# A Stückelberg Approach to Quadratic Curvature Gravity

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KH, Mehdi Saravani, arXiv:1508.02401

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$$S \sim \frac{1}{M_P^2} \int d^4x \sqrt{-g} \left[ R + \frac{1}{m^2} \left( a_2 \, R^2 + \cdots \right) + \frac{1}{m^4} \left( a_3 \, R^3 + \cdots \right) + \cdots \right]$$

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coefficients sensitive to details of new physics

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Observables are to be calculated *perturbatively*, order by order in powers of: $\frac{E}{M_P}, \quad \frac{E}{m}$

We do not worry about ghosts/superluminality/cauchy breakdown etc. associated with the higher derivative terms: these are non-perturbative in $E/m$

# Higher curvature gravity and effective field theory

Nevertheless, there is a long and fine tradition of ignoring the perturbativity requirement and asking what higher curvature terms have to say non-perturbatively e.g. Stelle (1976 — present)

Motivations:

- gain intuition about the gravitational effects higher-scale physics might produce
- display various pathologies that a UV completion must ultimately overcome
- try to find a UV complete theory of gravity

# Quadratic Curvature gravity

Most general action up to fourth order in derivatives:

$$S = M_P^2 \int d^4x \, \sqrt{-g} \left[ \frac{1}{2} R + \frac{1}{12m^2} R^2 + \frac{1}{4M^2} C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} \right]$$

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Degrees of freedom around Minkowski:

- massless graviton
- massive graviton
- massive scalar

one of these is always ghostly (pointing to massless graviton and massive graviton)

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Longitudinal modes of massive graviton described by a non-renormalizable galileon lagrangian:

$$-3(\partial\hat{\phi})^2 + \frac{6(6c_3-1)}{\Lambda_3^3}(\partial\hat{\phi})^2\Box\hat{\phi} - 4\frac{(6c_3-1)^2-4(8d_5+c_3)}{\Lambda_3^6}(\partial\hat{\phi})^2\left([\hat{\Pi}]^2-[\hat{\Pi}^2]\right)$$
$$-\frac{40(6c_3-1)(8d_5+c_3)}{\Lambda_3^9}(\partial\hat{\phi})^2\left([\hat{\Pi}]^3-3[\hat{\Pi}^2][\hat{\Pi}]+2[\hat{\Pi}^3]\right) + \text{scalar-tensor mixing}$$

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How does its massive graviton evade the ubiquitous $\Lambda_3$ strong coupling?

We will use the Stückelberg trick to easily see how

We use techniques developed in:

Claudia de Rham, Gregory Gabadadze, David Pirtskhalava, Andrew Tolley, Itay Yavin
"Nonlinear Dynamics of 3D Massive Gravity"
arXiv:1103.1351

# Re-write as a second-order theory

Quadratic gravity is a 4-th order theory:

$$M_P^2 \int d^4x \, \sqrt{-g} \left[ \frac{1}{2} R + \frac{1}{12m^2} R^2 + \frac{1}{4M^2} C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} \right]$$

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First eliminate f(R) part by introducing a scalar: $\phi = R$

$$M_P^2 \int d^4x \sqrt{-g} \left[ \frac{1}{2} \left( 1 + \frac{\phi}{3m^2} \right) R - \frac{1}{12m^2} \phi^2 + \frac{1}{4M^2} C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} \right]$$

Go to Einstein frame: $\phi = 3m^2 \left( e^{\psi} - 1 \right)$ $\qquad g_{\mu\nu} \to e^{-\psi} g_{\mu\nu}$

$$M_P^2 \int d^4x \sqrt{-g} \left[ \frac{1}{2} R - \frac{3}{4} (\partial\psi)^2 - \frac{3}{4} m^2 e^{-2\psi} \left( e^{\psi} - 1 \right)^2 + \frac{1}{4M^2} C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} \right]$$

