Germanium Detector Response to Low Energy Recoils for Dark Matter Searches

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Outline

- Ge Detector Energy Resolution Study
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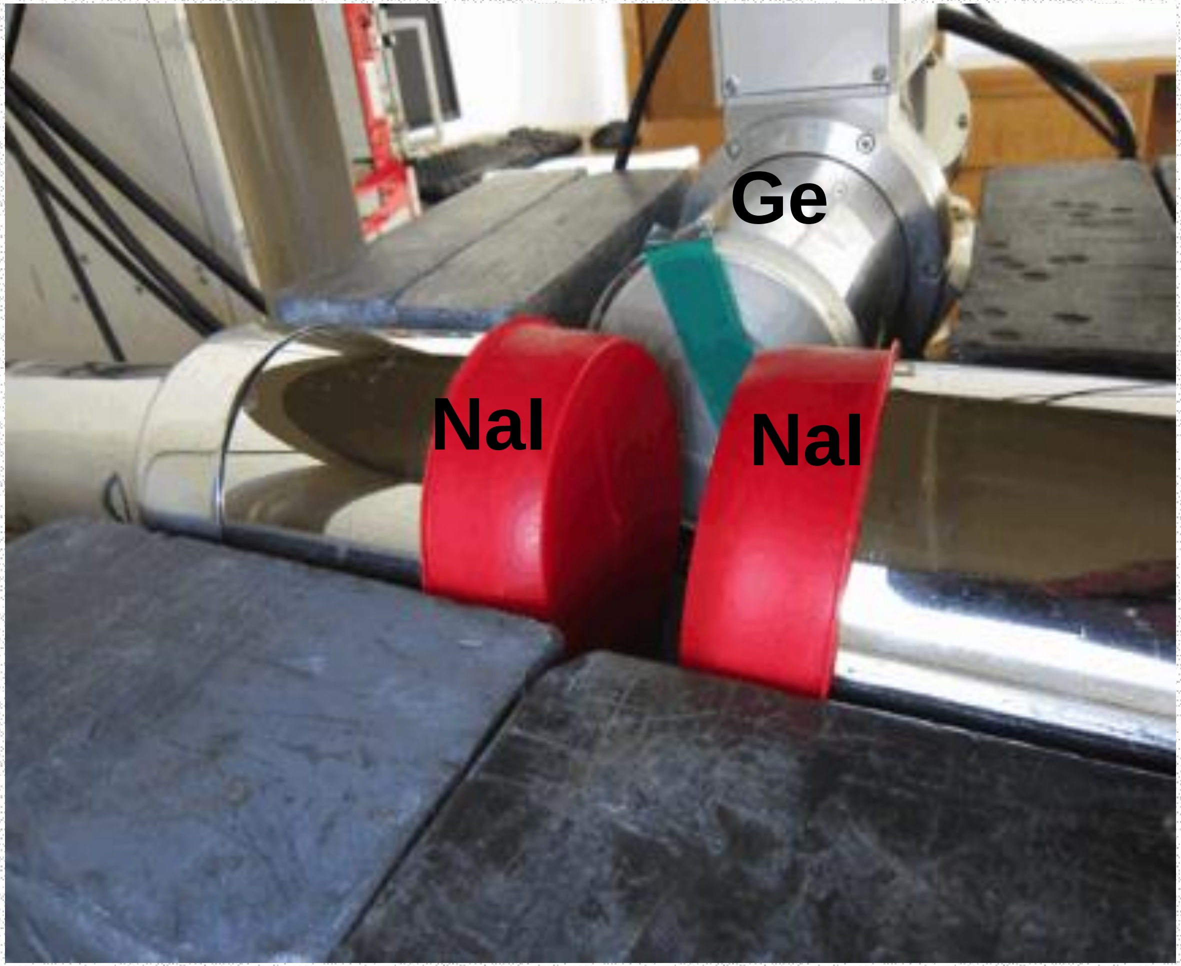
  - Energy resolution
  - Fano Factor
  - Temperature dependence of Fano factor
- Ge Detector Energy Response Study
  - Ionization efficiency
  - Birks' law in Ge detector
  - Relative calibration vs. Absolute calibration

