Massive gravity and cosmology

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Massive gravity: history

Simple question: Can graviton have mass?
May lead to acceleration without dark energy
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**Fierz-Pauli theory (1939)**
Unique linear theory without instabilities (ghosts)
Massive gravity: history

Simple question: Can graviton have mass?
May lead to acceleration without dark energy

Yes?
No?

Fierz-Pauli theory (1939)
Unique linear theory without instabilities (ghosts)

van Dam-Veltman-Zhakharov discontinuity (1970)
Massless limit $\neq$
General Relativity
Massive gravity: history

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Yes?  No?

Vainshtein mechanism (1972)
Nonlinearity $\rightarrow$ Massless limit $\neq$ General Relativity

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Massive gravity: history

Simple question: Can graviton have mass?
May lead to acceleration without dark energy

Yes?

- de Rham-Gabadadze-Tolley (2010)
  First example of nonlinear massive gravity without BD ghost since 1972

- Vainshtein mechanism (1972)
  Nonlinearity → Massless limit = General Relativity

- Fierz-Pauli theory (1939)
  Unique linear theory without instabilities (ghosts)

No?

- Boulware-Deser ghost (1972)
  6th d.o.f. @ Nonlinear level → Instability (ghost)

- van Dam-Veltman-Zhakharov discontinuity (1970)
  Massless limit ≠ General Relativity
Cosmological solutions in nonlinear massive gravity

Good?  

Bad?

D'Amico, et.al. (2011)  
Non-existence of flat FLRW (homogeneous isotropic) universe!
Cosmological solutions in nonlinear massive gravity

Good?

Open universes with self-acceleration
GLM (2011a)

Bad?

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GLM = Gumrukcuoglu-Lin-Mukohyama
Cosmological solutions in nonlinear massive gravity

Good?

Bad?

More general fiducial metric $f_{\nu\mu}$
Closed/flat/open FLRW universes allowed
GLM (2011b)

Open universes with self-acceleration
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Non-existence of flat FLRW (homogeneous isotropic) universe!

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Summary of introduction + $\alpha$

- Nonlinear massive gravity
  free from BD ghost
- FLRW background
  No closed/flat universe
  Open universes with self-acceleration!
- More general fiducial metric $f_{\mu\nu}$
  closed/flat/open FLRW universes allowed
  Friedmann eq does not depend on $f_{\mu\nu}$
- Cosmological linear perturbations
  Scalar/vector sectors $\rightarrow$ same as in GR
  Tensor sector $\rightarrow$ time-dependent mass
Nonlinear instability

DeFelice, Gumrukcuoglu, Mukohyama, arXiv: 1206.2080 [hep-th]

- de Sitter or FLRW fiducial metric
- Pure gravity + bare cc $\rightarrow$ FLRW sol = de Sitter
- Bianchi I universe with axisymmetry + linear perturbation (without decoupling limit)
- Small anisotropy expansion of Bianchi I + linear perturbation
  $\rightarrow$ nonlinear perturbation around flat FLRW

- **Odd-sector:**
  1 healthy mode + 1 healthy or ghosty mode

- **Even-sector:**
  2 healthy modes + 1 ghosty mode

- This is not BD ghost nor Higuchi ghost.
Cosmological solutions in nonlinear massive gravity

**Good?**

More general fiducial metric $f_{\mu\nu}$

- Closed/flat/open FLRW universes allowed
  - GLM (2011b)

- Open universes with self-acceleration
  - GLM (2011a)

**Bad?**

**NEW**

- Nonlinear instability of FLRW solutions
  - DGM (2012)

- D'Amico, et.al. (2011)
  - Non-existence of flat FLRW (homogeneous isotropic) universe!

