Antenna Response: Simulation of the Feed Radiation Pattern for MIDAS

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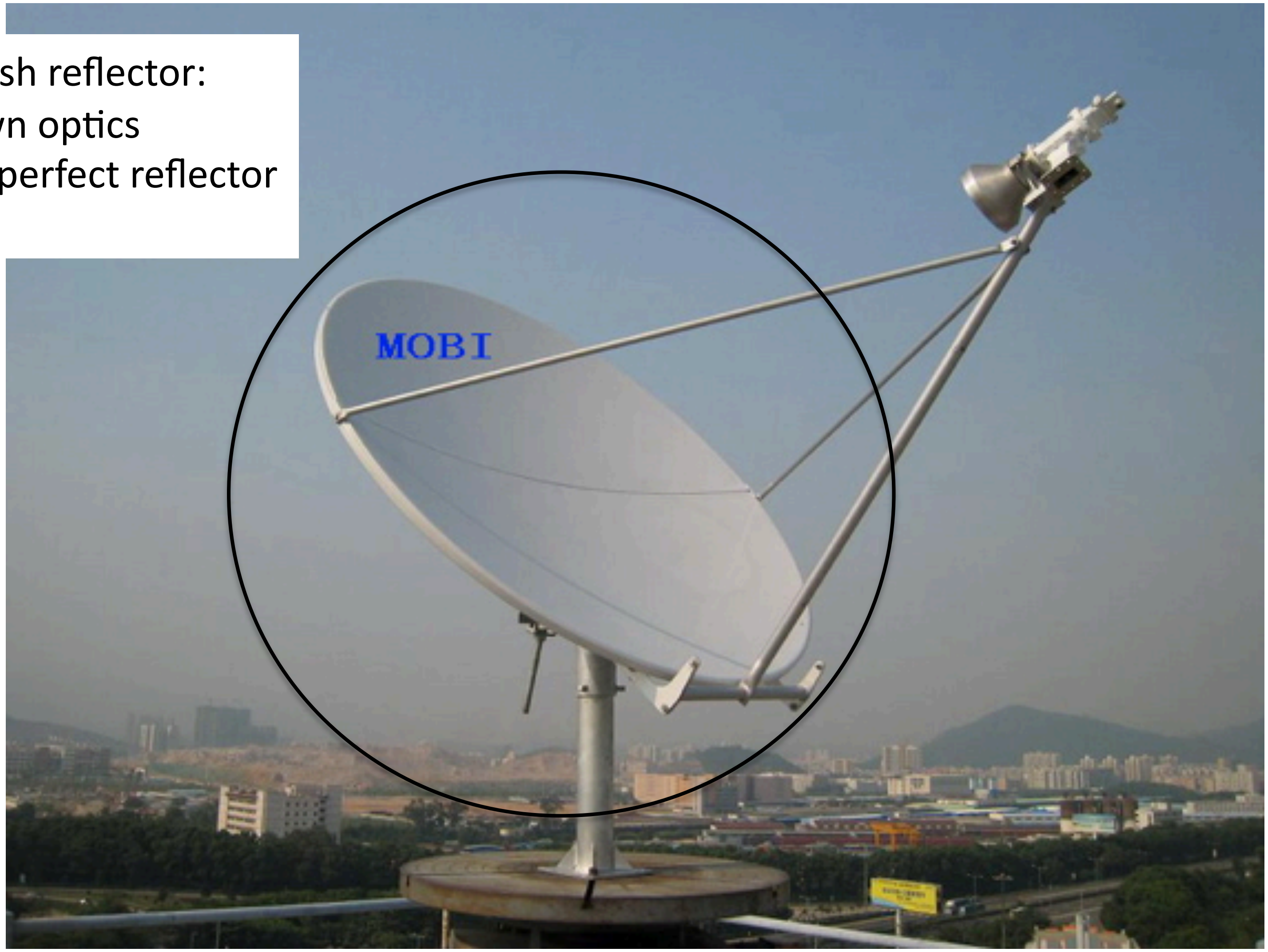
The Antenna System

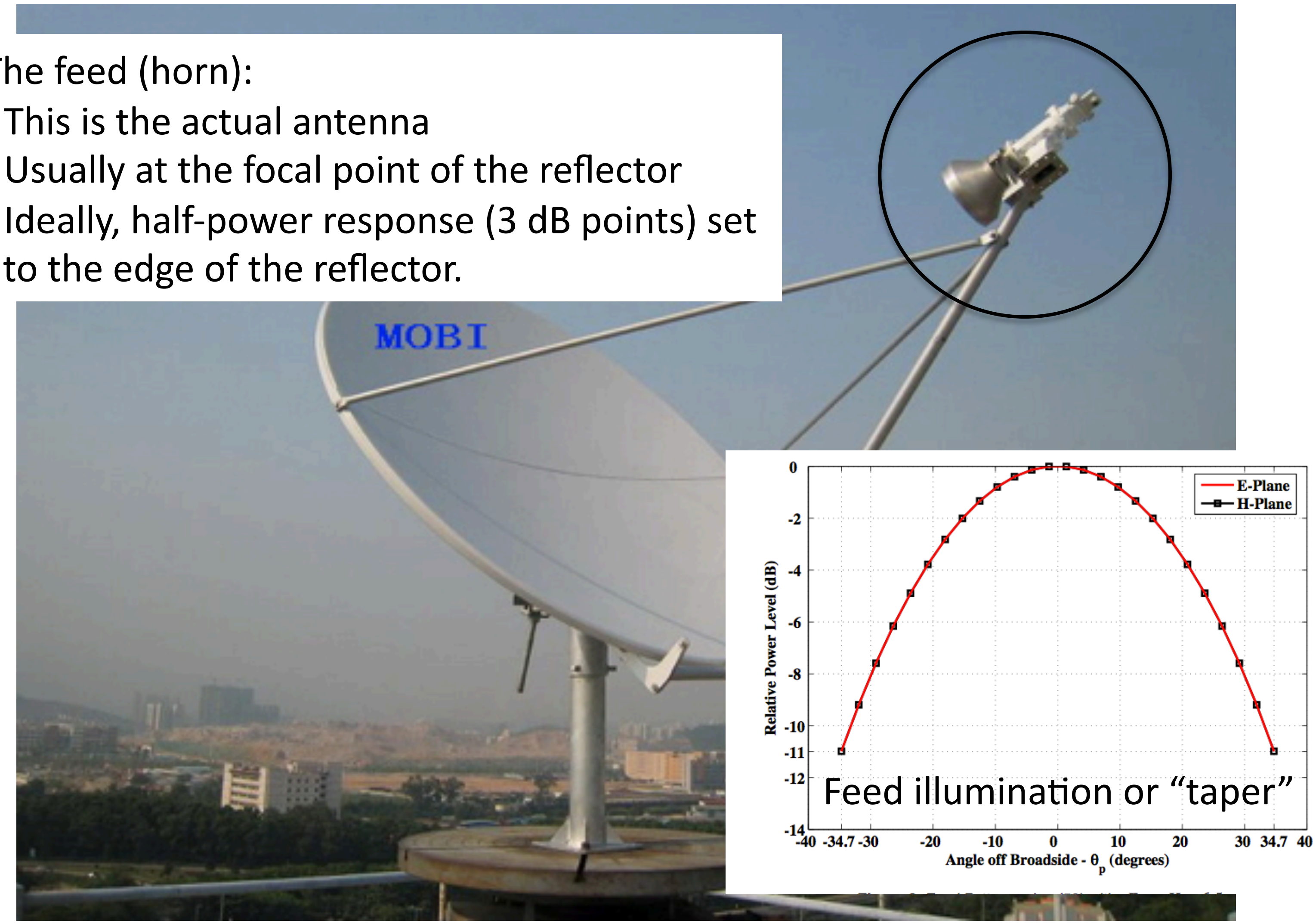
Parabolic dish reflector:
- Well-known optics
- Assume a perfect reflector



