Simulation of shower radio emission from first principles

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Summary

- Introduction
- ZHS Formalism
  - Time-domain
  - Frequency-domain
- ZHAireS implementation
- Some preliminary results
  - Cerenkov on AMY chamber
  - EAS
- Conclusions and outlook

Introduction

- Radio emission from EAS:
  - Synchrotron
  - Cherenkov
- ZHS algorithms (Zas, Halzen and Stanev, Phys.Rev.D V**45**, 362 (1992) and Phys.Rev.D81:123009,2010)
  - $e^{\pm}$ tracks - Length $L << MFP$ with constant $\vec{v}$
  - Based on Liénard-Wiechert potentials (disregards ϕ term)
  - First principles (Maxwell) - No emission model presuposed
- ZHAireS code (arXiv:1005.0552v1 [astro-ph.HE])
  - Uses expanded ZHS algorithms in conjunction with AIRES
  - Full shower simulation

Formalism: time-domain

- Maxwell $\longrightarrow$ Liénard-Wiechert potentials
  - $\phi = \frac{1}{4\pi\epsilon} \int \frac{\rho(\vec{x}',t')}{|\vec{x}-\vec{x}'|} d^3\vec{x}' dt'$ (disregarded)
  - $\vec{A} = \frac{\mu}{4\pi} \int \frac{\vec{J}_\perp(\vec{x}',t')}{|\vec{x}-\vec{x}'|} \delta(\sqrt{\mu\epsilon}|\vec{x}-\vec{x}'| - (t-t'))d^3\vec{x}' dt'$, where

    $\vec{J}_\perp = e\vec{v}_\perp \delta^3(\vec{x}' - \vec{x}_0 - \vec{v}t')[\Theta(t'-t1) - \Theta(t'-t2)]$ for $\vec{v} = const$.

- Integration inside track (δ functions):
  - nominator:

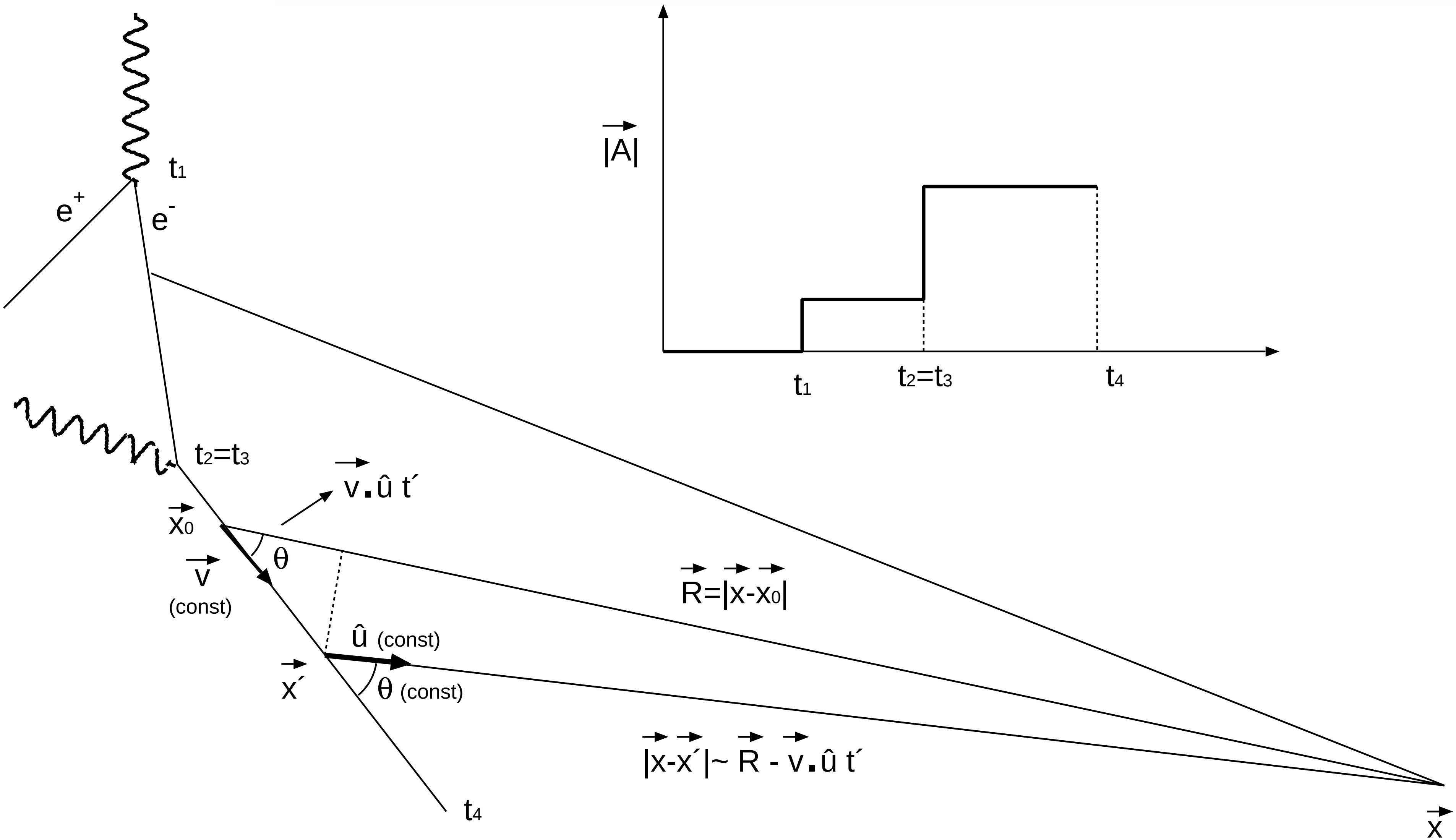
    $\vec{x}' \rightarrow \vec{x}_0 + \vec{v}t'$

    $|\vec{x} - \vec{x}'| = |\vec{x} - \vec{x}_0 - \vec{v}t'| \simeq R - \vec{v} \cdot \hat{u}t'$, where

    $R = |\vec{x} - \vec{x}_0|$ and $\hat{u} = \frac{\vec{x}-\vec{x}_0}{|\vec{x}-\vec{x}_0|} = const$ (Fraunhoffer approx inside a single track)
  - denominator:

    $|\vec{x} - \vec{x}'| \simeq R$

Formalism: path integration



$$|\vec{x} - \vec{x}'| = |\vec{x} - \vec{x}_0 - \vec{v}t'| \simeq R - \vec{v} \cdot \hat{u}t'$$

Formalism: time-domain

$$\delta(\tfrac{n}{c}|\vec{x}-\vec{x}'| - (t-t'))) \rightarrow \frac{1}{|1-n\beta\cos\theta|}\delta\left(t' - \frac{t-\frac{nR}{c}}{1-n\beta\cos\theta}\right) \Rightarrow$$

