Generalized Slow Roll and CMB Constraints on the Inflaton Potential

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References:

- **C. Dvorkin and W. Hu, astro-ph/0910.2237, PRD (2009).**
- **M. Mortonson, C. Dvorkin, H.V.Peiris, W.Hu, astro-ph/0903.4920 , PRD (2009).**
- **C. Dvorkin and W. Hu (2010, in preparation).**

Ordinary slow roll (brief background)

Technique for computing the initial curvature power spectrum for inflationary models where the scalar field potential is sufficiently flat and slowly varying.

$$\epsilon_H \equiv \frac{1}{2}\left(\frac{\dot{\phi}}{H}\right)^2$$

$$\eta_H \equiv -\left(\frac{\ddot{\phi}}{H\dot{\phi}}\right)$$

$$\delta_2 \equiv \frac{\dddot{\phi}}{H^2\dot{\phi}}$$

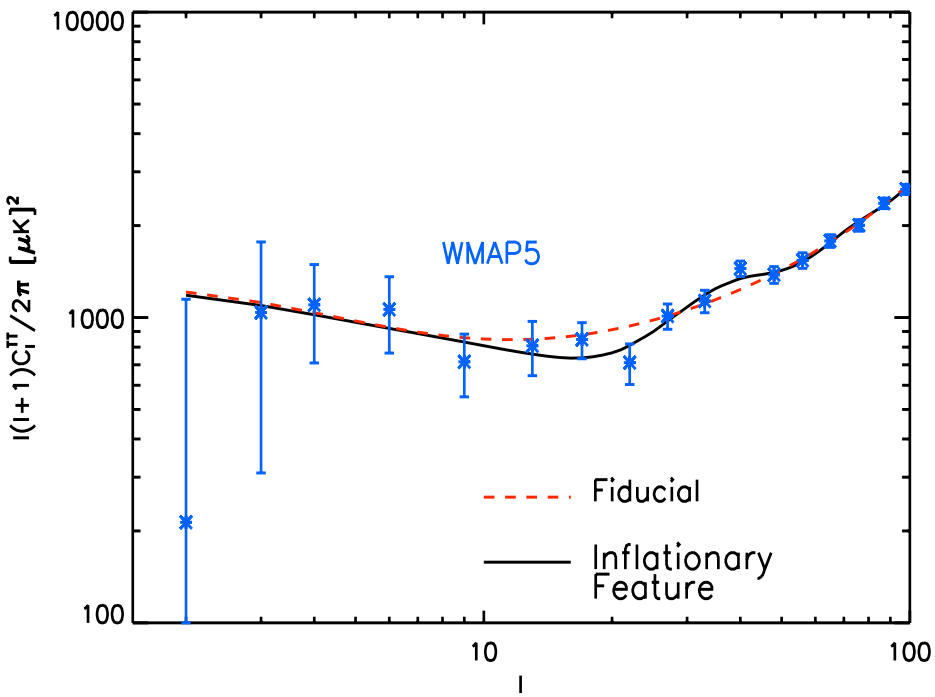
Linked to the **shape of the potential**

Slow-roll parameters:

<< 1 and slowly varying

Slow roll approximation: $\Delta^2_{\mathcal{R}} \approx \left[\left(1-(2C+1)\epsilon_H - C\eta_H\right)\frac{H^2}{2\pi|\dot{\phi}|}\right]^2_{k\eta\approx 1}$

Why go further?



