Massive gravity and cosmology

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Based on collaboration with
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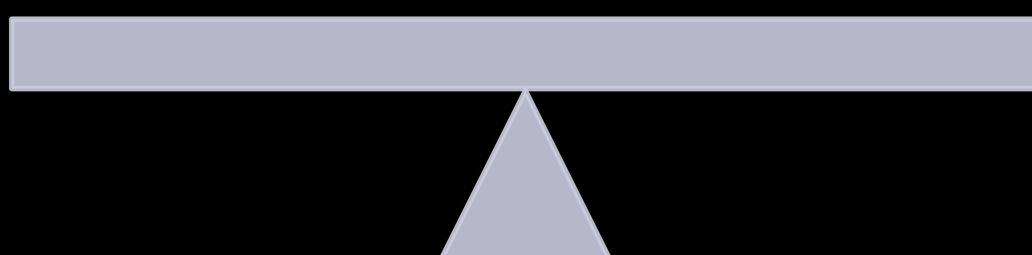
Massive gravity: history

Simple question: Can graviton have mass?

May lead to acceleration without dark energy

Yes?

No?



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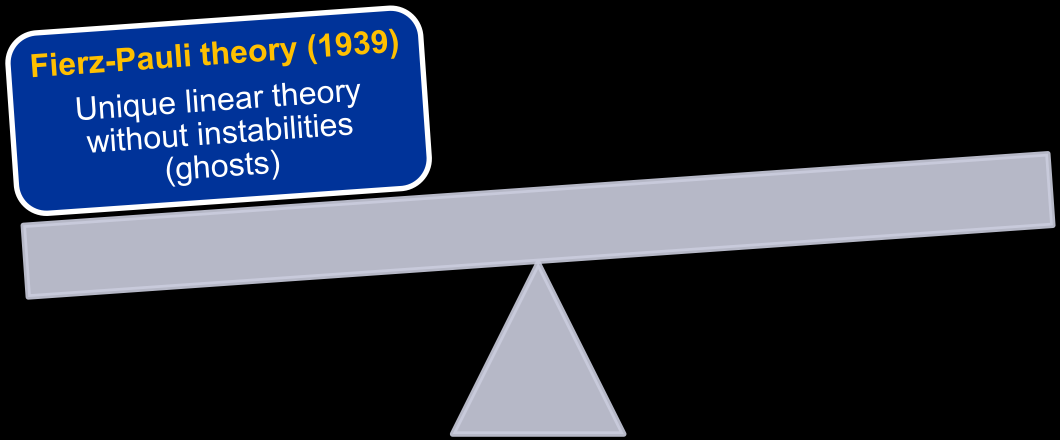

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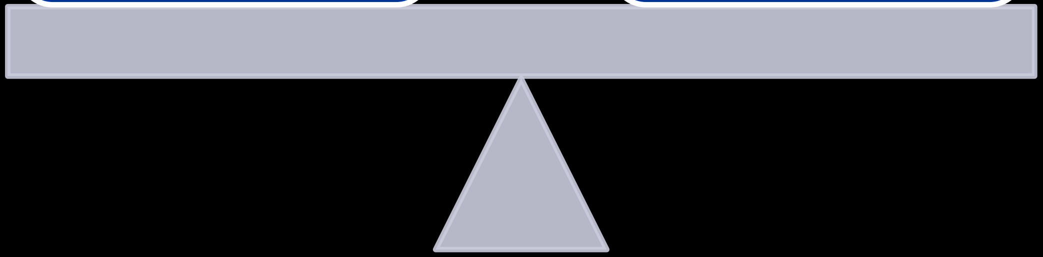
Fierz-Pauli theory (1939)

Unique linear theory without instabilities (ghosts)

van Dam-Veltman-Zhakharov discontinuity (1970)