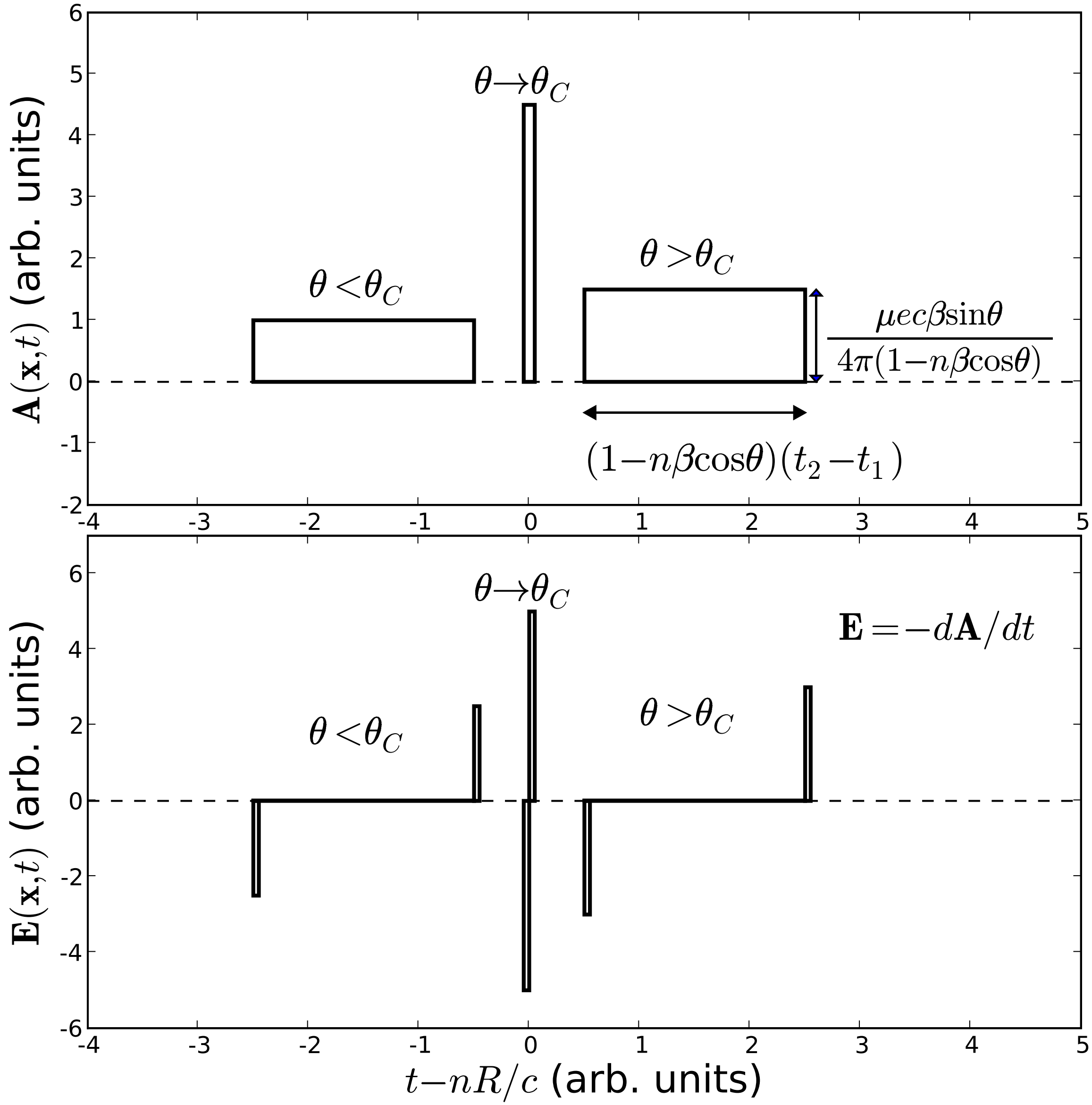
$$\boxed{\vec{A} = \frac{\mu\epsilon}{4\pi R}\vec{v}_\perp \frac{\Theta(t-\frac{nR}{c}-(1-n\beta\cos\theta)t_1) - \Theta(t-\frac{nR}{c}-(1-n\beta\cos\theta)t_2)}{(1-n\beta\cos\theta)}}$$

In the limit $\theta \rightarrow \theta_C$, $(1-n\beta\cos\theta)\delta t \rightarrow 0$.

$$\boxed{R\vec{A} = \frac{\mu_r}{4\pi\epsilon_0 c^2}\delta\left(t-\frac{nR}{c}\right)\vec{v}_\perp \delta t}$$

Formalism: single track field

(Phys.Rev.D81:123009,2010)



Formalism: frequency-domain

- Also derived from Liénard-Wiechert potentials

  Zas, Halzen and Stanev, Phys.Rev.D V**45**, 362 (1992)

- Can also be derived from the time-domain results, using

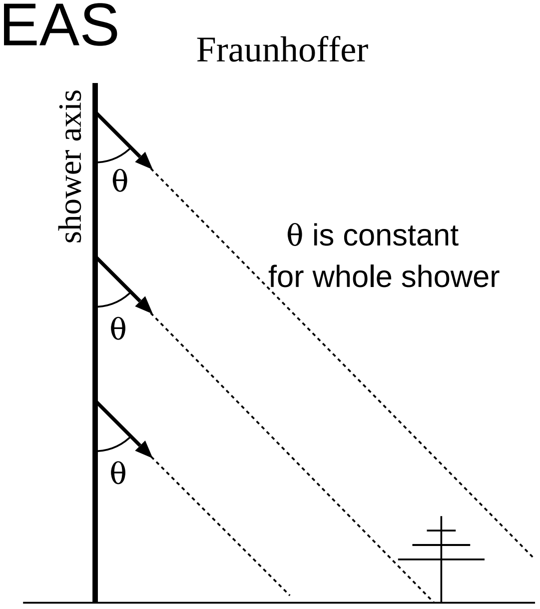
$$\vec{E} = -\frac{\partial \vec{A}}{\partial t} - \nabla \phi \quad \text{and} \quad \tilde{f}(\omega) = 2 \int_{-\infty}^{\infty} f(t) e^{i\omega t} dt$$

$$\boxed{\vec{E}(\omega, \vec{x}) = \frac{e\mu_r}{2\pi\epsilon_0 c^2} i\omega \frac{e^{ikR}}{R} e^{i(\omega - \vec{k}\cdot\vec{v})t_1} \vec{v}_\perp \left[\frac{e^{i(\omega-\vec{k}\cdot\vec{v})\delta t} - 1}{i(\omega - \vec{k}\cdot\vec{v})} \right]}$$

$$\boxed{R\vec{E}(\omega, \vec{x}) = \frac{e\mu_r i\omega}{2\pi\epsilon_0 c^2} \vec{v}_\perp \delta t \, e^{i(\omega t_1 - \vec{k}\cdot\vec{r}_1)} e^{ikR}}$$

