Non-trivial vacua

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think beyond the possible™

Σ-model

d-dim spacetime
D fields ϕ^a

$$\mathcal{L} = -\frac{1}{2}\eta^{\mu\nu}\partial_\mu\phi^a\partial_\nu\phi^b f_{ab}(\phi)$$

- If $\mathrm{sign}\,(f_{ab}) = (+ + \cdots +)$ well-defined

  (usually associated with compact space)

- If $\mathrm{sign}\,(f_{ab}) = (- + \cdots +)$ *NOT* well-defined

  (usually associated with non-compact space)

mode associated with the negative direction is a *ghost*

Σ-model (for non-compact groups)

$$\eta_{\mu\nu} \to g_{\mu\nu}(x)$$

$$\mathcal{L}_{\text{Polyakov}} = -\frac{1}{2}\sqrt{-g(x)}g^{\mu\nu}(x)\partial_\mu\phi^a\partial_\nu\phi^b f_{ab}(\phi)$$

Diff invariance → would-be ghost is gauge mode

In *2d* integrating out $g_{\mu\nu}$ leads to the Nambu-Goto action

$$\mathcal{L}_{\text{Nambu-Goto}} = -\frac{1}{2}\sqrt{-\det\left(\partial_\mu\phi^a\partial_\nu\phi^b f_{ab}(\phi)\right)}$$

Nambu-Goto action

$$\mathcal{L}_{\rm NG} = \det K = \mathcal{E}\mathcal{E}K^d$$

If K is diagonalizable

with $K^\mu_\nu = \sqrt{\eta^{\mu\alpha}\partial_\alpha\phi^a\partial_\nu\phi^b f_{ab}(\phi)}$

$$\det\left(\frac{\delta^2\mathcal{L}_{\rm NG}}{\delta\dot{\phi}^a\delta\dot{\phi}^b}\right) = 0 \quad \longrightarrow \quad \text{would-be ghost is absent}$$

Generalization…

$$\mathcal{L}_{\rm NG} = \mathcal{E}\mathcal{E}K^d \longrightarrow \mathcal{L}_n = \mathcal{E}\mathcal{E}K^n$$

with $K^\mu_\nu = \sqrt{\eta^{\mu\alpha}\partial_\alpha\phi^a\partial_\nu\phi^b f_{ab}(\phi)}$

$$\det\left(\frac{\delta^2\mathcal{L}_n}{\delta\dot{\phi}^a\delta\dot{\phi}^b}\right) = 0$$

→ would-be ghost is also absent

Generalization…

$$\mathcal{L}_{\text{NG}} = \mathcal{E}\mathcal{E}K^d \longrightarrow \mathcal{L}_n = \mathcal{E}\mathcal{E}K^n$$

The difference between $\mathcal{L}_n$ and the NG action is that for $n \neq 0, d$ the constraint is not associated with a symmetry

for $n = d$ there are $(D - d)$ dynamical dof

for $n \neq 0, d$ there are $(D - 1)$ dynamical dof (if $D \geq d$)

Field space: D-dim
Spacetime: d-dim

$D \geq d > 2$

$K^{\mu}_{\nu} = \sqrt{\eta^{\mu\alpha}\partial_\alpha\phi^a\partial_\nu\phi^b\eta_{ab}}$

Perturbative dofs

- Focus on $\mathcal{L}_2$ in $d = D = 4$ dimensions with $f_{ab} = \eta_{ab}$

$$\mathcal{L}_2 = -\frac{1}{8}\left(\mathrm{Tr}\left(K^2\right) - (\mathrm{Tr}K)^2\right)$$

This theory has $D - 1 = 3$ dofs

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$$\mathcal{L}_2 = -\frac{1}{4}F^2_{ab} + \mathcal{O}\left(\partial A\right)^3$$

$$F_{ab} = \partial_a A_b - \partial_b A_a$$

$U(1)$-symmetry $\longrightarrow$ only 2 propagating dofs…

Consequences for Gravity

$$\mathcal{L} = \frac{M_{\rm Pl}^2}{2}\sqrt{-g}\left(R[g] - \frac{1}{4}m^2\left(\mathrm{Tr}\left(K^2\right) - (\mathrm{Tr}K)^2\right)\right)$$