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Next eliminate Weyl part by introducing a tensor: $f_{\mu\nu} = \frac{1}{M^2} \left( R_{\mu\nu} - \frac{1}{6} R g_{\mu\nu} \right)$

$$M_P^2 \int d^4x \, \sqrt{-g} \left[ \frac{1}{2} R - \frac{3}{4} (\partial\psi)^2 - \frac{3}{4} m^2 e^{-2\psi} \left( e^{\psi} - 1 \right)^2 + f^{\mu\nu} G_{\mu\nu} - \frac{1}{2} M^2 \left( f_{\mu\nu} f^{\mu\nu} - f^2 \right) \right]$$

# Linear spectrum

Easiest way to see the spectrum: expand in linear fluctuations about flat space and diagonalize:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$h_{\mu\nu} = 2\left(h'_{\mu\nu} + f_{\mu\nu}\right)$$

Action to second order in fluctuations: $h'_{\mu\nu},\ f_{\mu\nu},\ \psi$

$$M_P^2 \int d^4x \ -\frac{3}{4}\left((\partial\psi)^2 - m^2\psi^2\right) + \frac{1}{2}h'^{\mu\nu}\left(\mathcal{E}h'\right)_{\mu\nu} - \frac{1}{2}f^{\mu\nu}\left(\mathcal{E}f\right)_{\mu\nu} - \frac{1}{2}M^2\left(f_{\mu\nu}f^{\mu\nu} - f^2\right)$$

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massive scalar field $\psi$, with mass squared $m^2$

massless spin-2 field $h'_{\mu\nu}$

massive (ghost) spin-2 field $f_{\mu\nu}$, with mass squared $M^2$

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$$f_{\mu\nu} \to f_{\mu\nu} + \nabla_\mu \tilde{V}_\nu + \nabla_\nu \tilde{V}_\mu, \quad \tilde{V}_\mu = V_\mu + \partial_\mu \pi$$

⬇

$$M_P^{D-2} \int d^D x \, \sqrt{-g} \left[ \frac{1}{2} R + f^{\mu\nu} G_{\mu\nu} - \frac{1}{2} M^2 \left( f_{\mu\nu} f^{\mu\nu} - f^2 \right) - \frac{1}{2} M^2 F_{\mu\nu}^2 \right.$$
$$\left. + 2M^2 R_{\mu\nu} \tilde{V}^\mu \tilde{V}^\nu - 2M^2 f^{\mu\nu} \left( \nabla_\mu \tilde{V}_\nu - g_{\mu\nu} \nabla \cdot \tilde{V} \right) \right],$$

second diff invariance: $\delta f_{\mu\nu} = \nabla_\mu \Lambda_\nu + \nabla_\nu \Lambda_\mu, \quad \delta V_\mu = -\Lambda_\mu + \partial_\mu \Lambda, \quad \delta\pi = \Lambda$

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$$+ 2M^2 R_{\mu\nu} \tilde{V}^\mu \tilde{V}^\nu - 2M^2 f^{\mu\nu} \left( \nabla_\mu \tilde{V}_\nu - g_{\mu\nu} \nabla \cdot \tilde{V} \right) \Big],$$

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Full non-linear DOF count ($D$=4):

(25 fields) − 2 × (9 gauge symmetries) = (7 degrees of freedom)

| | |
|---|---|
| $f_{\mu\nu}$ | 10 |
| $g_{\mu\nu}$ | 10 |
| $V_\mu$ | 4 |
| $\pi$ | 1 |

| | |
|---|---|
| $\xi^\mu$ | 4 |
| $\Lambda_\nu$ | 4 |
| $\Lambda$ | 1 |

| | |
|---|---|
| massless graviton | 2 |
| massive graviton | 5 |

No Boulware-Deser ghost associated with the massive graviton ⇒ expect a $\Lambda_3$ strong coupling scale

# Decoupling limit

Canonically normalize:

$$(h_{\mu\nu}, f_{\mu\nu}) \sim \frac{1}{M_P^{\frac{D}{2}-1}}(\hat{h}_{\mu\nu}, \hat{f}_{\mu\nu}), \quad V_\mu \sim \frac{1}{M_P^{\frac{D}{2}-1}M}\hat{V}_\mu, \quad \pi \sim \frac{1}{M_P^{\frac{D}{2}-1}M^2}\hat{\pi}$$

