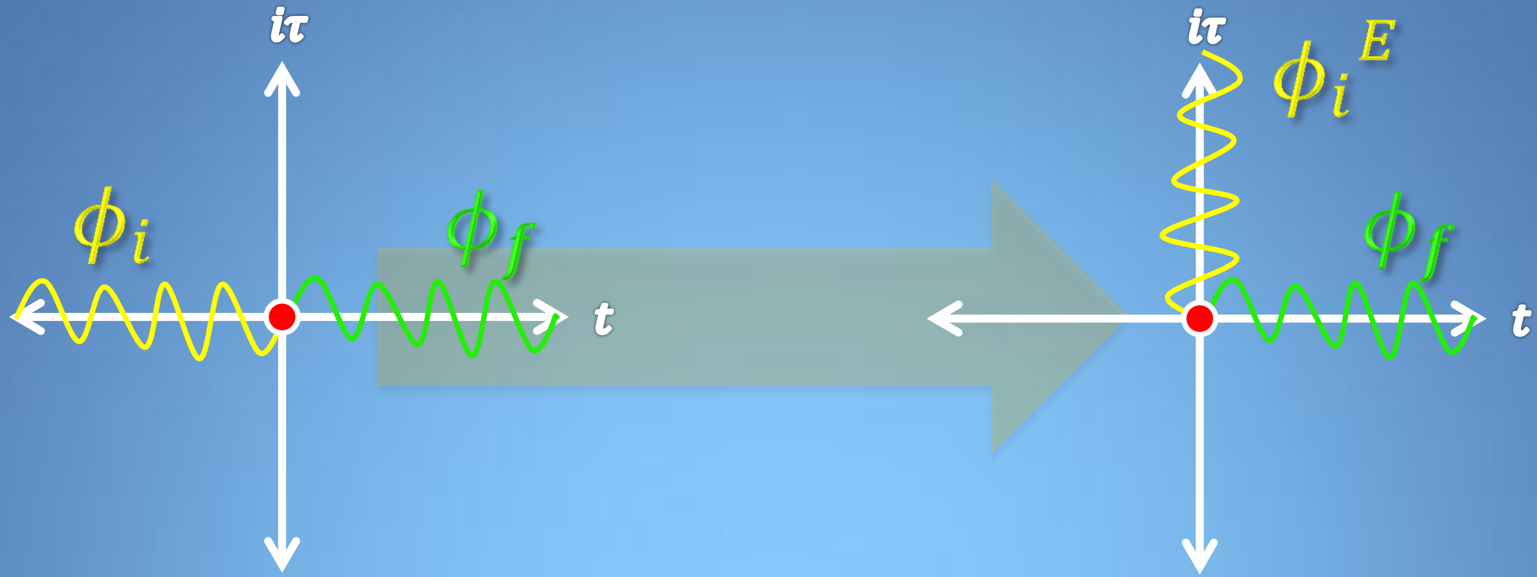

$$\phi_f = \phi_i + \delta\phi$$

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