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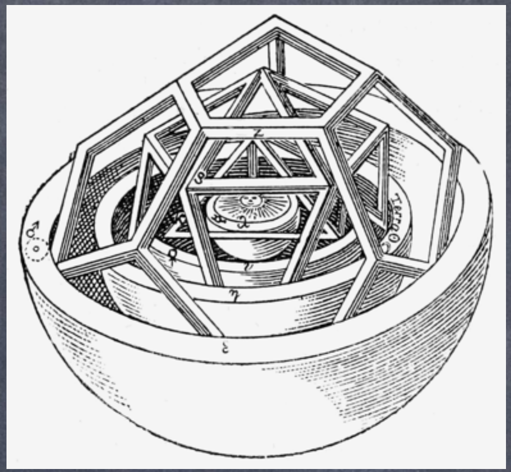

Icosahedral Inflation

w/ Jonghee Kang

(w/ Solomon Endlich and Junpu Wang, 2012)

Inflation: usual story

- The early universe: homogeneous and isotropic
- Usually modeled via $\varphi_a = \varphi_a(t)$
- Time-translations spontaneously broken

⇨ Goldstone excitation = adiabatic perturbations

- Systematic effective field theory

(Creminelli, Luty, Nicolis, Senatore 2006
Cheung, Creminelli, Fitzpatrick, Kaplan, Senatore 2007)

Solid inflation

(Endlich, Nicolis, Wang 2012)
(Gruzinov 2004)

- t-independent, x-dependent fields: $\varphi_a = \varphi_a(\vec{x})$
- time-translations unbroken
- spatial translations and rotations, broken

Apparently violates:

1. homogeneity and isotropy

2. the need for a physical "clock"

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Apparently violates:

1. homogeneity and isotropy $\Longrightarrow$ internal symmetries
2. the need for a physical "clock" $\Longrightarrow$ gravity

Homogeneity and isotropy

- Ex: one scalar w/ vev $\langle\varphi\rangle = x$
- If it has a shift symmetry $\varphi \to \varphi + a$
- unbroken diagonal translation

$$\begin{cases} x \to x - a \\ \varphi \to \varphi + a \end{cases}$$

Rotations still broken

Homogeneity and isotropy

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$$\begin{cases} x \to x - a \\ \varphi \to \varphi + a \end{cases}$$

Rotations still broken $\Longrightarrow$ need 3 fields

3 scalars: $\phi^I(\vec{x}, t)$ $I = 1, 2, 3$

vevs: $\langle \phi^I \rangle = x^I$

If internal symmetries:

$$\phi^I \to \phi^I + a^I$$
$$\phi^I \to SO(3)\, \phi^I$$

then unbroken diagonal subgroups

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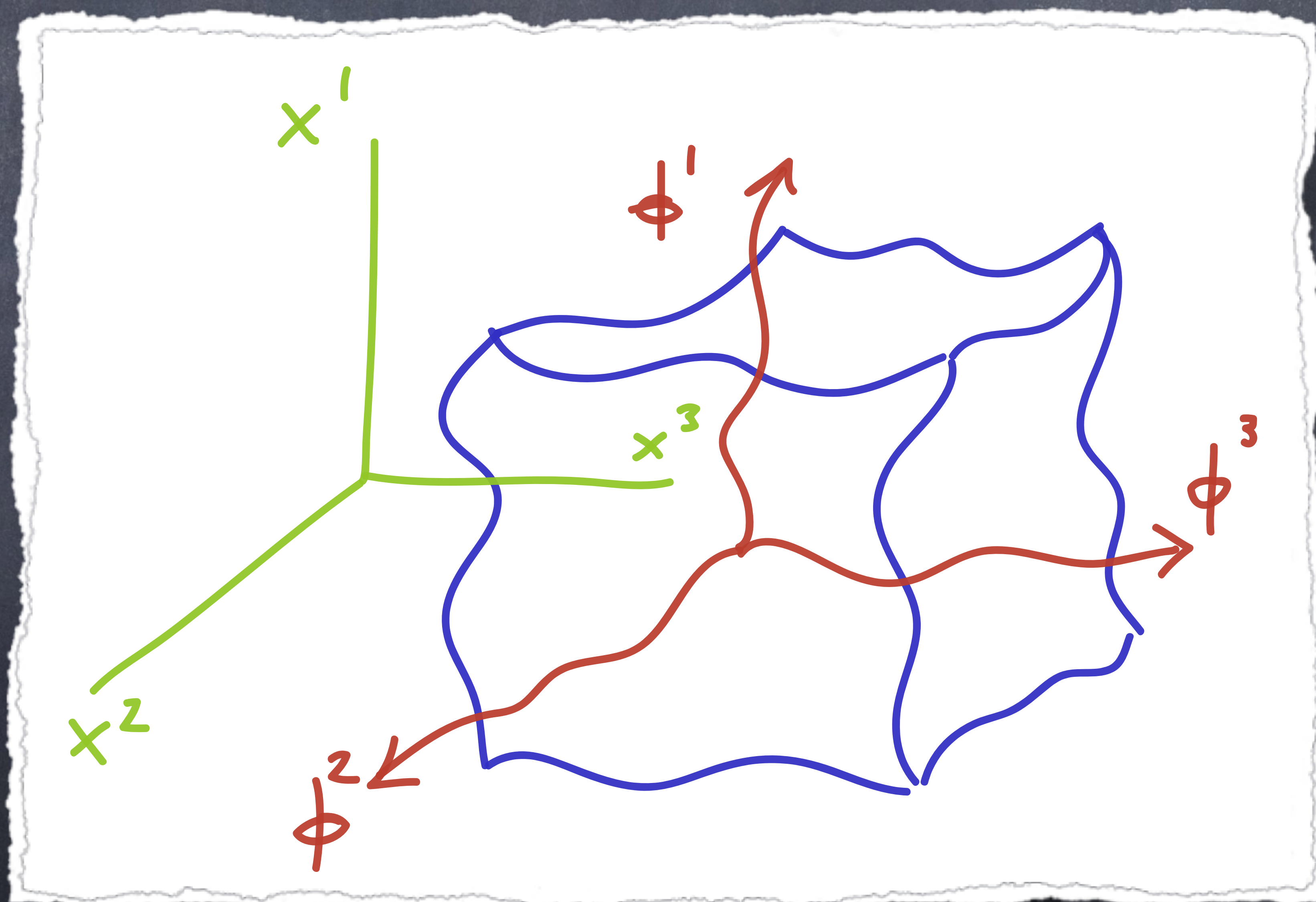
This is a solid

