Unified Field Equations Coupling Four Forces and Theory of Dark Matter and Dark Energy

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http://www.indiana.edu/~fluid

Outline

References:

1. Tian Ma & Shouhong Wang, Gravitational Field Equations and Theory of Dark Matter and Dark Energy, June, 2012, http://arxiv.org/abs/1206.5078

2. Tian Ma & Shouhong Wang, Unified Field Equations Coupling Four Forces and Principle of Interaction Dynamics, September, 2012, http://www.ma.utexas.edu/mp_arc-bin/mpa?yn=12-103

I. Motivations

- Mystery of dark matter and dark energy
- Grand challenges for coupling all four interactions

II. Principle of Interaction Dynamics (PID)

Due to the presence of dark energy and dark matter, the energy-momentum tensor T_{ij} of normal matter is no longer conserved: $D^i(T_{ij}) \neq 0$.

By an orthogonal decomposition theorem, there is a scalar function $\varphi : M \to R$ such that

$$T_{ij} = \widetilde{T}_{ij} - \frac{c^4}{8\pi G} D_i D_j \varphi, \qquad D^i \widetilde{T}_{ij} = 0$$

$$L_{EH} = \int_M \left(R + \frac{8\pi G}{c^4} S \right) \sqrt{-g}\, dx,$$

$$D^i \left[R_{ij} - \frac{1}{2} g_{ij} R \right] = 0 \quad \Longrightarrow \qquad R_{ij} - \frac{1}{2} g_{ij} R + \frac{8\pi G}{c^4} \widetilde{T}_{ij} = 0$$

Namely

(1) $$R_{ij} - \frac{1}{2} g_{ij} R = -\frac{8\pi G}{c^4} T_{ij} + D_i D_j \varphi, \qquad D^i \left(D_i D_j \varphi + \frac{8\pi G}{c^4} T_{ij} \right) = 0$$

Equivalently: The new gravitational field equations (1) can be derived with **constraint least action principle**:

$$\lim_{\lambda \to 0} \frac{1}{\lambda}[L_{EH}(g_{ij} + \lambda X_{ij}) - L_{EH}(g_{ij})] = (\delta L_{EH}(g_{ij}), X) = 0 \quad \forall\, D^i X_{ij} = 0$$

This constraint least action leads us to postulate:

Principle of Interaction Dynamics (PID): *For all physical interactions there are Lagrangian actions*

(2) $$L(g, A, \psi) = \int_M \mathcal{L}(g_{ij}, A, \psi)\sqrt{g}dx,$$

where A is a set of vector fields representing the gauge potentials, and ψ are the wave functions of particles. The states (g, A, ψ) are the extremum points of (2) with the $(D + A)$-free constraint.

III. Unified Field Equations Coupling Four Forces

Lagrangian action functional L is the sum of the Einstein-Hilbert $\mathcal{L}_{EH}$, the GWS $\mathcal{L}_{GWS}$ without Higgs field terms, and the standard QCD $\mathcal{L}_{QCD}$.

(3)
$$L = \int \left[\mathcal{L}_{EH} + \mathcal{L}_{GWS} + \mathcal{L}_{QCD}\right] \sqrt{-g}dx$$

$$\mathcal{L}_{EH} = R + \frac{8\pi G}{c^4}S,$$

$$\begin{aligned}\mathcal{L}_{GWS} = & -\frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ & + \sum_{k=1}^{6}\left\{\bar{L}_k(i\gamma^\mu\widetilde{D}_\mu - m_k)L_k + \bar{\psi}^R_k(i\gamma^\mu\widetilde{D}_\mu - m^l_k)\psi^R_k\right\},\end{aligned}$$

$$\mathcal{L}_{QCD} = -\frac{1}{4}F^b_{\mu\nu}F^{b\mu\nu} + \sum_{k=1}^{6}\bar{q}_k(i\gamma^\mu\widetilde{D}_\mu - m^q_k)q_k$$

$$
\begin{aligned}
W^a_{\mu\nu} &= D_\mu W^a_\nu - D_\nu W^a_\mu + g_1 f^{abc} W^b_\mu W^c_\nu, \\
F_{\mu\nu} &= D_\mu A_\nu - D_\nu A_\mu, \\
F^b_{\mu\nu} &= D_\mu B^b_\nu - D_\nu B^b_\mu + g_2 g^{bcd} B^c_\mu B^d_\nu, \\
\widetilde{D}_\mu L_k &= (\nabla_\mu - i\frac{g_1}{2} W^a_\mu \sigma_a + i\frac{e}{2} A_\mu) L_k, \\
\widetilde{D}_\mu \psi^R_k &= (\nabla_\mu + ieA_\mu)\psi^R_k, \\
\widetilde{D}_\mu q_k &= (\nabla_\mu + i\frac{g_2}{2} B^b_\mu \lambda_b) q_k,
\end{aligned} \tag{4}
$$

