Detector Response to Low Energy Recoils

--Low Energy Calibration Workshop at U. of Chicago

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Important Issues with Calibrations

- Detector energy response functions to various physics processes
  - **Relative calibration (Common in all experiments)**
    - Can we make visible energy of nuclear recoils equivalent to visible energy of electronic recoils?
      - We must calibrate the average energy expended per e-h pair
  - **Absolute calibration (How and Is it needed?)**
    - How the energy is dissipated with respect to different incoming particles?
      - Alphas, neutrons, electrons, gamma rays, X-rays, muons, etc
- Detector energy resolution for various physics processes
  - **Do nuclear recoils have the same energy resolution as electronic recoils?**
    - If noise does not dominate energy resolution for a given detector, is the internal statistical variation (Fano factor) the same for all physics processes? We must measure Fano factor for low energy recoils

Outline

- Energy partition in energy Loss
  - Energy Loss
  - Signal generation
  - Average Ionization energy
  - Fano Factor
- Energy response to low energy recoils
  - Relative calibration vs absolute calibration
- Impact of Energy Resolution on discrimination of nuclear recoils
  - Fano Factor for electronic recoils and nuclear recoils
- Conclusion

Energy Loss

- Ionization mechanism
  - Two types of energy loss of a charged particle in matter
    1) **Excitation:** $p + X \rightarrow p + X^*$

       $\sigma \approx 10^{-17} cm^2$, exact resonant energy required
    2) **Ionization:** $p + X \rightarrow e^- + X^+ + p$

       $\sigma \approx 10^{-16} cm^2$, not exact energy required
  - Primary Ionization and secondary ionization
    - **Primary ionization: 2)**
    - **Secondary ionization:** Sufficient energy is transferred to the electrons (delta-rays) so that it creates electron-ion pairs itself

Signal Generation

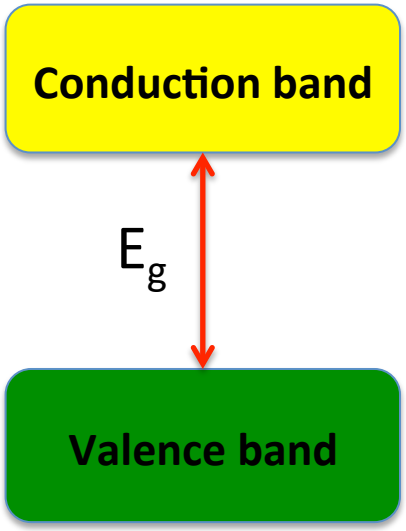
- An electrical signal is generated by ionization with formation of electron-ion (e-h) pairs **(Solid state detectors)**
- Excitation of optical states **(Scintillators)**
  - Direct excitation
  - Recombination
- Excitation of lattice vibrations **(Phonons)**
- Breakup of Cooper pairs **(Superconductors)**

Average Ionization Energy

- Energy required for creation of e-h pairs
  - Ionization energy > Band gap energy
  - Formations of e-h pairs require both
    - Conservation of energy
    - Conservation of momentum

    e-h pairs + phonons

    $$\varepsilon_i = E_g + 2E_{e-h} + \alpha\varepsilon_x$$
- Band gap energy and phonon energy
  - Temperature dependent
  - E_g = 0.73 eV at 77 K for Ge



ε_i = 2.96 eV at 77 K for Ge , only ~25% energy loss produces e-h pairs

Energy Partition in Energy Loss

- Average ionization energy > Band gap – because conservation of momentum requires excitation of phonons
  - $\varepsilon_i > E_g$

Only a fraction of energy is visible!!!

- Upon deposition of E_0, two excitations are possible
  - Lattice excitation with no formation of mobile charge
    - N_x excitations produce N_p phonons of energy E_x
  - Ionization with formation of a mobile charge pair
    - N_i ionizations form N_Q charge pairs of energy E_i

In other words:

$$E_0 = E_i N_i + E_x N_x$$

The total differential: $$dE_0 = \frac{\partial E_0}{\partial N_i} dN_i + \frac{\partial E_0}{\partial N_x} dN_x = 0$$

Thus, $$E_i \Delta N_i + E_x \Delta N_x = 0$$

For a given event, energy deposition is partitioned between charge formation and excitation of phonons, more energy goes into charge creation and less energy will be available for excitation

The variance from averaging over many events: $$E_i \sigma_i = E_x \sigma_x$$

With $\sigma_x = \sqrt{N_x}$ (assuming Gaussian statistics)

