A New Approach to the Cosmological Constant Problem, and Dark Energy

Gregory Gabadadze

New York University

GG, Phys. Lett. B, 2014,

and work in preparation with Siqing Yu

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Einstein's equations

$$G_{\mu\nu} = \Lambda\, g_{\mu\nu} + 8\pi G_N\, T_{\mu\nu}^{\mathrm{dm,m,rad,..}}$$

Λ doesn't redshift (this defines it). Λ is power-sensitive to short distance physics at diverse scales; its natural value way too large. If it's reduced dynamically, perhaps easier to eliminate it entirely.

Then, the scale of Dark Energy, $10^{-33}\ eV$, might be a stable scale where GR is modified – technical naturalness

$$G_{\mu\nu} = \left(10^{-33}\,\mathrm{eV}\right)^2 X_{\mu\nu} + 8\pi G_N\, T_{\mu\nu}^{\mathrm{dm,m,rad,..}}$$

Goals: (A) eliminate Λ entirely (B) get technically natural DE

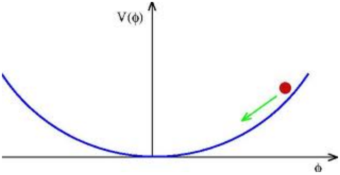
(A): Eliminating the big CC, *Tseytlin '90, motivated by Linde '88:*

The modified action principle:

$$\bar{S} = \frac{S}{V_g} = \frac{1}{V_g}\int d^4x\sqrt{g}\left(\frac{1}{2}R + L(g,\psi_n)\right)$$

where $V_g = \int d^4x\sqrt{g}$. Any constant shift, $L \to L + \Lambda$, gives rise to a shift of the new action by the same constant, $\bar{S} \to \bar{S} + \Lambda$, that does not affect equations of motion.

Subtracts a constant from a scalar potential



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Eliminates the "future value" of the stress tensor

The Einstein equations:

$$R_{\mu\nu} - \frac{1}{4} g_{\mu\nu} R = T_{\mu\nu} - \frac{1}{4} g_{\mu\nu} T \,, \qquad R + T = 0 \,.$$

Tseytlin's proposal for the trace equation:

$$R + T = \langle T \rangle - 2 \langle g^{\mu\nu} \frac{\partial L}{\partial g^{\mu\nu}} \rangle$$

where $< \cdots >$ denotes a certain space-time average defined as follows:

$$\langle \cdots \rangle \equiv \frac{\int d^4x \sqrt{g} (\cdots)}{\int d^4x \sqrt{g}} \equiv \frac{[\cdots]}{V_g} \,. \qquad (1)$$

Local quantities are affected by global ones – non-locality

This non-locality is operative only for vacuum energy, nothing else

Problems with the loops, Tseytlin '90

The $1/V_g$ factor gives an effective rescaling of the Planck's constant, $\hbar \to \hbar V_g$

$$\bar{S}_{Ren} = \frac{1}{V_g} \int d^4x \sqrt{g} \left(\frac{1}{2} R + L(g, \psi_n) + V_g L_1(g, \psi_n) + \mathcal{O}(V_g^2) \right)$$

where $L_1, L_2, ..$ contain all possible terms consistent with diffeomorphism and internal symmetries. This ruins the solution!

Same could be seen by defining an extended action:

$$\bar{S}_{q,\lambda} = \frac{1}{q} \int d^4x \sqrt{g} \left(\frac{1}{2} R + L \right) + \lambda (V_g - q) \, ,$$

and writing down the path integral for gravity as follows

$$Z_g = \mathrm{const} \int d\mu(g) \, dq \, d\lambda \exp \left(\frac{i}{\hbar} \bar{S}_{q,\lambda} \right) \, ,$$

Dealing with the loop problems: *GG '14*

The main idea – global bigravity:

$$A = \frac{V_f}{V_g} S + \int d^D y \sqrt{f} \left(\frac{M_f^{D-2}}{2} R(y) + c_0 M^D \cdots \right)$$

where $f_{AB}(y)$ is another metric, and $V_f = \int d^D y \sqrt{f(y)}$.