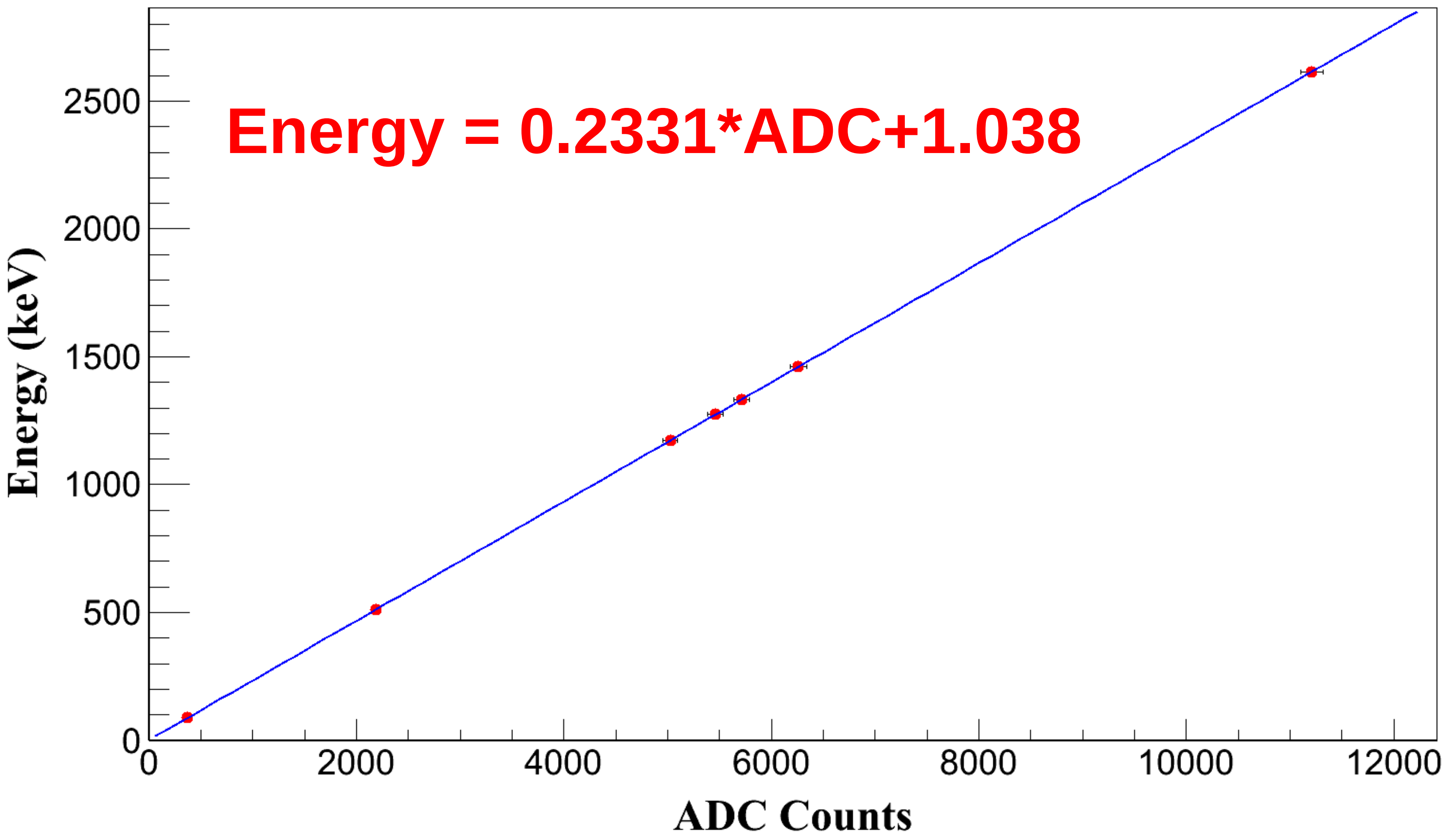
Part 1: Ge Detector Energy Resolution Study

Experimental Setup



- **Data Acquisition:** National Instruments PXI-1031 system and Igor Pro 4.07 software
- **Calibration Source:**
  Co-60: 1173 keV and 1332 keV
  Na-22: 511 keV and 1275 keV
  Cd-109: 88 keV
  Background **K-40**: 1461 keV
  Background **Th-232**: 2614 keV