Massless limit ≠ General Relativity



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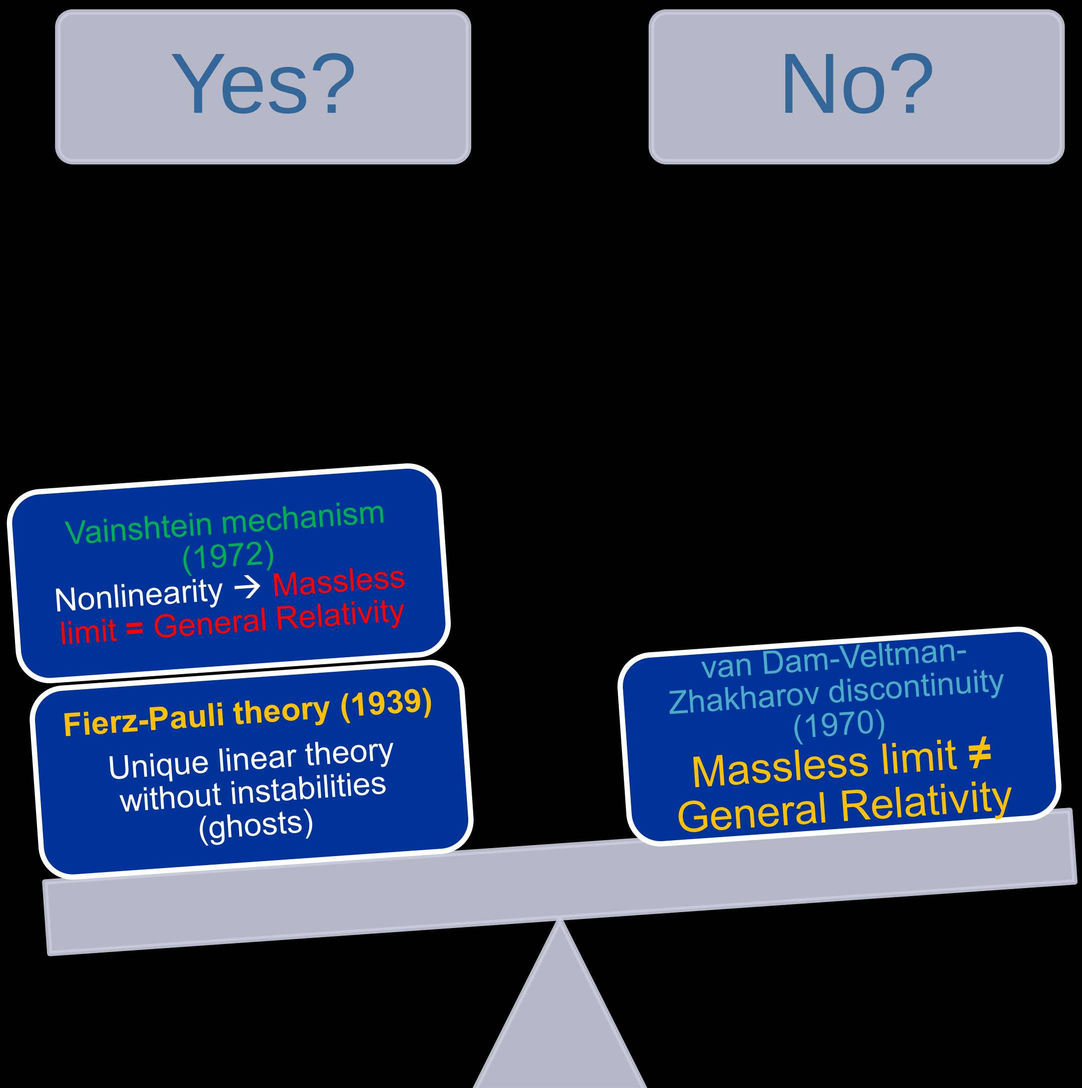

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Simple question: Can graviton have mass?

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Vainshtein mechanism (1972)

Nonlinearity → Massless limit = General Relativity

Fierz-Pauli theory (1939)

Unique linear theory without instabilities (ghosts)

Boulware-Deser ghost (1972)

6th d.o.f.@Nonlinear level → Instability (ghost)