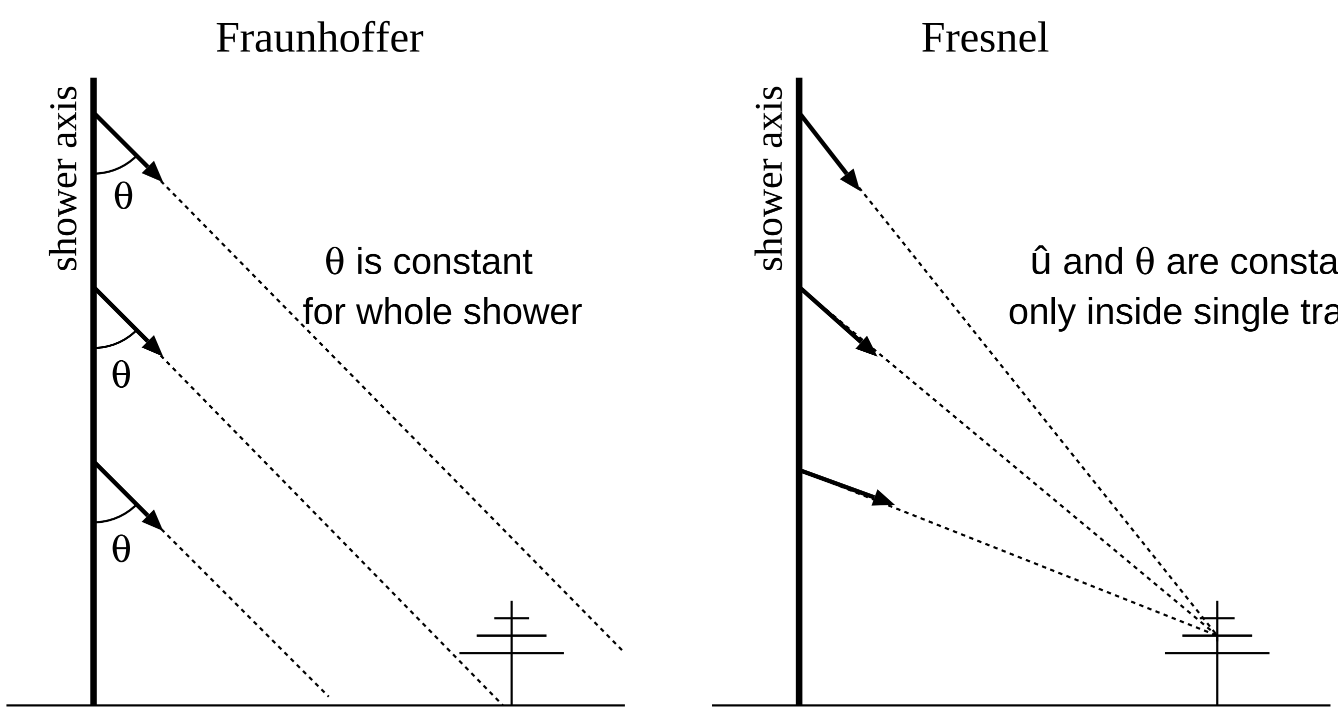
Original ZHS algorithms in ZHAireS

- ZHAireS: ZHS + AIRES
- Originally developed for ice using TIERRAS. Full Fraunhoffer approximation in the frequency-domain.
  - Observation angle θ i.r.t. shower axis is fixed, $R = 1$
    - calculates $R\vec{E}$ for each θ_{obs}
  - Valid for small shower size and distant observer (ice).
  - Not suitable for EAS



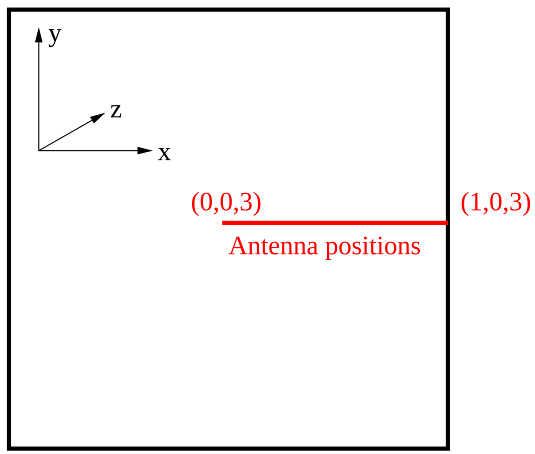
ZHS Algorithms - extensions

- Extended to time-domain and Fresnel approximation
  - $\hat{u}$ is fixed only inside single tracks
  - R is calculated distance from each track to antenna
  - Valid for track length $L^2/R < \lambda/2\pi$. (In AIRES $L << MFP$)