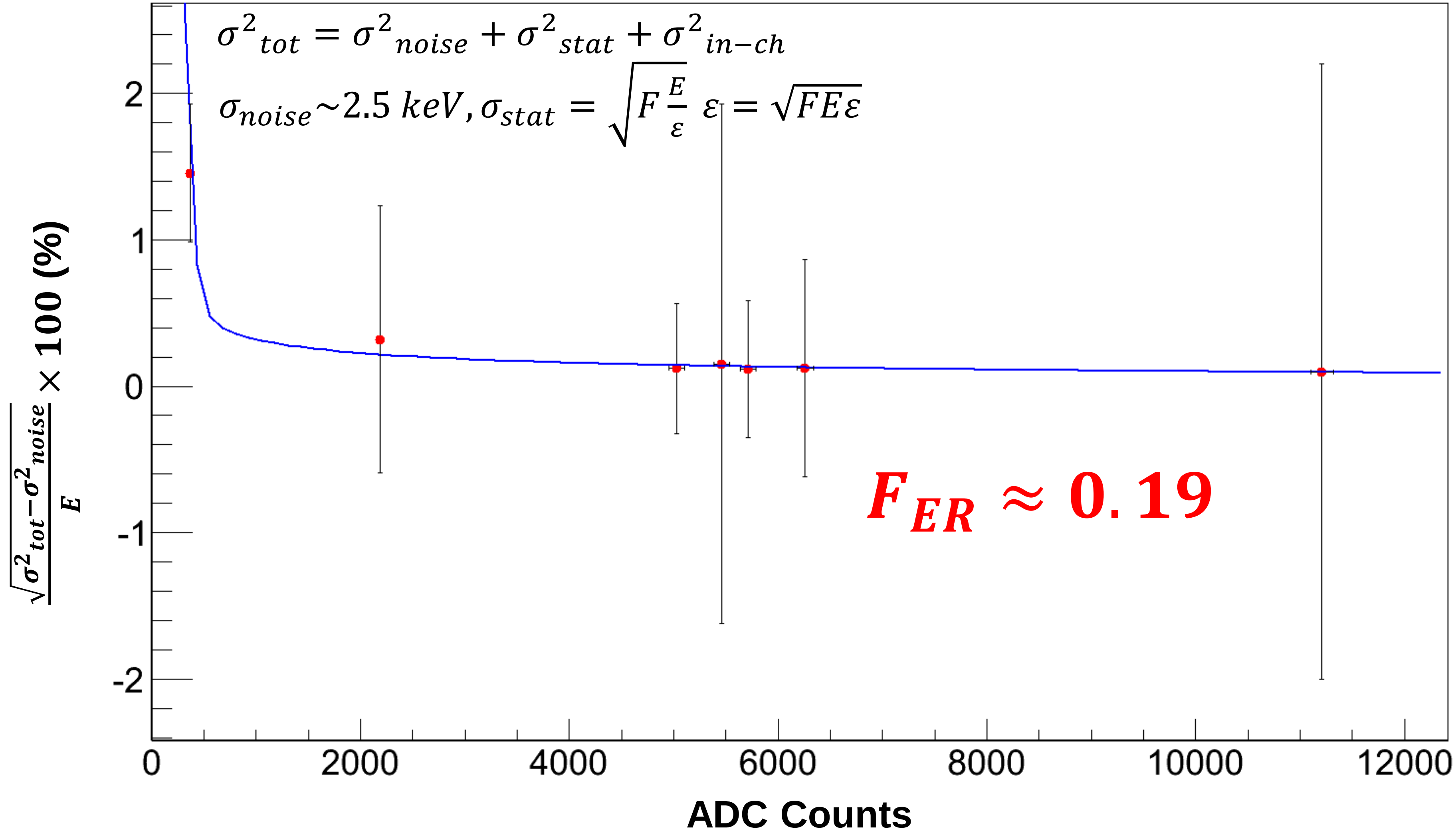
Energy Response



Energy (keV)	ADC Counts
88	373
511	2188
1173	5029
1275	5458
1332	5713
1461	6260
2614	11210

Energy Resolution VS. Energy

Fitting Function: $\frac{\sqrt{\sigma^2{}_{tot}-\sigma^2{}_{noise}}}{E}\times 100 = 0.004712\sqrt{\frac{4628}{0.001E}+8.107\times(0.001E)^{-9.364}}$



Fano Factor for NR

- Fano factor for NR is: $F_{NR} = \sqrt{\frac{E_x}{E_g}\left(\frac{\varepsilon_{avg}^{NR}}{E_g} - 1\right)}$ (See Dr. Mei's slides for details)

  Where, E_x is the average phonon energy;

  ε_{avg}^{NR} is the average energy expended per e-h pair;

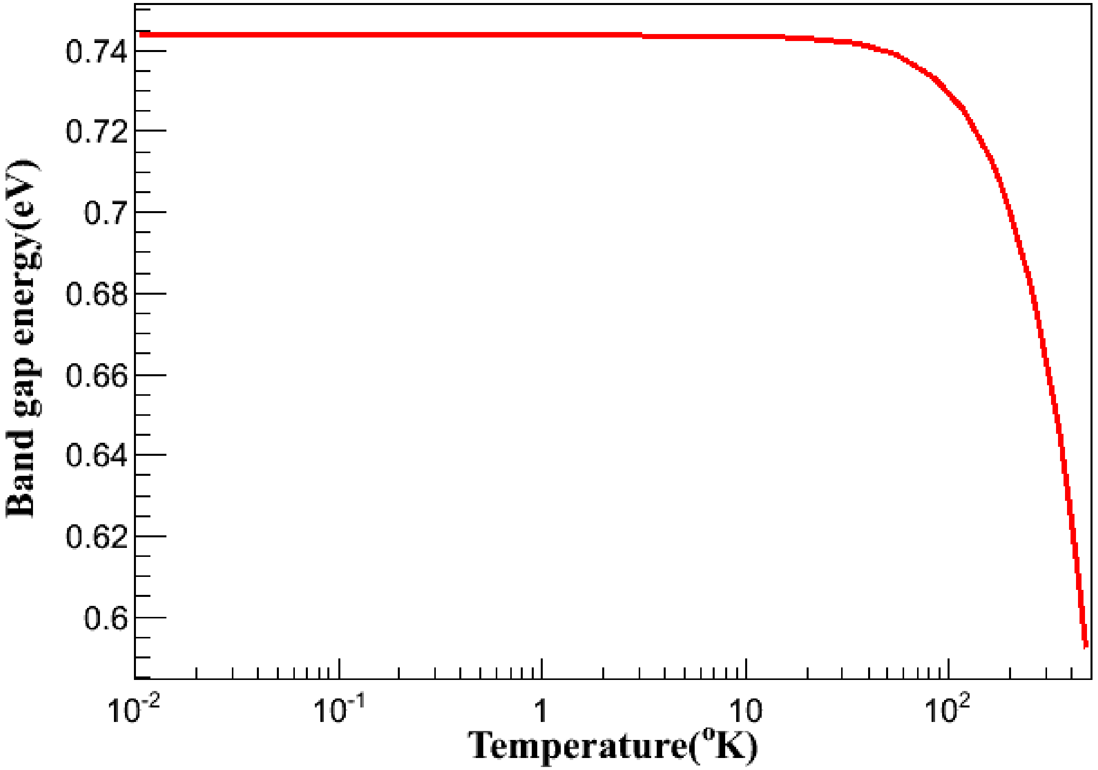
  E_g is the band gap energy;

- F_{NR} is both temperature and energy dependent.