The CC of our universe renormalizes CC in the other universe

$$\Delta A_{CC} = \frac{V_f}{V_g} \int d^4 x \sqrt{g} \Lambda = \int d^D y \sqrt{f} \Lambda$$

1. Our vacuum energy curves the other space-time; hence no old CC problem in our universe

2. If $V_f >> V_g$, then, $\hbar \to \hbar (V_g / V_f)$ loop effects suppressed

Defining the path integral for quantized SM:

$$Z(g, J_n) \sim \int d\mu(\tilde{\psi}_n) \mathrm{exp}\left(i \int d^4x \sqrt{g} \left(\mathcal{L}(g, \tilde{\psi}_n) + J_n \tilde{\psi}_n \right) \right)$$

The metric g is an external field, and so are the sources, J_n's. Then, the effective Lagrangian $L(g, \psi_n)$ is defined as a Legandre transform of $W(g, J_n) = -i \ln Z(g, J_n)$

$$\int d^4x \sqrt{g} L(g, \psi_n) \equiv W(g, J_n) - \int d^4x \sqrt{g} J_n \psi_n$$

where $\sqrt{g}\psi_n \equiv -i\delta \ln Z(g, J_n)/\delta J_n$. The obtained quantum effective action is a 1PI action. Thus, all the quantum corrections due to non-gravitational interactions are already taken into account in the effective Lagrangian L.

Thus, we define an effective generating functional

$$Z_{\rm SM}(g, \psi_n) \equiv \exp\left(i \int d^4x \sqrt{g} L(g, \psi_n)\right)$$

that includes all the SM loops, but not quantized gravity.

In the end, $g_{\mu\nu}$ should also be quantized. Corrections are large at scales M_{QG}, and they should be taken care of by a putative UV completion of the theory. However, irrespective of a UV completion the quantum gravity corrections should be small at scales well below M_{QG} for our approximations above to be meaningful. Hence one first needs to define the rules of calculation of the gravity loops given that the classical action has an unusual form.

We define an *extended* action:

$$\bar{A}_{q,\lambda} = \frac{1}{q}\int d^4x\sqrt{g}\left(\frac{1}{2}R + L\right) + \lambda\left(\frac{V_g}{V_f} - q\right) + S_f$$

and the path integral for gravity as follows

$$Z_g \sim \int d\mu(g)\, d\mu(f)\, dq\, d\lambda \exp\left(i\bar{A}_{q,\lambda}\right)$$

where we also integrates w.r.t. the *parameters* q and λ. This can be rewritten in terms of the path integral for the SM fields Z_{SM}:

$$Z_g \sim \int d\mu(g) d\mu(f)\, dq\, d\lambda \left(e^{iS_{\mathrm{EH}}}\, Z_{\mathrm{SM}}(g,\psi_n)\right)^{\frac{1}{q}}\; e^{i\lambda\left(\frac{V_g}{V_f}-q\right)+iS_f}$$

The SM loops done in a conventional way, gravity loops via an unconventional prescription specified above.

How do we achieve the condition $V_f / V_g >> 1$?

Assume that the g-universe has supersymmetry broken at some high scale, and therefore, there is a natural value of its vacuum energy density proportional to E_{vac}^4.

The scale E_{vac} can be anywhere between a few TeV and the GUT scale, $\mu_{GUT} \sim 10^{16}$ GeV.

As to the f-universe, $M_f \sim M_{\rm pl}$, but also we'd need the scale M to be somewhat higher than E_{vac}.

The latter condition should be natural, since without special arrangements one would expect $M \sim M_f \sim M_{\rm pl}$, and since $E_{vac} << M_{\rm pl}$, one would also get $E_{vac} < M$. If so, then the vacuum energy of the g-universe, E_{vac}^4, would make a small contribution to the pre-existing vacuum energy of the f-universe.