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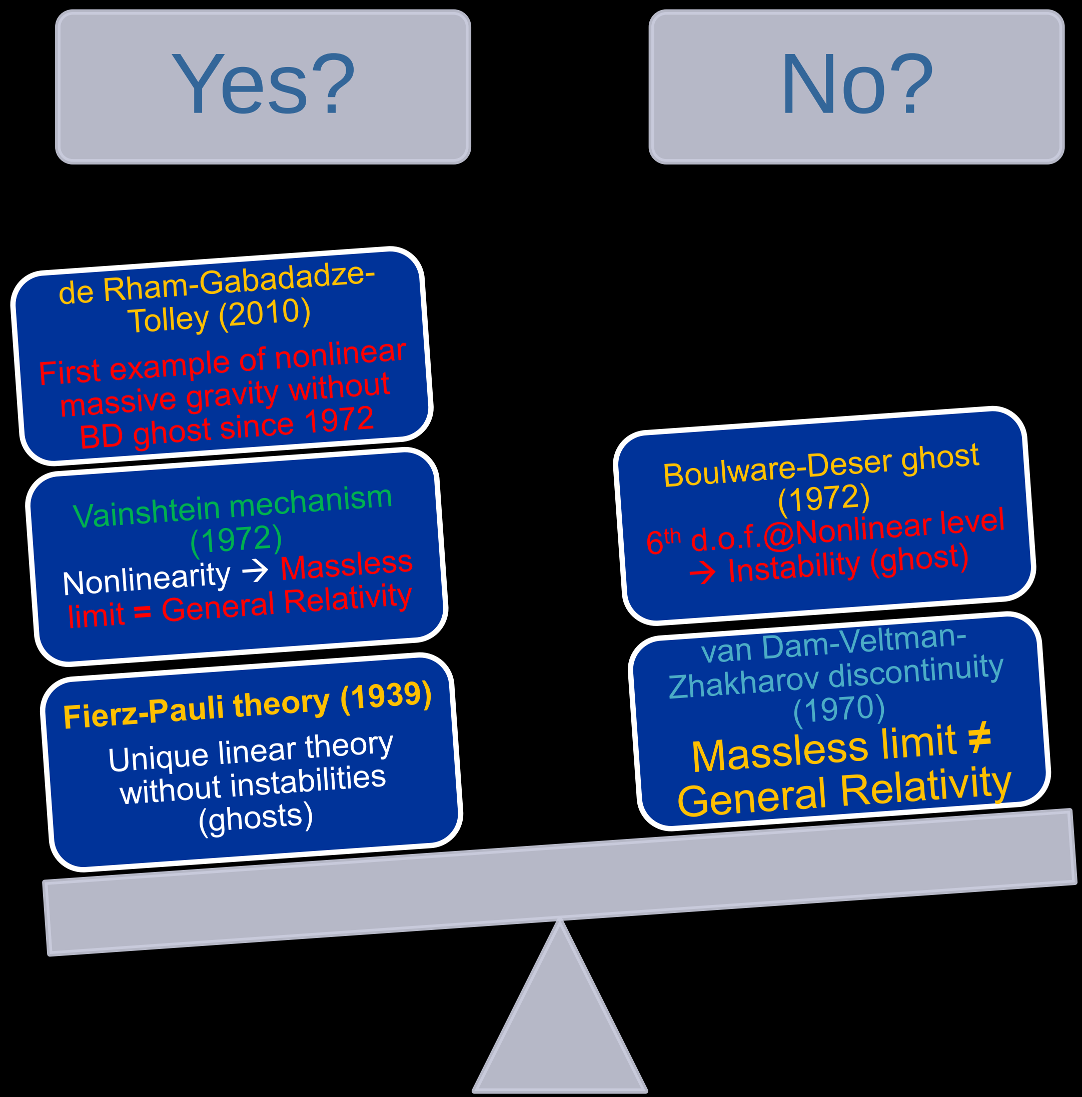
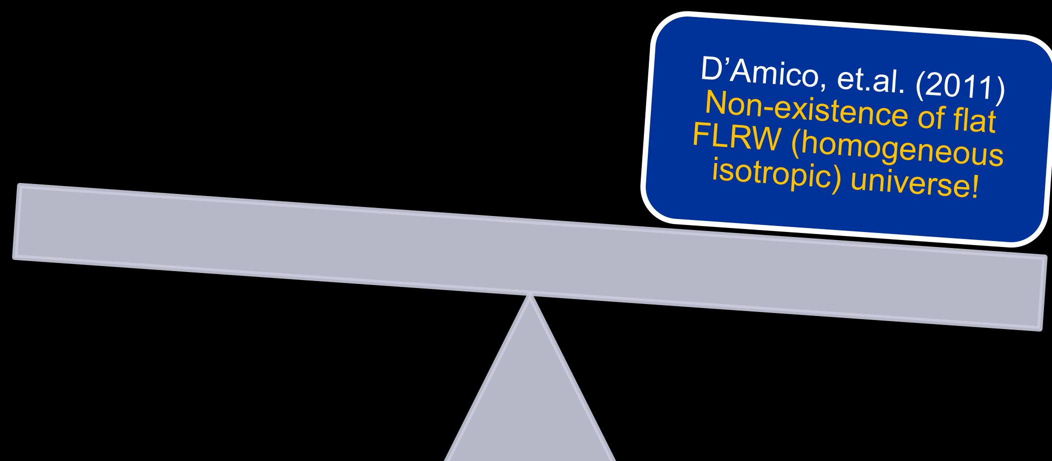


Cosmological solutions in nonlinear massive gravity

Good?

Bad?



Cosmological solutions in nonlinear massive gravity

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Open universes with self-acceleration
GLM (2011a)

D'Amico, et.al. (2011)
Non-existence of flat FLRW (homogeneous isotropic) universe!

GLM = Gumrukcuoglu-Lin-Mukohyama

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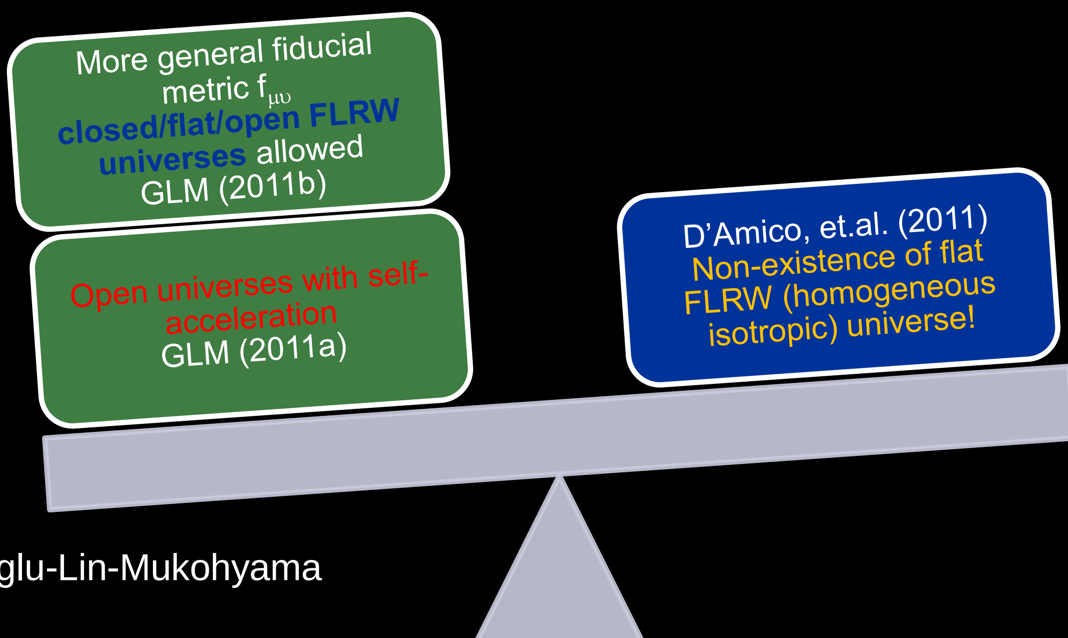