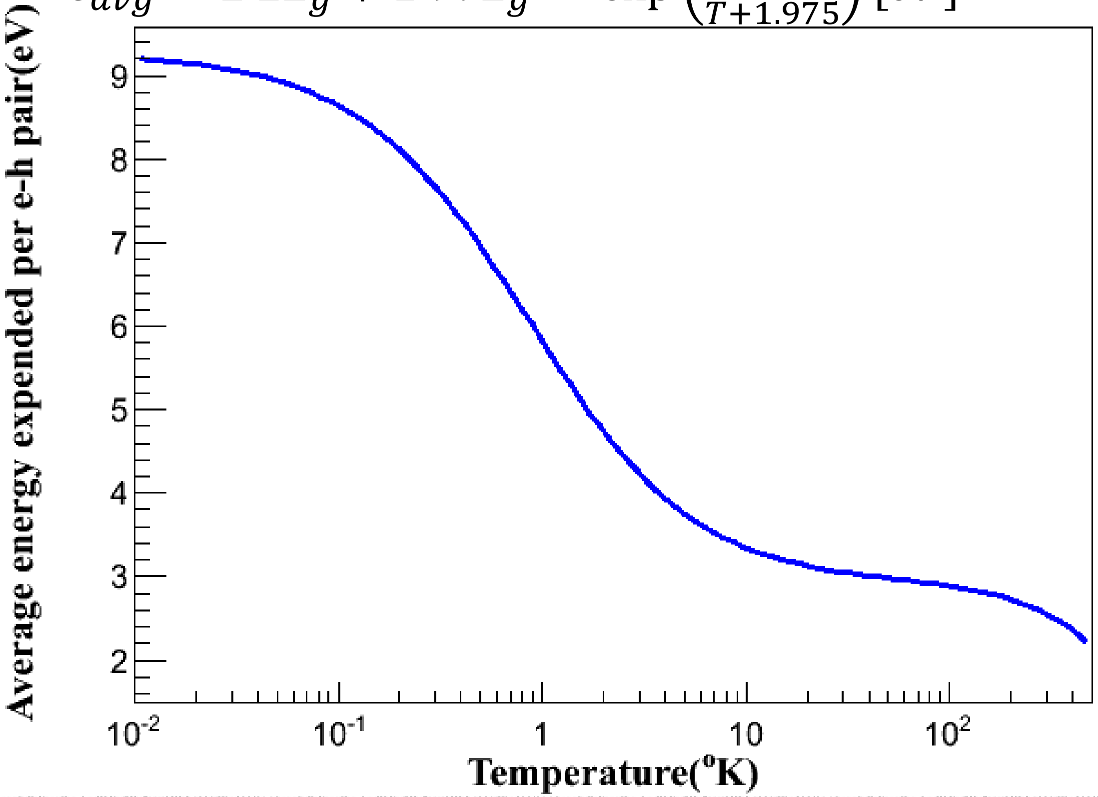
F_{NR} is quite different from F_{ER}.

Temperature Dependence of E_g and ε_{avg}

$$E_g = 0.7437 - \frac{4.77\times10^{-4}T^2}{T+235}\ [eV]$$ [1, 2]



$$\varepsilon_{avg} = 2.2E_g + 1.99{E_g}^{3/2}\exp\left(\frac{4.75E_g}{T+1.975}\right)[eV]$$ [2]



	50 mK	77 K	300 K
Band Gap Energy (E_g) (eV)	0.74	0.73	0.66
Avg. Energy Expended Per e-h Pair (ε_{avg}) (eV)	8.94	2.93	2.55

[1] R. A. Smith, Semiconductor (Cambridge University Press, London, 1960).
[2] F. E. Emery and T. A. Rabson, Physics Review, Volume 140, Number 6A (1965).

Part 2: Ge Detector Energy Response Study

Ionization Efficiency

- **Ionization efficiency** is defined as the ratio of the nuclear recoil visible energy using the electron-equivalent energy calibration to the true recoil energy.

- **D. Barker and D.-M. Mei: Astroparticle Physics 38 (2012) 1-6**

$$\eta = \frac{0.14476 \cdot E_R^{\,0.697747}}{-1.8728 + \exp[E_R^{\,0.211349}]} \quad \text{(Barker-Mei Model)}$$

- **New Model for Absolute Ionization Efficiency:**

$$\varepsilon_{tot} = \boxed{\frac{\alpha}{1+\beta \frac{\mathrm{d}E_{eff}^{NR}}{\mathrm{d}x}}} \times \eta$$

Where, α and β are constants at given temperature; E_R is recoil energy; This model works for Ge detector only and it's valid when E_R is from 1 keV to 100 keV.

Ge Detector Energy Response

- For ER, $E_{vis} = \alpha E_R$ (Linear Response) ----------- (1)

The total differential: $\frac{\mathrm{d}E_{vis}}{\mathrm{d}x} = \alpha \frac{\mathrm{d}E_R}{\mathrm{d}x}$ ----------- (2)

The linear response will not hold if any primary e-h pairs or excitations are lost due to high ionization density (charge recombination). A correction to Eq. (2) can be made by using Birks' law:

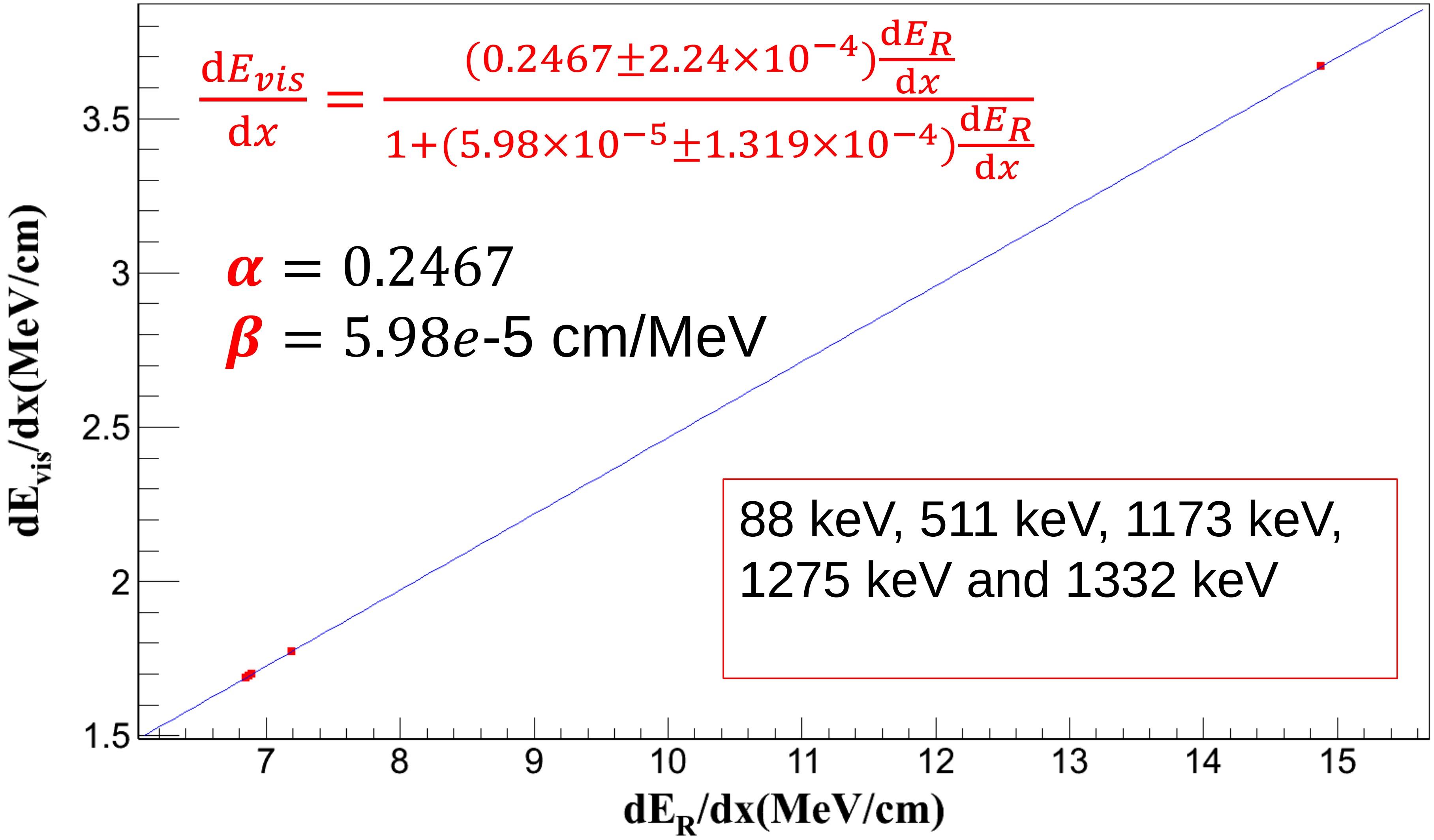
$\frac{\mathrm{d}E_{vis}}{\mathrm{d}x} = \frac{\alpha \frac{\mathrm{d}E_R}{\mathrm{d}x}}{1+\beta \frac{\mathrm{d}E_R}{\mathrm{d}x}}$, multiply dx on both sides and integrate by parts, we have:

$E_{vis} = \frac{\alpha E_R}{1+\beta \frac{\mathrm{d}E_R}{\mathrm{d}x}}$, with $E_{vis} = \frac{E_R}{\varepsilon_{avg}} \times E_g$, i.e. $\boxed{E_{vis} = \frac{E_R}{\varepsilon_{avg}} \times E_g = \frac{\alpha E_R}{1+\beta \frac{\mathrm{d}E_R}{\mathrm{d}x}}}$ ----------- (3)

where α and β are both constants and can be measured through ERs; ε_{avg} is the average energy expended per e-h pair and E_g is the bandgap energy; For Ge at 77K, $\varepsilon_{avg}^{ER} \approx 2.96\ eV$, $E_g \approx 0.73\ eV$

Eq. (3) is an extension of Eq. (1) and can be linear or non-linear depending on $\beta \frac{\mathrm{d}E_R}{\mathrm{d}x}$.

Determination of α and β



Ge Detector Energy Response

- For ERs, $\beta \frac{\mathrm{d}E_R}{\mathrm{d}x} \ll 1$, then Eq. (3) becomes the linear case, which is Eq. (1).