Thus, $$\sigma_i = \frac{E_x}{E_i} \sqrt{N_x}$$

From the total energy

$$E_0 = E_i N_i + E_x N_x$$

It follows

$$N_x = \frac{E_0 - E_i N_i}{E_x}$$

$$\sigma_i = \frac{E_x}{E_i} \sqrt{\frac{E_0}{E_x} - \frac{E_i}{E_x} N_i}$$

The number of charge pairs that contribute to the signal

$$N_i = N_Q = \frac{E_0}{\varepsilon_i}$$

and:

$$\sigma_i = \frac{E_x}{E_i}\sqrt{\frac{E_0}{E_x} - \frac{E_i}{E_x}\frac{E_0}{\varepsilon_i}} = \sqrt{\frac{E_0}{\varepsilon_i}}\sqrt{\underline{\frac{E_x}{E_i}\left(\frac{\varepsilon_i}{E_i} - 1\right)}}$$

Fano Factor?

Recall, the variance in signal charge Q is given by :

$$F = \frac{\text{Observed variance in N}}{\text{Poisson predicted variance}}$$

$$\sigma_Q = \sigma_i = \sqrt{FN_i} = \sqrt{F\frac{E_0}{\varepsilon_i}}$$

Thus, we find:

$$\boxed{F = \sqrt{\frac{E_x}{E_i}\left(\frac{\varepsilon_i}{E_i} - 1\right)}}$$

Fano Factor *F*

For Ge at 77 K:

$E_x = 0.0066$ *eV*

$E_i = E_g = 0.73$ *eV*

$\varepsilon_i = 2.96$ *eV*

$F = 0.166$

Experimental value is ~0.13

Do nuclear recoils have the same Fano Factor as electronic recoils?

Energy Response to Low Energy Recoils - Germanium Detectors

- Measurable quantity
  - Charge only – generic detectors
  - Both charge and phonons – SuperCDMS type
- Energy Response

Linear response for electronic recoils: $E_{vis} = \alpha E_R$

The total differential: $\frac{dE_{vis}}{dx} = \alpha \frac{dE_R}{dx}$

$$\frac{dE_{vis}}{dx} = \frac{\alpha \frac{dE_R}{dx}}{1 + \beta \frac{dE_R}{dx}}$$

If any primary e-h pairs or excitations are lost due to high ionization density – Birks' Law

Both α and β can be measured using electronic recoils – Wenzhao Wei's presentation for details

Multiply *dx* on both sides and integrating by parts:

$$E_{vis} = \frac{\alpha E_R}{1+\beta \frac{dE_R}{dx}}$$

Electronic recoils:

If: $\beta \frac{dE_R}{dx} << 1$

$$\frac{E_{vis}}{E_R} \approx \alpha$$

Measurable energy: $E_{vis} = \frac{E_R}{\varepsilon_i} \times E_g$ $\quad \alpha = \frac{E_g}{\varepsilon_i}$

If: $\beta \frac{dE_R}{dx}$ is significant

$$\frac{E_{vis}}{E_R} = \frac{\alpha}{1+\beta \frac{dE_R}{dx}}$$

Very low energy recoils with large stopping power, non-linear response

Nuclear recoils:

$$E_{eff} = E_R \times \eta$$

Ionization efficiency

$$E_{vis} = \frac{E_{eff}}{\varepsilon_i} \times E_g$$

$$E_{vis} = \frac{\alpha}{1+\beta \frac{dE_{eff}}{dx}} E_{eff} = \boxed{\frac{\alpha}{1+\beta \frac{dE_{eff}}{dx}}} E_R \times \eta$$