EFT for solids (and fluids)

Dof: volume elements' positions

$$\phi^I(\vec{x},t) \qquad I = 1,2,3$$



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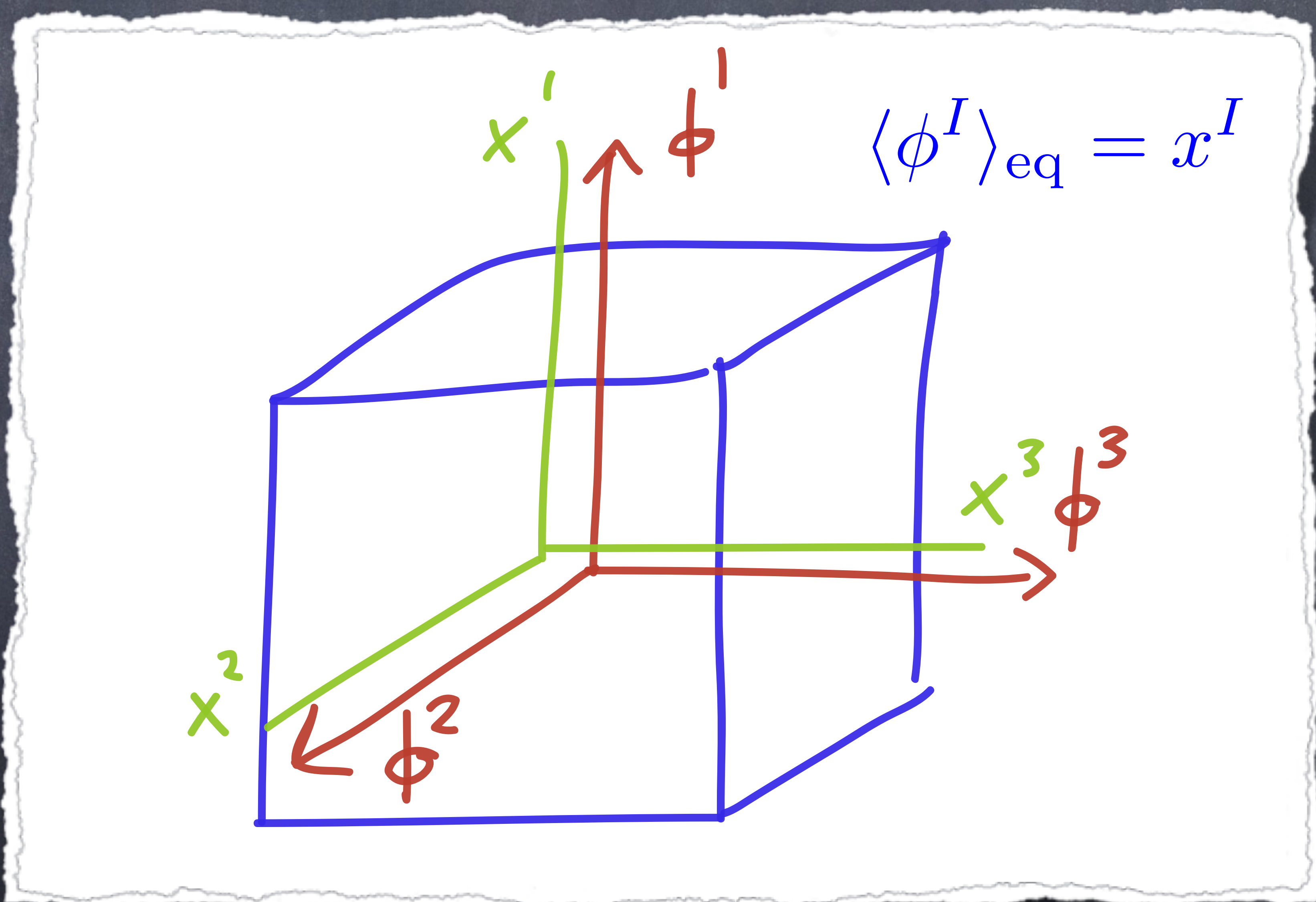