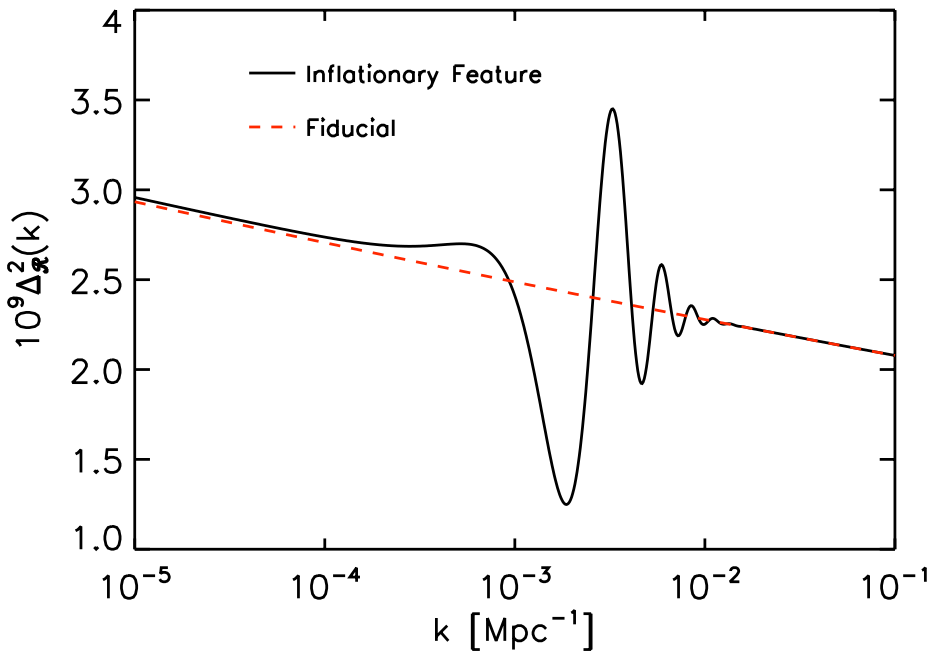
- Glitches in the temperature power spectrum of the CMB have led to interest in exploring models with features in the inflaton potential.

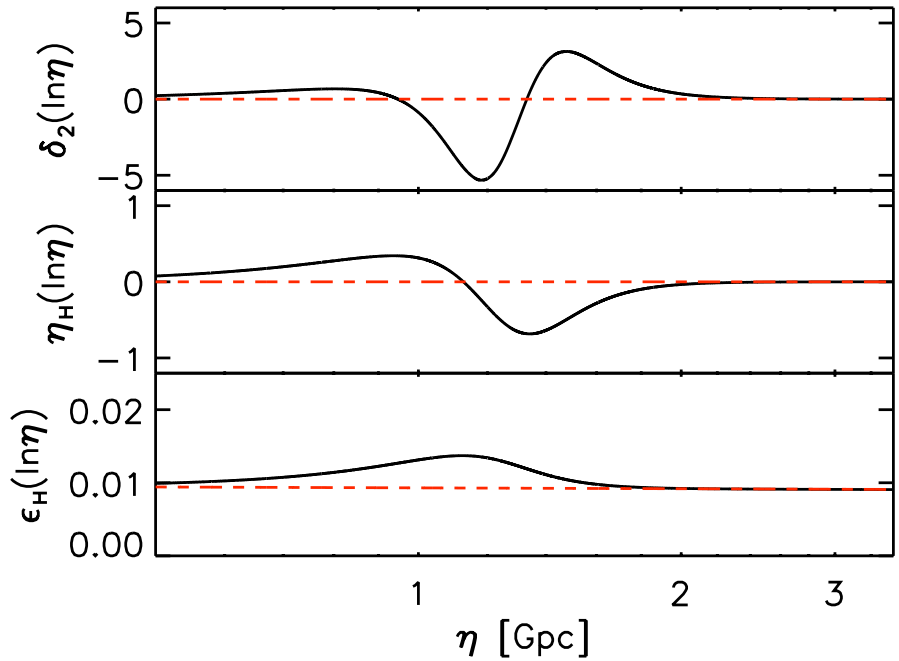
  - *L. Covi, J.Hamann, A. Melchiorri, A. Slozar and I. Sorbera, (2006).*
  - ***M. Mortonson, C. Dvorkin, H.V. Peiris and W. Hu, PRD (2009).***

- These models require order unity variations in the curvature power spectrum: slow-roll parameters are not necessarily small or slowly varying.

Inflationary features

A step in the inflationary potential generates an oscillatory initial curvature power spectrum.