  In this case, we have: $\alpha \approx \frac{E_{vis}^{ER}}{E_R^{ER}} \approx \frac{E_g}{\varepsilon_{avg}^{ER}} \approx \frac{0.73\ eV}{2.96\ eV} \approx 0.2466$

- However, for very low-energy NRs with large stopping power, $\beta \frac{\mathrm{d}E_R}{\mathrm{d}x}$ can be significant and hence cannot be ignored. So,

$$\frac{E_R^{eff}}{\varepsilon_{avg}^{ER}} \times E_g = \frac{\alpha}{1+\beta\frac{\mathrm{d}E_{eff}^{NR}}{\mathrm{d}x}} E_R^{eff} \qquad \text{----------} \quad (4)$$

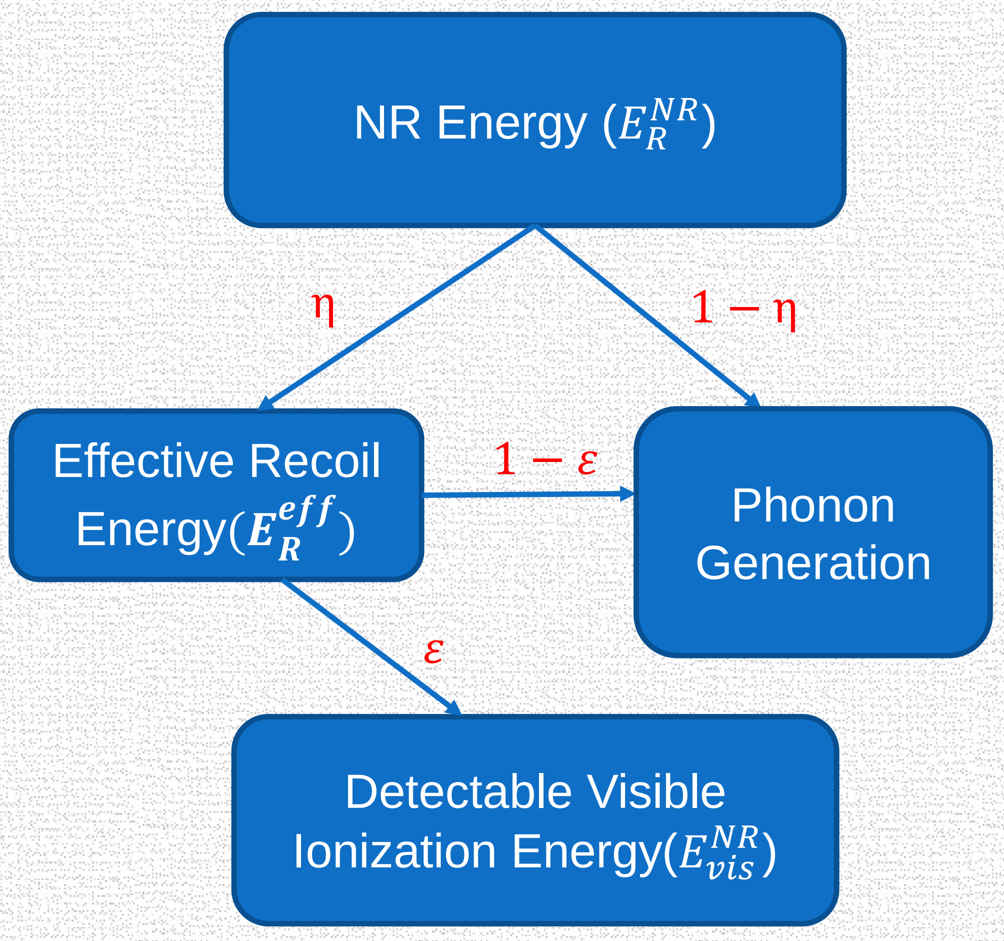
Where, $E_R^{eff} = \eta \times E_R^{NR}$ (η is the traditional ionization efficiency) ----------- (5)

$$\frac{E_R^{eff}}{\varepsilon_{avg}^{ER}} \times E_g = \frac{E_R^{eff}}{\varepsilon_{avg}^{NR} \times \eta} \times E_g = \frac{E_R^{eff}/\eta}{\varepsilon_{avg}^{NR}} \times E_g = \frac{E_R^{NR}}{\varepsilon_{avg}^{NR}} \times E_g = E_{vis}^{NR} \qquad \text{-----------} \quad (6)$$

Plug (5) and (6) into (4), Eq. (4) becomes:

$E_{vis}^{NR} = \frac{\alpha}{1+\beta\frac{\mathrm{d}E_{eff}^{NR}}{\mathrm{d}x}} \times \eta \times E_R^{NR}$, thus, the total ionization efficiency is:

$$\boxed{\boldsymbol{\varepsilon_{tot} = \frac{\alpha}{1+\beta\frac{\mathrm{d}E_{eff}^{NR}}{\mathrm{d}x}} \times \eta}}$$



$$\varepsilon = \frac{\alpha}{1+\beta\frac{\mathrm{d}E_{eff}^{NR}}{\mathrm{d}x}}$$

New Model

$$\varepsilon_{tot} = \frac{\alpha}{1+\beta\frac{dE_{eff}^{NR}}{dx}} \times \frac{0.14476 \cdot E_R^{0.697747}}{-1.8728 + exp[E_R^{0.211349}]}$$

Where, $\alpha \approx \frac{E_g}{\varepsilon_{avg}^{ER}}$ (Temperature dependent) and $\beta = 5.98\text{e-}5$ cm/MeV

At 77K, $\alpha \approx \frac{0.73\ eV}{2.96\ eV} \approx 0.2466$

noise stat.