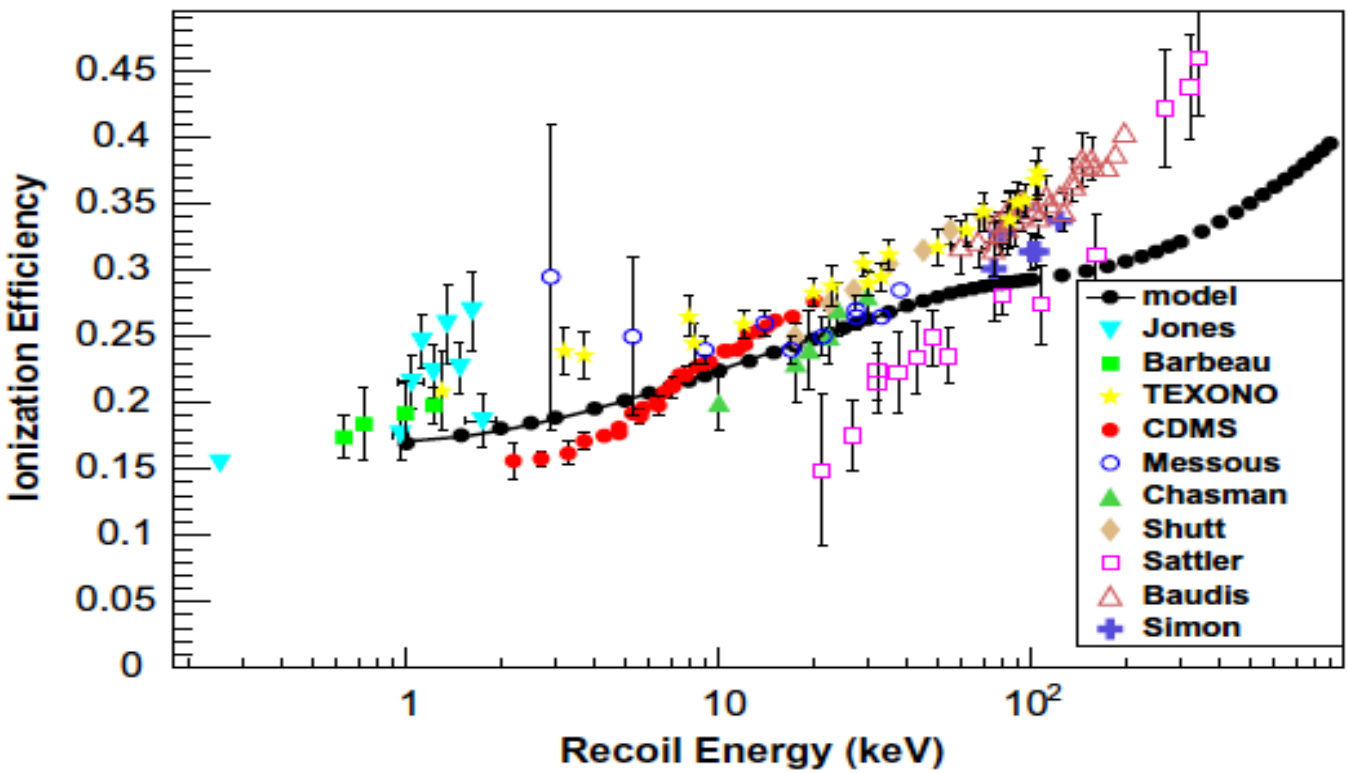
Additional ionization efficiency

Relative Ionization Efficiency:

(1) D. Barker and Mei: Astroparticle Physics 38 (2012) 1-6

$$\varepsilon_c = \frac{0.14476 \cdot E_r^{0.697747}}{-1.8728 + exp[E_r^{0.211349}]}$$