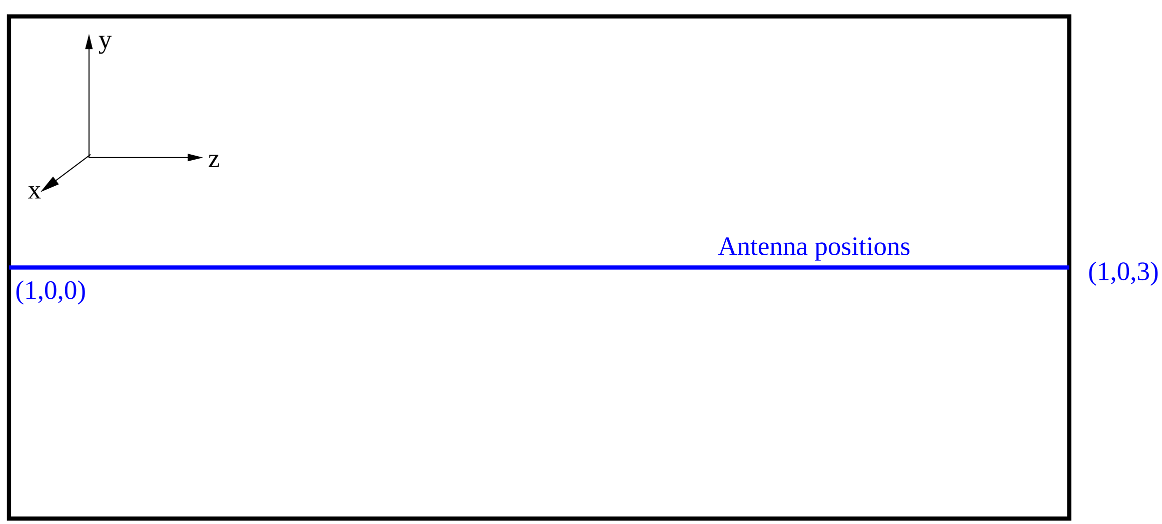
AMY chamber

End view of camera centered at (x,y,z) = (0,0,3) m

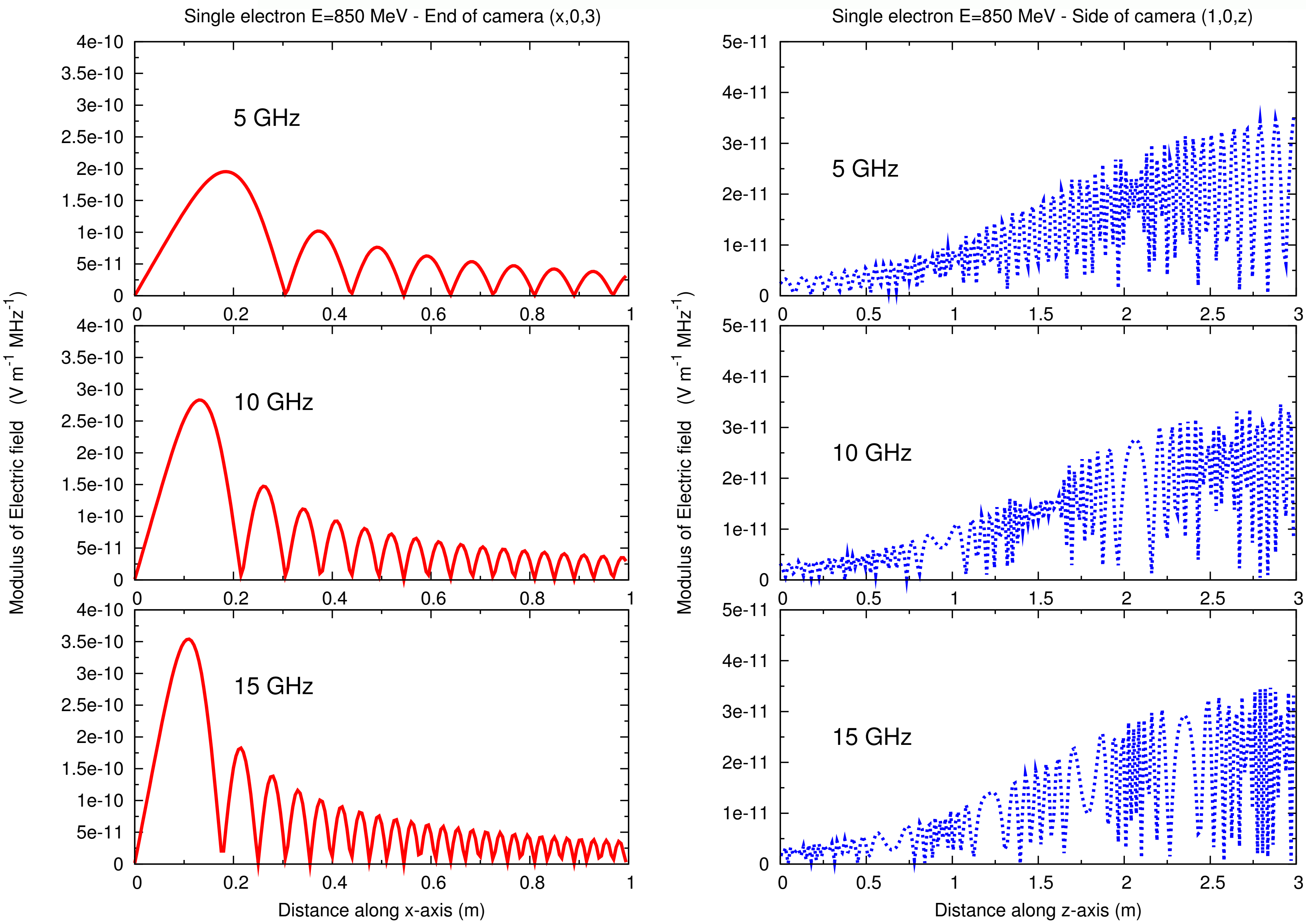


Beam injected at (x,y,z) = (0,0,0) along positive z−axis

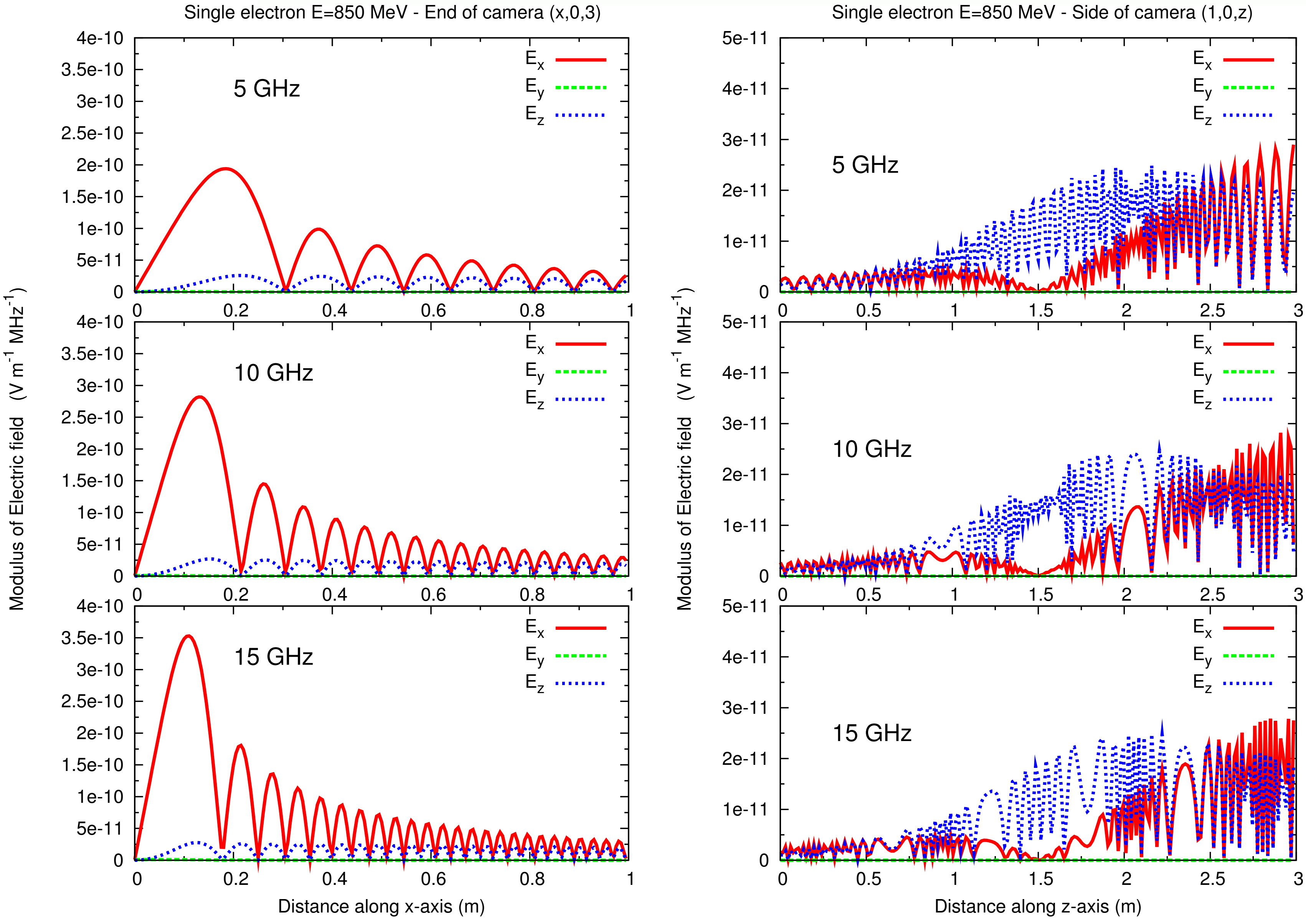
Side of camera at x=1 m



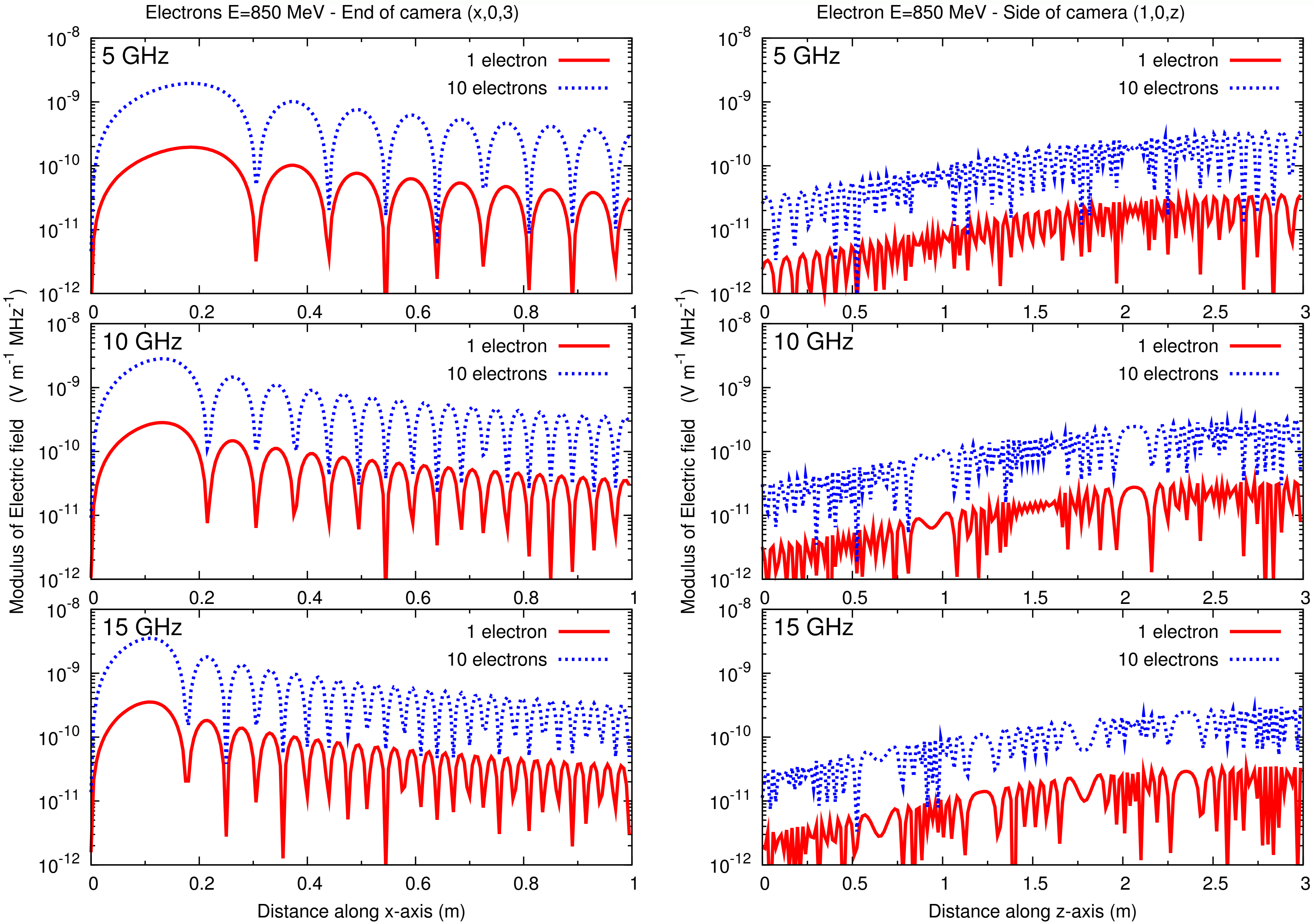
Preliminary Results: AMY chamber



Preliminary Results: AMY chamber



Preliminary Results: AMY chamber