GLM = Gumrukcuoglu-Lin-Mukohyama
DGM = DeFelice-Gumrukcuoglu-Mukohyama
Generic vs degenerate solutions

- Self-accelerating FLRW solution [GLM 2011a,b]
  \[(3-2X) + (3-X)(1-X)\alpha_3 + (1-X)^2\alpha_4 = 0, \quad X = \frac{a_f}{a_g}\]
  \[\rightarrow \text{generically two solutions } X = X_{\pm}\]
- Generic case with \(X_- \neq X_+\) [DGM 2012]

\[\text{Quadratic kinetic terms } = 0 \quad [\text{GLM 2011b}]
\text{Cubic kinetic terms } \neq 0 \rightarrow \text{nonlinear ghost}\]
- Anyway, 3 modes are infinitely strongly coupled
Generic vs degenerate solutions

• Self-accelerating solution [GLM 2011a]
  
  $$(3-2X) + (3-X)(1-X)\alpha_3 + (1-X)^2\alpha_4 = 0, \quad X = \frac{a_f}{a_g}$$

  $\Rightarrow$ generically two solutions $X = X_{\pm}$

• Degenerate case with $X_- = X_+$ [Masahide’s talk]

  Quadratic kinetic terms = 0 [GLM 2011b]
  Cubic kinetic terms = 0 $\Rightarrow$ Quartic?

• Anyway, 3 modes are infinitely strongly coupled
Cosmological solutions in nonlinear massive gravity

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New backgrounds or Extended theories

• New nonlinear instability [DeFelice, Gumrukcuoglu, Mukohyama 2012] \rightarrow (i) new backgrounds, or (ii) extended theories

• (i) Anisotropic FLRW (Gumrukcuoglu, Lin, Mukohyama 2012): physical metric is isotropic but fiducial metric is anisotropic

• (ii) Extended quasidilaton (DeFelice&Mukohyama 2013), Bimetric theory (Hassan, Rosen 2011; DeFelice, Nakamura, Tanaka 2013; DeFelice, Gumrukcuoglu, Mukohyama, Tanahashi, Tanaka 2014), Rotation-invariant theory (Rubakov 2004; Dubovsky 2004; Blas, Comelli, Pilo 2009; Comelli, Nesti, Pilo 2012; Langlois, Mukohyama, Namba, Naruko 2014), Composite metric (de Rham, Heisenberg, Ribeiro 2014; Gumrukcuoglu, Heisenberg, Mukohyama 2014, 2015), New quasidilaton (Mukohyama 2014), …

• They provide stable cosmology.
Cosmological solutions in nonlinear massive gravity

Good?

- NEW Class of Solutions
  - Anisotropic FLRW universe
    - GLM (2012)
  - More general fiducial metric $f_{\mu\nu}$
    - closed/flat/open FLRW universes allowed
      - GLM (2011b)
  - Open universes with self-acceleration
    - GLM (2011a)

Bad?

- NEW Nonlinear instability of FLRW solutions
  - D'Amico, et.al. (2011)
  - Non-existence of flat FLRW (homogeneous isotropic) universe!

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Good?

Extended theories:
- Extended quasidilatonic biometric theory, rotation-invariant theory, composite metric,

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More recent development

**Minimal Theory of Massive Gravity**

De Felice & Mukohyama, arXiv: 1506.01594

- 2 physical dof only = massive gravitational waves
- exactly same FLRW background as in dRGT
- no BD ghost, no Higuchi ghost, no nonlinear ghost

Three steps to the Minimal Theory

1. Fix local Lorentz to realize ADM vielbein in dRGT
2. Switch to Hamiltonian
3. Add 2 additional constraints
Step 1. Fix local Lorentz to realize ADM vielbein in dRGT

\[ \|e^A_\mu\| = \begin{pmatrix} N & \vec{0}^T \\ e^I_i N^i & e^I_j \end{pmatrix} \quad \|E^A_\mu\| = \begin{pmatrix} M & \vec{0}^T \\ E^I_i M^i & E^I_j \end{pmatrix} \]

\[ S_{\text{pre}} = \frac{M_p^2}{2} \int d^4 x \sqrt{-g} \mathcal{R}[g_{\mu\nu}] \]