The Antenna System

The feed (horn):

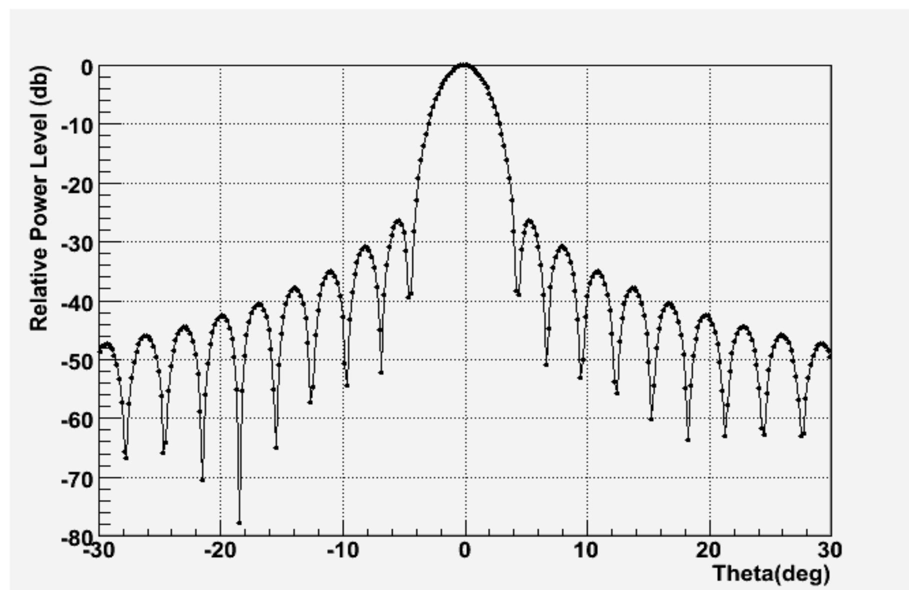
- This is the actual antenna
- Usually at the focal point of the reflector
- Ideally, half-power response (3 dB points) set to the edge of the reflector.



Feed illumination or “taper”

The Feed Radiation Pattern

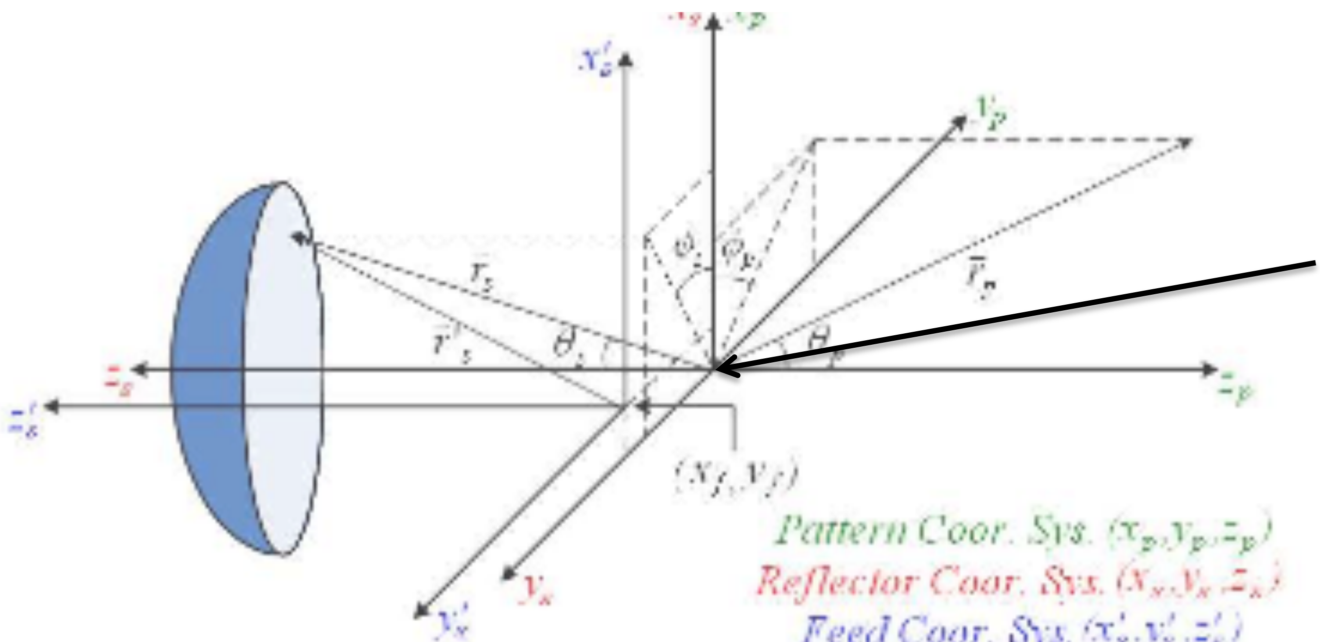
- The feed radiation pattern (aka far-field pattern) refers to the directional (angular) dependence of the radiation from the antenna
- Usual graphical representation:
  - Plot of the field strength at a constant (large) radius.
  - Normalized to the amplitude on the antenna boresight.
  - Plotted in dB

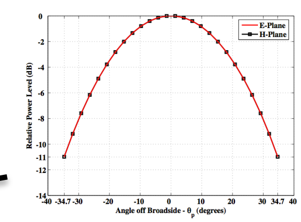


Calculating the Feed Radiation Pattern

As with any EM problem, the physics of the problem are simple, the trick is to solve it given the boundary conditions.

$$\bar{E}^{rad}(\theta_p,\phi_p) = -jkZ_o\frac{e^{-jk|\bar{r}_p|}}{|\bar{r}_p|}(\bar{\bar{I}}-\hat{r}_p\hat{r}_p)\int\int_S \bar{J}_s e^{jk\bar{r}_s\cdot\hat{r}_p}ds$$





Calculating the Feed Radiation Pattern

- Reasonable assumptions we can make:
  - The reflector has a large radius of curvature (compared to λ)
  - The incident field at each reflection point can be viewed as a plane wave.
  - The reflector is considered to be a perfectly conducting surface

- Complications:
  - N-1 feeds of the MIDAS antenna are displaced from the focus
  - Feed taper is not well known since feed base has been removed and adjacent feeds change the taper.