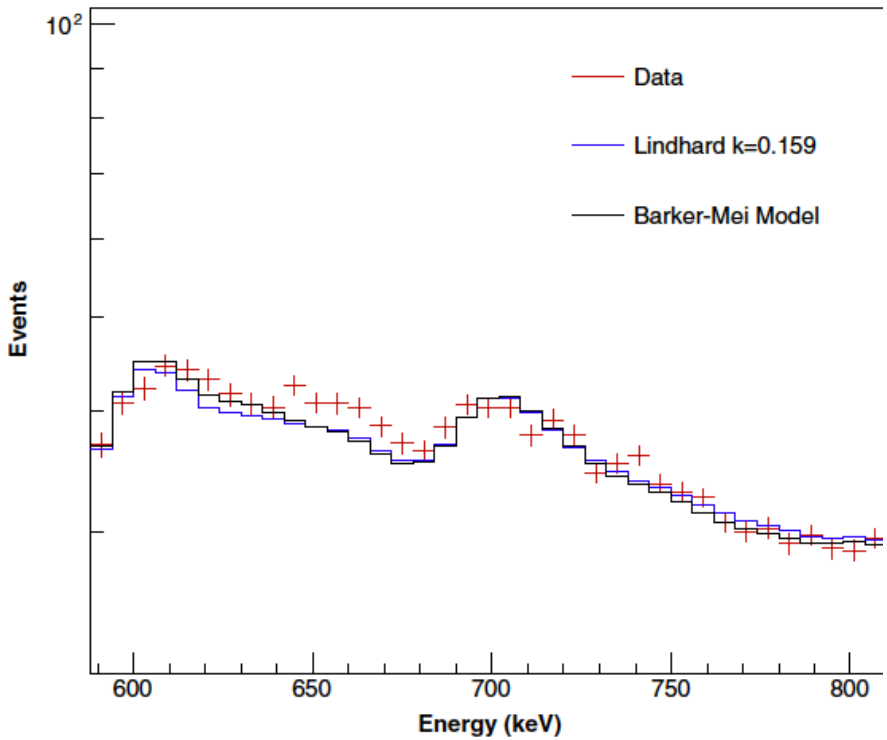
Corrected for phonon-electron coupling

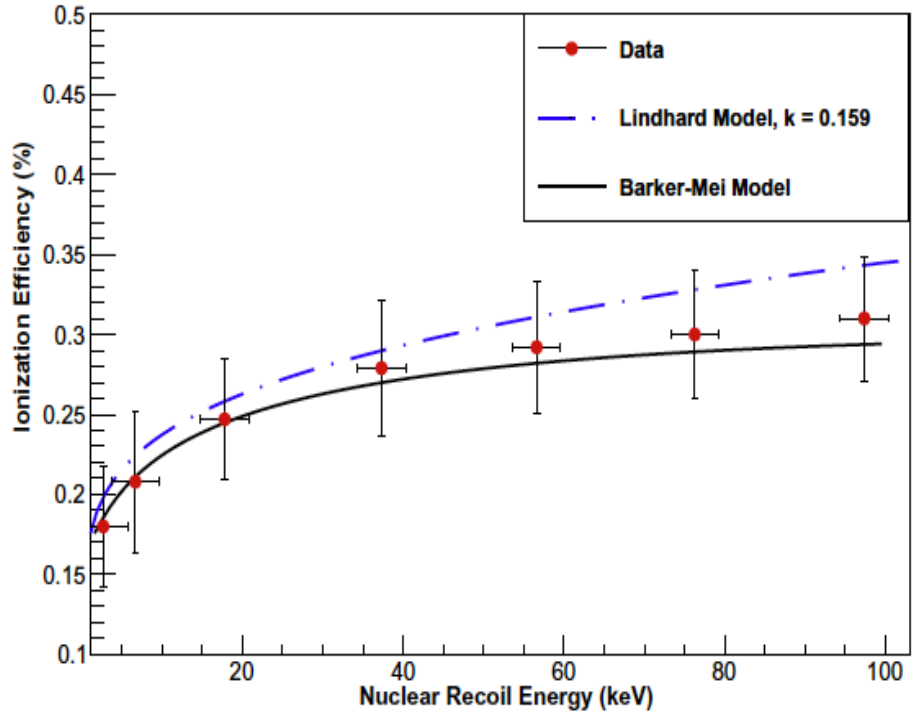


Relative Ionization efficiency:

(2) D. Barker, W.-Z. Wei, D.-M. Mei, C. Zhang
Astroparticle Physics 48 (2013) 8-15

