O(NlogN) Curved-sky Weak Lensing Simulations

Matthew R. Becker and Andrey V. Kravtsov
University of Chicago

Why should you care about weak lensing (and simulating it)?

Weak lensing measurements are an essential component of all current and upcoming large-area optical imaging surveys (e.g. DES, LSST, JDEM, etc.) aimed (at least partially) at understanding dark energy.

Specifically, weak lensing will be used for both cosmic shear measurements and to measure the masses of galaxies and clusters of galaxies.

Unfortunately, there are a number of systematic effects associated with the interpretation of weak lensing signals.

Large-area simulations of weak lensing applied to mock galaxy catalogs and based on N-body structure formation models are needed to address these issues.

Multiple-plane Ray Tracing

Pseudocode for a multiple-plane ray tracing algorithm:

```
initialize ray locations at the observer
FOR i=1,# of lens planes DO
   compute 2D mass density of the lens plane
   solve 2D Poisson equation for the lensing potential
   compute ray deflections and lensing Jacobian at ray positions from the
   lensing potential
   using ray deflections and the lensing Jacobian advance rays to next lens
   plane
ENDFOR
```

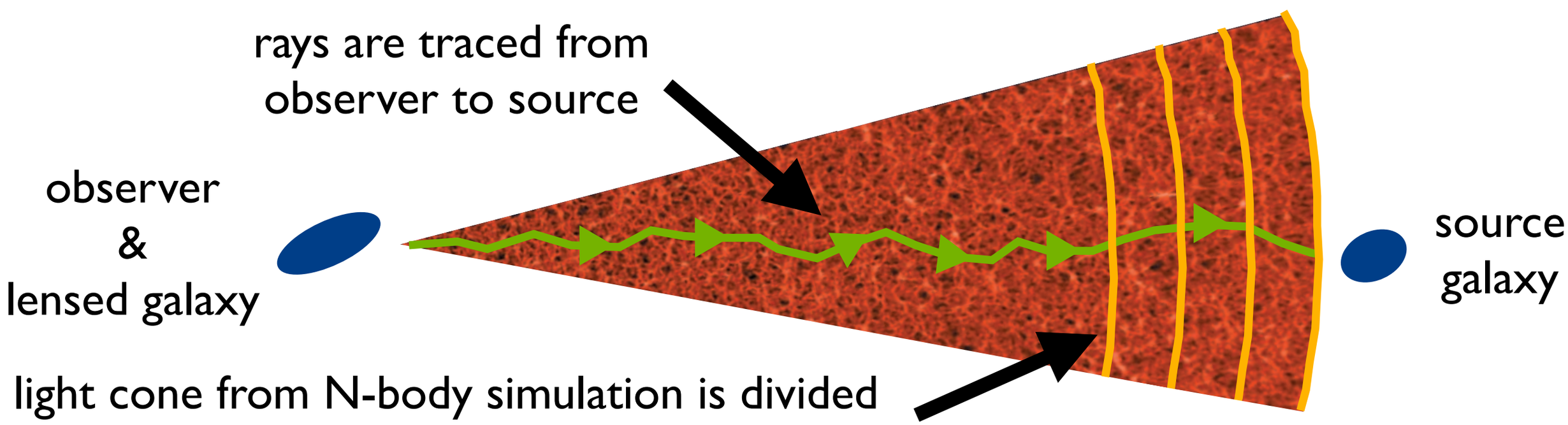


image from Evrard et al. 2002

Computational Challenges in Weak Lensing Simulations

- requirements for weak lensing simulations (in order to make mock galaxy catalogs):

  1) self-consistently handle large, contiguous areas of sky (e.g. DES, LSST, JDEM, etc. will cover $\geq$5000 deg^2)

  2) resolve both the relevant physical (~50 kpc/h) and angular (5-10 arcsecond) scales