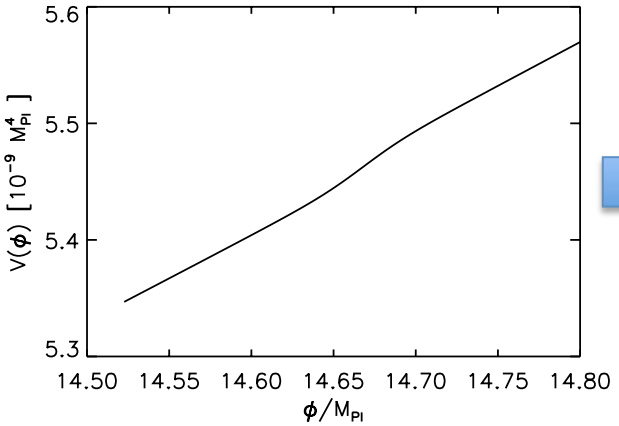




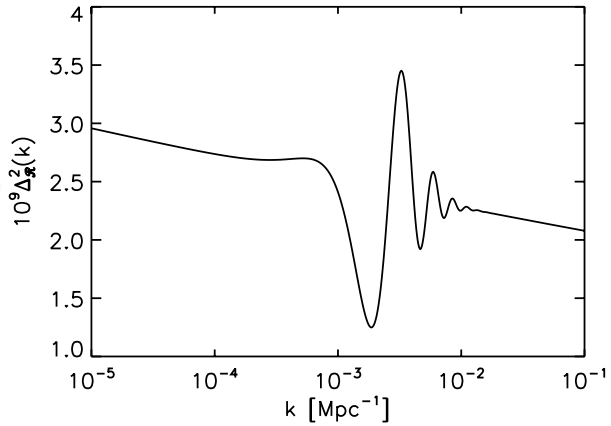
C. Dvorkin, W.Hu, PRD (2009)

The slow-roll parameters are *neither small nor slowly varying*.