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First non-trivial Non-renormalizable operator appears at $\Lambda_3$ . Decoupling limit:

$$M \to 0, \quad M_P \to \infty, \quad \Lambda_{\frac{D+2}{D-2}} \text{ fixed} \qquad \Lambda_{\frac{D+2}{D-2}} = \left(M^{\frac{4}{D-2}} M_P\right)^{\frac{D-2}{D+2}}$$

$$M_P^{D-2}\int d^Dx \left[ \quad \frac{1}{8}h^{\mu\nu}\left(\mathcal{E}h\right)_{\mu\nu} - \frac{1}{2}f^{\mu\nu}\left(\mathcal{E}h\right)_{\mu\nu} - \frac{1}{2}M^2F_{\mu\nu}^2 - 2M^2f^{\mu\nu}\left(\partial_\mu\partial_\nu\pi - \eta_{\mu\nu}\Box\pi\right) + 2M^2R^{\mathrm{L}}_{\mu\nu}(h)\partial^\mu\pi\partial^\nu\pi\right]$$

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Diagonalize:

$$h_{\mu\nu} \to 2\left(h'_{\mu\nu} + f'_{\mu\nu}\right) - \frac{4}{D-2}M^2\eta_{\mu\nu}\pi,$$

$$f_{\mu\nu} \to f'_{\mu\nu} - \frac{2}{D-2}M^2\eta_{\mu\nu}\pi - 2M^2\left[\partial_\mu\pi\partial_\nu\pi - \frac{1}{D-2}(\partial\pi)^2\eta_{\mu\nu}\right]$$

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↑ only interaction is a cubic galileon

Becomes strongly coupled at dRGT scale when $D \neq 4$:

- New Massive gravity non-renormalizable ($D$=3)
- Quadratic curvature gravity non-renormalizable in $D \geq 5$

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It turns out there are none.

Field re-definition removes all operators suppressed by scales $< M_P$

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No obstruction to taking a straight massless $M \to 0$ limit:

$$\begin{aligned} M_P^2 \int d^4x \, \sqrt{-g} \Big[ & - \frac{1}{2} M^2 F_{\mu\nu}^2 + 3M^4 (\partial\pi)^2 \\ & + f'^{\mu\nu} \left( G_{\mu\nu} - 2M^2 \left( \nabla_\mu \nabla_\nu \pi - g_{\mu\nu} \Box \pi \right) + 2M^4 \left( \nabla_\mu \pi \nabla_\nu \pi + \frac{1}{2} g_{\mu\nu} (\partial\pi)^2 \right) \right) \Big] \end{aligned}$$

Weyl transformation: $g_{\mu\nu} \to e^{-2M^2\pi} g_{\mu\nu}$

$$M_P^2 \int d^4x \, \sqrt{-g} \left[ f'^{\mu\nu} G_{\mu\nu} - \frac{1}{2} M^2 F_{\mu\nu}^2 + 3M^4 e^{-2M^2\pi} (\partial\pi)^2 \right]$$

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If the theory is renormalizable, there should be no strong coupling at all, even at $M_P$

# D=4 massless limit

Expand in fluctuations and diagonalize:

$$h_{\mu\nu} = \tilde{h}_{\mu\nu} + \tilde{f}_{\mu\nu}, \quad \delta f_{\mu\nu} = \tilde{f}_{\mu\nu} - \frac{1}{2}\tilde{h}_{\mu\nu}$$