GLM = Gumrukcuoglu-Lin-Mukohyama

Summary of introduction + α

- Nonlinear massive gravity
  free from BD ghost
- FLRW background
  No closed/flat universe
  Open universes with self-acceleration!
- More general fiducial metric $f_{\mu\upsilon}$
  closed/flat/open FLRW universes allowed
  Friedmann eq does not depend on $f_{\mu\upsilon}$
- Cosmological linear perturbations
  Scalar/vector sectors → same as in GR
  Tensor sector → time-dependent mass

Nonlinear instability

DeFelice, Gumrukcuoglu, Mukohyama, arXiv: 1206.2080 [hep-th]

- de Sitter or FLRW fiducial metric
- Pure gravity + bare cc → FLRW sol = de Sitter
- Bianchi I universe with axisymmetry + linear perturbation (without decoupling limit)
- Small anisotropy expansion of Bianchi I + linear perturbation
  → nonlinear perturbation around flat FLRW
- Odd-sector:
  1 healthy mode + 1 healthy or ghosty mode
- Even-sector:
  2 healthy modes + 1 ghosty mode
- This is not BD ghost nor Higuchi ghost.

Cosmological solutions in nonlinear massive gravity

Good?

Bad?

More general fiducial metric $f_{\mu\upsilon}$
closed/flat/open FLRW universes allowed
GLM (2011b)

Open universes with self-acceleration
GLM (2011a)

NEW
Nonlinear instability of FLRW solutions
DGM (2012)

D'Amico, et.al. (2011)
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GLM = Gumrukcuoglu-Lin-Mukohyama
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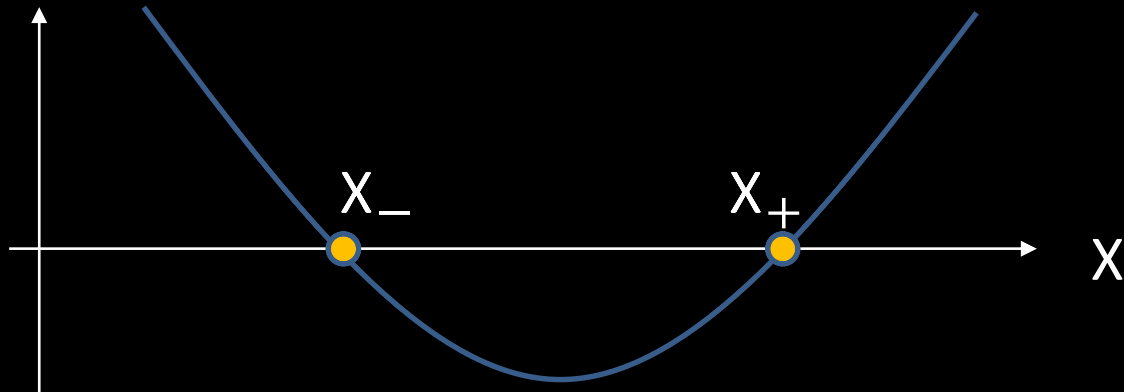
Generic vs degenerate solutions

- Self-accelerating FLRW solution [GLM 2011a,b]
  $(3-2X) + (3-X)(1-X)\alpha_3 + (1-X)^2\alpha_4 = 0, \quad X = a_f/a_g$
  → generically two solutions $X = X_\pm$
- Generic case with $X_- \neq X_+$ [DGM 2012]



  Quadratic kinetic terms = 0 [GLM 2011b]
  Cubic kinetic terms ≠ 0 → nonlinear ghost
- Anyway, 3 modes are infinitely strongly coupled

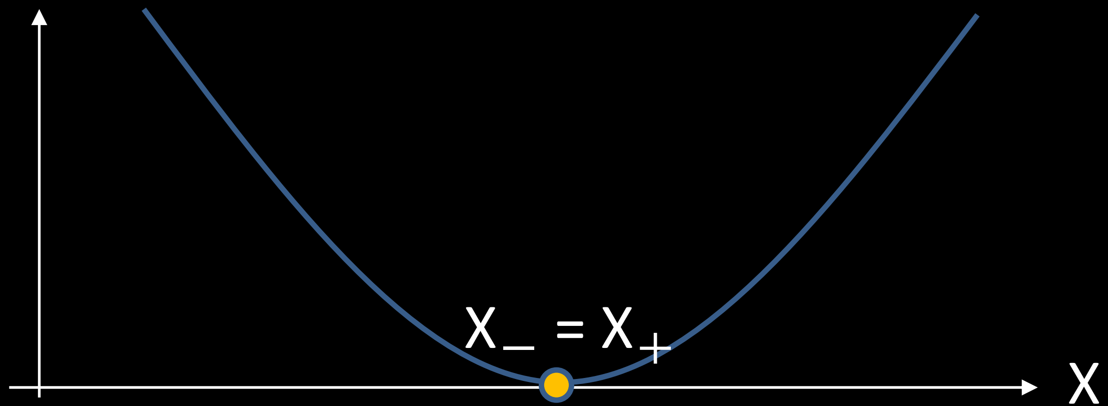
Generic vs degenerate solutions

- Self-accelerating solution [GLM 2011a]

  $(3-2X) + (3-X)(1-X)\alpha_3 + (1-X)^2\alpha_4 = 0$, $X = a_f/a_g$

  → generically two solutions $X = X_\pm$

- Degenerate case with $X_- = X_+$ [Masahide's talk]



Quadratic kinetic terms = 0 [GLM 2011b]

Cubic kinetic terms = 0 → Quartic?