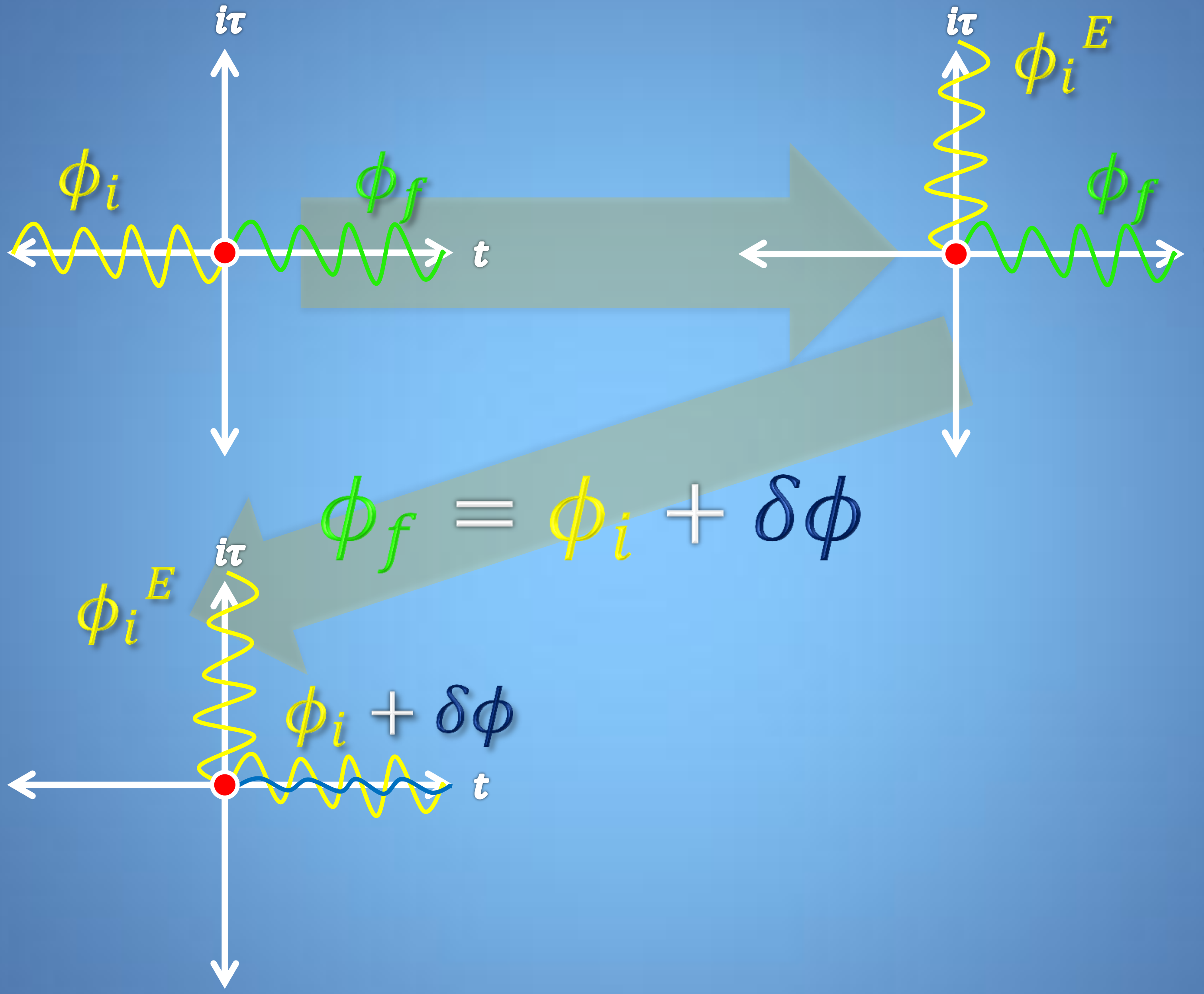

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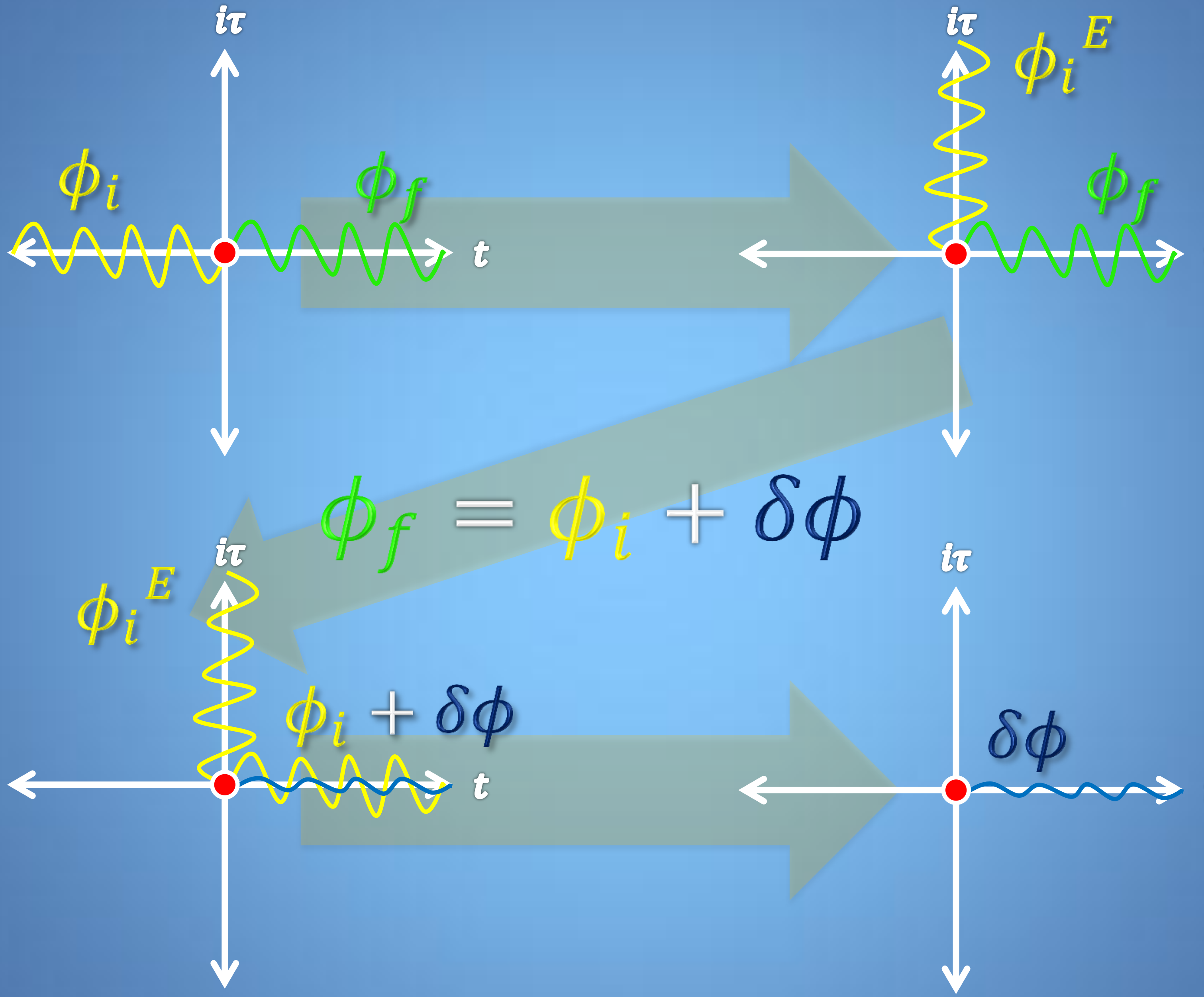