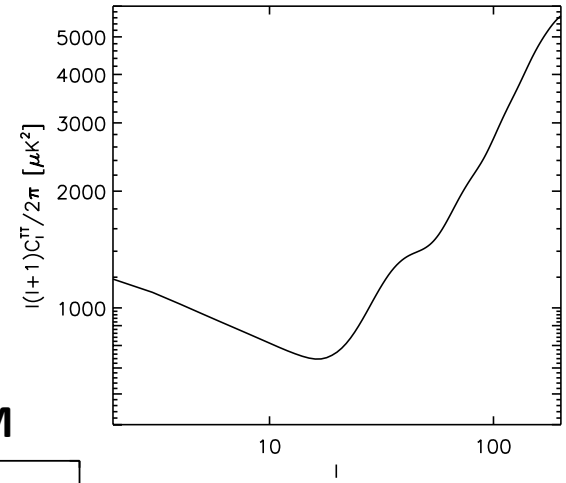
INFLATIONARY POTENTIAL



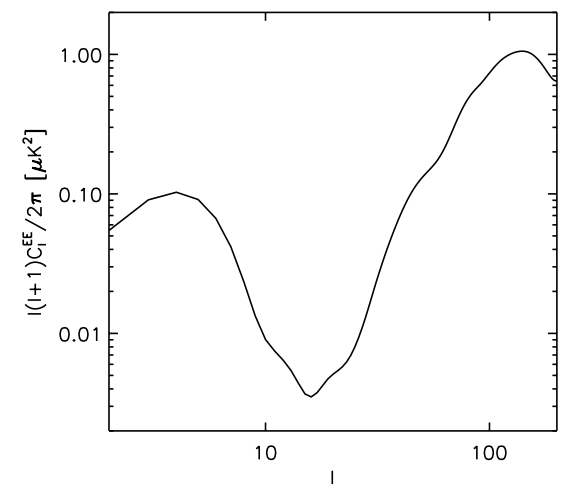
POWER SPECTRUM

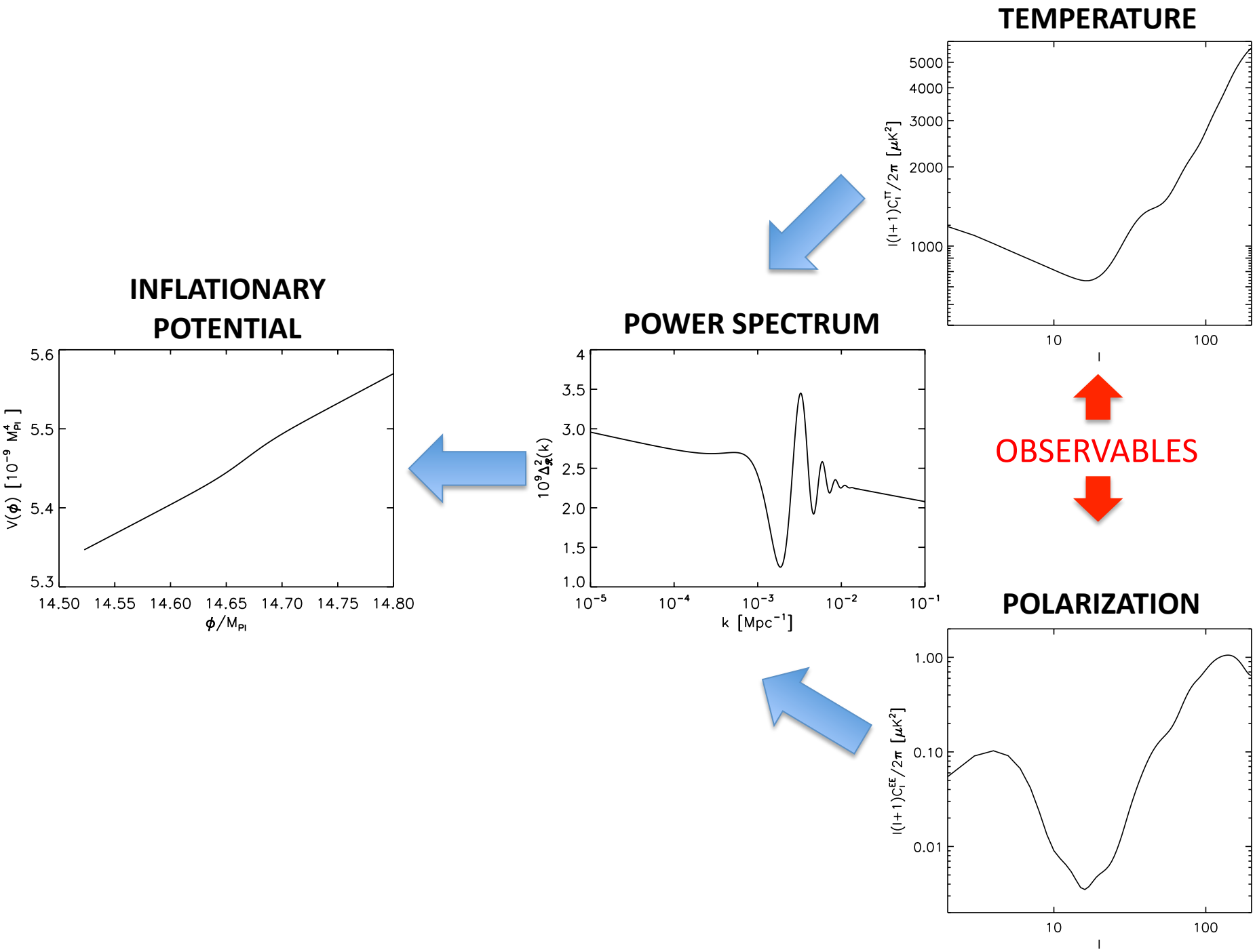


TEMPERATURE



POLARIZATION




TEMPERATURE
5000
4000
3000
2000
1000
l(l+1)C_l^TT/2π [μK²]
10
100
l
OBSERVABLES
POLARIZATION
1.00
0.10
0.01
l(l+1)C_l^EE/2π [μK²]
10
100
l
POWER SPECTRUM
4
3.5
3.0
2.5
2.0
1.5
1.0
10⁹Δ²_ζ(k)
10⁻⁵
10⁻⁴
10⁻³
10⁻²
10⁻¹
k [Mpc⁻¹]
INFLATIONARY POTENTIAL
5.6
5.5
5.4
5.3
V(φ) [10⁻⁹ M_Pl⁴]
14.50
14.55
14.60
14.65
14.70
14.75
14.80
φ/M_Pl

How does the Generalized Slow Roll approximation work?

E. Stewart, PRD (2002).

- Field equation: $$\frac{d^2y}{dx^2} + \left(1 - \frac{2}{x^2}\right) y = \frac{g(\ln x)}{x^2} y$$

  $(y = \sqrt{2k} u_k;\ x = k\eta)$

- Perfect slow roll: $$\frac{d^2y_0}{dx^2} + \left(1 - \frac{2}{x^2}\right) y_0 = 0$$

- GSR approximation: $$\frac{d^2y}{dx^2} + \left(1 - \frac{2}{x^2}\right) y = \frac{g(\ln x)}{x^2} y_0$$

Source function (linear in slow-roll parameters)

Solution can be constructed with Green function technique.

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Solution can be constructed with Green function technique.