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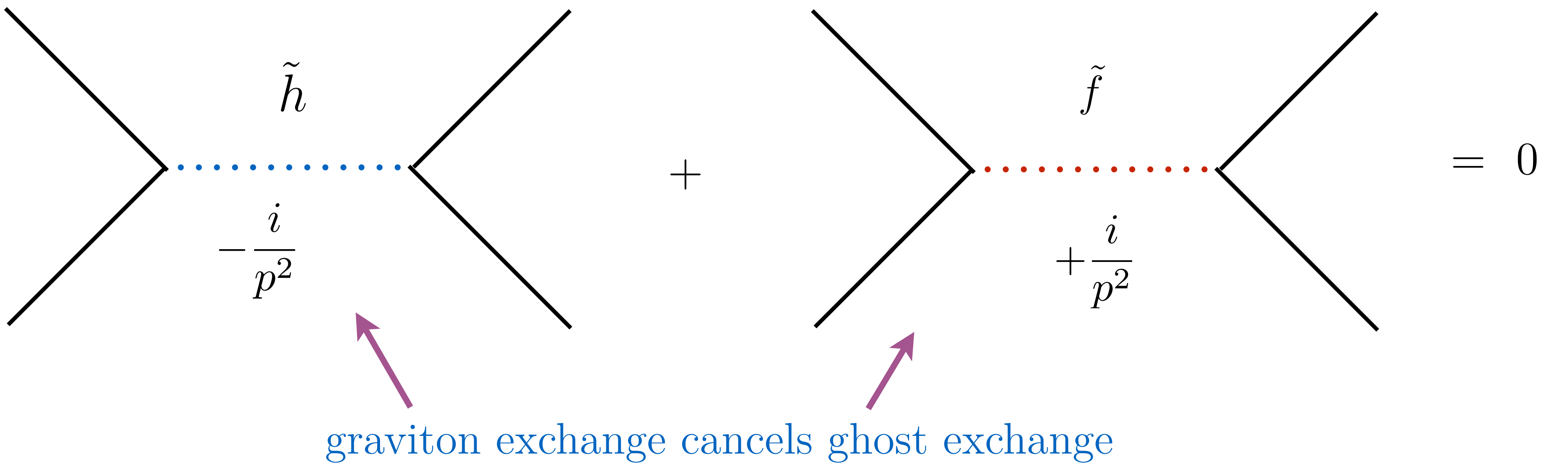
$$+\left(\tilde{f}_{\mu\nu} - \frac{1}{2}\tilde{h}_{\mu\nu}\right)\sqrt{-g}G^{(\geq 2)\mu\nu}\left[\tilde{h}+\tilde{f}\right] + \mathcal{L}_{V,\pi}^{(\geq 1)}\left[\tilde{h}+\tilde{f}, V, \pi\right]\Big]$$

Coupling is only through the combination $h$+$f$



# D=4 massless limit

Field re-definition completely eliminates interactions:

$$\begin{pmatrix} \tilde{h}_{\mu\nu} \\ \tilde{f}_{\mu\nu} \end{pmatrix} = \begin{pmatrix} \cosh\alpha & \sinh\alpha \\ \sinh\alpha & \cosh\alpha \end{pmatrix} \begin{pmatrix} \tilde{h}^{(\alpha)}_{\mu\nu} \\ \tilde{f}^{(\alpha)}_{\mu\nu} \end{pmatrix}$$

exploit so(1,1) invariance of ghostly kinetic terms

$$M_P^2 \int d^4x \Big[ \overbrace{\frac{3}{8}\tilde{h}^{(\alpha)\mu\nu}\left(\mathcal{E}\tilde{h}^{(\alpha)}\right)_{\mu\nu} - \frac{3}{8}\tilde{f}^{(\alpha)\mu\nu}\left(\mathcal{E}\tilde{f}^{(\alpha)}\right)_{\mu\nu}} - \frac{1}{2}M^2 F_{\mu\nu}^2 + 3M^4 e^{-2M^2\pi}(\partial\pi)^2$$

$$+ \left(\tilde{f} - \frac{1}{2}\tilde{h}\right)_{\mu\nu} \sqrt{-g}\, G^{(\geq 2)\mu\nu}\left[e^{\alpha}\left(\tilde{h}^{(\alpha)} + \tilde{f}^{(\alpha)}\right)\right] + \mathcal{L}^{(\geq 1)}_{V,\pi}\left[e^{\alpha}\left(\tilde{h}^{(\alpha)} + \tilde{f}^{(\alpha)}\right), V, \pi\right] \Big]$$

