Germanium Detector Response to Low Energy Recoils for Dark Matter Searches

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Low Energy Calibration Particle Detector Workshop
U. of Chicago, IL

September 23-25, 2015

Acknowledgement
DOE Funding: DE-FG02-10ER46709
NSF Funding: OIA-14342421
The State of South Dakota
Outline

- Ge Detector Energy Resolution Study
  - Experimental setup
  - Energy resolution
  - Fano Factor
  - Temperature dependence of Fano factor

- Ge Detector Energy Response Study
  - Ionization efficiency
  - Birks’ law in Ge detector
  - Relative calibration vs. Absolute calibration
Part 1: Ge Detector Energy Resolution Study
Data Acquisition: National Instruments PXI-1031 system and Igor Pro 4.07 software

Calibration Source:
- **Co-60**: 1173 keV and 1332 keV
- **Na-22**: 511 keV and 1275 keV
- **Cd-109**: 88 keV
- Background **K-40**: 1461 keV
- Background **Th-232**: 2614 keV
Energy Response

\[ \text{Energy} = 0.2331 \times \text{ADC} + 1.038 \]

<table>
<thead>
<tr>
<th>Energy (keV)</th>
<th>ADC Counts</th>
</tr>
</thead>
<tbody>
<tr>
<td>88</td>
<td>373</td>
</tr>
<tr>
<td>511</td>
<td>2188</td>
</tr>
<tr>
<td>1173</td>
<td>5029</td>
</tr>
<tr>
<td>1275</td>
<td>5458</td>
</tr>
<tr>
<td>1332</td>
<td>5713</td>
</tr>
<tr>
<td>1461</td>
<td>6260</td>
</tr>
<tr>
<td>2614</td>
<td>11210</td>
</tr>
</tbody>
</table>
Energy Resolution VS. Energy

Fitting Function: \( \sqrt{\frac{\sigma_{tot}^2 - \sigma_{noise}^2}{E}} \times 100 = 0.004712 \sqrt{\frac{462.8}{0.001E}} + 8.107 \times (0.001E)^{-9.364} \)

\( \sigma_{tot}^2 = \sigma_{noise}^2 + \sigma_{stat}^2 + \sigma_{in-ch}^2 \)

\( \sigma_{noise} \sim 2.5 \text{ keV}, \sigma_{stat} = \sqrt{\frac{F_E}{\varepsilon}} \), \( \sigma = \sqrt{F E \varepsilon} \)

\( F_{ER} \approx 0.19 \)
Fano Factor for NR

- Fano factor for NR is: \( F_{NR} = \sqrt{\frac{E_x}{E_g} \left( \frac{\varepsilon_{avg}^{NR}}{E_g} - 1 \right)} \) (See Dr. Mei’s slides for details)

Where, \( E_x \) is the average phonon energy;
\( \varepsilon_{avg}^{NR} \) is the average energy expended per e-h pair;
\( E_g \) is the band gap energy;

- \( F_{NR} \) is both temperature and energy dependent.

\( F_{NR} \) is quite different from \( F_{ER} \).
Temperature Dependence of $E_g$ and $\varepsilon_{avg}$

\[ E_g = 0.7437 - \frac{4.77 \times 10^{-4} T^2}{T+235} \text{[eV]}^{[1,2]} \]

\[ \varepsilon_{avg} = 2.2E_g + 1.99E_g^{3/2} \exp \left( \frac{4.75E_g}{T+1.975} \right) \text{[eV]}^{[2]} \]

<table>
<thead>
<tr>
<th>Temperature (°K)</th>
<th>50 mK</th>
<th>77 K</th>
<th>300 K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Band Gap Energy ($E_g$) (eV)</td>
<td>0.74</td>
<td>0.73</td>
<td>0.66</td>
</tr>
<tr>
<td>Avg. Energy Expended Per e-h Pair ($\varepsilon_{avg}$) (eV)</td>
<td>8.94</td>
<td>2.93</td>
<td>2.55</td>
</tr>
</tbody>
</table>

Part 2: Ge Detector Energy Response Study
Ionization Efficiency

- **Ionization efficiency** is defined as the ratio of the nuclear recoil visible energy using the electron-equivalent energy calibration to the true recoil energy.