- Anyway, 3 modes are infinitely strongly coupled

Cosmological solutions in nonlinear massive gravity

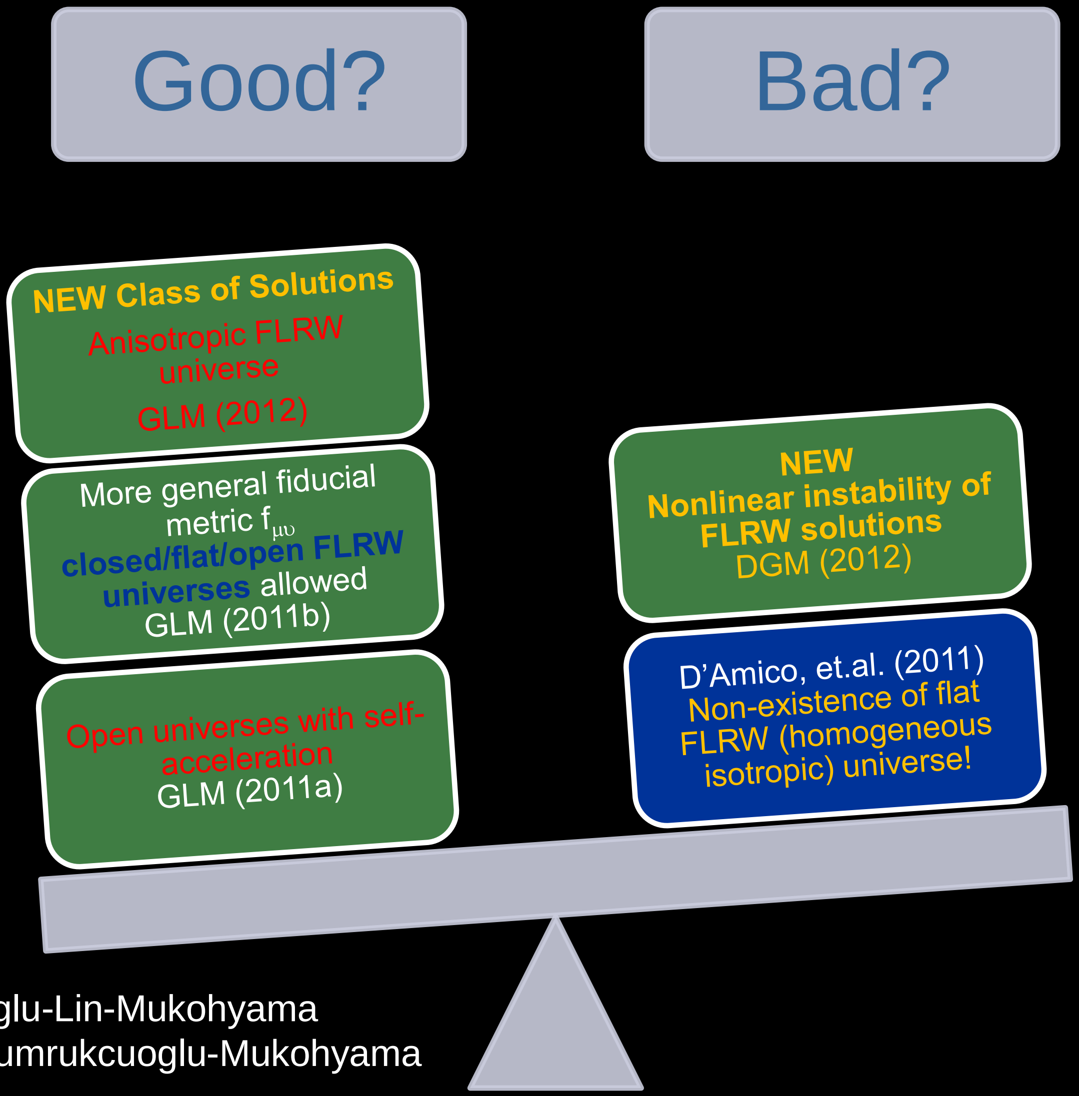
Good?	Bad?
More general fiducial metric $f_{\mu\nu}$ **closed/flat/open FLRW universes** allowed GLM (2011b)	**NEW** **Nonlinear instability of FLRW solutions** DGM (2012)
Open universes with self-acceleration GLM (2011a)	D'Amico, et.al. (2011) Non-existence of flat FLRW (homogeneous isotropic) universe!