The f-universe can be exactly supersymmetric, described, for instance, by unbroken AdS supergravity.

The new terms do not affect the trace equations, except that they just introduce a overall multiplier V_f. Thus, the cosmological constant is eliminated from the g-universe. There is, however, a new equation due to variation w.r.t. f:

$$M_f^{D-2}(R_{AB}(y) - \frac{1}{2} f_{AB} R(y)) = f_{AB}(\bar{S} + c_0 M^D) + \cdots . \qquad (2)$$

The right hand side contains a vacuum energy generated in our universe, $\bar{S} = \frac{[E^4_{vac}]}{V_g} = E^4_{vac}$, as well as that of the f-universe.

According to our construction, the net energy density is negative, so that the f-universe has an AdS curvature. If so, then $V_f = \infty$.

Still need to produce $V_f/V_g >>> 1$; use massive gravity – and its extensions with quasidilaton – instead of GR in the g-universe:

(B): GR Extended by Mass and Potential Terms

Previous no-go statements invalid: *de Rham, GG, '10*

The Lagrangian of the theory: *de Rham, GG, Tolley, '11*

Using $g_{\mu\nu}(x)$ and 4 scalars $\phi^a(x)$, $a = 0, 1, 2, 3$, define

$$\mathcal{K}^\mu_\nu(g, \phi) = \delta^\mu_\nu - \sqrt{g^{\mu\alpha}\tilde{f}_{\alpha\nu}} \qquad \tilde{f}_{\alpha\nu} \equiv \partial_\alpha\phi^a\partial_\nu\phi^b\eta_{ab}$$

The Lagrangian is written using notation $tr(\mathcal{K}) \equiv [\mathcal{K}]$:

$$\mathcal{L} = M^2_{\rm pl}\sqrt{g}\left(R + m^2\left(\mathcal{U}_2 + \alpha_3\,\mathcal{U}_3 + \alpha_4\,\mathcal{U}_4\right)\right)$$

$$\mathcal{U}_2 = [\mathcal{K}]^2 - [\mathcal{K}^2] \sim det_2(\mathcal{K})$$

$$\mathcal{U}_3 = [\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3] \sim det_3(\mathcal{K})$$

$$\mathcal{U}_4 = [\mathcal{K}]^4 - 6[\mathcal{K}^2][\mathcal{K}]^2 + 8[\mathcal{K}^3][\mathcal{K}] + 3[\mathcal{K}^2]^2 - 6[\mathcal{K}^4] \sim det_4(\mathcal{K})$$

Theory of Quasi-Dilaton: *D'Amico, GG, Hui, Pirtskhalava, '12*

Invariance of the action to rescaling of the reference frame coordinates ϕ^a w.r.t. the physical space coordinates, x^a, requires a field σ. In the Einstein frame:

$$\phi^a \to e^\alpha \ \phi^a, \qquad \sigma \to \sigma - \alpha M_{\rm Pl}$$

Hence we can construct the invariant action by replacing $\mathcal{K}$ by $\bar{K}$

$$\bar{K}^\mu_\nu = \delta^\mu_\nu - \sqrt{g^{\mu\alpha}\bar{f}_{\alpha\nu}} \qquad \bar{f}_{\alpha_\nu} = e^{2\sigma/M_{\rm Pl}}\partial_\alpha\phi^a\partial_\nu\phi^b\eta_{ab}$$

and adding the sigma kinetic term

$$\mathcal{L} = \mathcal{L}_{dRGT}\left(\mathcal{K} \to \bar{K}\right) - \omega\sqrt{g}(\partial\sigma)^2 + \text{Galileons of } \sigma$$

and the term $\int d^4x\sqrt{-det\bar{f}}$ can also be added. In the Einstein frame σ does not couple to matter, but it does in the Jordan frame

Extensions of massive gravity (subjective and incomplete list):

Extended Quasidilaton: De Felice, Mukohyama, '13; Mukohyama, '13; De Felice, Gümrükcüoglu, Mukohyama, '13, Mukohyama, 14; GG, Kimura, Pirtskhalava, '14,'15

Bigravity: Hassan, R.A. Rosen, '11, ... Cosmology e.g., De Felice, Gümrükc|uoglu, Mukohyama, Tanahashi, Tanaka, 14, ...