- **D. Barker and D.-M. Mei: Astroparticle Physics 38 (2012) 1-6**

\[
\eta = \frac{0.14476 \cdot E_R^{0.697747}}{-1.8728 + \exp[E_R^{0.211349}]} \quad \text{(Barker-Mei Model)}
\]

- **New Model for Absolute Ionization Efficiency:**

\[
\varepsilon_{tot} = \frac{\alpha}{1 + \beta \frac{dE_{NR}^{eff}}{dx}} \times \eta
\]

Where, \(\alpha\) and \(\beta\) are constants at given temperature; \(E_R\) is recoil energy; This model works for Ge detector only and it’s valid when \(E_R\) is from 1 keV to 100 keV.
Ge Detector Energy Response

For ER, $E_{vis} = \alpha E_R$ (Linear Response)  

\[ E_{vis} = \alpha E_R \]  

----------------- (1)

The total differential: \[ \frac{dE_{vis}}{dx} = \alpha \frac{dE_R}{dx} \]  

----------------- (2)

The linear response will not hold if any primary e-h pairs or excitations are lost due to high ionization density (charge recombination). A correction to Eq. (2) can be made by using Birks’ law:

\[ \frac{dE_{vis}}{dx} = \frac{\alpha dE_R}{1 + \beta dE_R} \] , multiply dx on both sides and integrate by parts, we have:

\[ E_{vis} = \frac{\alpha E_R}{1 + \beta \frac{dE_R}{dx}} \]

with \[ E_{vis} = \frac{E_R}{\epsilon_{avg}} \times E_g \] , i.e. \[ E_{vis} = \frac{E_R}{\epsilon_{avg}} \times E_g = \frac{\alpha E_R}{1 + \beta \frac{dE_R}{dx}} \]  

----------------- (3)

where $\alpha$ and $\beta$ are both constants and can be measured through ERs; $\epsilon_{avg}$ is the average energy expended per e-h pair and $E_g$ is the bandgap energy; For Ge at 77K, $\epsilon_{avg}^{ER} \approx 2.96 \text{ eV}$, $E_g \approx 0.73 \text{ eV}$

Eq. (3) is an extension of Eq. (1) and can be linear or non-linear depending on $\beta \frac{dE_R}{dx}$. 
Determination of $\alpha$ and $\beta$

\[
\frac{dE_{\text{vis}}}{dx} = \frac{(0.2467 \pm 2.24 \times 10^{-4}) \frac{dE_R}{dx}}{1 + (5.98 \times 10^{-5} \pm 1.319 \times 10^{-4}) \frac{dE_R}{dx}}
\]

$\alpha = 0.2467$

$\beta = 5.98 \times 10^{-5} \text{ cm/MeV}$

88 keV, 511 keV, 1173 keV, 1275 keV and 1332 keV
Ge Detector Energy Response

- For ERs, $\beta \frac{dE_R}{dx} \ll 1$, then Eq. (3) becomes the linear case, which is Eq. (1).

  In this case, we have: $\alpha \approx \frac{E_{vis}}{E_R} \approx \frac{E_g}{E_{avg}} \approx \frac{0.73 \text{ eV}}{2.96 \text{ eV}} \approx 0.2466$

- However, for very low-energy NRs with large stopping power, $\beta \frac{dE_R}{dx}$ can be significant and hence cannot be ignored. So,

$$\frac{E_{eff}}{E_{avg}} \times E_g = \frac{\alpha}{1 + \beta \frac{dE_{eff}}{dx}} E_{eff}^{NR}$$  \hspace{1cm} (4)

Where, $E_{eff}^{NR} = \eta \times E_{NR}$ (\(\eta\) is the traditional ionization efficiency)  \hspace{1cm} (5)

$$\frac{E_{eff}}{E_{avg}} \times E_g = \frac{E_{eff}}{E_{avg}} \times E_g = \frac{E_{eff}}{E_{avg}} \times E_{g}^{NR} = E_{vis}^{NR}$$  \hspace{1cm} (6)

Plug (5) and (6) into (4), Eq. (4) becomes:

$$E_{vis}^{NR} = \frac{\alpha}{1 + \beta \frac{dE_{eff}}{dx}} \times \eta \times E_{NR}^{NR}, \text{ thus, the total ionization efficiency is:}$$

$$\varepsilon_{tot} = \frac{\alpha}{1 + \beta \frac{dE_{eff}}{dx}} \times \eta$$
New Model

\[ \varepsilon_{\text{tot}} = \frac{\alpha}{1 + \beta \frac{dE_{\text{eff}}}{dx}} \times \frac{0.14476 \cdot E_R^{0.697747}}{-1.8728 + \exp[E_R^{0.211349}]} \]