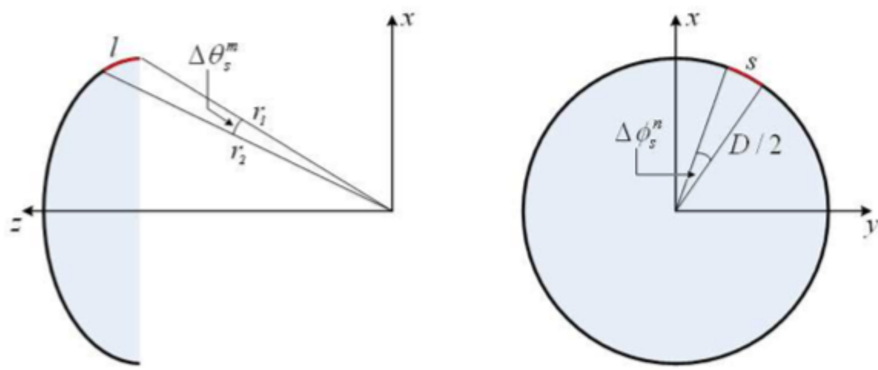
$$E_x^{rad}(\theta_p,\phi_p) = -jkZ_o\frac{e^{-jkr_p}}{r_p}[(\bar{F}-\hat{r}_p(\hat{r}_p\cdot\bar{F}))\cdot\hat{x}_p]$$

$$F_x = \int\int_S J_x e^{jk\bar{r}_s\cdot\hat{r}_p}ds$$

Ludwig's Method

Ludwig, A.C. (1968). Computation of Radiation Patterns Involving Numerical Double Integration.
IEEE Trans. Antenna & Prop., ***Vol. AP-16, pp. 767–769.***

$$\bar{F}(\theta_p,\phi_p) = \int\int_S \bar{K}(\theta_s,\phi_s)e^{jk\gamma(\theta_s,\phi_s,\theta_p,\phi_p)}d\theta d\phi$$



Since typically electromagnetic fields do not have abrupt changes over a distance on the order of a wavelength, we can then say that the integrand, *K* will not vary abruptly and is well behaved over the incremental area ΔS_{mn}. Furthermore, it can be approximated by a linear function.

$$K(\theta_s,\phi_s) \simeq a_{mn} + b_{mn}(\theta_s - \theta_s^m) \;+\; c_{mn}(\phi_s - \phi_s^n)$$

Ludwig's Method

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The method used by Ludwig for determining the coefficients a_m, b_m, c_m is to use a best fit mean- squared plane to the values of the function K*, at the corners of ΔS_{mn}:*

$$
\begin{aligned}
a_{mn} &= \frac{1}{4}\left[3K(\theta_s^m,\phi_s^n) - K(\theta_s^{m+1},\phi_s^{n+1})\right. \\
&\quad + \left. K(\theta_s^{m+1},\phi_s^n) + K(\theta_s^m,\phi_s^{n+1})\right] \\
b_{mn} &= \frac{1}{2\Delta\theta_s^m}\left[K(\theta_s^{m+1},\phi_s^n) - K(\theta_s^m,\phi_s^n)\right. \\
&\quad + \left. K(\theta_s^{m+1},\phi_s^{n+1}) - K(\theta_s^m,\phi_s^{n+1})\right] \\
c_{mn} &= \frac{1}{2\Delta\phi_s^n}\left[K(\theta_s^m,\phi_s^{n+1}) - K(\theta_s^m,\phi_s^n)\right. \\
&\quad + \left. K(\theta_s^{m+1},\phi_s^{n+1}) - K(\theta_s^{m+1},\phi_s^n)\right]
\end{aligned}
$$

The integral (i.e. sum over the incremental surface elements ΔS_{mn}) consists of the contribution F_{mn} from every surface element:

$$
\begin{aligned}
\Delta F_{mn} &= \left\{ a_{mn}\left[\frac{e^{jk\beta_{mn}\Delta\theta_s^m}-1}{jk\beta_{mn}}\right]\left[\frac{e^{jk\xi_{mn}\Delta\phi_s^n}-1}{jk\xi_{mn}}\right]\right. \\
&\quad + b_{mn}\left[\frac{\Delta\theta_s^m}{jk\beta_{mn}}e^{jk\beta_{mn}\Delta\theta_s^m} - \frac{e^{jk\beta_{mn}\Delta\theta_s^m}-1}{(jk\beta_{mn})^2}\right]\left[\frac{e^{jk\xi_{mn}\Delta\phi_s^n}-1}{jk\xi_{mn}}\right] \\
&\quad + \left. c_{mn}\left[\frac{e^{jk\beta_{mn}\Delta\theta_s^m}-1}{jk\beta_{mn}}\right]\left[\frac{\Delta\phi_s^n}{jk\xi_{mn}}e^{jk\xi_{mn}\Delta\phi_s^n} - \frac{e^{jk\xi_{mn}\Delta\phi_s^n}-1}{(jk\xi_{mn})^2}\right]\right\}
\end{aligned}
$$

Advantages of Ludwig's Method

- No numerical integration is required, just simple sums
- Coefficients a_m,b_m, $c_{m,}$ etc. (i.e. Ludwig's coefficients) are independent of the observation point, thus only one calculation is required.
- From the coding point of view, the method is easy to be implemented as a function call with the feed location and the observation angle as input variables. Having this routine coded as a function significantly increases the computational efficiency because all the variables of the script, including the large variables containing the coefficient information, are local to the function.