BUT…

- Nodes in the power spectrum.
- Curvature is not constant for modes outside the horizon.

Generalized Slow Roll for Large Deviations

The curvature power spectrum only depends on a single source function:

$$\ln \Delta^2_{\mathcal{R}}(k) = G(\ln \eta_{\rm min}) + \int_{\eta_{\rm min}}^{\eta_{\rm max}} \frac{d\eta}{\eta} W(k\eta) G'(\ln \eta) + \ln \left[1 + \frac{1}{2} \left(\int_{\eta_{\rm min}}^{\eta_{\rm max}} \frac{d\eta}{\eta} X(k\eta) G'(\ln \eta) \right)^2 \right]$$

C. Dvorkin, W. Hu, PRD (2009)

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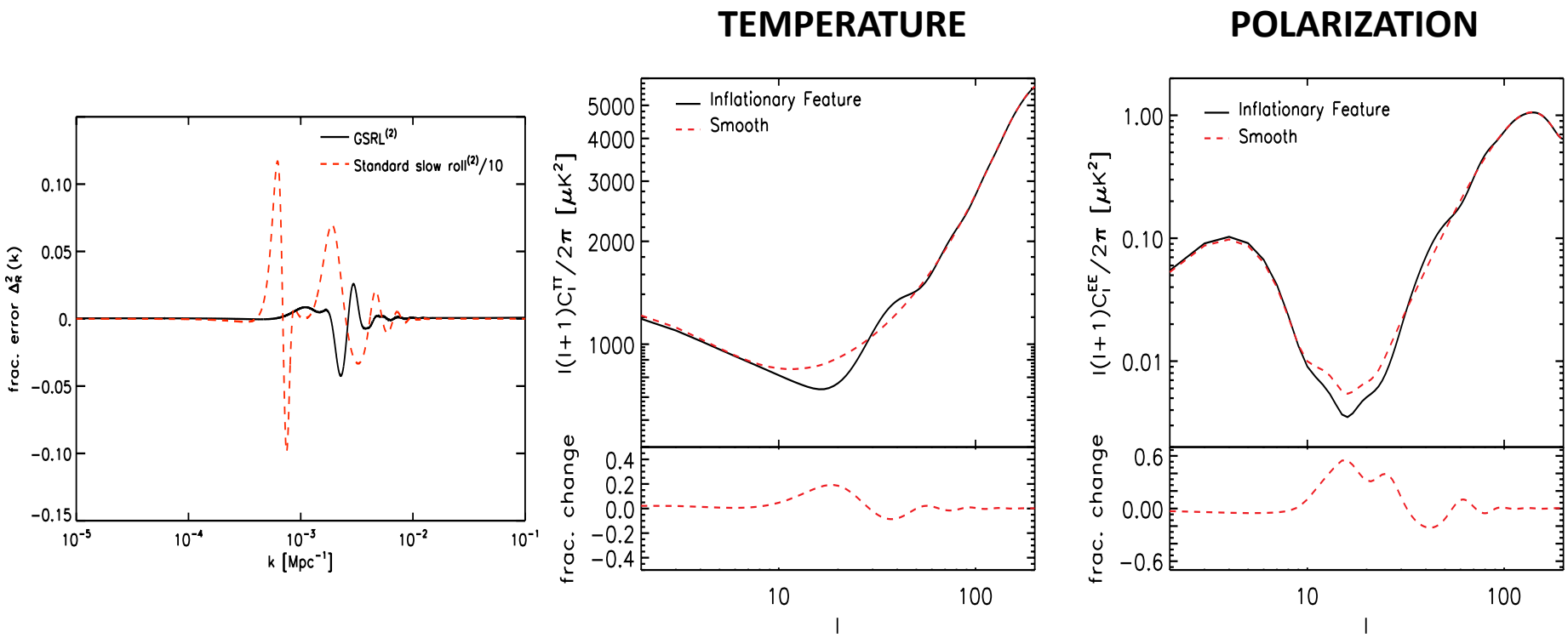
C. Dvorkin, W. Hu, PRD (2009)

- ✓ Constant curvature for modes outside the horizon.
- ✓ Well controlled at large values of the source: percent level errors. (time integrals are small even if the source is not small everywhere).
- ✓ Simple to relate to the Inflaton Potential: $G' \approx 3 \left(\frac{V_{,\phi}}{V} \right)^2 - 2 \left(\frac{V_{,\phi\phi}}{V} \right)$