  3) produce proper redshift dependence of shear and magnification bias effects

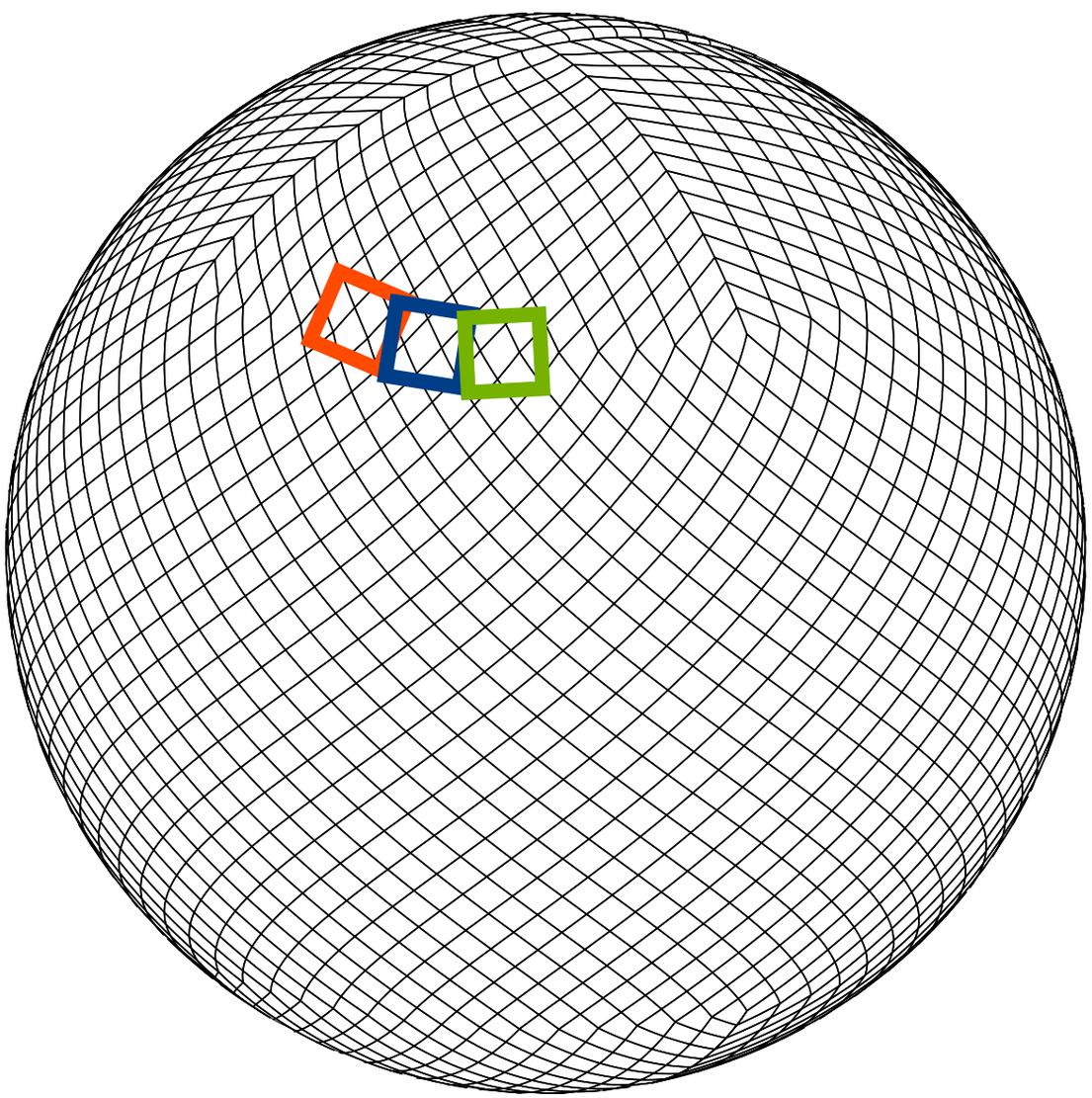
- current algorithms either scale nicely with resolution (e.g. O(NlogN) as opposed to O($N^{3/2}$)) or handle a curved-sky, but **cannot do both**

- distributed or shared memory computers are a must - a full-sky map of the shear, convergence, and lensed positions at 12.9 arcsecond resolution in single precision takes up 60 GB of RAM

The Essential Idea

We want to divide the sphere into a union of patches which can be ray traced in parallel using the flat-sky approximation.

Order 4 HEALPix Map w/ Patches



Why?