\[ + \frac{M_p^2}{2} m^2 \int d^4 x \left[ \frac{c_0}{24} \epsilon_{ABCD} \epsilon^{\alpha\beta\gamma\delta} E^A_\alpha E^B_\beta E^C_\gamma E^D_\delta \right. \]

\[ + \frac{c_1}{6} \epsilon_{ABCD} \epsilon^{\alpha\beta\gamma\delta} E^A_\alpha E^B_\beta E^C_\gamma E^D_\delta \]

\[ + \frac{c_2}{4} \epsilon_{ABCD} \epsilon^{\alpha\beta\gamma\delta} E^A_\alpha E^B_\beta E^C_\gamma e^D_\delta \]

\[ + \frac{c_3}{6} \epsilon_{ABCD} \epsilon^{\alpha\beta\gamma\delta} E^A_\alpha e^B_\beta e^C_\gamma e^D_\delta \]

dRGT potential
Step 2. Switch to Hamiltonian

\[ H_{\text{pre}} = \int d^3x \left[ -N R_0 - N^i R_i \right] \]

- linear in lapse and shift
- \( \rightarrow 4 \) primary constraints

\[ + m^2 \mathcal{M} \mathcal{H}_1 + \left[ \tilde{\lambda}^\alpha \tilde{C}_\alpha \right] \]

- 2 secondary constraints \((\alpha = 1, 2)\)

\[ + \alpha_{MN} \mathcal{P}^{[MN]} + \beta_{MN} Y^{[MN]} \]

- 6 \((= 3 \text{ primary } + 3 \text{ secondary})\) constraints associated with symmetry of spatial vielbein

\[ 9 \times 2 - 4 - 2 - 6 = 6 \rightarrow 3 \text{ d.o.f.} \]

c.f. consistent with the analysis by Comelli, Nesti and Pilo 2014
Step2. Switch to Hamiltonian

\[ H_{pre} = \int d^3 x \left[ -N R_0 - N^i R_i \right] \]

linear in lapse and shift
\[ \rightarrow 4 \text{ primary constraints} \]

Precursor theory with 3 d.o.f.

\[ \begin{align*}
-\frac{m^2}{16\pi} M \mathcal{H}_1 &+ \sum_{\alpha=1,2} \lambda \mathcal{C}_\alpha \\
+ \alpha \epsilon_{MN} F_{MN} Y^{[MN]} &+ \beta \epsilon_{MN} \tilde{Y}^{[MN]} \end{align*} \]

6 ( = 3 primary + 3 secondary) constraints associated with symmetry of spatial vielbein

\[ 9 \times 2 - 4 - 2 - 6 = 6 \rightarrow 3 \text{ d.o.f.} \]

c.f. consistent with the analysis by Comelli, Nesti and Pilo 2014
Step 3. Add 2 additional constraints

\[ H = \int d^3x \left[ -NR_0 - N^i R_i \right. \]
\[ + m^2 M H_1 + \lambda C_0 + \lambda^i C_i \]
\[ + \alpha_{MN} P^{[MN]} + \beta_{MN} Y^{[MN]} \]
\[ C_0 \overset{\dot{}}{=} \{ R_0 , H_1 \} + \frac{\partial R_0}{\partial t} \quad C_l \overset{\dot{}}{=} \{ R_l , H_1 \} \]

Only 2 among \((C_0, C_i)\) are new

6 (from precursor theory) – 2 (additional constraints) = 4 \(\Rightarrow\) 2 d.o.f.
Phenomenology of the minimal theory

• The remaining 2 d.o.f. = massive gravitational waves
• FLRW cosmology: exactly same as dRGT $\rightarrow$ self-accelerating solution
• Absolutely stable: no BD ghost, no Higuchi ghost, no nonlinear ghost
• Constraint from binary pulsar $m_{gw} < 10^{-5}$ Hz
• Stochastic GW? CMB B-mode?
Cosmological solutions in nonlinear massive gravity

Good?