# D=4 massless limit

Field re-definition completely eliminates interactions:

$$\begin{pmatrix} \tilde{h}_{\mu\nu} \\ \tilde{f}_{\mu\nu} \end{pmatrix} = \begin{pmatrix} \cosh\alpha & \sinh\alpha \\ \sinh\alpha & \cosh\alpha \end{pmatrix} \begin{pmatrix} \tilde{h}^{(\alpha)}_{\mu\nu} \\ \tilde{f}^{(\alpha)}_{\mu\nu} \end{pmatrix}$$

exploit so(1,1) invariance of ghostly kinetic terms

$$M_P^2 \int d^4x \Big[ \overbrace{\frac{3}{8}\tilde{h}^{(\alpha)\mu\nu}\left(\mathcal{E}\tilde{h}^{(\alpha)}\right)_{\mu\nu} - \frac{3}{8}\tilde{f}^{(\alpha)\mu\nu}\left(\mathcal{E}\tilde{f}^{(\alpha)}\right)_{\mu\nu}} - \frac{1}{2}M^2 F_{\mu\nu}^2 + 3M^4 e^{-2M^2\pi}(\partial\pi)^2$$
$$+ \left(\tilde{f} - \frac{1}{2}\tilde{h}\right)_{\mu\nu} \sqrt{-g}\, G^{(\geq 2)\mu\nu}\left[e^{\alpha}\left(\tilde{h}^{(\alpha)} + \tilde{f}^{(\alpha)}\right)\right] + \mathcal{L}_{V,\pi}^{(\geq 1)}\left[e^{\alpha}\left(\tilde{h}^{(\alpha)} + \tilde{f}^{(\alpha)}\right), V, \pi\right]\Big]$$

$\alpha \to -\infty \quad \Rightarrow$ Theory is trivial at high energies (renormalizable & asymptotically free)

$$M_P^2 \int d^4x\, \frac{3}{8}\tilde{h}^{(\alpha)\mu\nu}\left(\mathcal{E}\tilde{h}^{(\alpha)}\right)_{\mu\nu} - \frac{3}{8}\tilde{f}^{(\alpha)\mu\nu}\left(\mathcal{E}\tilde{f}^{(\alpha)}\right)_{\mu\nu} - \frac{1}{2}M^2 F_{\mu\nu}^2 + 3M^4 e^{-2M^2\pi}(\partial\pi)^2$$

# D=4 massless limit

Bringing back the scalar:

$$M_P^2 \int d^4x \left[\frac{3}{4}e^{-\pi}(\partial\pi)^2 - \frac{3}{4}e^{-\pi}(\partial\psi)^2 - \frac{3}{4}m^2 e^{-2(\psi+\pi)}\left(e^{\psi}-1\right)^2\right]$$

We can field re-define to find an explicitly renormalizable interaction

$$\pi = -\log\left(\tilde{\pi}^2 - \tilde{\psi}^2\right), \quad \psi = \log\left(\frac{\tilde{\pi}+\tilde{\psi}}{\tilde{\pi}-\tilde{\psi}}\right)$$

$$\Downarrow$$

$$3M_P^2 \int d^4x \; (\partial\tilde{\pi})^2 - (\partial\tilde{\psi})^2 + m^2\tilde{\psi}^2\left(\tilde{\pi}-\tilde{\psi}\right)^2$$

Renormalizable $\phi^4$ -type potential with coupling $\lambda \sim \dfrac{m^2}{M_P^2}$

# Conclusions

- Quadratic curvature gravity in the high-energy limit is greatly simplified using the Stückelberg trick

- Makes the renormalizability/asymptotic freedom of the theory easy to see

- Shows how the massive graviton overcomes the $\Lambda_3$ strong coupling scale

- Shows the necessity of the ghost in making this possible