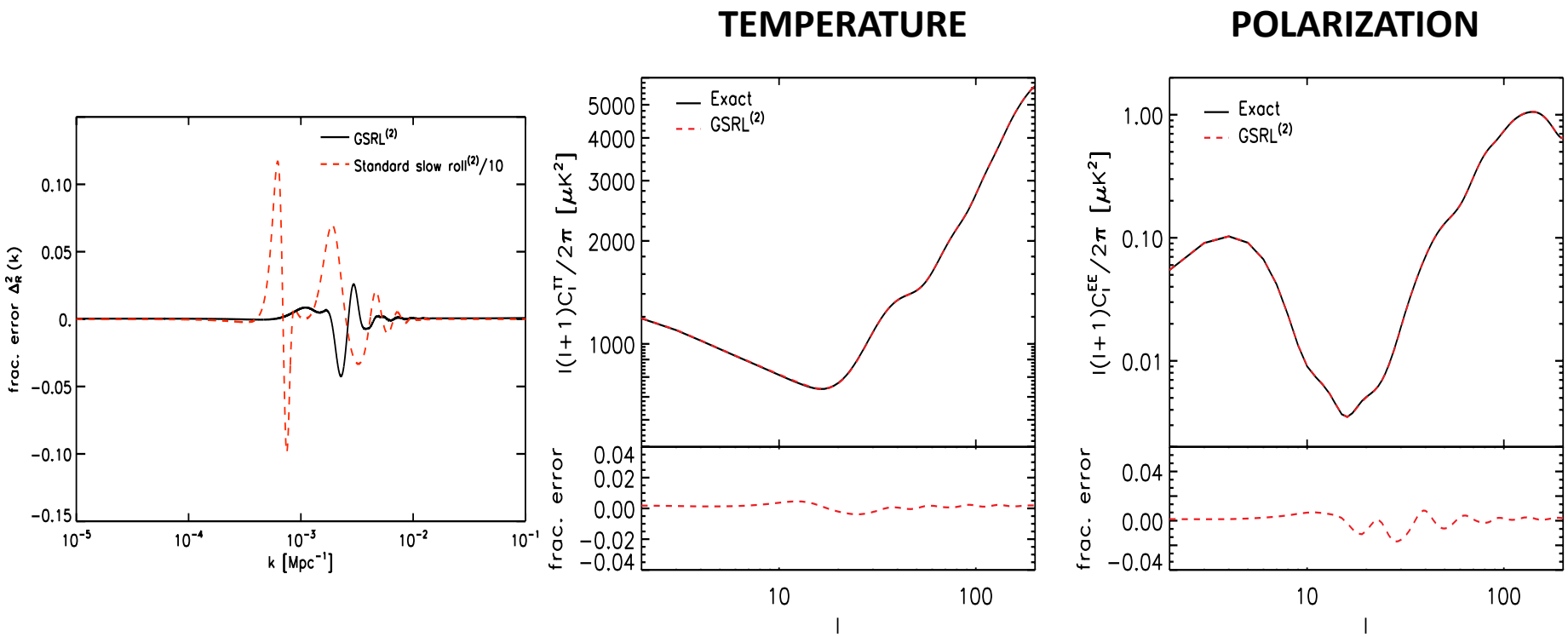
Second order Generalized Slow Roll: Well controlled



M.Mortonson, C. Dvorkin, H.V.Peiris, W.Hu, PRD (2009)

Polarization can test the feature hypothesis
at ***2.5 sigma with Planck, 5-6 sigma with CMBpol****.*

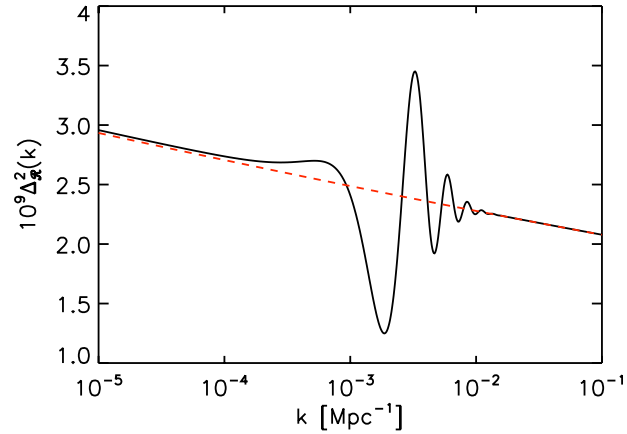
Second order Generalized Slow Roll: Well controlled