- Minimal Theory of Massive Gravity
  DeFelice&Mukohyama
  (2015)

- More general fiducial metric $f_{\mu\nu}$
  closed/flat/open FLRW universes allowed
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DGHM = DeFelice-Gumrukcuoglu-Heisenberg-Mukohyama
Summary

• Nonlinear massive gravity free from BD ghost
• FLRW background No closed/flat universe
  Open universes with self-acceleration!
• More general fiducial metric $f_{\mu\nu}$
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  Scalar/vector sectors $\rightarrow$ same as in GR
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• All homogeneous and isotropic FLRW solutions in the original dRGT theory have ghost
• Stable cosmology realized in (i) new class of cosmological solution or (ii) extended theories
• Minimal theory of massive gravity with 2dof results in stable self-accelerating cosmology
More recent development

**Composite vielbein**

- Composite metric in terms of vielbain
  \[ e^a_{(\text{eff})\mu} = \alpha e^a_{(g)\mu} + \beta e^a_{(f)\mu} \]

- Hamiltonian is linear in N after eliminating local boost \( w_{0I} \) and \( N^i \) [Hinterbichler & Rosen 2015]

- However, Hamiltonian becomes nonlinear after eliminating local rotation \( w_{IJ} \) [de Rham & Tolley 2015]

- Partially constrained vielbein [De Felice, Gumrukcuoglu, Heisenberg, Mukohyama 2015]

  let \( w_{0I} \) be determined by eom while
  let \( w_{IJ} \) be fixed by 3d symmetric condition

  → BD ghost free & stable cosmological sol
New quasidilaton theory


\[ I_{\text{newQD}}[g_{\mu\nu}, f_{\mu\nu}, \sigma] = M_{\text{Pl}}^2 m_g^2 \int d^4 x \sqrt{-g} \left[ \mathcal{L}_2(\bar{\mathcal{K}}) + \alpha_3 \mathcal{L}_3(\bar{\mathcal{K}}) + \alpha_4 \mathcal{L}_4(\bar{\mathcal{K}}) \right] \]

\[ - \frac{\omega}{2} \int d^4 x \sqrt{-g_{\text{eff}}} g_{\mu\nu}^{\text{eff}} \partial_\mu \sigma \partial_\nu \sigma \]

\[ g_{\mu\nu}^{\text{eff}} = g_{\mu\nu} + 2\beta e^{\sigma/M_{\text{Pl}}} g_{\mu\rho}(\sqrt{g^{-1}f})^\rho_\nu + \beta^2 e^{2\sigma/M_{\text{Pl}}} f_{\mu\nu} \]

- Quasidilaton kinetic term is now defined on the effective metric \( \rightarrow \) new parameter \( \beta \)

- Self-acccerating de Sitter solution is stable in a range of parameters if \( \beta \) is non-zero
New quasidilaton theory
based on composite metric \( \text{Mukohyama, arXiv: 1410.1996} \)

\[ I_{\text{new QD}}[g_{\mu\nu}, f_{\mu\nu}, \sigma] = M_{\text{Pl}}^2 m^2 \int d^4x \sqrt{-g} \left[ \mathcal{L}_2(\tilde{K}) + \mathcal{L}_3(\tilde{K}) + \mathcal{L}_4(\tilde{K}) \right] \]

Upgrade to partially constrained vielbein formulation?

\[ g^{\text{eff}}_{\mu\nu} = g_{\mu\nu} + 2\beta e^{\sigma/M_{\text{Pl}}} \left[ g_{\mu\nu} \left( \sqrt{g^{-1} f} \right)^{\rho} + \beta^2 e^{2\sigma/M_{\text{Pl}}} f_{\mu\nu} \right] \]

• [DeFelice, Gumrukcuoglu, Heisenberg, Mukohyama, Tanahashi, to appear soon]

• Self-accerating de Sitter solution is stable in a range of parameters if \( \beta \) is non-zero