GLM = Gumrukcuoglu-Lin-Mukohyama
DGM = DeFelice-Gumrukcuoglu-Mukohyama

New backgrounds or Extended theories

- New nonlinear instability [DeFelice, Gumrukcuoglu, Mukohyama 2012] → (i) new backgrounds, or (ii) extended theories
- (i) Anisotropic FLRW (Gumrukcuoglu, Lin, Mukohyama 2012): physical metric is isotropic but fiducial metric is anisotropic
- (ii) Extended quasidilaton (De Felice&Mukohyama 2013), Bimetric theory (Hassan, Rosen 2011; DeFelice, Nakamura, Tanaka 2013; DeFelice, Gumrukcuoglu, Mukohyama, Tanahashi, Tanaka 2014), Rotation-invariant theory (Rubakov 2004; Dubovsky 2004; Blas, Comelli, Pilo 2009; Comelli, Nesti, Pilo 2012; Langlois, Mukohyama, Namba, Naruko 2014), Composite metric (de Rham, Heisenberg, Ribeiro 2014; Gumrukcuoglu, Heisenberg, Mukohyama 2014, 2015), New quasidilaton (Mukohyama 2014), ...
- They provide stable cosmology.

Cosmological solutions in nonlinear massive gravity



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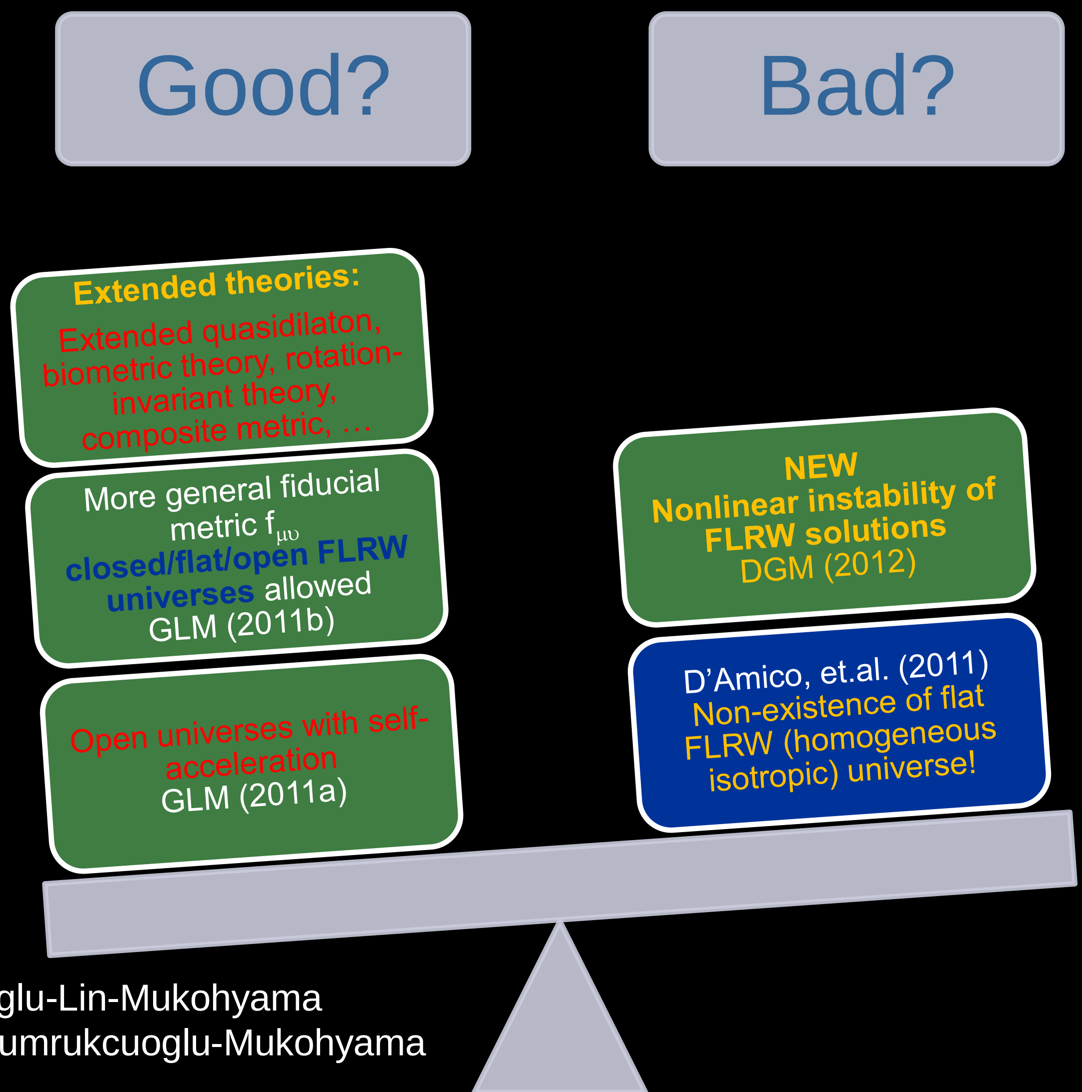

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More recent development

Minimal Theory of Massive Gravity

De Felice & Mukohyama, arXiv: 1506.01594

- 2 physical dof only = massive gravitational waves
- exactly same FLRW background as in dRGT
- no BD ghost, no Higuchi ghost, no nonlinear ghost