Symmetries: Poincaré + internal

$$\left.\begin{aligned} \phi^I &\to \phi^I + a^I \\ \phi^I &\to SO(3)\,\phi^I \end{aligned}\right\} \text{ recover homogeneity/isotropy}$$

$$\phi^I \to \xi^I(\phi) \qquad \det \frac{\partial \xi^I}{\partial \phi^J} = 1$$

fluid vs solid

Action

$$B^{IJ} \equiv \partial_\mu \phi^I \partial^\mu \phi^J$$

$$\mathcal{L} = F\Big([B], \underbrace{\frac{[B^2]}{[B]^2}, \frac{[B^3]}{[B]^3}}\Big) + \dots \qquad [\dots] = \mathrm{Tr}(\dots)$$

$$(X, Y, Z)$$

(For the fluid $\mathcal{L} = F(\det B) + \dots$)

(Dubovsky, Gregoire, Nicolis, Rattazzi 2006)
(Son 2005)

Stress-energy tensor

$$T_{\mu\nu} \sim (F, F') \times (g_{\mu\nu}, \partial_\mu \phi^I \partial_\nu \phi^J) \times (\delta^{IJ}, B^{IJ}, B^{IK} B^{KJ})$$

On the background $B^{IJ} = \delta^{IJ}$

$$T_{\mu\nu} \to \begin{cases} \rho = -F \\ \rho + p = -2\, X F_X \end{cases}$$

inflation ("slow roll") ⇒ small $F_X = \mathcal{O}(\epsilon)$

⇒ Approximate internal scale invariance $\phi^I \to \lambda \phi^I$

The "clock"

$$B^{IJ} \equiv g^{\mu\nu}\partial_\mu\phi^I\partial_\nu\phi^J \to \frac{1}{a^2(t)}\delta^{IJ}$$

$$X \to 1/a^6$$

$$Y, Z \to 1$$

time-dependence from the metric

⇨ no associated Goldstone boson

(no equivalence theorem-like limit)

Reheating = Melting

- Solid/fluid transition at some critical det(B) (or Tr(B), or ...)
- Similar to solid He at 0K and 25bar (30% compressible, we need e^60...)
- Fluid: same dof, more symmetries
- Sharp feature in F(X, Y, Z) -- region of enhanced symmetry in X, Y, Z space.

Cosmological perturbations

$$\phi^I = x^I + \pi^I$$

$$g_{\mu\nu} = g_{\mu\nu}^{\rm FRW} + \delta g_{\mu\nu}$$

U.G.: $F(B^{IJ}) \to F(g^{IJ})$

Lorentz violating
massive gravity

Very roughly:

$$\mathcal{L}_2 \sim F_X \cdot (\partial\pi)^2$$

$$\mathcal{L}_3 \sim F \cdot (\partial\pi)^3$$

$$\zeta \sim \vec{\nabla} \cdot \vec{\pi}$$

$$\Rightarrow \begin{cases} \langle\zeta\zeta\rangle \sim \dfrac{1}{\epsilon}\dfrac{1}{c_L^5}\dfrac{H^2}{M_{\rm Pl}^2} \\[2ex] \dfrac{\mathcal{L}_3}{\mathcal{L}_2} \sim \dfrac{1}{\epsilon}\dfrac{1}{c_L^2}\zeta \end{cases}$$

(cf. $\dfrac{1}{\epsilon}\dfrac{1}{c_L}\dfrac{H^2}{M_{\rm Pl}^2}$)

(cf. $\dfrac{1}{c_L^2}\zeta$)