Goldstone

$$\sigma(E, N) = \left(\frac{N_{\text{crit}}}{N}\right)^{3N}$$

$$N_{\text{crit}}^{3} = c^{3}\left(\frac{E}{\Lambda}\right)^{4}$$

Goldstone

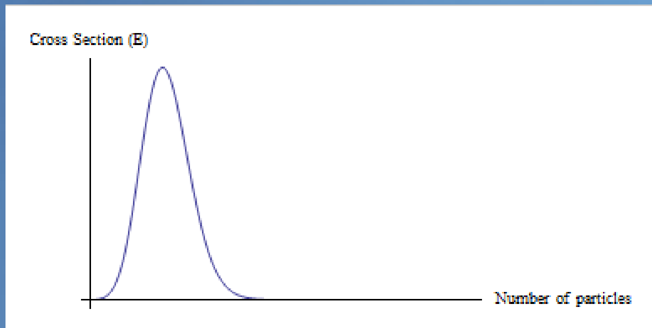
Classical length scale r_* dominates behavior of quantum scattering cross section.

$$\sigma(E,N) = \left(\frac{N_{\rm crit}}{N}\right)^{3N}$$

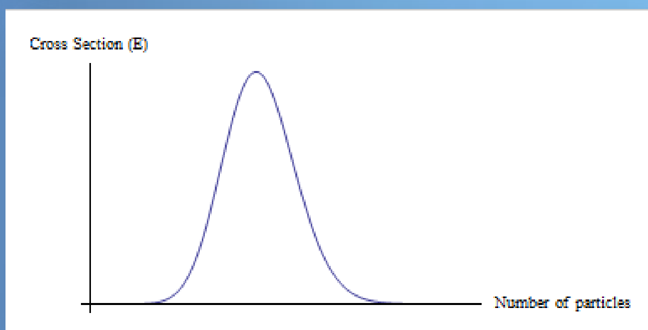
$$N_{\rm crit}^3 = c^3 \left(\frac{E}{\Lambda}\right)^4$$

$$\frac{1}{r_{\rm crit}} = \frac{E}{N_{\rm crit}}$$

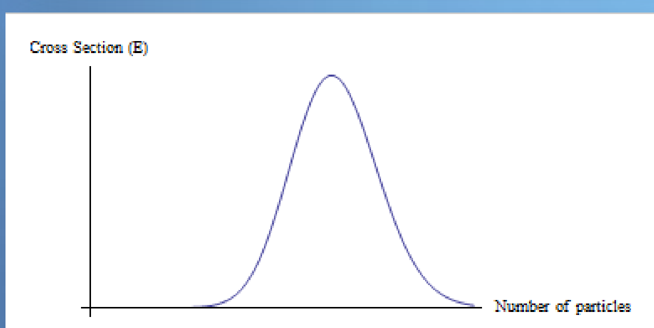
$$r_{\rm crit} = \frac{c}{\Lambda}\left(\frac{E}{\Lambda}\right)^{1/3} = r_*$$



E



E



E

As energy increases, the **preferred cross section** is the one that **has more particles**, each with low energy, as oppose to a few particles with large energy.

Galileon

$$\sigma(E,N) = \left(\frac{N_{\text{crit}}}{N}\right)^{5N}$$

$$N_{\text{crit}}^3 = c^5 \left(\frac{E}{\Lambda}\right)^6$$

$$\frac{1}{r_{\text{crit}}} = \frac{E}{N_{\text{crit}}}$$

$$r_{\text{crit}} = \frac{c}{\Lambda}\left(\frac{E}{\Lambda}\right)^{1/5} = r_*$$

Summary

- r_* phenomenon is important for Classicalization and needs to be taken into account for scattering process. Perturbation theory can't see this effect.
- These models admit positive finite self energy.
- Many soft quanta are preferred over few hard quanta.

Future Work

- Construct path integral formalism to capture classicalization effect.