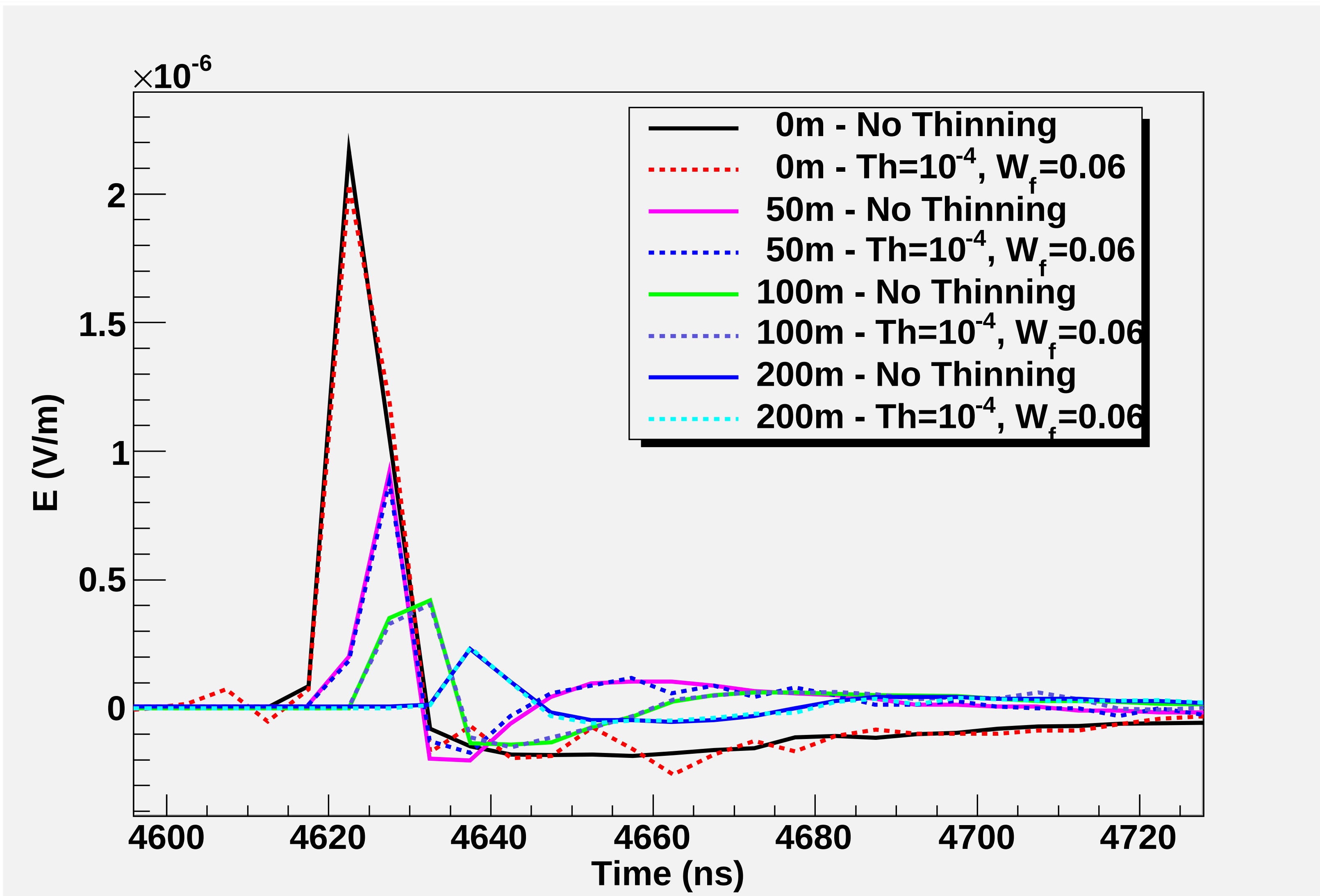


ZHAireS implementation

- Full track information extracted directly from AIRES
  - Track start and end 4-vectors $(\vec{x}_i, ct_i)$ and $(\vec{x}_f, ct_f)$
  - Average energy of track is used for field calculations
  - Particle type ($e^{\pm}, \mu^{\pm}, \pi^{\pm}, p, \bar{p}$) and particle weight.
- Uses original ZHS field calculation algorithms and Fresnel extensions.
  - Time and frequency domains, Fresnel or Fraunhoffer
  - Takes refractive index n of medium into account
  - Adjustable sampling time bin width ΔT (time-domain)
- Can be used to calculate field for any primary particle
- Calculates the resulting field due to all emission mechanisms

Preliminary Results ZHAireS: signal

proton $1PeV$ at Auger site. $t = 0$ at first interaction.