For SuperCDMS at 50mK, $\sigma_E = \sqrt{(0.293)^2+(0.056)^2E}$ (keV) for E<10 keV [3]

So, $\sigma^2{}_{stat} = (0.056)^2E$, or $\sqrt{FE\varepsilon_{avg}}^2 = (0.056)^2E$, or $\varepsilon_{avg}F \times 10^{-3} = 0.056^2$ -------- (1)

Also, we have: $F_{ER} = \sqrt{\frac{E_x}{E_g}(\frac{\varepsilon_{avg}}{E_g} - 1)}$ -------- (2)

For SuperCDMS, $E_x \approx 0.00828$ eV with a frequency of 2 THz; $E_g \approx 0.74\ eV$ at 50mK;

Eqs. (1) and (2) will yield: $\varepsilon_{avg} \approx 8.94\ eV$ and $F_{ER} \approx 0.36$

Hence, at 50mK, $\alpha \approx \frac{0.744\ eV}{8.94\ eV} \approx 0.08277$

[3] SuperCDMS, Physical Review D 81, 042002 (2010)

CoGeNT VS. SuperCDMS

$$\frac{E_R^{eff}}{\varepsilon_{avg}^{ER}} \times E_g = \frac{\alpha}{1+\beta\frac{dE_{eff}^{NR}}{dx}} \times \eta \times E_R^{NR}$$

For CoGeNT $(T = 77K)$:

- $E_R^{eff} = 0.5\ keV$
- $E_g = 0.73\ eV$
- $\varepsilon_{avg}^{ER} = 2.96\ eV$
- $\alpha = 0.2467$
- $\beta = 5.98 \times 10^{-5} \frac{cm}{MeV}$
- $\frac{dE_{eff}^{NR}}{dx} \approx 21.519\ MeV/cm$

$$\boxed{E_R^{NR} = 2.55\ keV}$$

CoGeNT result: 2.27 keV

For SuperCDMS $(T = 50\ mK)$:

ER: $P_t = E_R(1 + \frac{eV_b}{\varepsilon_{avg}^{ER}})$ **NR:** $P_t = E_R(1 + \frac{eV_b}{\varepsilon_{avg}^{ER}} Y(E_R))$

$At\ V_b = 69\ V, if\ \varepsilon_{avg}^{ER} \approx 3\ eV, then:$

ER: $P_t = 24 \times E_R$ **NR:** $P_t = E_R(1+23\ Y(E_R))$

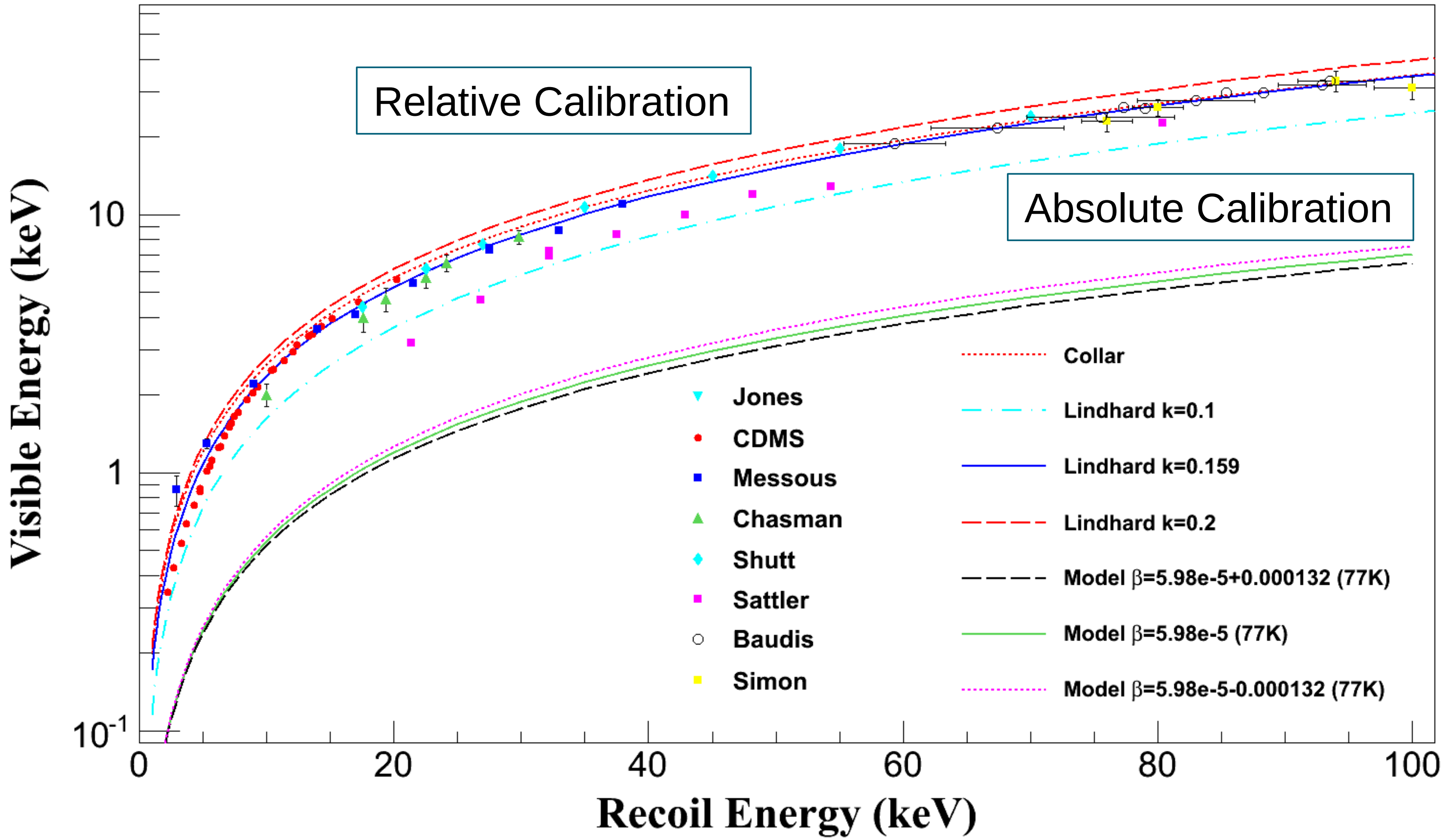
SuperCDMS claims that $E_R \approx 2.214\ keV$ *(energy threshold), which corresponds to* $P_t \approx 11.814\ keV$.