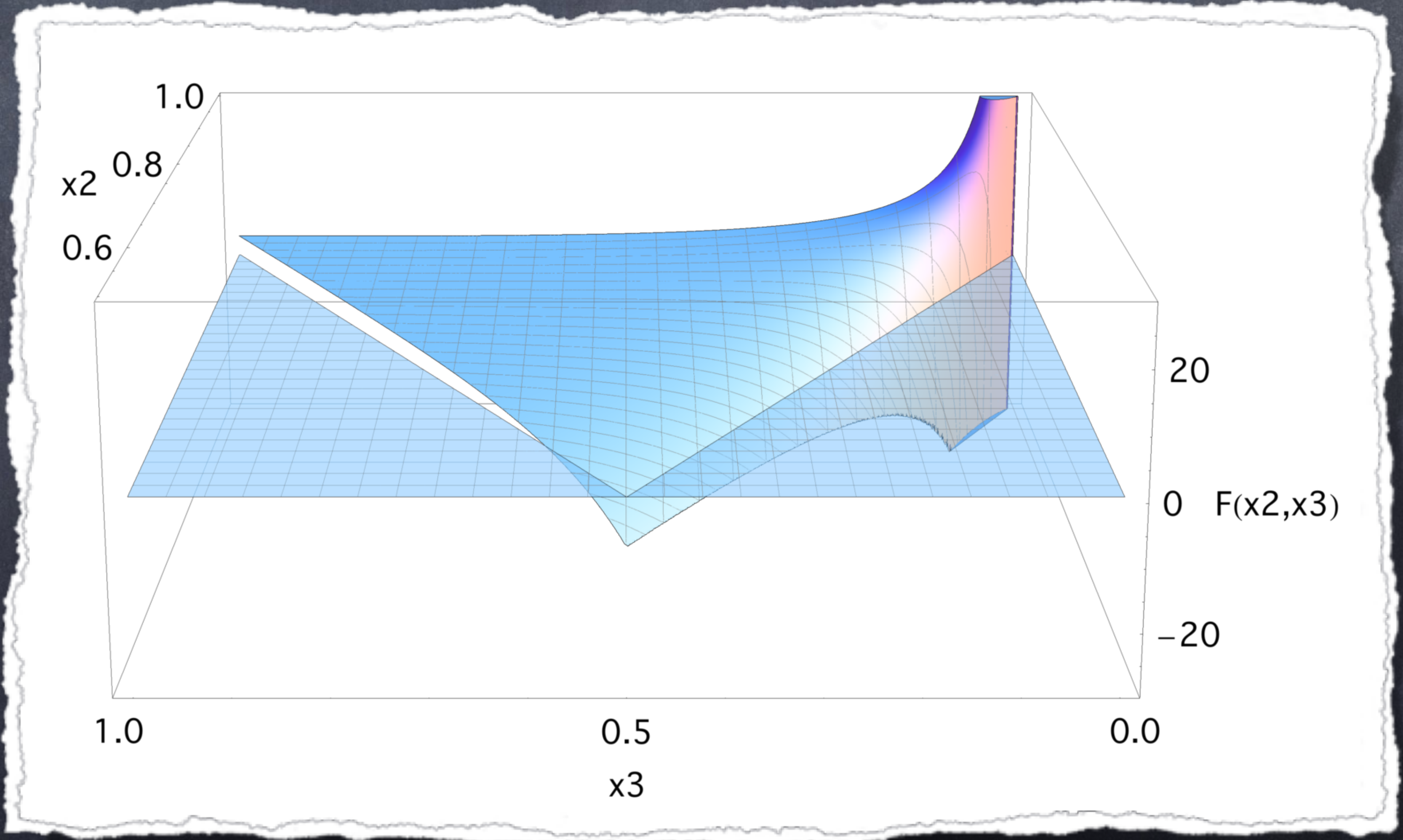
Observables

$$n_S - 1 = 2\epsilon\, c_L^2 - \eta - 5s$$

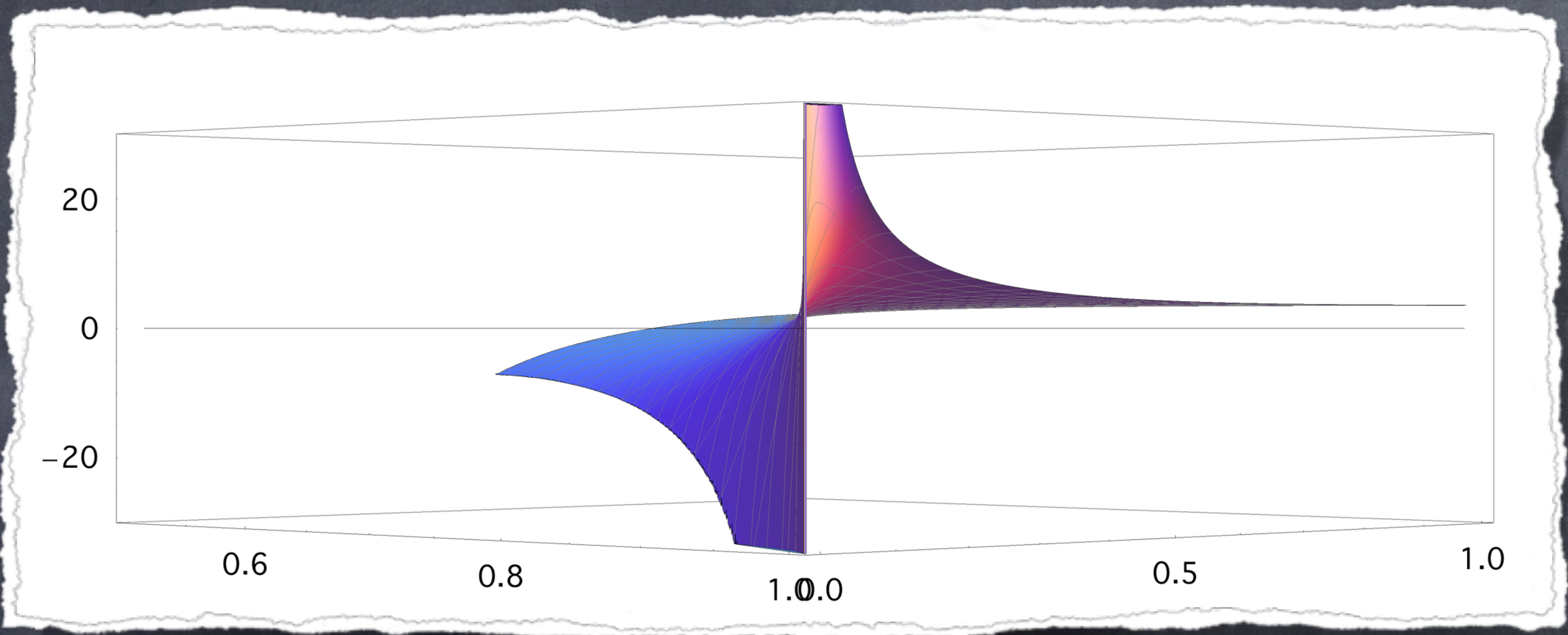
$$n_T - 1 = 2\epsilon\, c_L^2 \qquad \text{(mass term } \sim c_T^2\text{)}$$