Preliminary Results ZHAireS: signal

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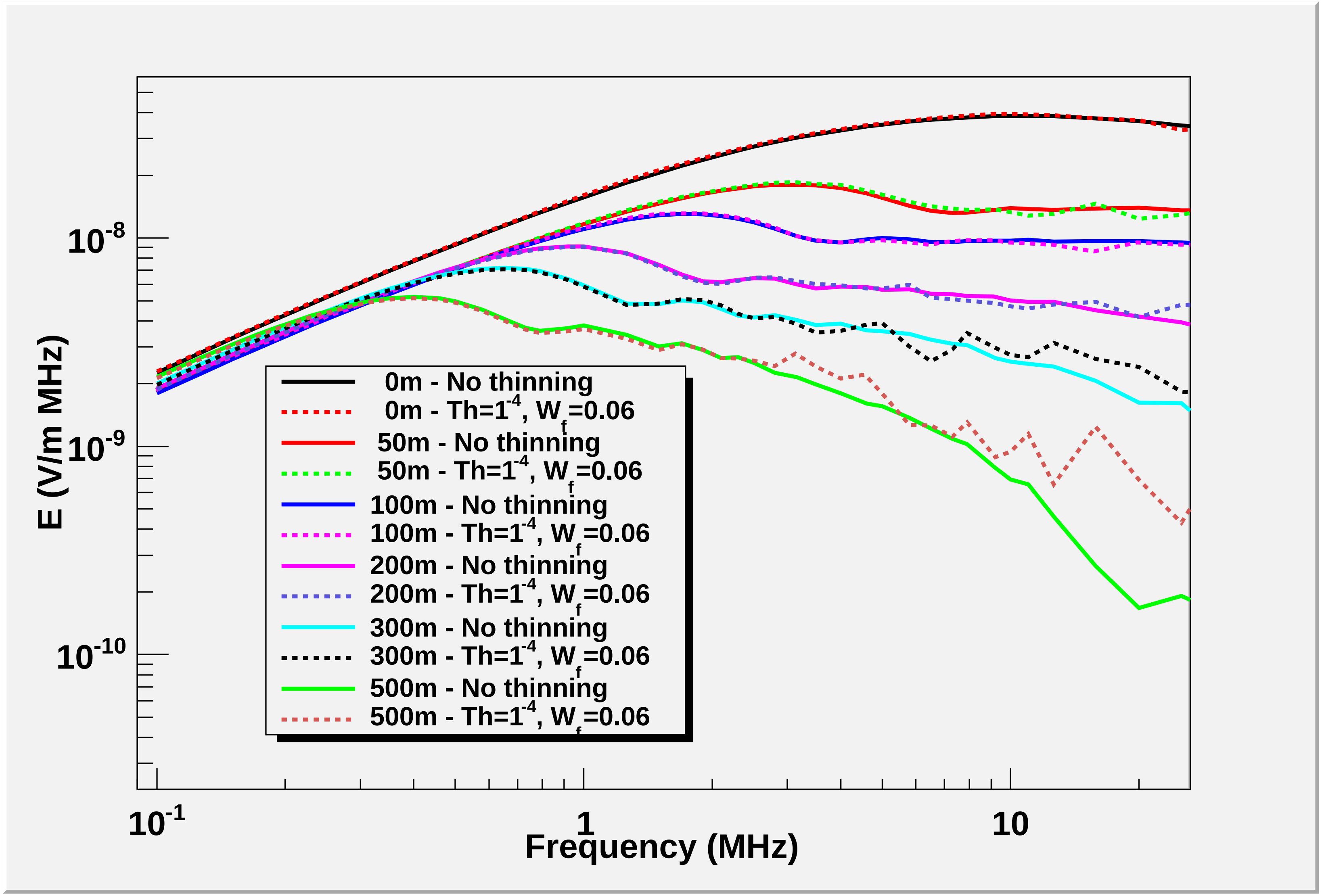
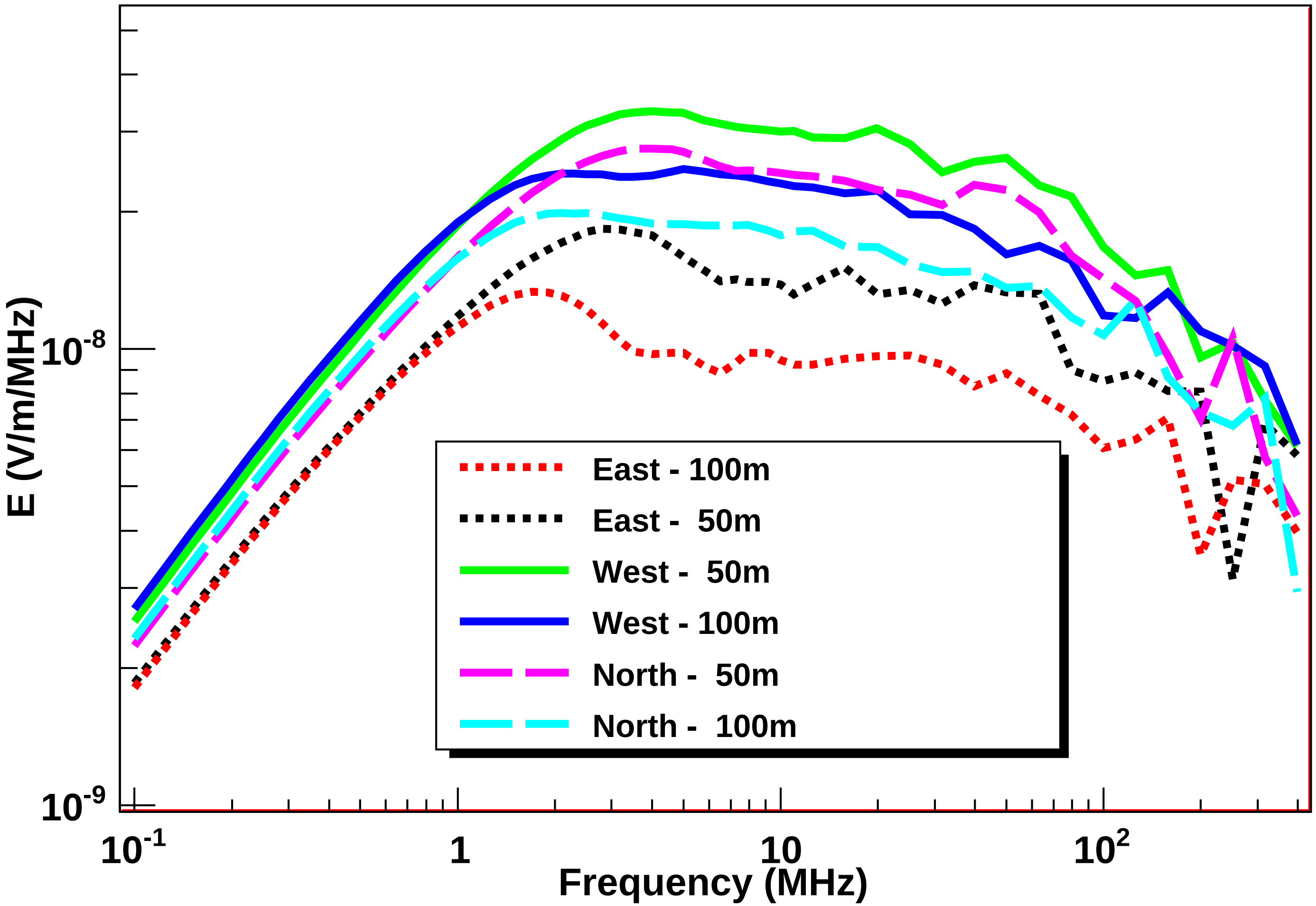


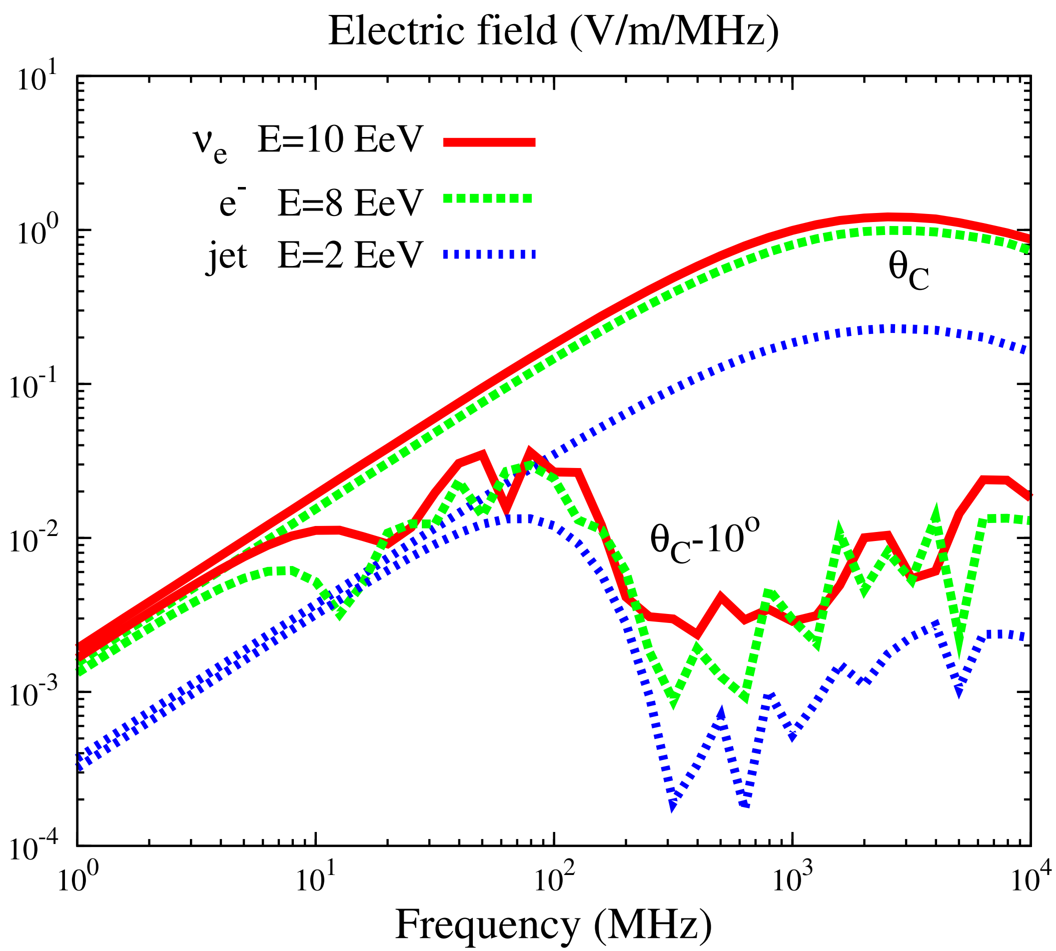
Preliminary Results ZHAireS: spectrum

proton $1PeV$ at Auger site.