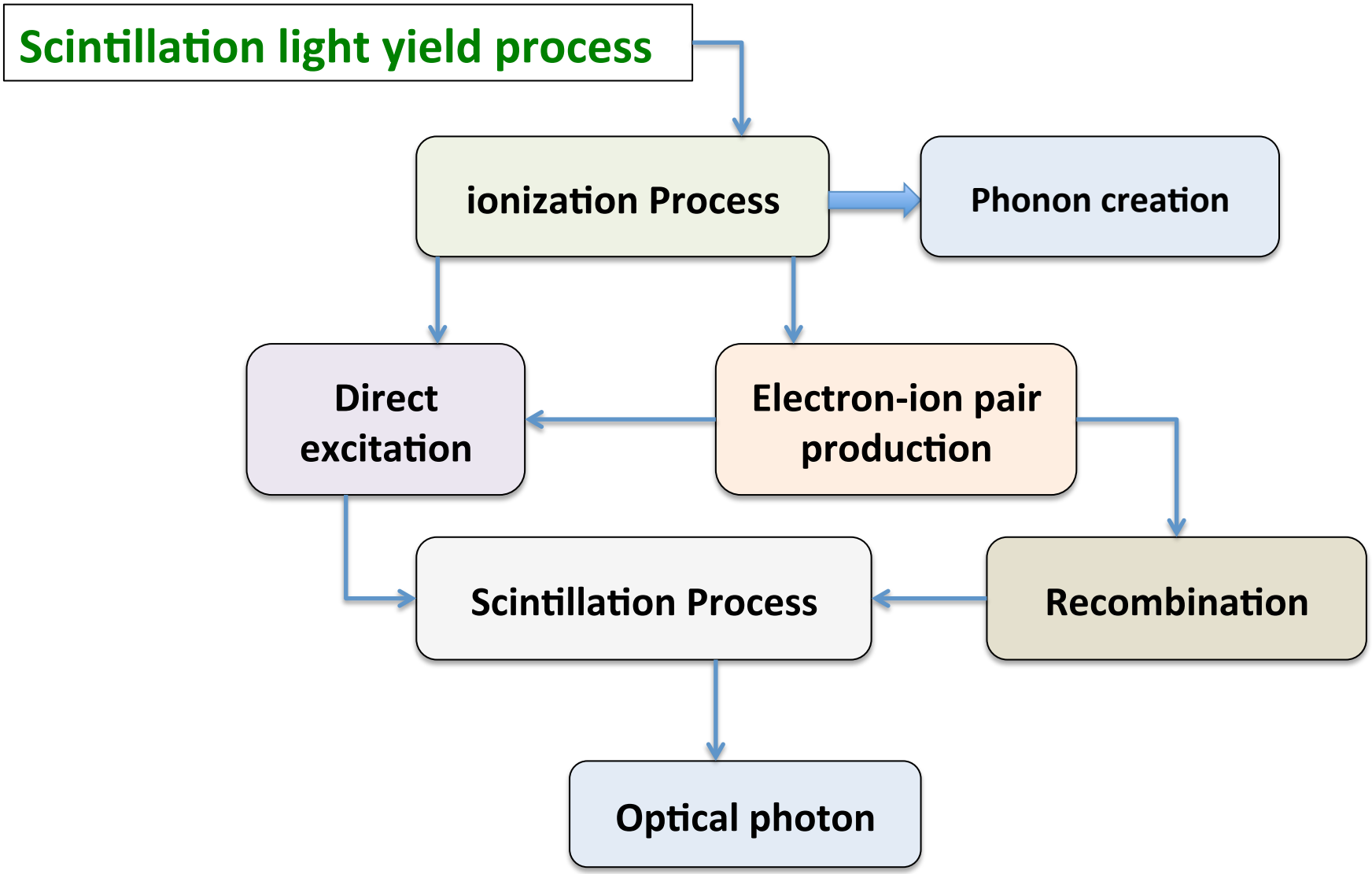
Verification of Model using E0 transition





Though a relative calibration is OK in the case of Ge, however, an absolute calibration is preferred. See Wenzhao's Wei's presentation

Energy Response to Low Energy Recoils - Scintillation Detectors



Ionization process: Energy partition in the energy loss process is similar to Ge detector

$$E_{eff} = E_R \times \eta$$

→ **Ionization efficiency**

E_{eff}: Effective energy creates e-h pairs and direct excitation for optical photon production

η: Lindhard's model describes the portion of energy loss to phonon generation, which is energy dependent

- see Lu Wang's presentation for details

Scintillation Process:

Electronic recoils:

$$E_{eff} = E_R$$

Nuclear recoils:

$$E_{eff} = E_R \times \eta$$

- Measurable quantity
  - Optical photons
    - Emission processes
      - Direct excitation – ionization without formation of charge pairs
      - recombination – ionization with formation of charge pairs
- Energy response

For ideal scintillator and low ionization density, Luminescence ~ Energy dissipated in scintillator

$$L = SE_{eff}$$

Or in the differential form:

$$\frac{dL}{dx} = S\frac{dE_{eff}}{dx}$$

The specific density of ionized and excited molecules along the particle track is

$$B\frac{dE_{eff}}{dx}$$

Assume that a portion of the primary excitation is lost at high ionization density (ionization quenching) and introduce a quenching parameter *k*. Then

$$\frac{dL}{dx} = \frac{S\frac{dE_{eff}}{dx}}{1 + kB\frac{dE_{eff}}{dx}}$$

kB: Birks' constant

For a given energy dissipated in scintillator

$$v = \frac{S}{1 + kB\frac{dE_{eff}}{dx}}$$

If $kB\frac{dE_{eff}}{dx} << 1$ $\qquad \frac{dL}{dx} = S\frac{dE_{eff}}{dx}$

If $kB\frac{dE_{eff}}{dx}$ is significant, luminescence is quenched

Relative Calibration:

Electronic Recoils:

$$E_{eff} = \alpha E_R \qquad \alpha = 1$$

Nuclear Recoils:

$$L_{eff} = \eta \times \mu$$

$$E_{eff} = L_{eff} \times E_R \qquad L_{eff} = \frac{\# N_{Pe}^{NR} / E_{NR}}{\# N_{pe}^{ER} / E_{ER}}$$

$$E_{ER} = 122 keV$$

η: Lindhard, μ: Hitachi

Absolute Calibration:

Electronic Recoils:

$$\frac{E_{eff} \times E_g}{W_i} = \frac{\alpha E_R}{1 + kB \frac{dE_{eff}}{dx}}$$

E_g: Band gap energy

W_i: Average energy required per e-h pair

Nuclear recoils:

η: Lindhard's function

$$\frac{E_{eff} \times E_g}{W_i} = \boxed{\frac{\alpha}{1 + kB \frac{dE_{eff}}{dx}}} \times E_R \times \eta = \nu \times E_R \times \eta$$

ν

$$\frac{dE_{eff}}{dx}$$: **Electronic stopping power in nuclear recoils**

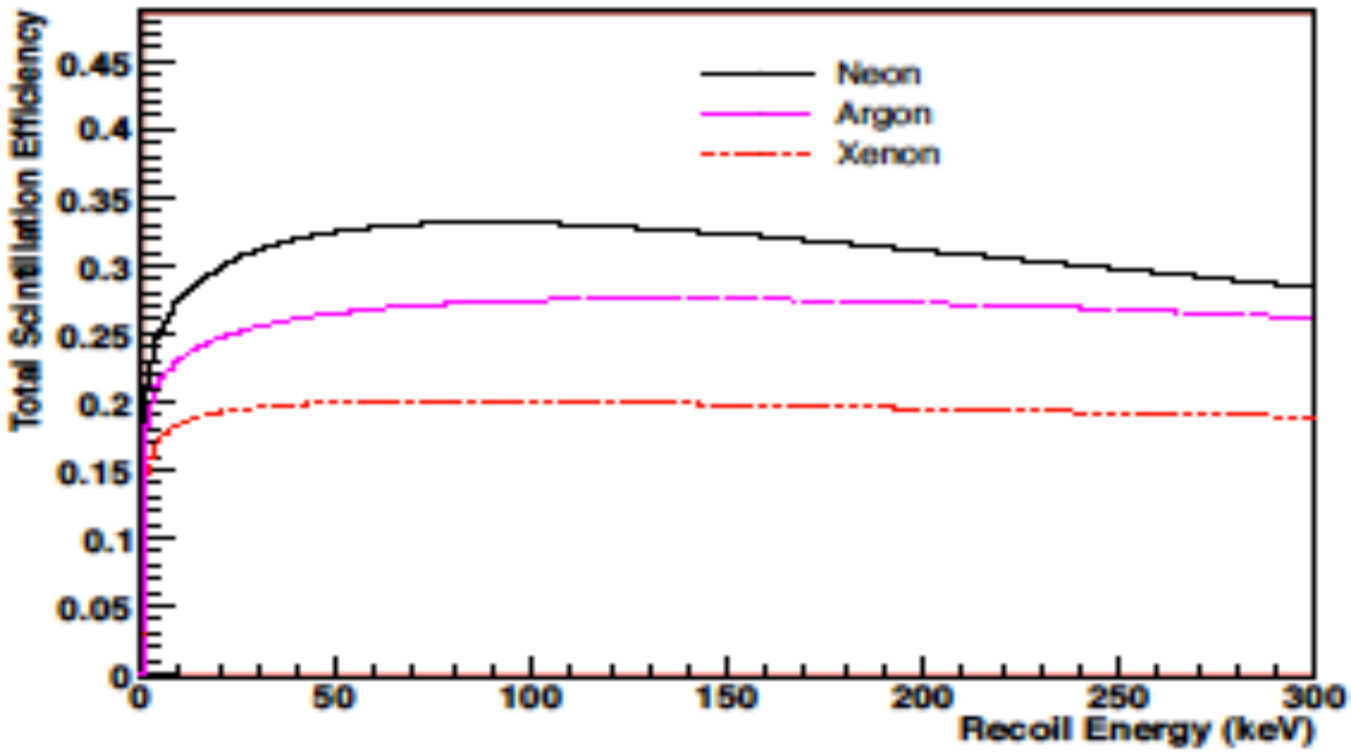
See Lu Wang's presentation Fro details

Total scintillation efficiency:

D.-M. Mei, Z.-B. Yin, C. Stonehill, A. Hime, Astroparticle Physics 30 (2008) 12-17.

$$\varepsilon_{tot} = \eta \times f_l$$

If an absolute calibration today: $f_l = \alpha \times \nu$



Impact of Fano Factor on Discrimination of Nuclear Recoils

Germanium Detectors at 50 mK:

Fano factor:

E_x: 1.242x10^3 eV

ε_i: 8.9 eV for ER

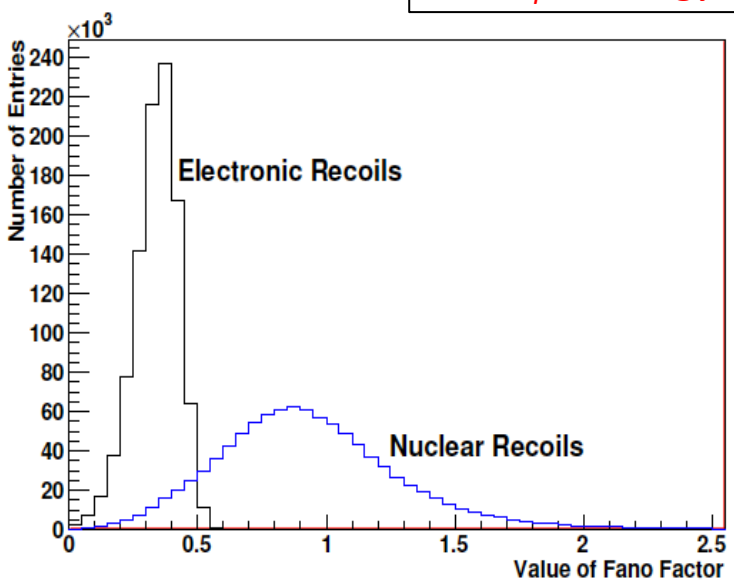
$$F = \sqrt{\frac{E_x}{E_g}\left(\frac{\varepsilon_i}{E_g} - 1\right)}$$