Checking the implementation of the Method

From Frederic Arpin's:
Parabolic Reflector Modelling Techniques
with a Laterally Displaced Feed

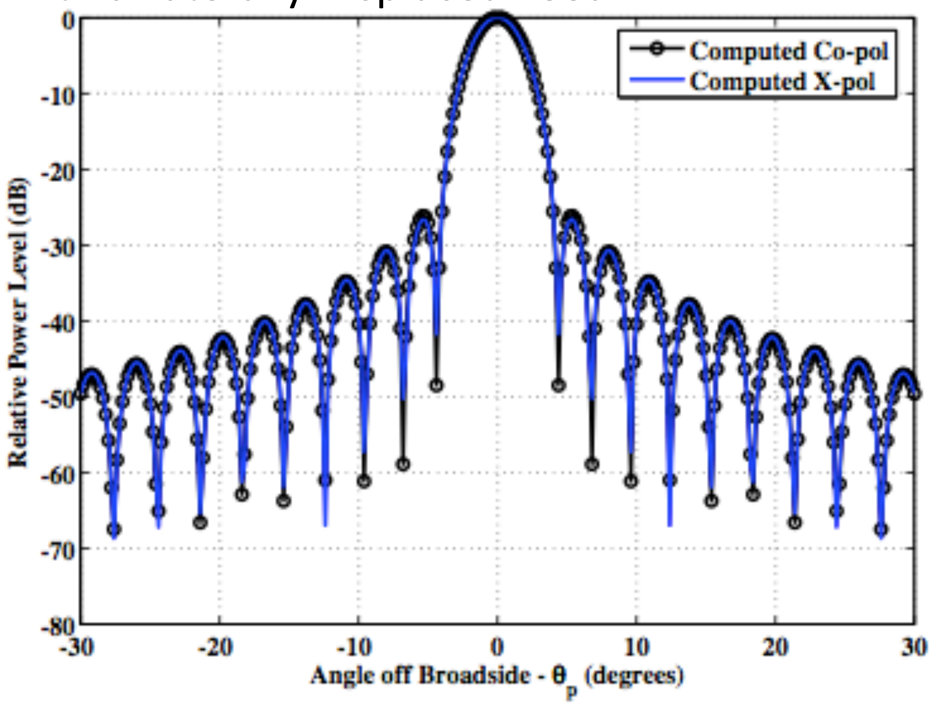
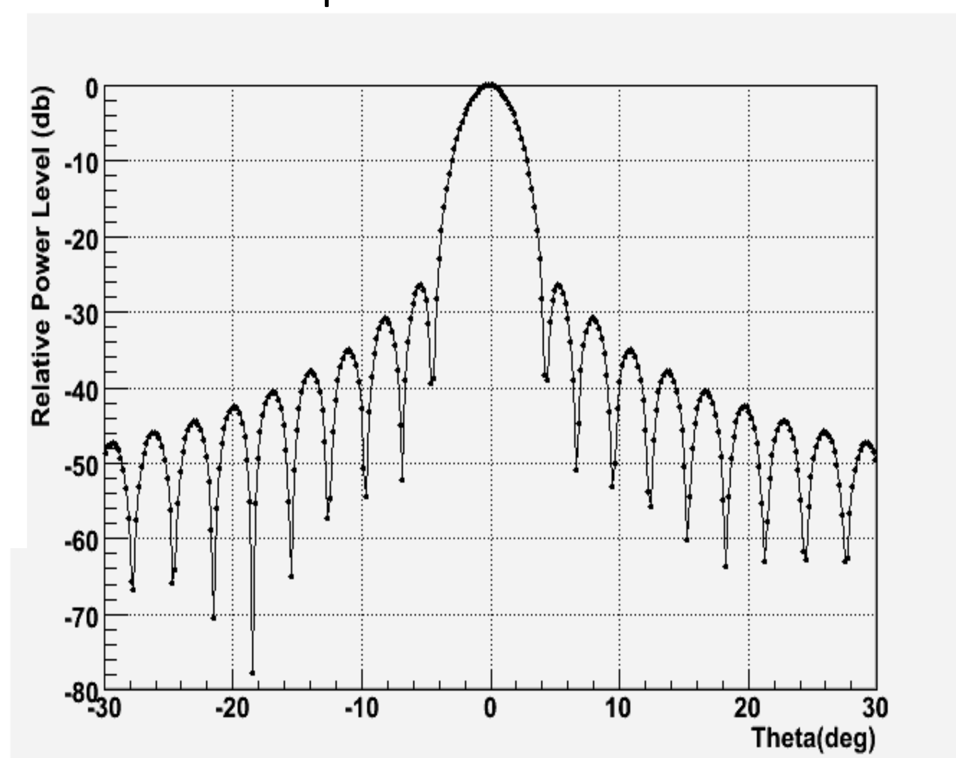
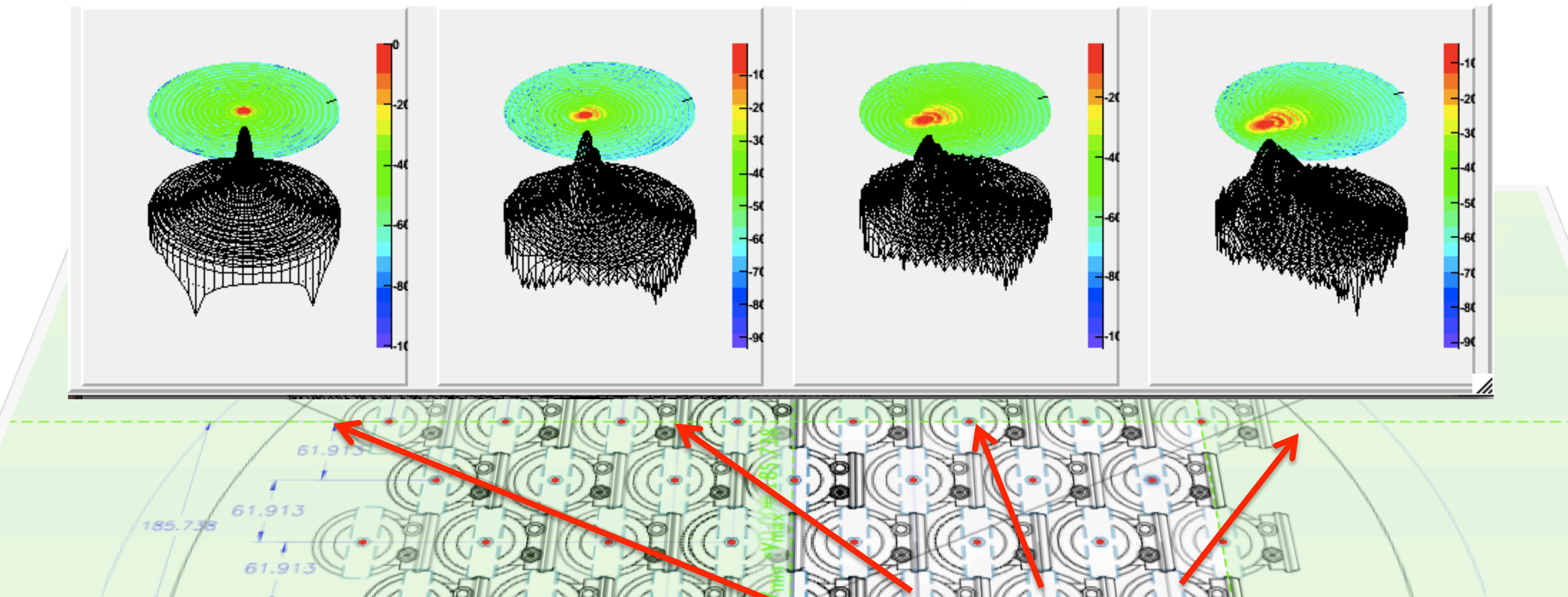