where g_1 and g_2 are constants, f^{abc} and g^{bcd} are the structure constants of $SU(2)$ and $SU(3)$, σ_a are the Pauli matrices, λ_b are the Gell-Mann matrices, $L_k = (\psi_{\nu_k}, \psi^L_k)^t$ are the wave functions of left-hand lepton and quark pairs (each has 3 generations), ψ^R_k are the right-hand leptons and quarks, $q_k = (q_{k1}, q_{k2}, q_{k3})^t$ is the k-th flavored quark, and ∇_μ is the Lorentz Vierbein covariant derivative.

- Thanks to the div_A-free constraint, the Euler-Lagrangian of the action functional is balanced by gradient fields, resulting the unified field equations (5) coupling all four forces.

- The gradient fields break spontaneously the gauge-symmetries. This gives rise to a complete new mechanism for spontaneously breaking gauge-symmetries and for energy and mass generation, which provides similar outcomes as the Higgs mechanism.

Unified Field Equations Coupling Four Forces:

(5)
$$
\begin{aligned}
& R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \frac{8\pi G}{c^4} T_{\mu\nu} = (D_\mu + \alpha_0 A_\mu + \alpha_{Wa} W^a_\mu + \alpha_{Bj} B^j_\mu) \Phi_\nu, \\
& \partial^\mu (\partial_\mu A_\nu - \partial_\nu A_\mu) + ie \sum_{k=1}^{6} \left[\frac{1}{2} \bar{L}_k \gamma^\mu L_k + \bar{\psi}^R_k \gamma^\mu \psi^R_k \right] \\
& \qquad = (D_\nu + \beta_0 A_\nu + \beta_{Wa} W^a_\nu + \beta_{Bj} B^j_\nu) \phi^0, \\
& \partial^\mu W^a_{\mu\nu} - \frac{g_1}{2} \sum_{b=1}^{3} f^{bca} g^{\mu\alpha} W^a_{\mu\nu} W^c_\alpha + i \frac{g_1}{2} \sum_{k=1}^{6} \bar{L}_k \gamma^\nu \sigma_a L_k \\
& \qquad = (D_\nu + \kappa^a_0 A_\nu + \kappa^a_{Wb} W^b_\nu + \kappa^a_{Bj} B^j_\nu) \phi^a_W, \\
& \partial^\mu F^b_{\mu\nu} - \frac{g_2}{2} \sum_{k=1}^{8} g^{klb} g^{\mu\alpha} B^b_{\mu\nu} B^l_\alpha + i \frac{g_2}{2} \sum_{k=1}^{6} \bar{q}_k \gamma^\nu \lambda_b q_k \\
& \qquad = (D_\nu + \theta^b_0 A_\nu + \theta^b_{Wl} W^l_\nu + \theta^b_{Bl} B^l_\nu) \phi^b_B, \\
& (i\gamma^\mu \widetilde{D}_\mu - m_k) L_k = 0 \qquad \text{for } k = 1, 2, 3, \\
& (i\gamma^\mu \widetilde{D}_\mu - m^l_k) \psi^R_k = 0 \qquad \text{for } k = 1, 2, 3, \\
& (i\gamma^\mu \widetilde{D}_\mu - m^q_k) q_k = 0 \qquad \text{for } k = 1, \cdots, 8.
\end{aligned}
$$

$$T_{\mu\nu} = \frac{\delta S}{\delta g_{ij}} + \frac{c^4}{16\pi G} g^{\alpha\beta} \left[W^a_{\alpha\mu} W^a_{\beta\nu} + F_{\alpha\mu} F_{\beta\nu} + F^b_{\alpha\mu} F^b_{\beta\nu} \right] - \frac{c^4}{16\pi G} g_{\mu\nu} (\mathcal{L}_{GWS} + \mathcal{L}_{QCD}).$$

IV. Duality

The unified field equations (5) coupling all four forces induce a **natural duality**:

$$
(6) \qquad \begin{aligned}
&\{g_{\mu\nu}\} && \longleftrightarrow \Phi_\mu, \\
&A_\mu && \longleftrightarrow \phi^0, \\
&W_\mu^a && \longleftrightarrow \phi_W^a \quad \text{for } a = 1, 2, 3, \\
&B_\mu^b && \longleftrightarrow \phi_B^b \quad \text{for } b = 1, \cdots, 8.
\end{aligned}
$$

Equivalently the graviton g, the photon γ, vector bosons $W^\pm$ and Z, and the gluons g_b, represented by the left-hand side of the above duality

corresponds to

the bosonic fields: vector boson Φ_μ (massless with spin 1), and scalar bosons ϕ^0 (massless with spin 0), ϕ_W^a, ϕ_B^b with spin 0.