Three steps to the Minimal Theory

1. Fix local Lorentz to realize ADM vielbein in dRGT
2. Switch to Hamiltonian
3. Add 2 additional constraints

Step 1. Fix local Lorentz to realize ADM vielbein in dRGT

$$\|e^{\mathcal{A}}{}_{\mu}\| = \begin{pmatrix} N & \boxed{\vec{0}^T} \\ e^I{}_i N^i & e^I{}_j \end{pmatrix} \qquad \|E^{\mathcal{A}}{}_{\mu}\| \doteq \begin{pmatrix} M & \boxed{\vec{0}^T} \\ E^I{}_i M^i & E^I{}_j \end{pmatrix}$$

physical fiducial

$$S_{\rm pre} = \frac{M_{\rm P}^2}{2}\int d^4x\sqrt{-g}\,\mathcal{R}[g_{\mu\nu}]$$
$$+\frac{M_{\rm P}^2}{2}m^2\int d^4x\left[\frac{c_0}{24}\epsilon_{\mathcal{ABCD}}\epsilon^{\alpha\beta\gamma\delta}E^{\mathcal{A}}{}_{\alpha}E^{\mathcal{B}}{}_{\beta}E^{\mathcal{C}}{}_{\gamma}E^{\mathcal{D}}{}_{\delta}\right.$$
$$+\frac{c_1}{6}\epsilon_{\mathcal{ABCD}}\epsilon^{\alpha\beta\gamma\delta}E^{\mathcal{A}}{}_{\alpha}E^{\mathcal{B}}{}_{\beta}E^{\mathcal{C}}{}_{\gamma}e^{\mathcal{D}}{}_{\delta}$$
$$+\frac{c_2}{4}\epsilon_{\mathcal{ABCD}}\epsilon^{\alpha\beta\gamma\delta}E^{\mathcal{A}}{}_{\alpha}E^{\mathcal{B}}{}_{\beta}e^{\mathcal{C}}{}_{\gamma}e^{\mathcal{D}}{}_{\delta}$$
$$+\frac{c_3}{6}\epsilon_{\mathcal{ABCD}}\epsilon^{\alpha\beta\gamma\delta}E^{\mathcal{A}}{}_{\alpha}e^{\mathcal{B}}{}_{\beta}e^{\mathcal{C}}{}_{\gamma}e^{\mathcal{D}}{}_{\delta}$$

dRGT potential

Step2. Switch to Hamiltonian

$$H_{\rm pre} = \int d^3x [-N\mathcal{R}_0 - N^i\mathcal{R}_i$$

linear in lapse and shift
→ 4 primary constraints

$$+ m^2 M\mathcal{H}_1 + \tilde{\lambda}^\alpha \tilde{\mathcal{C}}_\alpha$$

2 secondary
constraints (α=1,2)

$$+ \alpha_{MN}\mathcal{P}^{[MN]} + \beta_{MN}Y^{[MN]}]$$

6 (= 3 primary + 3 secondary) constraints
associated with symmetry of spatial vielbein

9 x 2 – 4 – 2 – 6 = 6 → 3 d.o.f.

c.f. consistent with the analysis by Comelli, Nesti and Pilo 2014

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Precursor theory with 3 d.o.f.

6 (= 3 primary + 3 secondary) constraints associated with symmetry of spatial vielbein

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Step3. Add 2 additional constraints

$$H = \int d^3x[-N\mathcal{R}_0 - N^i\mathcal{R}_i$$

$$+ m^2 M\mathcal{H}_1 + \lambda\mathcal{C}_0 + \lambda^i\mathcal{C}_i$$

4 constraints instead of 2

$$+ \alpha_{MN}\mathcal{P}^{[MN]} + \beta_{MN}Y^{[MN]}]$$

$$\mathcal{C}_0 \doteq \{\mathcal{R}_0, H_1\} + \frac{\partial\mathcal{R}_0}{\partial t} \qquad \mathcal{C}_l \doteq \{\mathcal{R}_l, H_1\}$$