$$r = 16\,\epsilon c_L^5$$

$$\langle \zeta\zeta\zeta \rangle \propto$$



Quadrupolar “squeezed limit”



$$\langle\zeta\zeta\zeta\rangle \to f_{NL} \times \langle\zeta\zeta\rangle\langle\zeta\zeta\rangle \times (1 - 3\cos^2\theta)$$

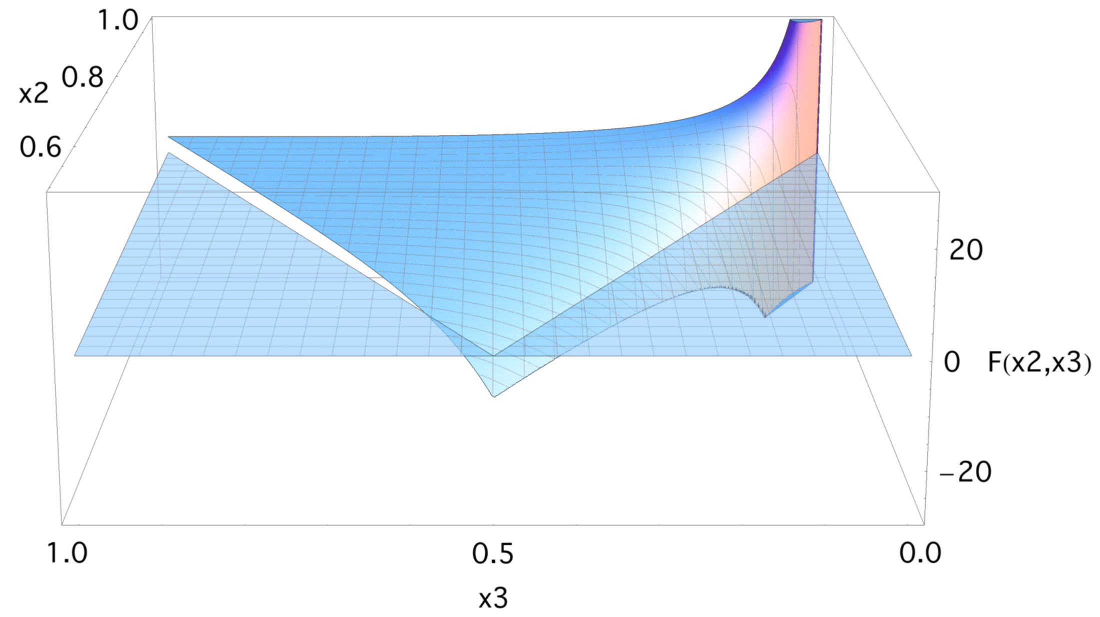
$$f_{NL} \sim \frac{1}{\epsilon}\frac{1}{c_L^2}$$

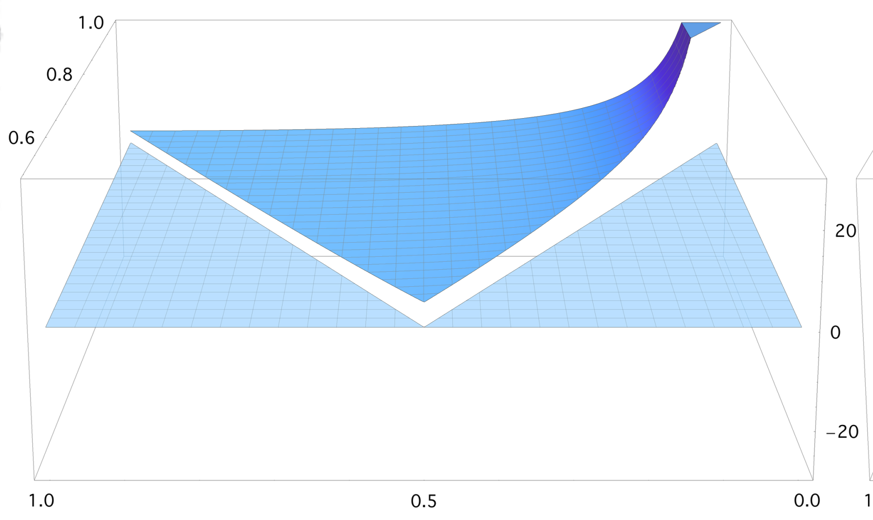
2% overlap w/ “local” shape

39% w/ “equilateral”