On the trivial background: $\phi^a = x^a + A^a + \partial^a\chi\,, \quad g_{\mu\nu} = \eta_{\mu\nu}$

χ is a gauge mode…

χ only becomes dynamical once interactions with the graviton are considered

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$h^{\mu\nu}\left(\partial_\mu\partial_\nu\chi - \Box\chi\eta_{\mu\nu}\right)$$

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At the origin of
vDVZ
Need for Vainshtein,
Strong coupling
no exact FRWL solutions,
SL (?)
…

Consequences for Gravity

M^2 (1)

On t

χ

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oling
solutions,

h^μ

MG on AdS

$$\mathcal{L} = \frac{M_{\rm Pl}^2}{2}\sqrt{-g}\left(R[g] - \frac{1}{4}m^2\left(\mathrm{Tr}\left(K^2\right) - (\mathrm{Tr}K)^2\right)\right)$$

On the AdS background: $\phi^a = x^a + A^a + \partial^a \chi$

$$g_{\mu\nu} = \gamma_{\mu\nu}^{(\rm AdS)} + h_{\mu\nu}$$

Helicity-0 mode is dynamical on that background

$$\mathcal{L}_\chi = -\frac{1}{2}\left(\frac{M_{\rm Pl}^2 m^2}{L^2} + M_{\rm Pl}^2 m^4\right)(\partial\chi)^2 + m^2 \chi T$$

MG on AdS

$$\mathcal{L} = \frac{M_{\rm Pl}^2}{2}\sqrt{-g}\left(R[g] - \frac{1}{4}m^2\left(\mathrm{Tr}\left(K^2\right) - (\mathrm{Tr}K)^2\right)\right)$$

$$L^{-1} \ll m \quad \Rightarrow \quad \hat{\chi} = M_{\rm Pl} m^2 \chi$$

$$\mathcal{L}_\chi = -\frac{1}{2}(\partial\hat{\chi})^2 + \frac{1}{M_{\rm Pl}}\chi T$$

Strong coupling scale:

$$\Lambda_3 = \left(M_{\rm Pl} m^2\right)^{1/3}$$

$$L^{-1} \gg m \quad \Rightarrow \quad \hat{\chi} = \frac{M_{\rm Pl} m}{L}\chi$$

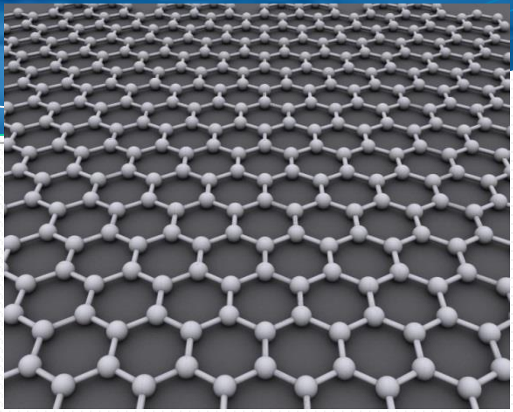
$$\mathcal{L}_\chi = -\frac{1}{2}(\partial\hat{\chi})^2 + \frac{\overbrace{mL}^{\ll 1}}{M_{\rm Pl}}\chi T$$

Strong coupling scale:

$$\Lambda_* = \left(\frac{M_{\rm Pl} m}{L}\right)^{1/3} \gg \Lambda_3$$

AdS curvature makes it much better defined…

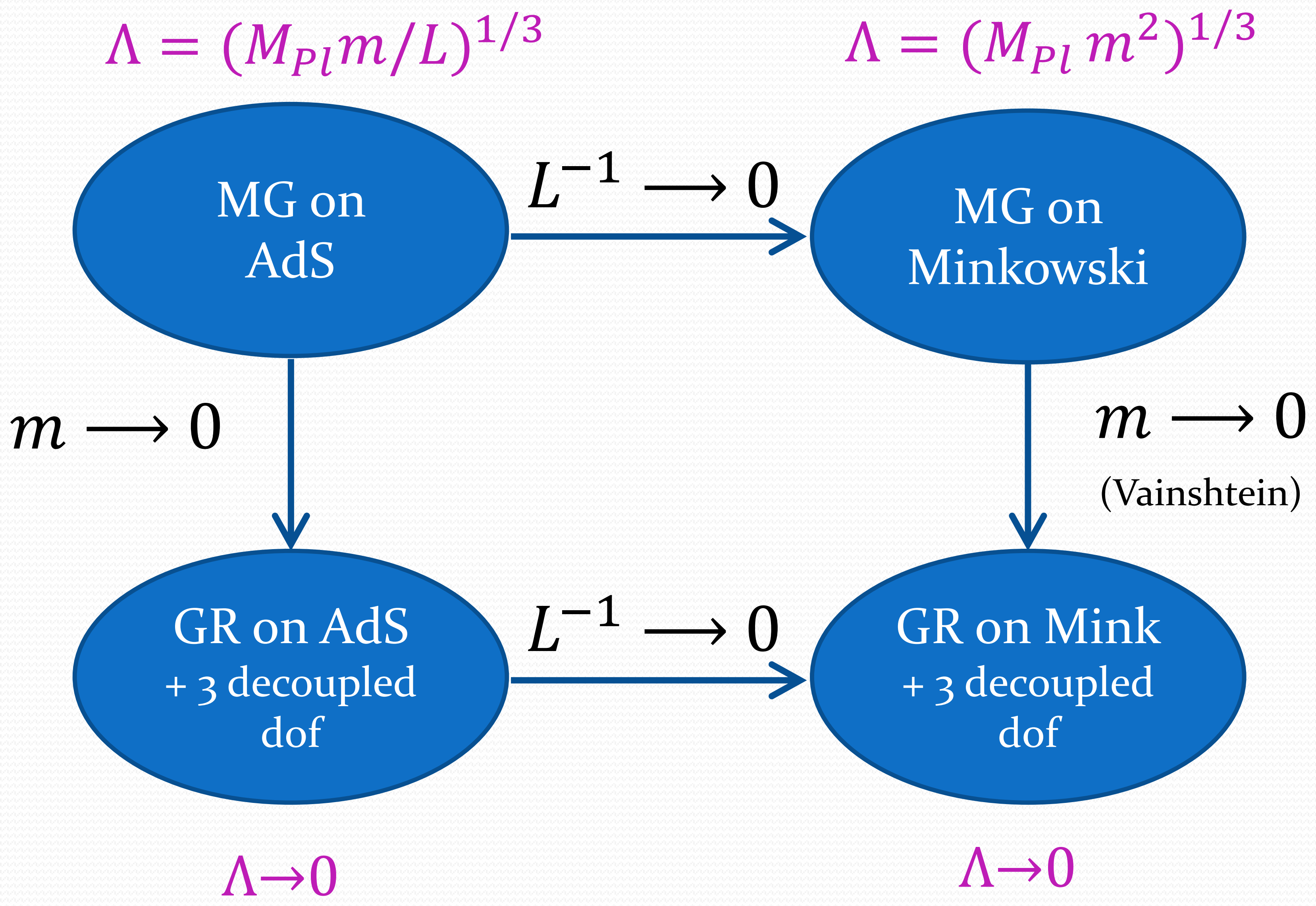
MG on AdS



- On AdS there is no linearized vDVZ discontinuity
- Strong coupling scale can be pushed higher
- There may be a local, causal, unitary UV completion of a single massive graviton

Porrati, 2001

- AdS/CMT may be one of massive gravity raison d’être



$\Lambda = (M_{Pl} m/L)^{1/3}$
$\Lambda = (M_{Pl}\, m^2)^{1/3}$
MG on AdS
$L^{-1} \longrightarrow 0$
MG on Minkowski
$m \longrightarrow 0$
$m \longrightarrow 0$
(Vainshtein)
GR on AdS + 3 decoupled dof
$L^{-1} \longrightarrow 0$
GR on Mink + 3 decoupled dof
$\Lambda \rightarrow 0$
$\Lambda \rightarrow 0$

Copping with Minkowski…

- Can we mimic the effect of AdS while remaining on Minkowski ?