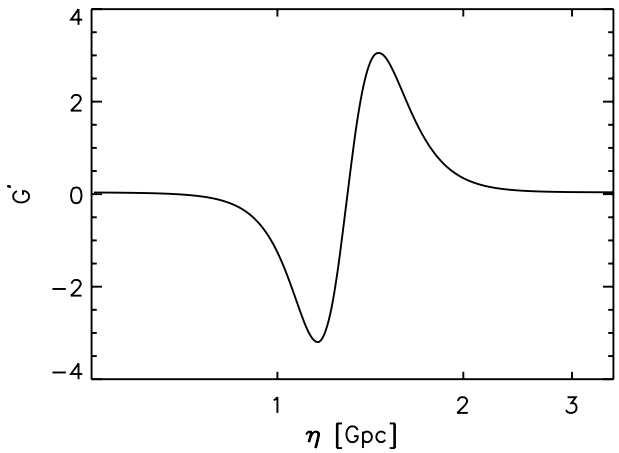
C. Dvorkin, W. Hu, PRD (2009)

We can use features in the power spectrum to directly constrain features in the potential.

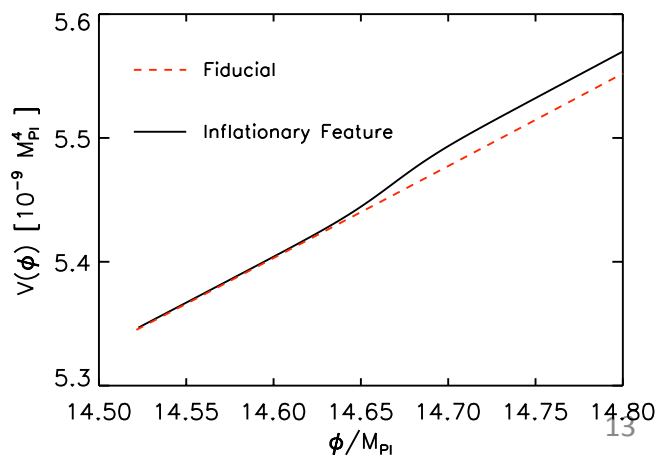
- Power spectrum



- Source



- Potential

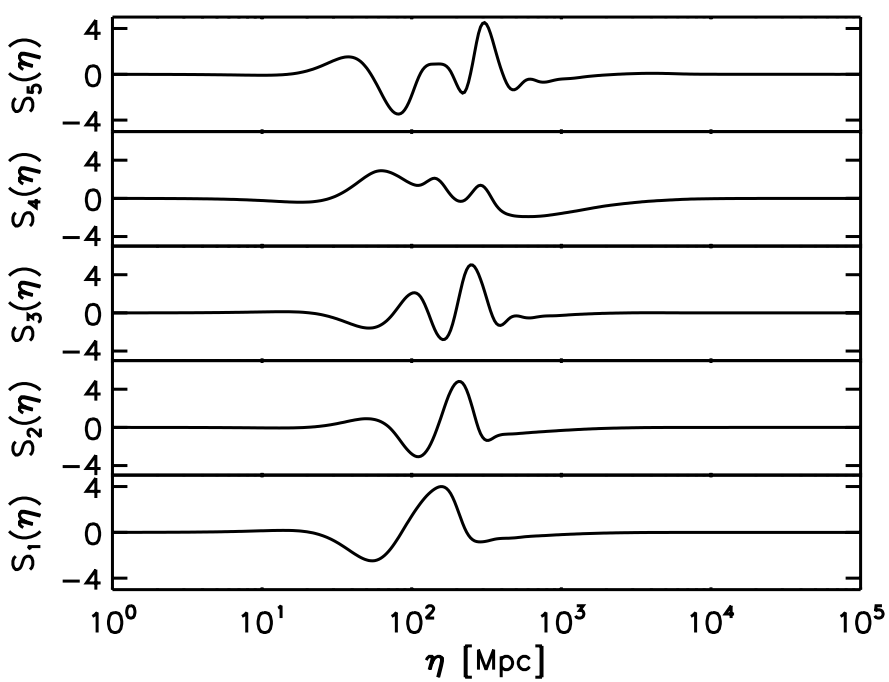


Model independent constraints (PC's) (Work in progress)

Principal components: Basis for a complete representation of observable properties of the source function.