32% w/ “orthogonal”

(see also Shiraishi et al. 2012, Barnaby et al. 2012, Bartolo et al. 2013)


1.0
0.8
x2
0.6
20
0
F(x2,x3)
−20
1.0
0.5
0.0
x3


1.0
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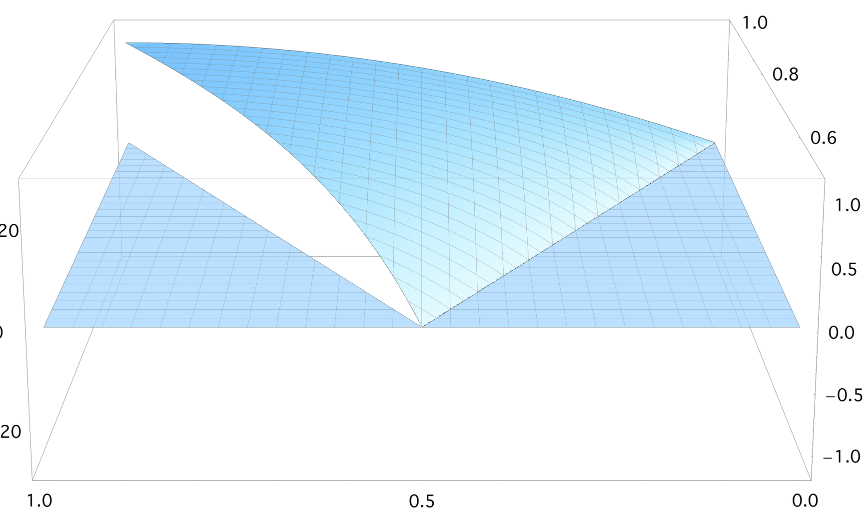
1.0
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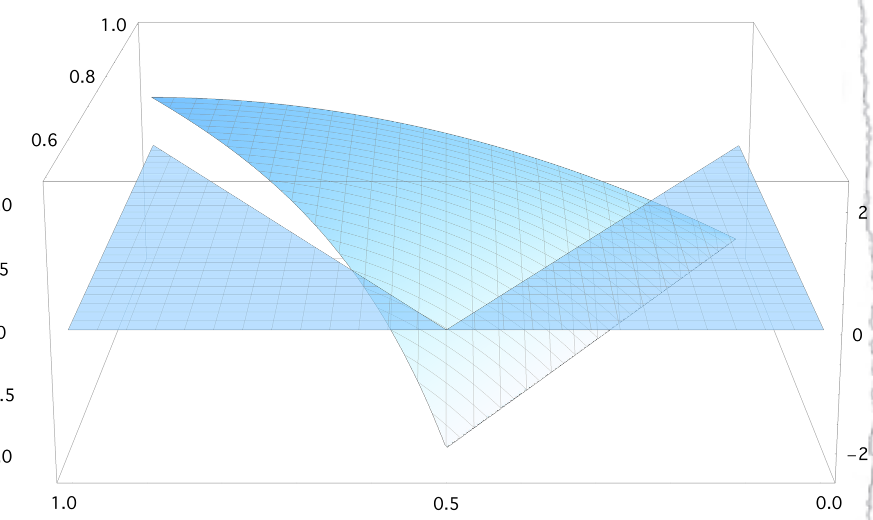
1.0
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Anisotropic generalizations

$$\phi^I \to \phi^I + a^I$$
$$\phi^I \to \cancel{SO(3)}\,\phi^I$$

discrete rotations

Yet, we want:

- isotropic background
- isotropic scalar spectrum

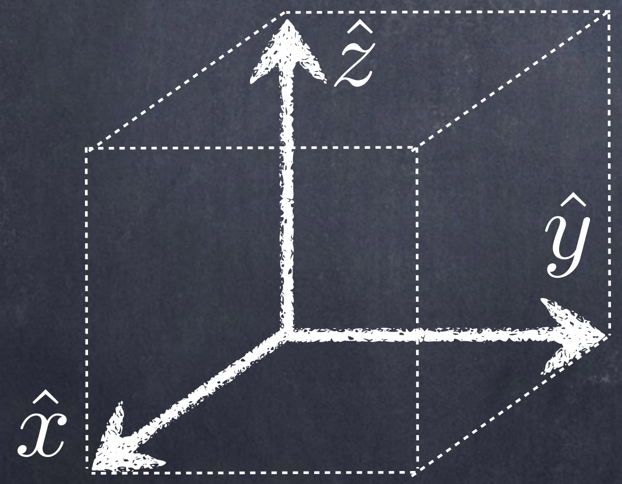
Background

T_{00}

$T_{ij} \propto \delta_{ij}$

Discrete subgroup of SO(3) with isotropic 2-index tensors?