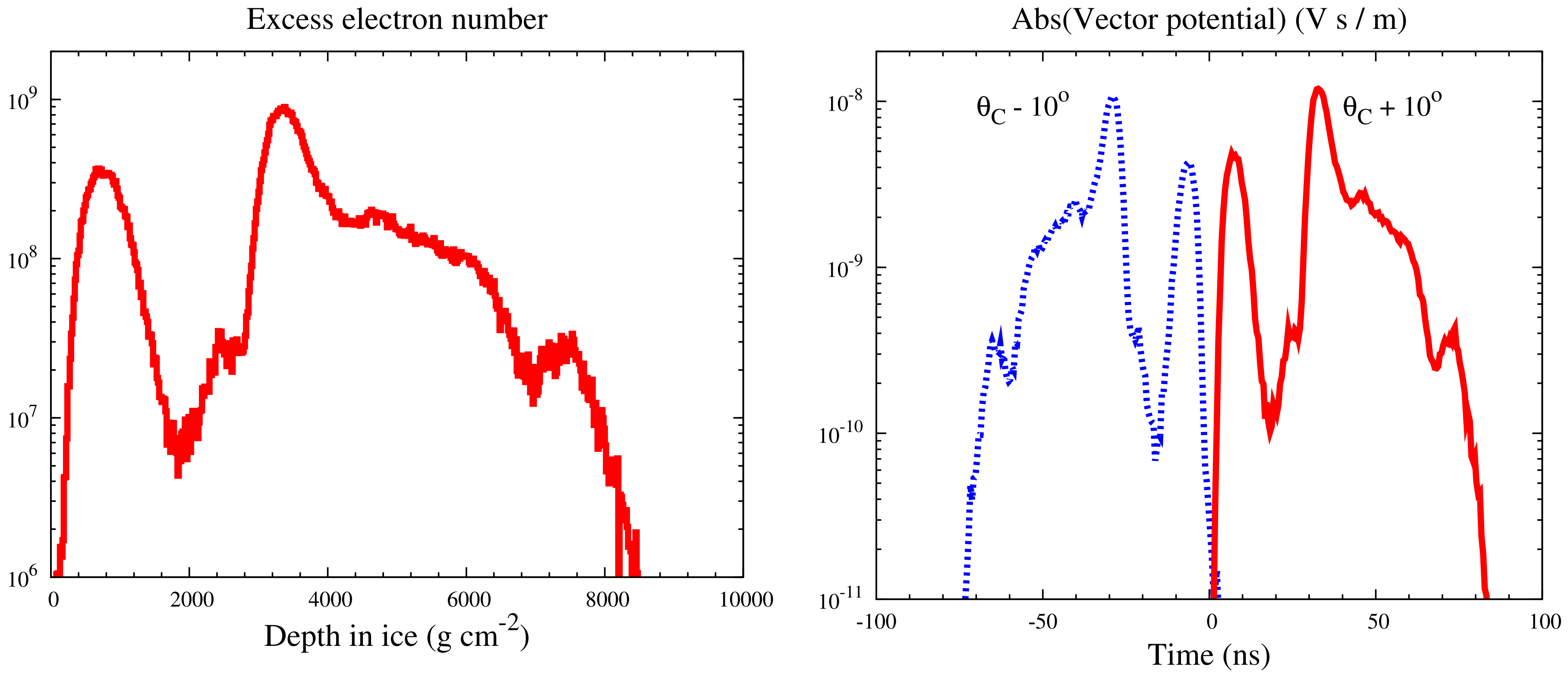


Preliminary Results ZHAireS: neutrinos in ice

$\nu_e 10EeV \rightarrow 20\% \;\; jet \quad 80\% e^-$ at Auger site.



Signal x Excess Profile



Conclusions and outlook

- Fresnel ZHAireS seems to be working
  - Time domain
  - Frequency domain
  - Fresnel or Fraunhoffer approximations
- Field very sensitive to geometry (EAS).
- Full paper on Fresnel extensions soon.

Questions?

Synchrotron: qualitative approach

- General formalism: no specific emission mechanism
  - ZHS in dense media: Askarian effect (Cherenkov)
  - But how does it behave in the case of synchrotron?

Feynman, Lecture on Physics, vol 2

$$t = \tau + \frac{R_0}{c} + \frac{z(\tau)}{c}, \; \frac{R_0}{c} = const \rightarrow ct = c\tau + z(\tau)$$

$$x'(t) = x(\tau), \qquad y'(t) = y(\tau). \qquad (34.4)$$

Now these are complicated equations, but it is easy enough to make a geometrical picture to describe their solution. This picture will give us a good qualitative feeling for how things work, but it still takes a lot of detailed mathematics to deduce the precise results of a complicated problem.

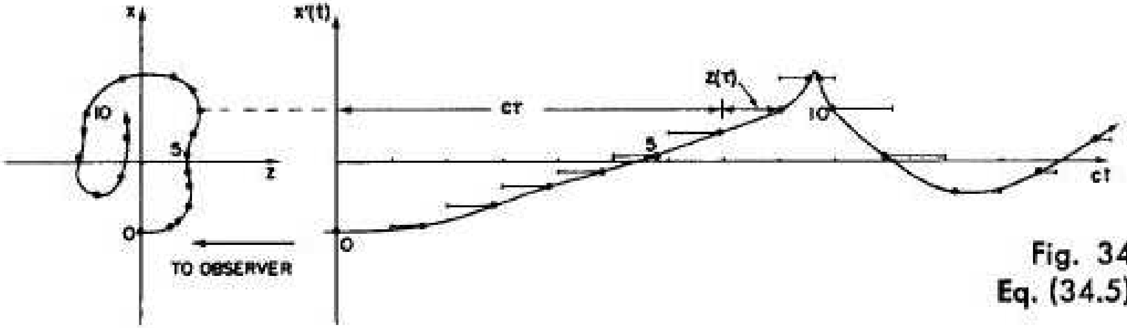


Fig. 34–2. A geometrical solution of Eq. (34.5) to find $x'(t)$.

cyclotron, synchrotron, “super” synchrotron