However, if $\varepsilon_{avg}^{ER} \approx 8.94\ eV$, *at* $69\ V$:

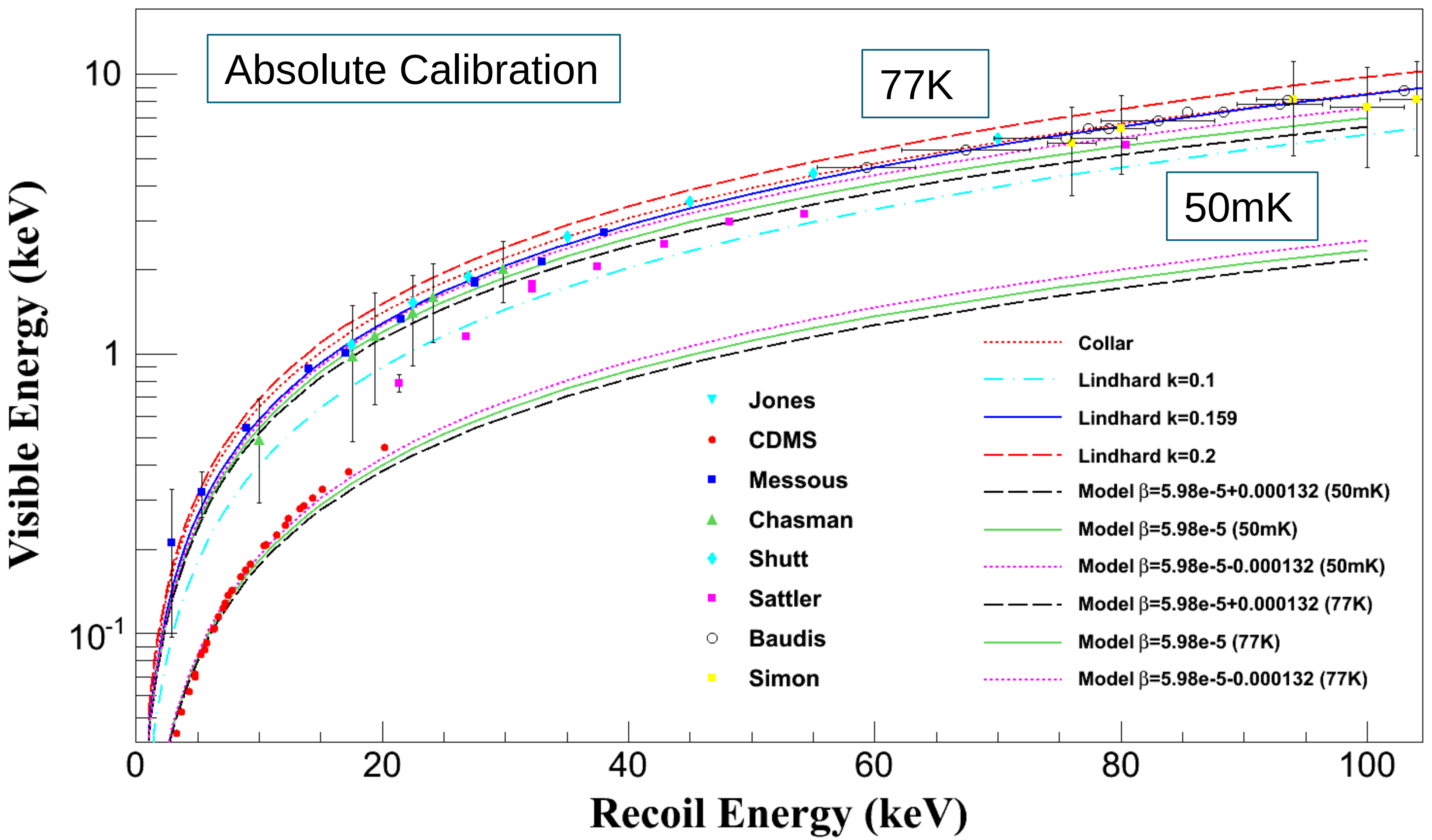
ER: $P_t = 8.718 \times E_R$ **NR:** $P_t = E_R(1+7.718\ Y(E_R))$

Given that $P_t \approx 11.814\ keV$, then the energy threshold is $E_R \approx 4.515\ keV$, *which is a factor of* ~ 2.04 *bigger than what SuperCDMS claims.*

Relative Calibration VS. Absolute Calibration



Comparison with Absolute Calibration



Conclusions

- The relationship between energy resolution and Fano factor has been studied, which shows that ER and NR have different energy resolution and Fano factor has impact on the discrimination of ER/NR.
- Fano factor should be measured for low energies since Fano factor can be used to determine the average energy expended per e-h pair.
- The average energy expended per e-h pair is temperature dependent.
- A new model of absolute ionization efficiency has been proposed by studying the physical process involved in energy response in Ge.
- A relative calibration agrees with our absolute calibration if the Ge detector energy response is completely linear.

Thank You!

Any Questions??