Gravitational waves from BH-NS binaries: Effective Fisher matrices and parameter estimation using higher-harmonics

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Outline

Motivation

- At leading order, the GW signal from a binary oscillates at twice the orbital frequency. Other harmonics of the orbital frequency enter at higher post-Newtonian (PN) order
- Are higher harmonics/amplitude corrections (ACs) important for parameter estimation, particularly for BH-NS systems?
- This can be addressed with Bayesian inference techniques like MCMC or nested sampling runs, but these are expensive
- We would like to get some insights with simpler, cheaper analytic methods
- The Fisher matrix is a standard approach, but can be perilous

Fisher matrix assumptions

- The Fisher matrix formalism is widely used because it is much cheaper than a full Bayesian analysis
- However, it makes several assumptions that are rather optimistic for ground-based GW data analysis:
  - High SNR
  - Stationary, Gaussian noise
  - Signal depends linearly on all of its parameters
- What could possibly go wrong?

The Fisher matrix

- Those assumptions are meant to justify approximating the likelihood (of an observed signal given some proposed waveform parameters) as a multivariate Gaussian in some neighborhood of the true parameters

$$\mathcal{L} \simeq e^{-\Gamma_{ij}\,\delta\lambda^i\,\delta\lambda^j/2} \qquad \text{where} \qquad \Gamma_{ij} = \left\langle \frac{\partial h}{\partial \lambda_i} \middle| \frac{\partial h}{\partial \lambda_j} \right\rangle$$

  is the Fisher matrix.

- The covariance matrix, the inverse of the Fisher matrix, will estimate the expected errors and correlations of the parameter measurements

$$\begin{aligned} \Sigma &= \Gamma^{-1} \\ \sigma_i &= \sqrt{\Sigma_{ii}} \\ c_{ij} &= \frac{\Sigma_{ij}}{\sqrt{\Sigma_{ii}\Sigma_{jj}}} \end{aligned}$$

Log Likelihood

- The likelihood that $h(\lambda')$ will produce a signal $s = h(\lambda) + n$ is:

$$\mathcal{L}(s|\lambda') = \exp\left[-\frac{1}{2}\langle s - h(\lambda')|s - h(\lambda')\rangle\right]$$

- Expanding s and neglecting the noise contribution to the likelihood, we get:

$$\mathcal{L} = \exp\left[-\frac{1}{2}\Big(\langle h(\lambda)|h(\lambda)\rangle + \langle h(\lambda')|h(\lambda')\rangle - 2\mathrm{Re}\langle h(\lambda)|h(\lambda')\rangle\Big)\right]$$

- Assuming $\rho^2 = \langle h(\lambda)|h(\lambda)\rangle = \langle h(\lambda')|h(\lambda')\rangle$,

$$\ln \mathcal{L} = -\rho^2 \left(1 - \frac{\mathrm{Re}\langle h(\lambda)|h(\lambda')\rangle}{\sqrt{\langle h(\lambda)|h(\lambda)\rangle\langle h(\lambda')|h(\lambda')\rangle}}\right)$$

Ambiguity function

- The ambiguity function, the normalized overlap between different points in the parameter space, is

$$P(\lambda, \lambda') = \frac{\mathrm{Re}\langle h(\lambda) | h(\lambda') \rangle}{\sqrt{\langle h(\lambda) | h(\lambda) \rangle \langle h(\lambda') | h(\lambda') \rangle}}$$

- Computing the ambiguity function allows us to map out the log likelihood, and it can be related to the Fisher matrix

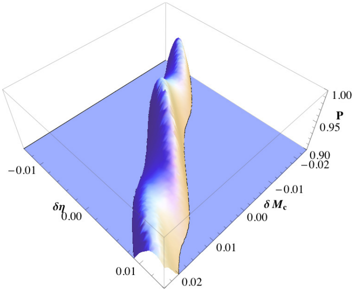
$$\ln \mathcal{L} = -\rho^2 \left(1 - P(\lambda, \lambda')\right) \simeq -\Gamma_{ij}\, \delta\lambda^i\, \delta\lambda^j / 2$$

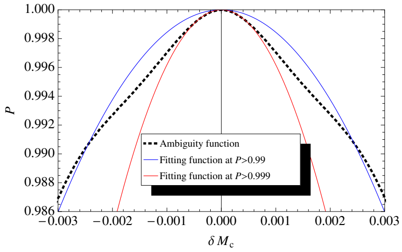
- The (normalized) *effective* Fisher matrix is obtained by fitting the ambiguity function with a multivariate quadratic

$$P_{\mathrm{fit}}(\delta\lambda) = 1 - \hat{\Gamma}^{\mathrm{eff}}_{ij} \delta\lambda^i\, \delta\lambda^j / 2$$

- The range over which you fit is determined by the expected SNR of your signal: $1 - P \leq 1/\rho^2$