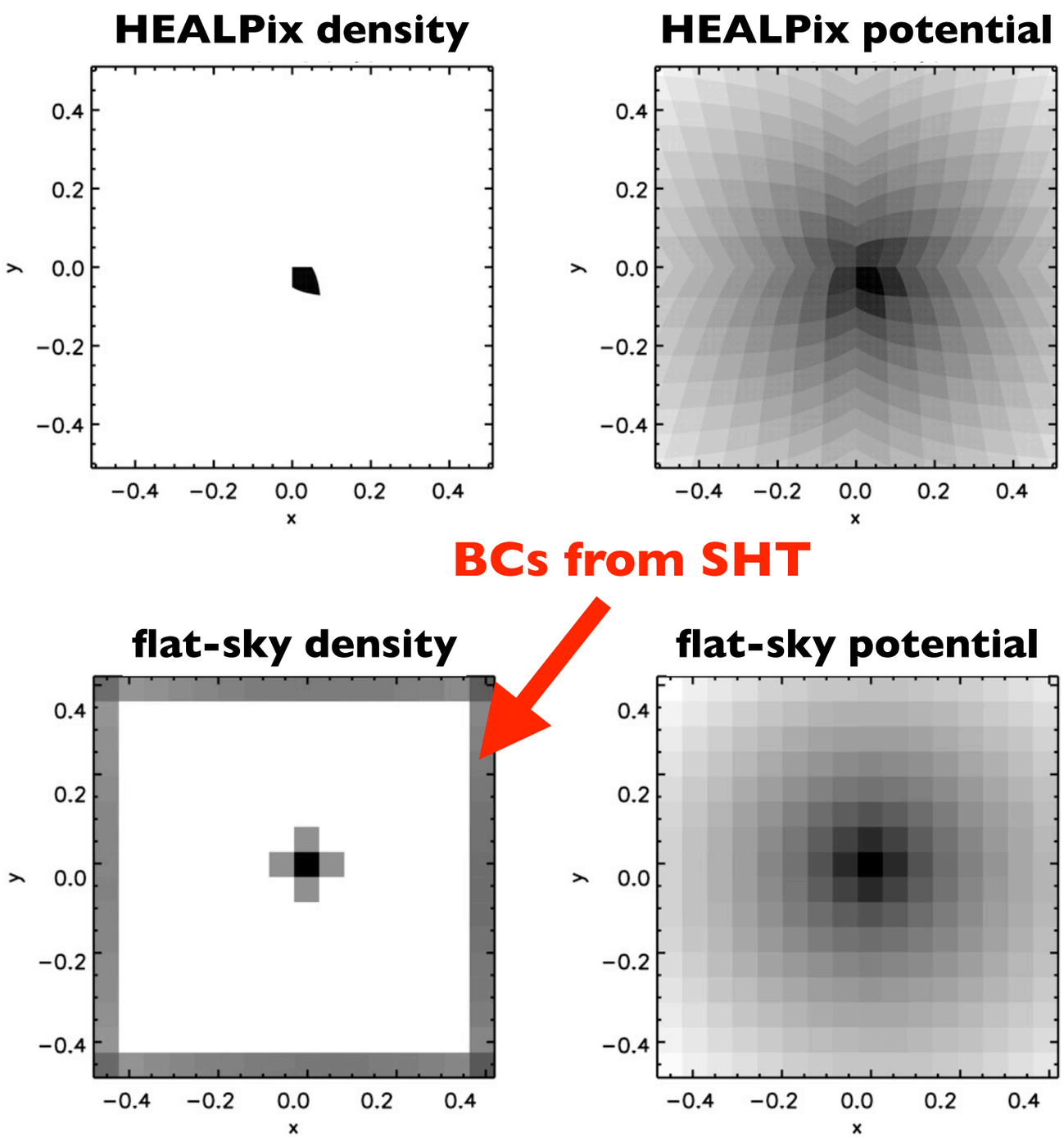
- flat-sky patches can be divided evenly between all of the processors
- thus the code can handle as many rays and N-body particles as you have RAM/ processors for
- as I will show later, this scheme can reduce algorithmic complexity of the Poisson solver to approximately O(NlogN) from $O(N^{3/2})$

Potential problem:

How do you maintain continuity in the lensing quantities between the flat-sky patches and also resolve larger-scale modes in the shear power spectrum?