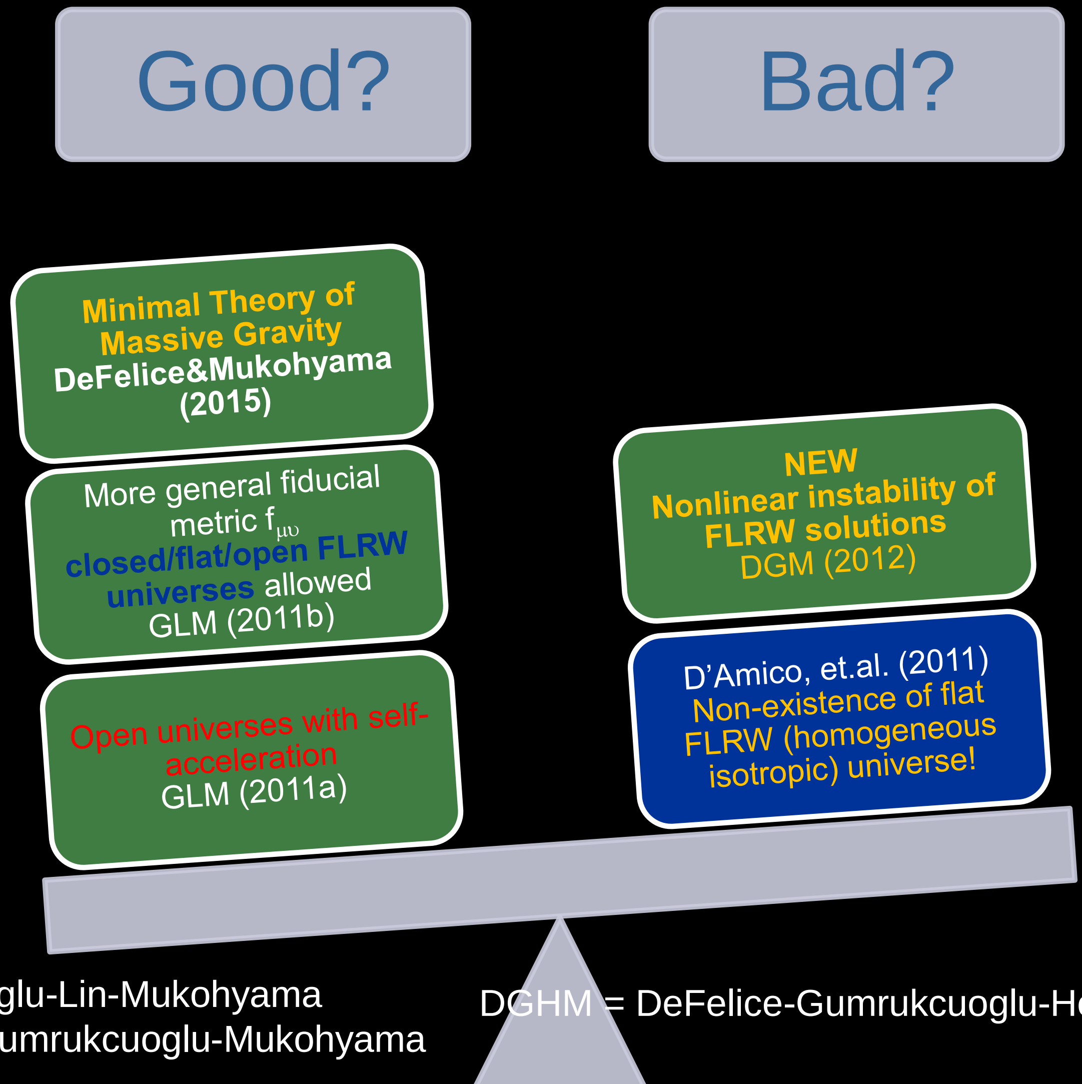
➡ Only 2 among (C_0, C_i) are new

6 (from precursor theory) – 2 (additional constraints) = 4 ➔ 2 d.o.f.

Phenomenology of the minimal theory

- The remaining 2 d.o.f. = massive gravitational waves
- FLRW cosmology: exactly same as dRGT → self-accelerating solution
- Absolutely stable: no BD ghost, no Higuchi ghost, no nonlinear ghost
- Constraint from binary pulsar $m_{gw} < 10^{-5}$ Hz
- Stochastic GW? CMB B-mode?

Cosmological solutions in nonlinear massive gravity



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DGHM = DeFelice-Gumrukcuoglu-Heisenberg-Mukohyama

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- More general fiducial metric $f_{\mu\upsilon}$
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- Cosmological linear perturbations
  Scalar/vector sectors → same as in GR
  Tensor sector → time-dependent mass
- All homogeneous and isotropic FLRW solutions in the original dRGT theory have ghost
- Stable cosmology realized in (i) new class of cosmological solution or (ii) extended theories
- Minimal theory of massive gravity with 2dof results in stable self-accelerating cosmology

More recent development

Composite vielbein

- Composite metric in terms of vielbain

$$e^{a}_{(eff)\mu} = \alpha e^{a}_{(g)\mu} + \beta e^{a}_{(\mathrm{f})\mu}$$

- Hamiltonian is linear in N after eliminating local boost w_{0I} and N^i [Hinterbichler & Rosen 2015]
- However, Hamiltonian becomes nonlinear after eliminating local rotation w_{IJ} [de Rham & Tolley 2015]
- Partially constrained vielbein [De Felice, Gumrukcuoglu, Heisenberg, Mukohyama 2015]

  let w_{0I} be determined by eom while
  let w_{IJ} be fixed by 3d symmetric condition

  ⇒ BD ghost free & stable cosmological sol

New quasidilaton theory

based on composite metric   Mukohyama, arXiv: 1410.1996

$$I_{\mathrm{newQD}}[g_{\mu\nu}, f_{\mu\nu}, \sigma] = M_{\mathrm{Pl}}^2 m_g^2 \int d^4x \sqrt{-g}\, \left[\mathcal{L}_2(\bar{\mathcal{K}}) + \alpha_3 \mathcal{L}_3(\bar{\mathcal{K}}) + \alpha_4 \mathcal{L}_4(\bar{\mathcal{K}})\right]$$

$$- \frac{\omega}{2} \int d^4x \sqrt{-g_{\mathrm{eff}}}\, g_{\mathrm{eff}}^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma$$

$$g_{\mu\nu}^{\mathrm{eff}} = g_{\mu\nu} + 2\beta \boxed{e^{\sigma/M_{\mathrm{Pl}}}} g_{\mu\rho} \left(\sqrt{g^{-1}f}\right)^{\rho}{}_{\nu} + \beta^2 \boxed{e^{2\sigma/M_{\mathrm{Pl}}}} f_{\mu\nu}$$

- Quasidilaton kinetic term is now defined on the effective metric → **new parameter β**
- Self-accerating de Sitter solution is stable in a range of parameters if β is non-zero

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Upgrade to partially constrained vielbein formulation?

[DeFelice, Gumrukcuoglu, Heisenberg, Mukohyama, Tanahashi, to appear soon]

jk

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