Extended and Generalized Massive Gravities: GG, Hinterbichler, Khoury, Pirtskhalava, Trodden, 13; Gümrükcüoglu, Hinterbichler, Lin, Mukohyama, Trodden 13; de Rham, Keltner, Tolley, 14, ...

Thus, the g-universe has dS metric, and f-universe has AdS metric. $q = V_g/V_f \to 0$, hence quantum gravity corrections in the g-universe are determined by positive powers of the parameter, $\hbar q \to 0$. Quantum gravity is present only in the f-universe!

Could f and the fiducial metric, $\tilde{f}$, be related?
Paper in preparation with Siqing Yu: The f-universe as AdS_5

$$ds^2 = f_{AB}dy^A dy^B = \frac{l^2}{z^2}\left(\eta_{ab}dy^a dy^b + dz^2\right), \quad a = 0,1,2,3; A = a,5$$

The AdS boundary coordinates x^μ, $\mu = 0,1,2,3$. Parametrization of the boundary located at $z = 0$, $y^a = \phi^a(x)$

$$ds^2 = \frac{l^2}{z^2}\left(\tilde{f}_{\mu\nu}dx^\mu dx^\nu + dz^2\right), \quad \tilde{f}_{\mu\nu} = \partial_\mu\phi^a\partial_\nu\phi^b\eta_{ab}$$

The fiducial metric, $\tilde{f}_{\mu\nu}$, as a non-dynamical pullback of the 5D AdS metric

$$A = \frac{V_f}{V_g}S_{mGR}(g,\tilde{f}) + S_{AdS_5}(f)$$

This removes our CC into the 5D AdS space (need a small hierarchy between 5D and 4D CC's, as before), and gives rise to dark energy via massive gravity or it extensions.

Removing the quantum strong coupling problem, in preparation with Siqing Yu: gravity in g-universe quantized with $\hbar q \to 0$

Decoupling limit: $M_{Pl} \to \infty$, $m \to 0$, $\Lambda_3 = (M_{Pl} m^2)^{1/3}$

$$S_\pi = \int d^4x \left(-\frac{1}{2}(\partial\pi)^2 + \alpha \frac{(\partial\pi)^2 \Box\pi}{\Lambda_3^3} + \frac{2\alpha^2}{3} G_4(\partial\pi) + ... \right)$$

The massive gravity Lagrangian is not quantized with $\hbar$, but instead with $\hbar q \to 0$. The respective part of the path integral reads schematically as follows

$$\exp\left(\frac{i}{\hbar q} S_\pi\right) .$$

Every π propagator proportional to $\hbar q$, every π vertex, to $1/\hbar q$. Normalize one-particle states, loop expansion, then all loop corrections proportional to positive powers of $\hbar q$, and vanish in the setup considered here. This does not obviate Λ_3, but makes all classical calculations in gravity exact (no quantum gravity effects in the g-universe, only in the f-universe.).

Conclusions:

- The big cosmological constant can be eliminated via a nonlocal mechanism that extends Tseytlin's approach. The proposed action is stable w.r.t. quantum gravity loop corrections. Embedding in SUGRA.
- Dark energy can be accommodated by various means, but not by means of CC. Using massive gravity and its extensions has virtues of: (a) ascribing origin to the fiducial metric, (b) removing the quantum strong coupling problem.
- Possible observational consequences from massive gravity and its extensions, due mainly to the astrophysical and cosmological Vainshtein mechamism.