A Hybrid SHT+FFT Poisson Solver

Point Mass Test of the Poisson Solver



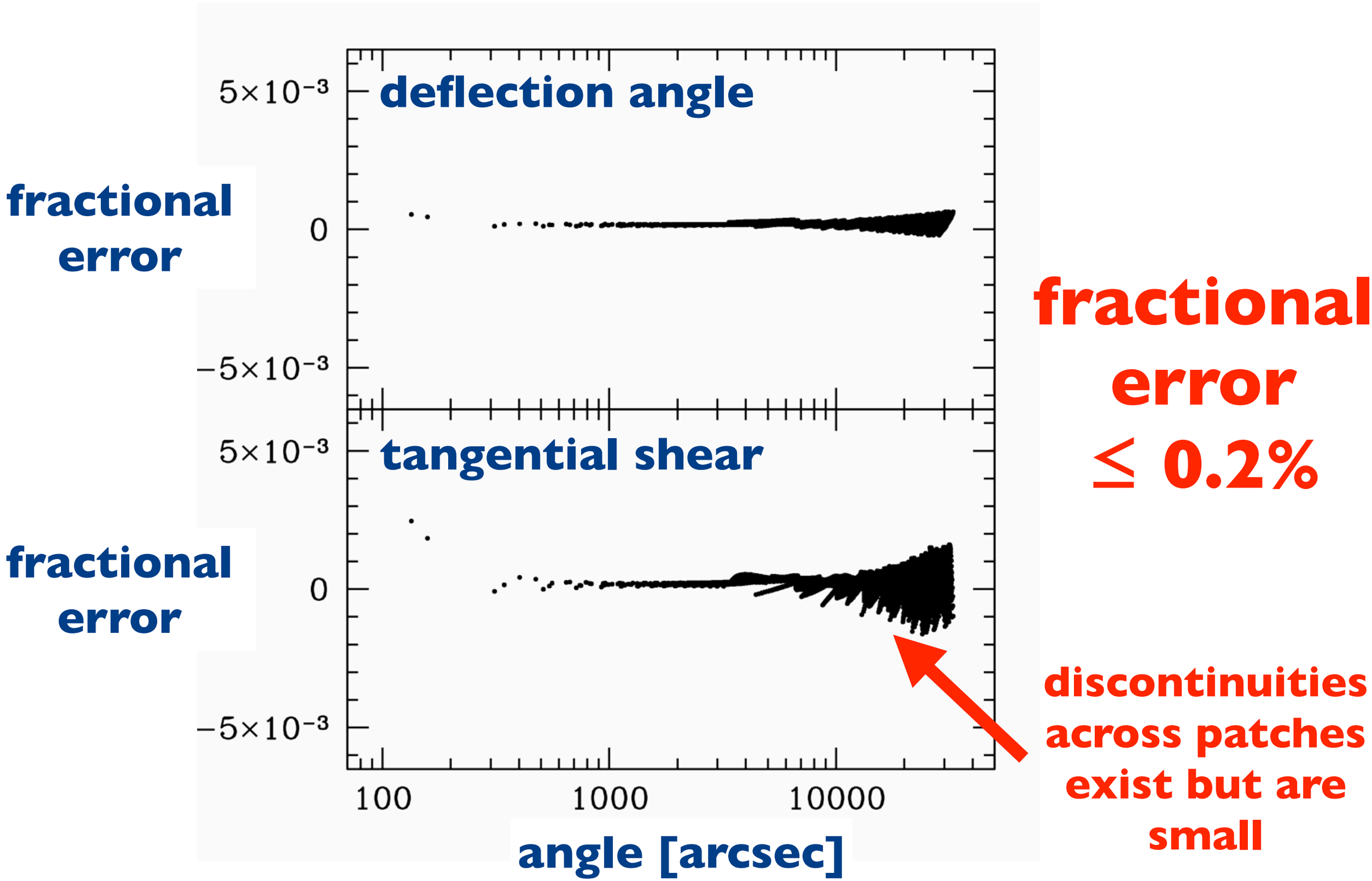
Key Idea:

Solution from SHT (via HEALPix) encodes the large-scale modes in the lensing potential, enforces the boundary conditions over the sphere, and enforces continuity across the flat-sky patches. The FFTs on the flat-sky patch resolve the small scale modes.