Ex: cubic group



$$O^{(2)}_{ij} = \hat{x}_i\hat{x}_j + \hat{y}_i\hat{y}_j + \hat{z}_i\hat{z}_j = \delta_{ij}$$

accidentally isotropic!

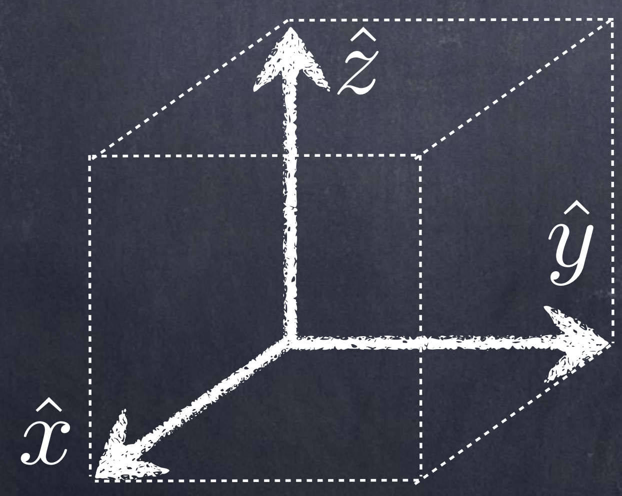
Scalar spectrum

$$\phi^I = x^I + \pi^I$$

$$\mathcal{L}_2 = O^{(2)}_{ij} \cdot \dot{\pi}_i \dot{\pi}_j + O^{(4)}_{ijkl} \cdot \partial_i \pi_j \partial_k \pi_l$$

Discrete subgroup of SO(3) with isotropic 4-index tensors?

Ex: cubic group



$$O^{(4)}_{ijkl} = \quad \delta_{ij}\delta_{kl}$$

$$\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}$$

$$\hat{x}_i \hat{x}_j \hat{x}_k \hat{x}_l + (\hat{x} \to \hat{y}, \hat{z})$$

not isotropic!

Scalar 3-pt function:

$$\mathcal{L}_3 \supset O^{(6)}_{ijklmn} \cdot \partial_i \pi_j \, \partial_k \pi_l \, \partial_m \pi_n$$

Tensor spectrum:

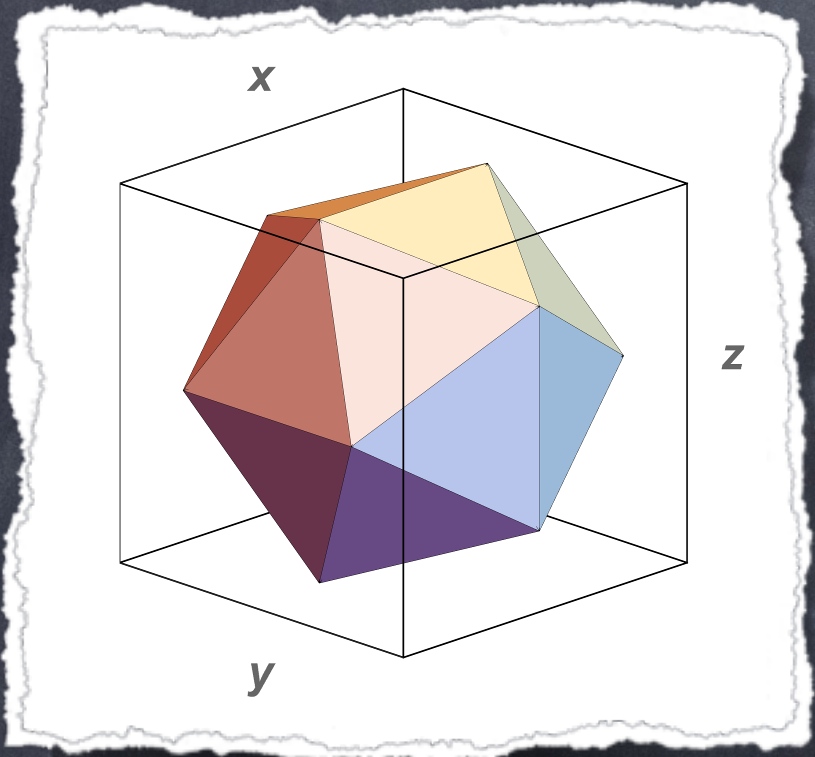
$$\mathcal{L}_2 = O^{(4)}_{ijkl} \cdot \dot{\gamma}_{ij} \, \dot{\gamma}_{kl} + O^{(6)}_{ijklmn} \cdot \partial_i \gamma_{jk} \, \partial_l \gamma_{mn}$$