Figure 21: Radiation Pattern for Reflector #1, $\phi_p = 0^\circ$, Ludwig Method

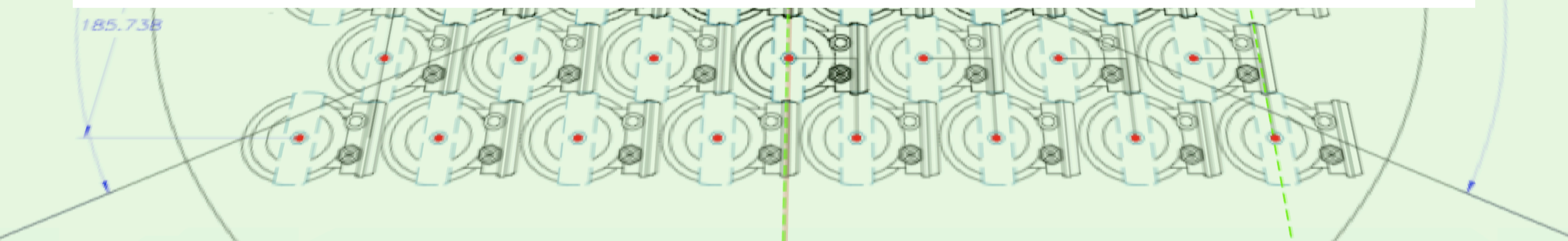
Radiation pattern obtained with the code:



Using the method:
Central Feed Response

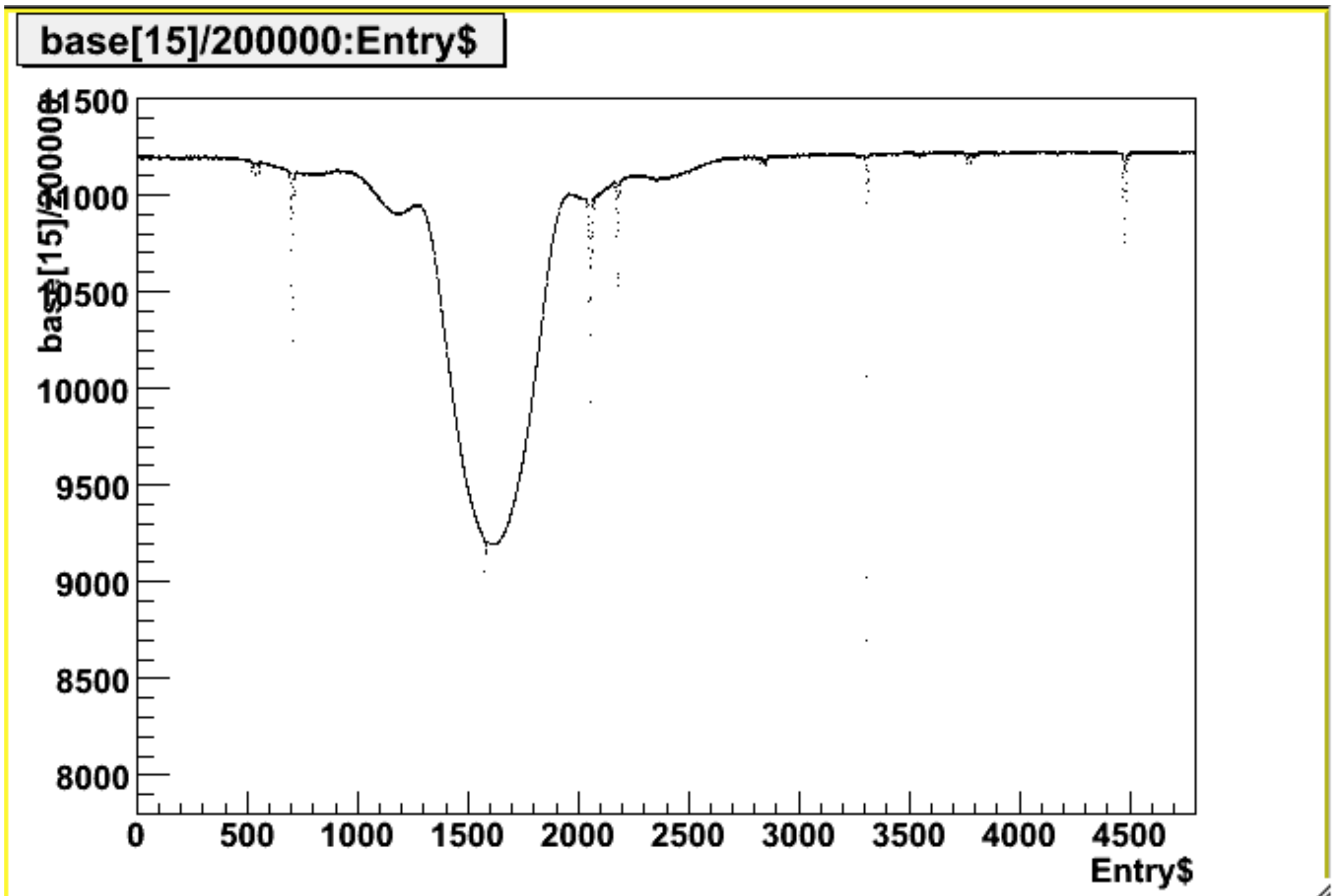


- Calculation of the coefficients takes about 2 minutes
- Map generation depends on the number of points in the grid, typically 0.1 seconds per point