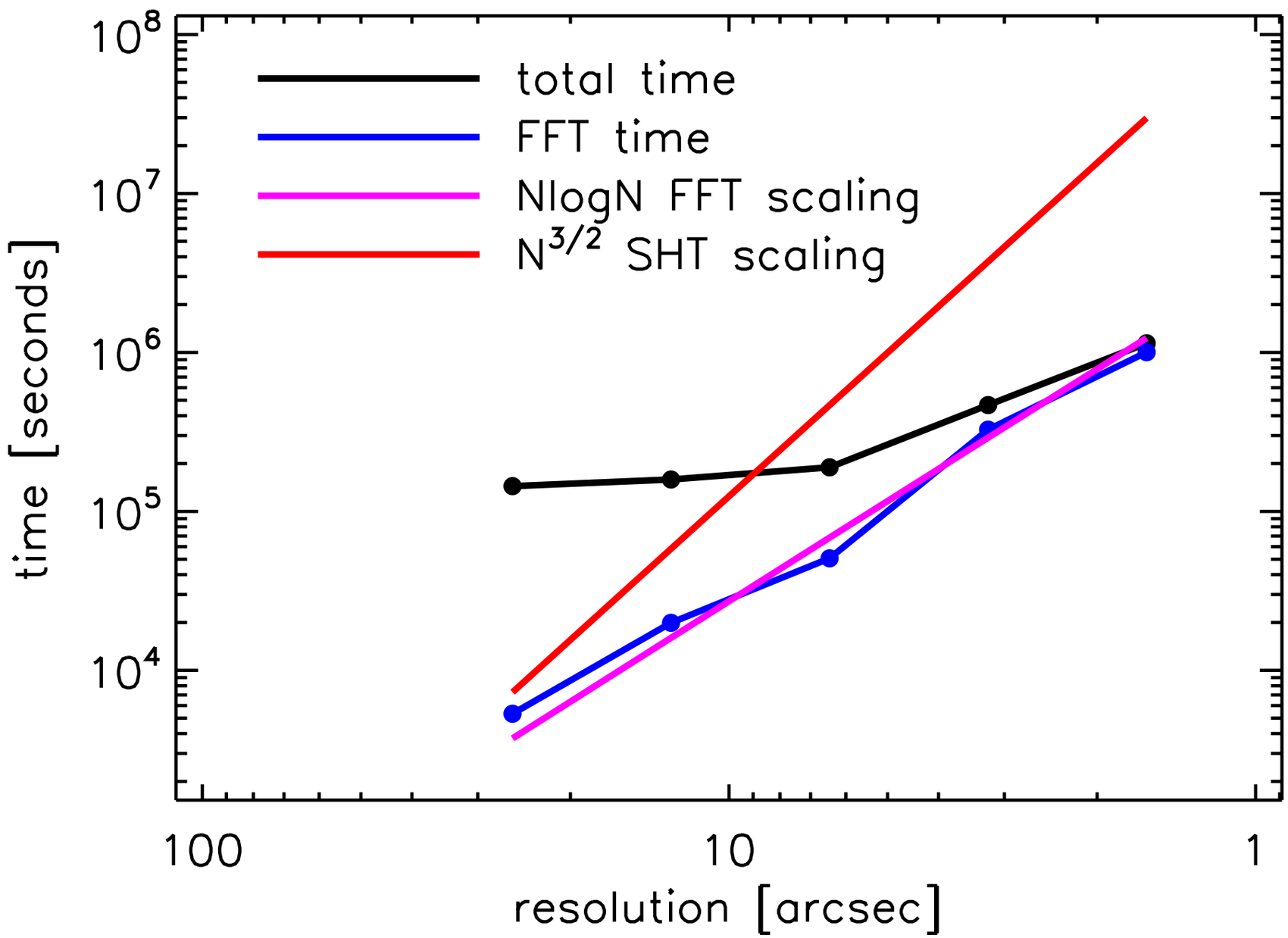
Other advantages:

- SHT is expensive $O(N^{3/2})$ and here we only have to do 1 analysis + 1 synthesis operation as opposed to 1 analysis + 5 synthesis operations
- very easy to parallelize on distributed memory systems
- can use any adaptive finite-volume Poisson solver - I have chosen a set of nested FFTs which is approximately O(NlogN).

Code Validation: A Point Mass



Code Scaling with Resolution (extrapolated to a full-sky)



Summary and Applications

- Summary:
  - this new algorithm enables accurate, efficient, high-resolution, full-sky lensing calculations on moderate sized machines (~100 processors/cores)
  - very portable - written in C with MPI and standard libraries like HDF5, GSL, and FFTW
- Some example applications:
  - code will first be used for the DES Blind Cosmology Challenge - will test if the DES can recover input cosmology from realistic mock galaxy catalogs
  - optical sky survey simulations with realistic weak lensing information - cosmic shear, galaxy-galaxy lensing, cluster lensing, etc.
  - also could be used for high-resolution CMB lensing calculations