Work in progress with Shuang-Yong Zhou and Andrew Tolley

$$K^\mu_\nu = \delta^\mu_\nu - \sqrt{\eta^{\mu\alpha}\partial_\alpha\phi^a\partial_\nu\phi^b\eta_{ab}}$$

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Switch gravity off

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Consider other vacuum solution

$$K^{\mu}_{\nu} = \delta^{\mu}_{\nu} - \sqrt{\eta^{\mu\alpha}\partial_{\alpha}\phi^{a}\partial_{\nu}\phi^{b}\eta_{ab}}$$

Non-trivial vacuum

$$\mathcal{L} = -\frac{1}{8}\left(\mathrm{Tr}\left(K^2\right) - (\mathrm{Tr}K)^2\right)$$

eg. Plane waves are exact vacuum solutions

$$\bar{\phi}^a = x^a + F(t-x)\delta^a_y$$

DOFs about that background

$$\phi^a = \bar{\phi}^a + V^a$$

$$\Rightarrow \quad \mathcal{L} = -M^{\mu\nu}_{ab}(F)\,\partial_\mu V^a \partial_\nu V^b + \mathcal{O}\left(\partial V\right)^3$$

Breaks the $U(1)$ symmetry $\longrightarrow$ 3 dof

Consistent vacuum

We can easily find local vacua

$$\bar{\phi}^a = \bar{M}^a{}_b x^b + \frac{1}{L}\bar{P}^a_{bc} x^b x^c + \cdots$$

On which there are 3 dofs and Hamiltonian is positive definite

Could be a consistent Σ-model on Minkowski
Without the need for gravity

With a strong coupling scale Λ, $\Lambda_3 \ll \Lambda \ll L^{-1}$

Consequences for MG

$$\mathcal{L}_{\rm MG} = \mathcal{L}_{\rm GR} - m^2 M_{\rm Pl}^2 \left({\rm Tr}\left(K^2\right) - ({\rm Tr} K)^2 \right) + \mathcal{L}_{\rm matter}(g_{\mu\nu}, \psi)$$

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By working about a non-trivial vacuum, we can easily take a Λ_2 decoupling limit of MG

$$M_{\rm Pl} \to \infty$$

$$\Lambda_2 = (m M_{\rm Pl})^{1/2} \to {\rm const}$$

$$\mathcal{L}_{\rm MG} \;\to\; h\partial^2 h + \frac{1}{M_{\rm Pl}} hT + \Lambda_2^4 \, \mathcal{L}\left(\eta_{\mu\nu}, V^a\right)$$

Partially Massless

- Massive Gravity on de Sitter

$$\mathcal{L}_\chi = -\frac{M_{\rm Pl}^2 m^2}{2}\left(m^2 - 2H^2\right)(\partial\chi)^2$$

- Helicity-0 mode is non-dynamical if $m^2 = 2H^2$
  (Higuchi bound 1986)

open the door for a potential PM gravity

(MG with additional symmetry with no helicity-0 mode)

But in MG it was shown that the symmetry does not remain at higher order and the helicity-0 mode reappears making the dS background infinitely strongly coupled

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Could the conclusions be different in Bi-Gravity ???

PM bi-Gravity

$$\mathcal{L}_{\rm BiG-PM} = \mathcal{L}_{\rm GR}(g) + \mathcal{L}_{\rm GR}(f) + m^2 M_{\rm Pl}^2 \mathcal{L}_K(\phi^a)$$

The helicity-0 mode acquires a kinetic term in non-trivial backgrounds

$$\bar{\phi}^a = \bar{M}^a{}_b x^b + \frac{1}{L} \bar{P}^a_{bc} x^b x^c + \cdots$$

$$\underbrace{\mathcal{L}_{\rm BiG-PM} = \mathcal{L}_{\rm GR}(g)}_{\text{2dof}} + \underbrace{\mathcal{L}_{\rm GR}(f)}_{\text{2dof}} + \underbrace{\mathcal{L}_F(V^i)}_{\text{3dof}} + \mathcal{O}(m^2)$$

The theory has 7 dofs on that vacuum. It cannot be enjoying a symmetry
(ie just like there is no non-linear PM gravity, there is no non-linear PM bi-gravity)

The End

- The potential of massive gravity can be thought of as a Σ-model for a non-compact group

- There are (local) non-trivial vacua for which all the dofs are healthy and stable

- On these vacua MG can have a much higher strong coupling scale that previously anticipated and no fifth force in the decoupling limit

- This also means that there cannot be a PM theory of neither gravity nor bi-Gravity or multi-Gravity.