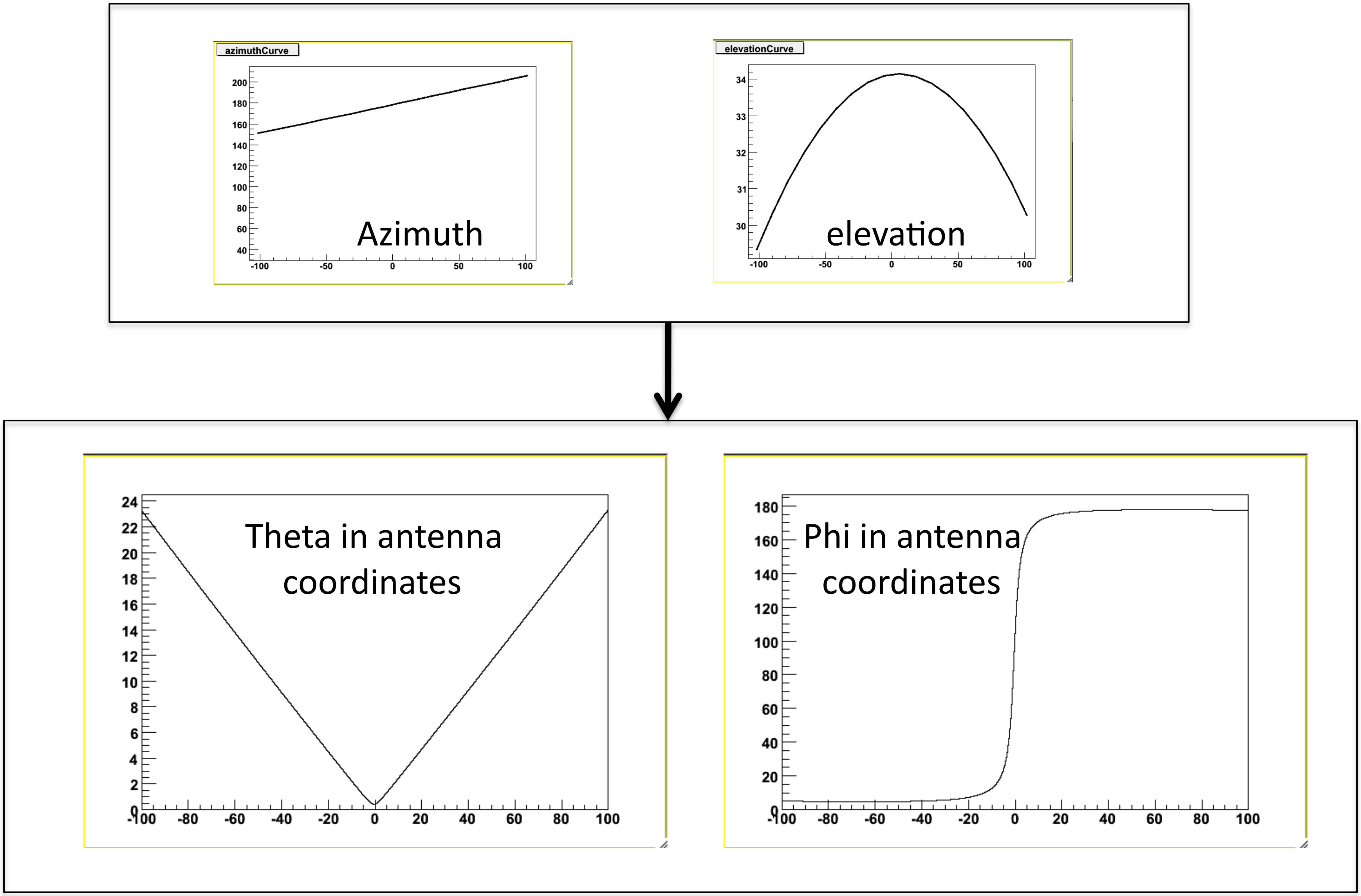


Reality check: Looking at the Sun

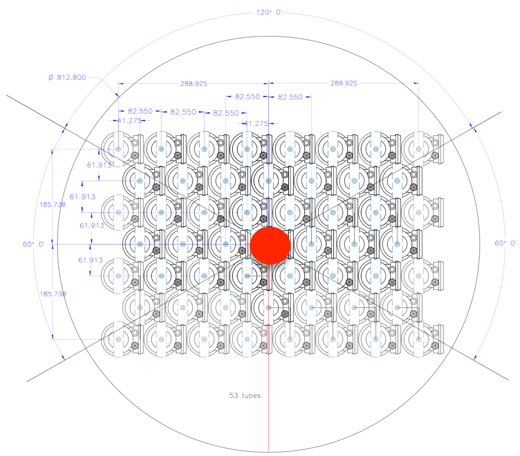
Can we reproduce this?

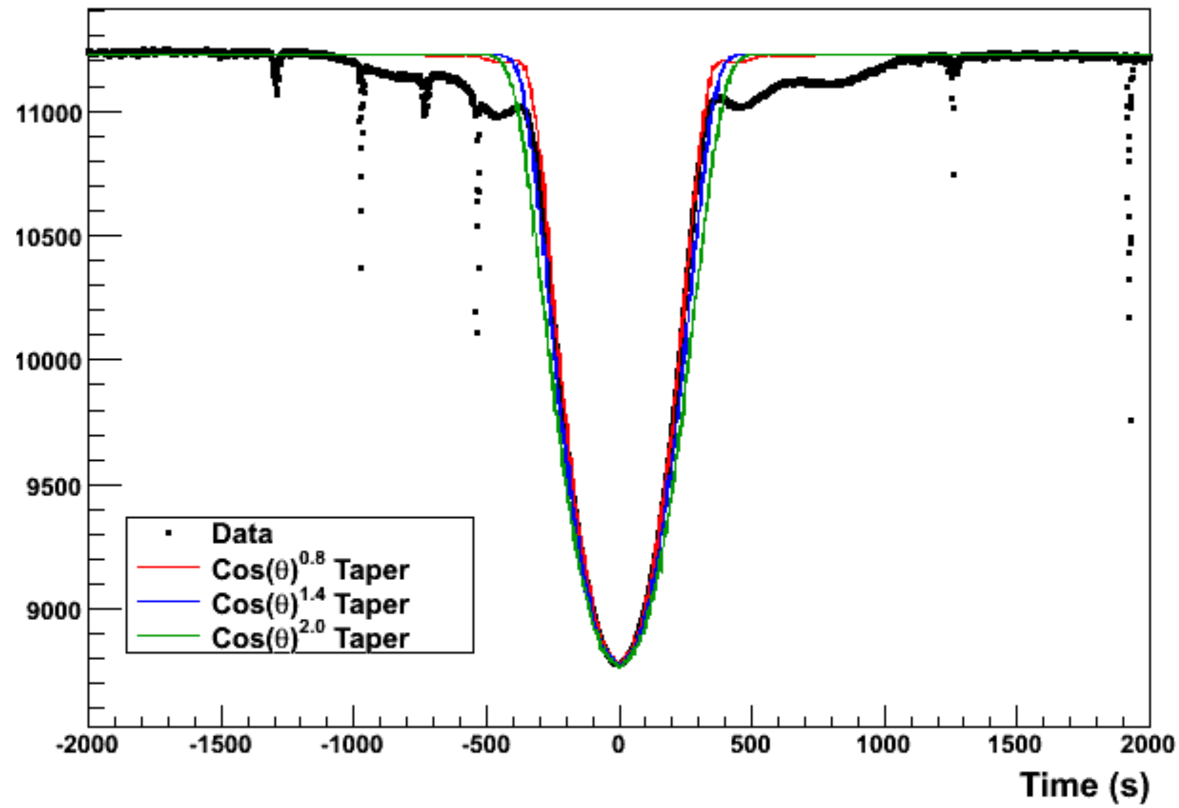


Sun trajectory



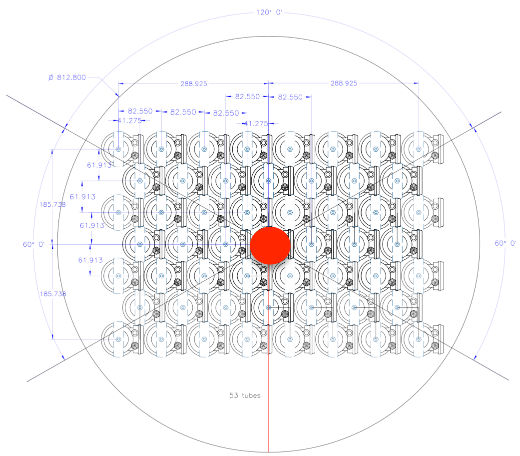
Central Feed:

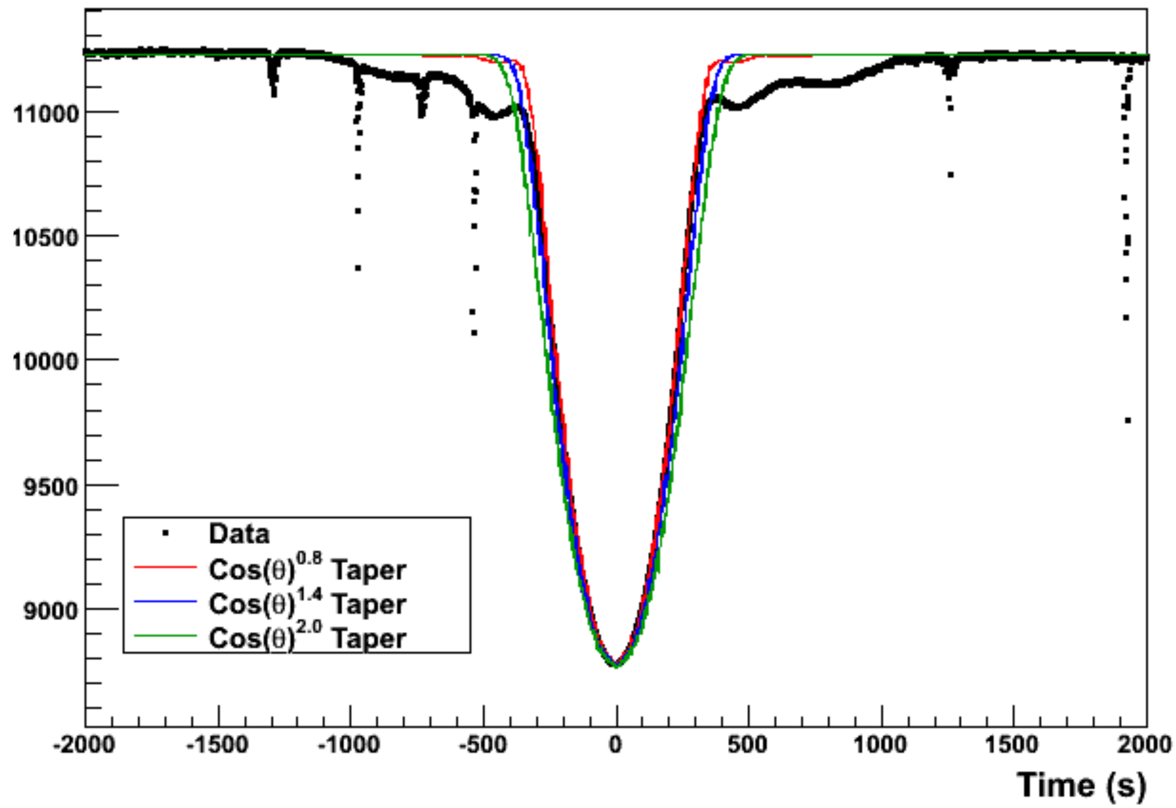




Only the normalization of the peak is scaled to the data. The profile of the signal is calculated from the response of the feed and the location of the Sun as a function of time.

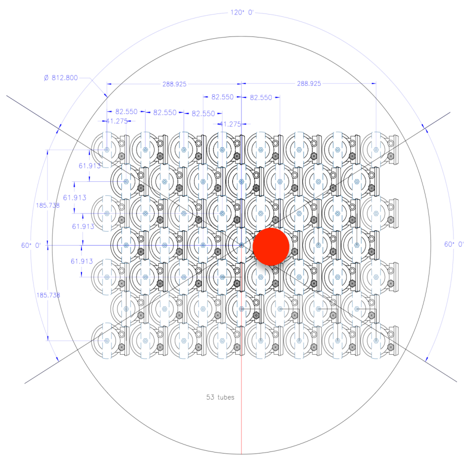
Central Feed:

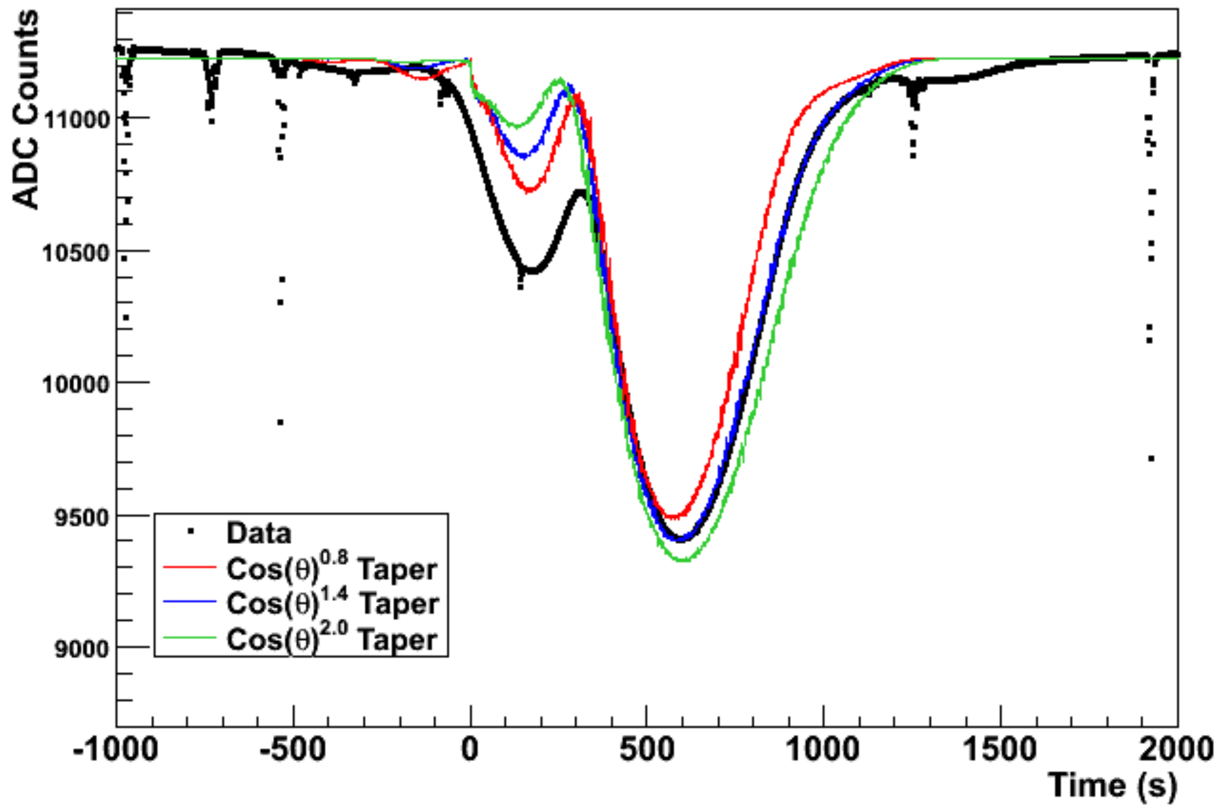




- Good agreement for the main lobe.
- Side lobe non-detectable according to the simulation. However, it's quite obvious in the data

Next-to-central Feed:





For this plot we use the same scale used in the previous plot. Again, the profile of the signal is obtained from the response of the feed and the location of the Sun as a function of time. The "feed taper" that agrees better with the data is within the expected range.

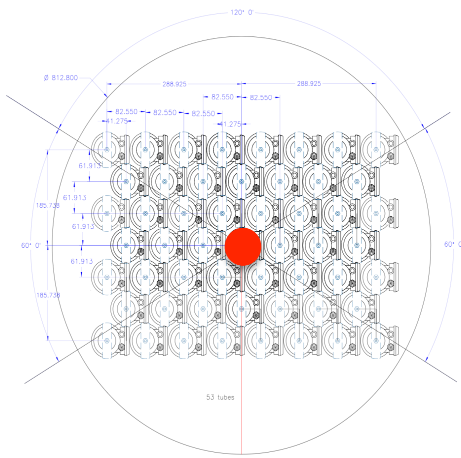
What are we ignoring?

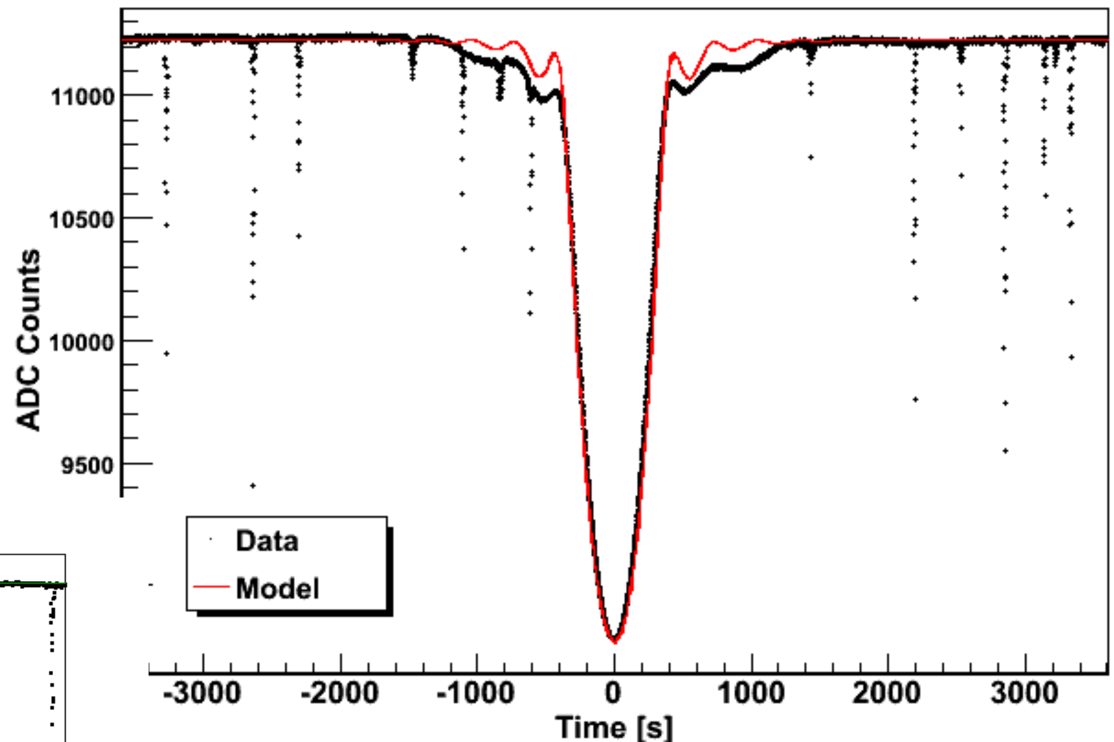
- Complex feed taper due to adjacent feeds and plate
- Inefficiencies:
  - Spillover of the feed
  - Ohmic loss in the reflector
  - Polarisation efficiency
  - Surface error in the dish (ie. irregular scattering)
  - Focus error (axial defocus)
  - **Blocking efficiency**

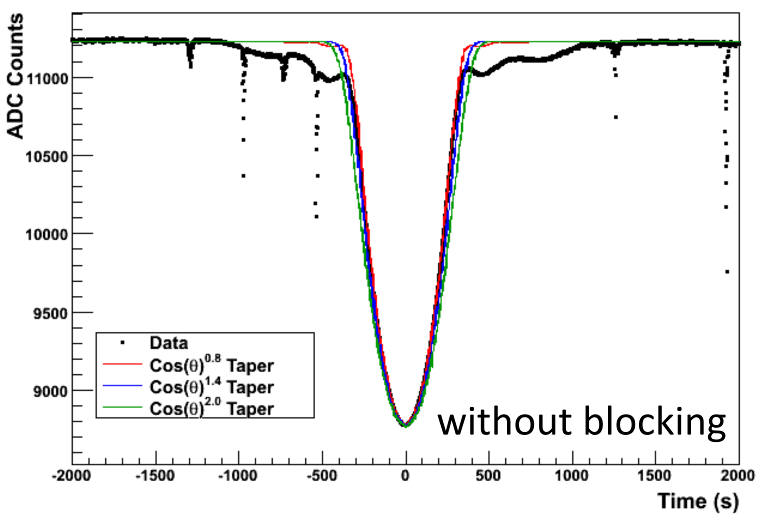
Blocking Efficiency

- The central shadow of the plate (plane)
- Shadow of the tripod to the incoming wave (plane)
- Shadow of the tripod to the reflected wave (spherical)

Central Feed with plate blocking:







- Significant improvement but side lobes still underestimated.

Summary

- Ludwig's method has been implemented as numerical code in the ROOT/CINT framework
- With minimal effort would be available for sharing. Looking forward to compare with results from other tools.
- We have reached the point where second order effects will be a lot harder to implement.