Zero-point energy:

- The field ϕ^0, adjoint to the electromagnetic potential A_μ, is a massless field with spin $s = 0$, and it needs to be confirmed experimentally.
- We think it is this particle field ϕ^0 that causes the vacuum fluctuation or zero-point energy.

Eightfold Way mesons: We conjecture that the combination of the eight scalar bosons ϕ_B^b corresponding to gluons g_b $(1 \leq b \leq 8)$ leads to the Eightfold Way mesons: π^+, π^-, π^0, K^+, K^-, K^0, $\bar{K}^0, \eta^0$.

V. Electroweak Theory

- **Higgs Bosons:** The combination of the three Higgs bosonic fields on the right hand side of the duality induces three Higgs boson particles given by

$$\phi^{\pm} = \frac{1}{\sqrt{2}}(\phi^1_W \pm i\phi^2_W), \qquad \phi^3_W$$

  with $\phi^{\pm}$ having $\pm$ electric charges, and with ϕ^3_W being neutral.

- Note that the classical Weinberg-Salam theory induces one neutral Higgs boson particle. All three Higgs bosons deduced here possess masses, generated by the new mechanism.

- In fact, consider only the electromagnetic and weak interactions and ignore the effect of other interactions, we derive a totally different electroweak theory.

  Again this electroweak theory produces the three vector bosons $W^{\pm}$ and Z, as well as the three Higgs bosons.

  The spontaneous gauge-symmetry breaking is achieved by the constraint action without Higgs terms in the action functional.

VI Unified Theory of Dark Energy and Dark Matter

New gravitational vector bosonic field:

- The new vector particle field Φ_μ is massless with spin $s = 1$.
- This particle field corresponds to the scalar potential field φ in (1), caused by the non-uniform distribution of matter in the universe.
- The interaction between this particle field Φ_μ and the graviton leads to a unified theory of dark matter and dark energy and explains the acceleration of expanding universe.

Consider gravity field alone:

$$R_{ij} - \frac{1}{2}g_{ij}R = -\frac{8\pi G}{c^4}T_{ij} - D_i D_j \varphi, \qquad D^i\left(\frac{8\pi G}{c^4}T_{ij} + D_i D_j \varphi\right) = 0$$

$$R = \frac{8\pi G}{c^4}T + \Phi, \qquad \int_M \Phi\sqrt{-g}dx = 0$$

Consider a spherically symmetric central matter field with mass M and radius r_0. With the new field equations, we derive that the force exerted on an object with mass m is given by

$$(7) \quad F = mMG\left[-\frac{1}{r^2} - \frac{c^2}{2GM}\left(2 + \frac{2GM}{c^2}\frac{1}{r}\right)\frac{d\varphi}{dr} + \frac{c^2}{2GM}\Phi\ r\right], \qquad \text{for } r > r_0.$$

The first term is the classical Newton gravitation, the second term is the coupling interaction between matter and the scalar potential φ, and the third term is the interaction generated by the scalar potential energy density Φ ($R = \Phi$ for $r > r_0$).

The sum $\varepsilon = \varepsilon_1 + \varepsilon_2$ of the scalar potential energy density

$$\varepsilon_1 = \frac{c^4}{8\pi G}\Phi$$

and the coupling interaction energy between the matter field and the scalar potential field φ

$$\varepsilon_2 = -\frac{c^4}{8\pi G}\left(\frac{2}{r} + \frac{2MG}{c^2 r^2}\right)\frac{d\varphi}{dr},$$

unifies dark matter and dark energy:

$\varepsilon > 0$	represents dark energy,
$\varepsilon < 0$	represents dark matter.

The interaction force formula (7) can be further simplified to derive the following approximate formula for $r_0 < r < r_1 \approx 10^{21} - 10^{22} km$:

$$F = mMG \left[-\frac{1}{r^2} - \frac{k_0}{r} + k_1 r \right], \tag{8}$$

where $k_0 = 4 \times 10^{-18} km^{-1}$ and $k_1 = 10^{-57} km^{-3}$, which are estimated using rotation curves of galactic motion.

Again, in (8), the first term represents the Newton gravitation, the attracting second term stands for dark matter and the repelling third term is the dark energy.