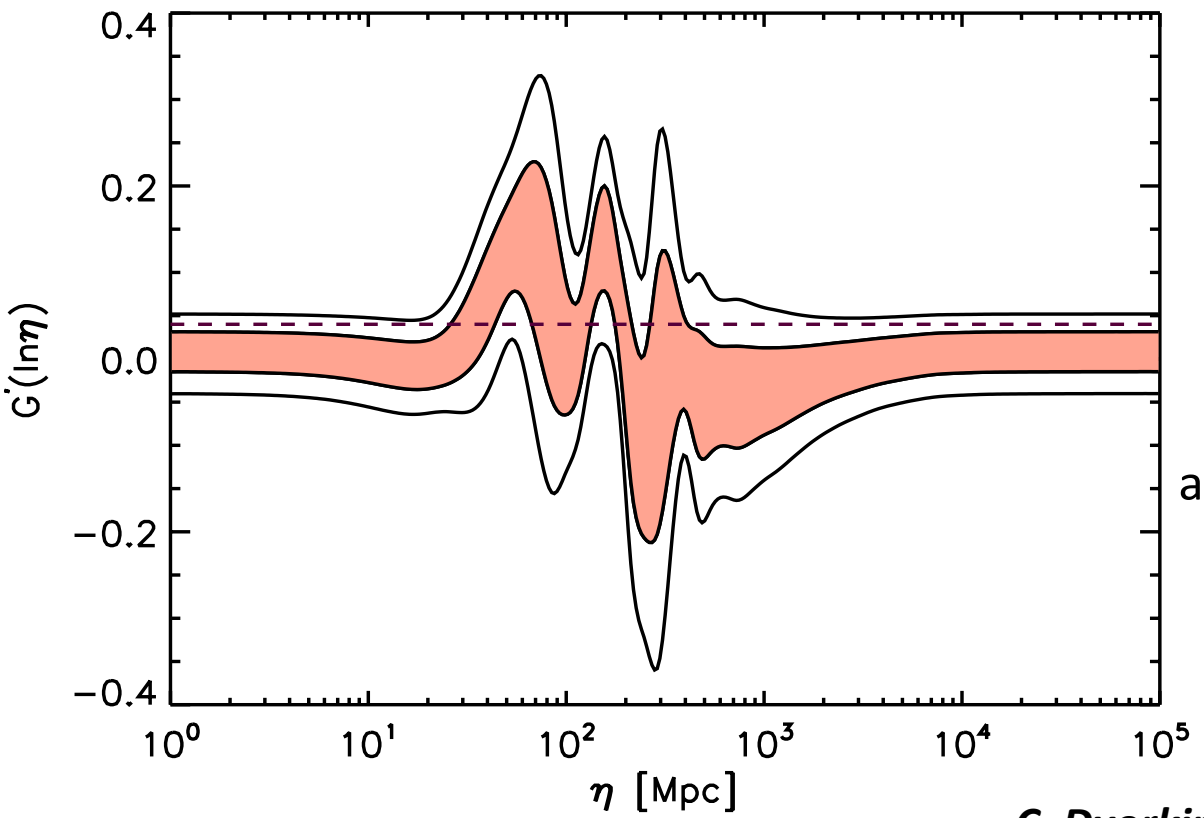
$$G' = 1 - n_s + \sum_{a=1}^{N} m_a S_a(\ln \eta)$$

C. Dvorkin, W. Hu (2010, in preparation)



GSR approximation allows us to go *beyond specific inflationary models* and think of the data as directly constraining the source function.

WMAP7 constraints from MCMC's (Work in progress)



As a by-product:
We parallelized the
WMAP likelihood evaluation
for multi-core systems
and we improved its **speed** by
~ 5*Ncore

C. Dvorkin, W. Hu (2010, in preparation)

Summary

- The Generalized Slow Roll approximation is accurate at the percent level for order unity deviations in the CMB temperature and polarization power spectra.
- There is a single source function responsible for observed features and it is simply related to the local slope and curvature of the Inflaton Potential.

- Applications (work in progress):

➢ Use this technique for model independent constraints on the Inflaton Potential: think of the data as directly constraining the source function.