Where, \( \alpha \approx \frac{E_g}{\varepsilon_{\text{avg}}} \) (Temperature dependent) and \( \beta = 5.98 \times 10^{-5} \text{ cm/MeV} \)

At 77K, \( \alpha \approx \frac{0.73 \text{ eV}}{2.96 \text{ eV}} \approx 0.2466 \)

For SuperCDMS at 50mK, \( \sigma_E = \sqrt{(0.293)^2 + (0.056)^2 E} \) (keV) for \( E < 10 \text{ keV} \)

So, \( \sigma_{\text{stat}}^2 = (0.056)^2 E \), or \( \sqrt{FE\varepsilon_{\text{avg}}^2} = (0.056)^2 E \), or \( \varepsilon_{\text{avg}} F \times 10^{-3} = 0.056^2 \) \quad \text{(1)}

Also, we have: \( F_{ER} = \frac{E_x}{\sqrt{E_g}} \left( \frac{\varepsilon_{\text{avg}}}{E_g} - 1 \right) \) \quad \text{(2)}

For SuperCDMS, \( E_x \approx 0.00828 \text{ eV} \) with a frequency of 2 THz; \( E_g \approx 0.74 \text{ eV} \) at 50mK;

Eqs. (1) and (2) will yield: \( \varepsilon_{\text{avg}} \approx 8.94 \text{ eV} \) and \( F_{ER} \approx 0.36 \)

Hence, at 50mK, \( \alpha \approx \frac{0.744 \text{ eV}}{8.94 \text{ eV}} \approx 0.08277 \)

**CoGeNT VS. SuperCDMS**

**For CoGeNT (T = 77K):**
- $E_{R}^{\text{eff}} = 0.5$ keV
- $E_g = 0.73$ eV
- $\varepsilon_{\text{avg}}^{ER} = 2.96$ eV
- $\alpha = 0.2467$
- $\beta = 5.98 \times 10^{-5}$ cm/MeV
- $\frac{dE_{R}^{NR}}{dx} \approx 21.519$ MeV/cm

**CoGeNT result:** 2.27 keV

**For SuperCDMS (T = 50 mK):**

**ER:** $P_t = E_R (1 + \frac{eV_b}{\varepsilon_{\text{avg}}^{ER}})$

**NR:** $P_t = E_R (1 + \frac{eV_b}{\varepsilon_{\text{avg}}^{ER}} Y(E_R))$

At $V_b = 69$ V, if $\varepsilon_{\text{avg}}^{ER} \approx 3$ eV, then:

**ER:** $P_t = 24 \times E_R$

**NR:** $P_t = E_R (1+23 Y(E_R))$

SuperCDMS claims that $E_R \approx 2.214$ keV (energy threshold), which corresponds to $P_t \approx 11.814$ keV.

However, if $\varepsilon_{\text{avg}}^{ER} \approx 8.94$ eV, at 69 V:

**ER:** $P_t = 8.718 \times E_R$

**NR:** $P_t = E_R (1+7.718 Y(E_R))$

Given that $P_t \approx 11.814$ keV, then the energy threshold is $E_R \approx 4.515$ keV, which is a factor of $\sim 2.04$ bigger than what SuperCDMS claims.
Relative Calibration VS. Absolute Calibration

![Graph showing the comparison between Relative Calibration and Absolute Calibration](image)

- **Relative Calibration**
- **Absolute Calibration**
Comparison with Absolute Calibration

[Graph showing comparison of absolute calibration at 77K and 50mK]
Conclusions

- The relationship between energy resolution and Fano factor has been studied, which shows that ER and NR have different energy resolution and Fano factor has impact on the discrimination of ER/NR.
- Fano factor should be measured for low energies since Fano factor can be used to determine the average energy expended per e-h pair.
- The average energy expended per e-h pair is temperature dependent.
- A new model of absolute ionization efficiency has been proposed by studying the physical process involved in energy response in Ge.
- A relative calibration agrees with our absolute calibration if the Ge detector energy response is completely linear.
Thank You!

Any Questions??