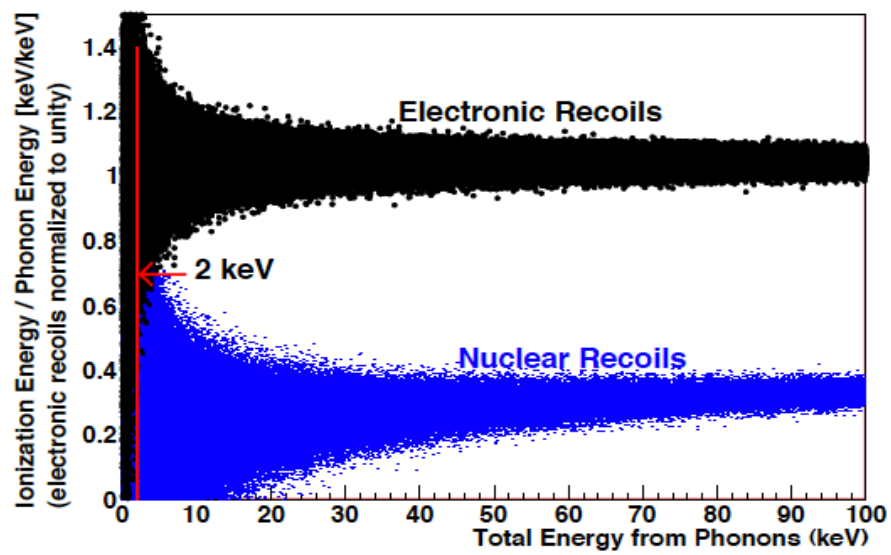
NR: *ε_i is energy dependent*

Ionization yield vs total energy

$$\sigma = \sqrt{\varepsilon_i F E_R}$$

Assuming Gaussian statistics





Impact of Fano Factor on Discrimination of Nuclear Recoils

Xenon Detectors at 165 K

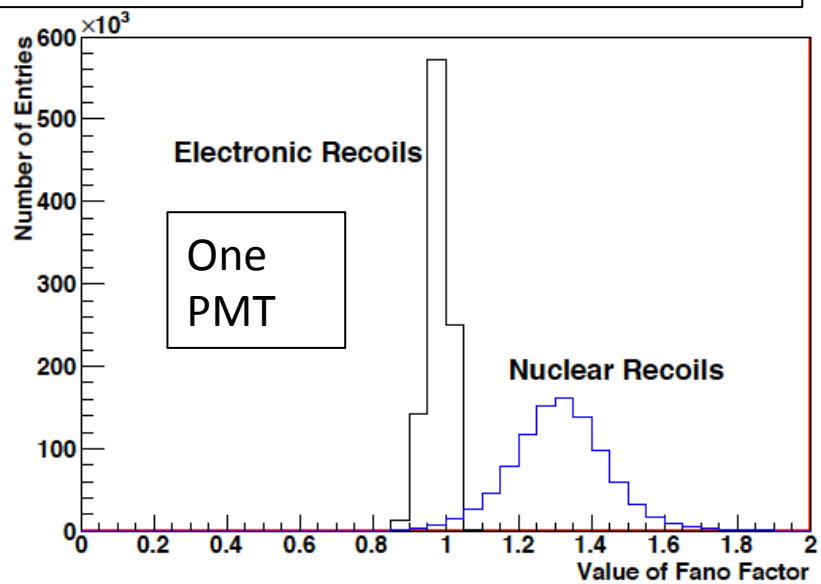
Fano factor: $$F_n = \sum_{i=1}^{m} (1 + \varepsilon(F_N - 1))$$

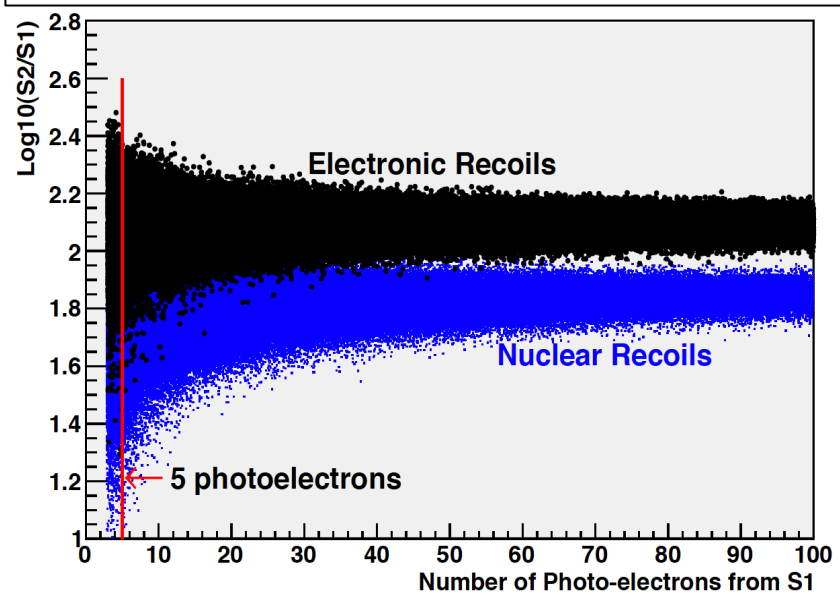
F_N: 0.059 Doke et al., 1976, Nucl. Instrum. Methods 134, 353.

ε: Photoelectron detection efficiency

Abdelkader Bousselham, Harrison H.Barrett, Vaibhav Bora, Kanai Shah, Nucl. Instrum. Methods in Phys. A 620 (2010) 359 - 362.

Extraction efficiency: 0.65 and multiplication factor is 25, without considering saturation in S2. W-value is 15.6 eV per e-h pair.





Conclusion

- **Energy response functions are different for different physics processes**
  - The detectable energy is only fraction of recoil energy and its large portion goes to phonon generation
  - Phonon creation is physics process and temperature dependent
  - Average energy per e-h pair is temperature dependent and also energy dependent for nuclear recoils
  - Fano factor of nuclear recoil is energy dependent
  - A generalized formula, **ABSOLUTE CALIBRATION**, can be used to describe energy response to both electronic and nuclear recoils
  - kB or β can be measured with gamma-ray sources
- **Energy resolution is different for electronic and nuclear recoils**
  - Energy resolution must be calibrated separately
  - Fano Factor can impact the discrimination of nuclear recoils from electronic recoils in low energy region
    - Internal statistical variation dominates the width of electronic recoil band and nuclear recoil band
    - Discrimination capability vanishes when nuclear recoil energy below 2 keV

See presentations from Lu Wang and Wenzhao Wei for detailed information