Advantages of the Effective Fisher matrix

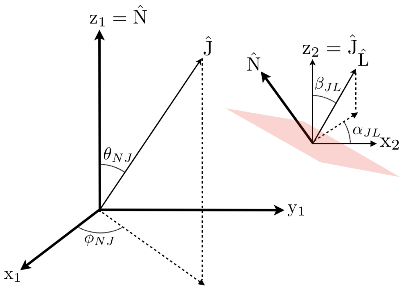




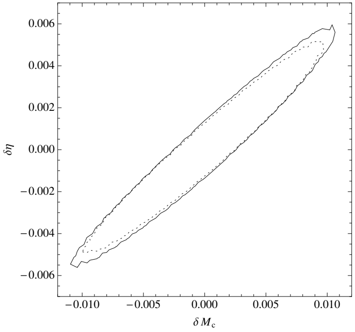
- Plotting the ambiguity function will tell you if the Fisher matrix approximation is at all reasonable
- Fitting to a scale set by the SNR makes it robust against fine-scale structure that is unobservable at that SNR

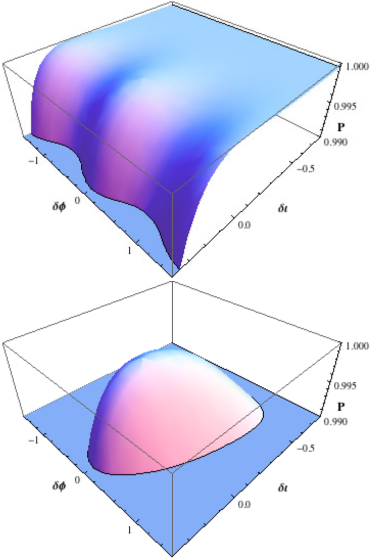
Fiducial Binary

- We study a $(10 + 1.4)\ M_\odot$ BH-NS binary ($M_c = 2.99$, $\eta = 0.1077$) in initial LIGO
- Use precessing PN waveforms (SpinTaylorT4) with 0PN or 1.5PN amplitude
- Non-spinning/Spin-aligned: L has an inclination of $\pi/4$ to line of sight N; BH has spin 0 or 1 along L; NS is non-spinning
- Precessing: J is inclined to line of sight by $\theta_{NJ} = \pi/4$; precession cone has an opening angle $\beta_{JL} = \pi/4$; L is initially either along N or perpendicular to it



Non-spinning results

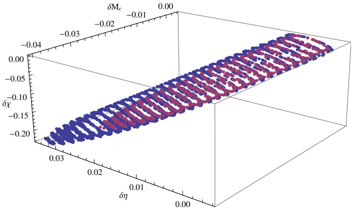




amp. order	0PN	1.5PN
$\Delta M_c/M_c$	.254%	.235%
$\Delta\eta/\eta$	3.83%	3.38%
$\Delta\iota_{LN}$	—	0.30
$\Delta\phi_{\text{ref}}$	—	0.585

Spin-aligned results

- Non-spinning: ACs do not improve measurement of intrinsic parameters
- Spin-aligned: ACs give a modest improvement to intrinsic parameter errors
- In both cases, extrinsic parameters are unmeasurable without amp. cor., and are measureable (but with large errors) with ACs



amp. order	0PN	1.5PN
$\Delta M_c/M_c$	1.08%	.795%
$\Delta\eta/\eta$	18.7%	12.7%
$\Delta\chi$	0.149	0.108
$\Delta\iota_{LN}$	—	0.282
$\Delta\phi_{\rm ref}$	—	0.821

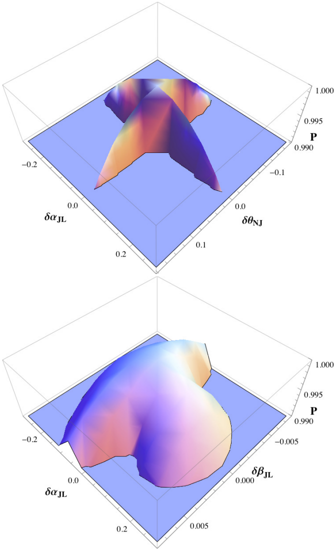
Precessing results

- The errors are smaller for precessing binaries than for spin-aligned
- The gains from ACs become less pronounced, however
- The divide between intrinsic and extrinsic parameters becomes blurred, as angles describing the precession cone correlate to both

amp. order	0PN	1.5PN
$\Delta M_c/M_c$	.208%	.195%
$\Delta\eta/\eta$	6.14%	4.91%
$\Delta\chi$	0.0495	0.0421
$\Delta\beta_{JL}$	0.0241	0.0210
$\Delta\theta_{JN}$	0.117	0.113
$\Delta\alpha_{JL}$	0.187	0.191
$\Delta\phi_{\mathrm{ref}}$	—	—

Caveats

- We see non-quadratic behavior in the precessing case for some orientations (but not others)
- We use a complex inner product s.t. Re $\langle h|h'\rangle = (h_+|h'_+) + (h_\times|h_\times)$ corresponding to an idealized network.
- The results should lie between those of a single-IFO and a real multi-detector network
- We cautiously believe the trends seen here and are working to confirm with MCMC



Conclusions

- The effective Fisher matrix allows one to see whether the Fisher matrix formalism is likely to be valid (and for which parameters)
- It is also robust against small-scale structure that can be problematic for the standard Fisher matrix
- Higher harmonics will likely have little effect on the measurement of intrinsic parameters, but they can improve the measurement of extrinsic parameters
- For non-precessing systems, there is a clear separation of intrinsic and extrinsic parameters, but this becomes blurred for precessing
- For more details, see arXiv:1209.4494
- Full MCMC runs underway to get more precise results and confirm the trends seen here