Looking for a discrete subgroup of SO(3) w/

- Isotropic $O^{(2)}$
- Isotropic $O^{(4)}$
- Anisotropic $O^{(6)}$

$\Rightarrow$ $\left\{ \begin{array}{l} \text{isotropic background, scalar spectrum} \\ \text{anisotropic scalar 3-pt function,} \\ \qquad \text{tensor spectrum} \end{array} \right.$

Only one possibility: icosahedral group



$$O^{(2)}_{ij} = \delta_{ij}$$

$$O^{(4)}_{ijkl} = \quad \delta_{ij}\delta_{kl}$$
$$\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}$$

$$\begin{aligned} O^{(6)}_{ijklmn} = \quad & 2(\gamma+2)\delta_{ijklmn} \\ & + (\gamma+1)(\delta_{ijkl}\delta_{mn}\delta_{m,i+1} + \dots) \\ & + (\delta_{ijkl}\delta_{mn}\delta_{m,i-1} + \dots) \end{aligned}$$

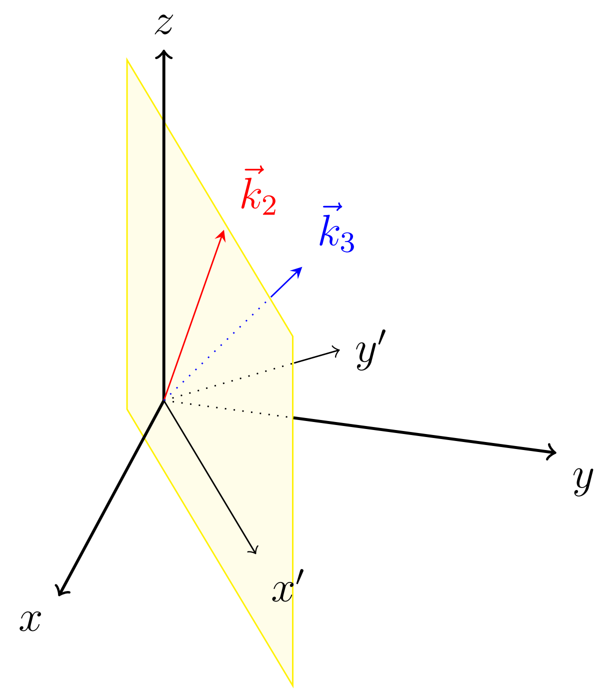
$$\gamma = (1+\sqrt{5})/2$$

Scalar 3-pt function

Messy expression — depends on vectors k2, k3

Two independent parameters α, β

Anisotropies $\propto (\beta - 9/2)$



Overlap with standard shapes $\propto (\beta - 8)$

$\Rightarrow$ $\beta = 8$ completely anisotropic case

$$\bar{f}_{\mathrm{NL}}(\theta_2) \;=\; -\frac{\alpha}{\epsilon c_L^2}\left[\tfrac{19415}{378}(\beta - 8) + \tfrac{104135}{6048}(2\beta - 9)P_6(\cos\theta_2)\right]$$

Tensor spectrum

No anisotropy to lowest-order in derivatives:

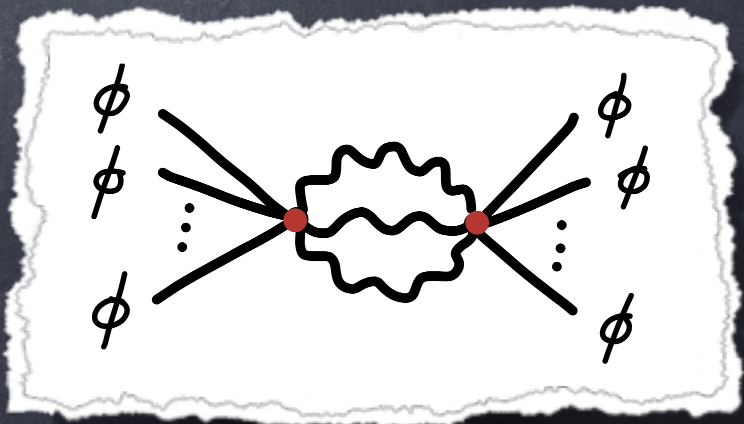
$$F(B^{IJ}) = F(g^{\mu\nu}\partial_\mu\phi^I\partial_\nu\phi^J) \quad \not\to \quad \partial\gamma\partial\gamma$$

$$\to \quad O^{(4)}_{ijkl}\cdot\gamma_{ij}\,\gamma_{kl}$$

Needs higher-derivative couplings — e.g.:

$$(R^{\mu\nu\rho\sigma}\partial_\mu\phi^I\partial_\nu\phi^J\partial_\rho\phi^K\partial_\sigma\phi^L)^3\cdot T_{\text{aniso}}$$

$\Rightarrow$ O(1) anisotropies within EFT



Systematics? in progress...

Conclusions

- Observed isotropy of the universe could be accidental
- Potentially anisotropic non-gaussianity
- Potentially anisotropic tensor modes
- Data?