On the Vainshtein Mechanism

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• The Vainshtein Mechanism
  – Basics
  – Exploring solutions and stability (perts)
  – Relaxing symmetry and strong gravity

• Toy model (without gravity)

• Conclusions
Why?

unnecessary
Why?
Why?

Screening Mechanism

*Cosmological modifications of order 1

*Strong gravity regime may also show something

Why?

necessary

Baker et al, 2014
2nd. Laboratory

- Derivative interactions arise from UV physics
- Strongly coupled regime
- Dualities & UV completion? (e.g. Keltner & Tolley, 1502.05706)
- Is it helpful for condensed matter or particle physics?
Vainshtein Mechanism (Review)

In massive gravity by Vainshtein, 1972 (see Babichev and Deffayet, 2013)

Start with

\[ S = \int d^4x \sqrt{-g} \left( M_p^2 R - \frac{1}{2} g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi - \frac{\alpha^2}{\Lambda^3} \Box \Phi g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi \right) \]

Spherically symmetric ansatz

\[ ds^2 = -e^{2\nu(r)} dt^2 + e^{2\lambda(r)} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \]

\[ \Phi = \Phi(r). \]
Vainshtein Mechanism (Review)

Scalar equation could be integrated once (Nicolis et al)

\[ e^{\nu - \lambda r^2} \Lambda^3 \Phi' + 2\alpha^2 r e^{\nu - 3\lambda} (2 + rv') \Phi'^2 = \Lambda^3 \zeta \]

with solution

\[ \Phi' = -e^{\nu - \lambda r^2} \Lambda^3 \pm \sqrt{e^{2(\nu - \lambda)} r^4 \Lambda^6 + 8\alpha^2 \Lambda^3 \zeta r e^{\nu - 3\lambda} (2 + rv')} \]

\[ 4\alpha^2 r e^{\nu - 3\lambda} (2 + rv') \]

Around flat space (test field) for large \( \alpha \)

\[ \Phi_{\text{flat}}' = \left( \frac{\Lambda^3 \zeta}{4r^2} \right)^{1/2} - \frac{\Lambda^3 r}{8\alpha^2} + \left( \frac{\Lambda^9 r^5}{2^{12} \zeta \alpha^6} \right)^{1/2} + O\left( \alpha^{-4} \right) \]

\[ r^3 \sim \frac{\alpha^2 \zeta}{\Lambda^3} \]

From

Galileon

From

Kinetic

~ Mass
Vainshtein Mechanism (Review)

\[ \Phi'_{\text{flat}} = \left( \frac{\Lambda^3 \zeta}{4ra^2} \right)^{1/2} - \frac{\Lambda^3 r}{8a^2} + \left( \frac{\Lambda^9 r^5}{2^{12} \zeta a^6} \right)^{1/2} + O(\alpha^{-4}) \]

1) Suggests 1/sqrt(coupling) expansion \ (Gabadadze el al, Padilla et al)
2) Should recover GR (Schwarzschild) at short scales.
3) Consistent construction with this expansion

New Vainshtein radius

\[ \frac{r - r_s}{r - \frac{3}{4} r_s} \sim \frac{\alpha^2 \xi_0}{\Lambda^3} \]

(larger than in Minkowski)
“Vainshtein” effective theory

Description that captures the Vainshtein effect from Horndeski's Lagrangian

(Koyama, Niz, Tasinato)

Truncate the theory so that

1) It is an expansion around flat space

\[ \phi = \phi_0 + \pi, \quad g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \]

2) Leading order in \( h_{\mu\nu} \) and \( \pi, \partial \pi \)

3) Keep all second order derivatives of \( \pi \), (i.e. \( \partial^2 \pi \)) with an associated scale \( \Lambda \)
“Vainshtein” effective theory

Remarks

• It is equivalent to demand the Galilean symmetry (c.f. Nicolis et al)
  \[ \pi \rightarrow \pi + c + b_\mu x^\mu \]

• Reduces to (limits of) well know theories:
  dRGT massive gravity, DGP, (restricted) Galileons, etc.
  (de Rham et al)     (Dvali et al)     (Berezhiani et al, Nicolis et al)

• Does not include other models from Horndeski which (may) present the Vainshtein mechanism,
  eg. the Fab Four (Charmousis al, Kaloper et al)
  or k-mouflage (Babichev et al)
"Vainshtein" effective theory

\[ \mathcal{L}_2 = K(\phi, X) \]

\[ \mathcal{L}_3 = -G_3(\phi, X) \Box \phi \]

\[ \mathcal{L}_4 = G_4(\phi, X) R + G_{4X}(\phi, X) [(\Box \phi)^2 - (\nabla_\mu \nabla_\nu \phi)^2] \]

\[ \mathcal{L}_5 = G_5(\phi, X) G^{\mu\nu} \nabla_\mu \nabla_\nu \phi - \frac{1}{6} G_{5X}(\phi, X) [(\Box \phi)^3 \]

\[ -3 \Box \phi (\nabla_\mu \nabla_\nu \phi)^2 + 2 (\nabla_\mu \nabla_\nu \phi)^3] \]

\[ X := - (\partial \phi)^2 / 2 \]

Under this decoupling limit

\[ \mathcal{L}_{\text{eff}} = -\frac{1}{4} \bar{h}^{\mu\nu} \mathcal{E}_{\alpha\beta} \mu_\nu h_{\alpha\beta} + \frac{\eta}{2} \pi \Box \pi + \frac{1}{2M_{pl}} \bar{h}^{\mu\nu} T_{\mu\nu} \]

\[ + \frac{\mu}{\Lambda^3} \mathcal{L}_3^{\text{gal}} + \frac{\nu}{\Lambda^6} \mathcal{L}_4^{\text{gal}} + \frac{\omega}{\Lambda^9} \mathcal{L}_5^{\text{gal}} \]

\[ -\xi \bar{h}^{\mu\nu} X^{(1)}_\mu_\nu - \frac{1}{\Lambda^3} \alpha \bar{h}^{\mu\nu} X^{(2)}_\mu_\nu + \frac{1}{2\Lambda^6} \beta \bar{h}^{\mu\nu} X^{(3)}_\mu_\nu \]
“Vainshtein” effective theory

\[ \mathcal{L}^{\text{eff}} = -\frac{1}{4} h_{\mu\nu} \mathcal{E}^{\alpha\beta}_{\mu\nu} h_{\alpha\beta} + \frac{\eta}{2} \pi \Box \pi + \frac{1}{2M_{\text{pl}}} h_{\mu\nu} T_{\mu\nu} \]

\[ + \frac{\mu}{\Lambda^3} \mathcal{L}^{\text{gal}}_3 + \frac{\nu}{\Lambda^6} \mathcal{L}^{\text{gal}}_4 + \frac{\varpi}{\Lambda^9} \mathcal{L}^{\text{gal}}_5 \]

\[ -\xi h_{\mu\nu}^{\mu\nu} X_{\mu\nu}^{(1)} - \frac{1}{\Lambda^3} \alpha h_{\mu\nu}^{\mu\nu} X_{\mu\nu}^{(2)} + \frac{1}{2\Lambda^6} \beta h_{\mu\nu}^{\mu\nu} X_{\mu\nu}^{(3)} \]

\[ X_{\mu\nu}^{(1)} = \eta_{\mu\nu} [\Pi] - \Pi_{\mu\nu} \]

\[ \Pi_{\mu\nu} = \partial_\mu \partial_\nu \pi \]

\[ X_{\mu\nu}^{(2)} = \Pi_{\mu\nu}^2 - [\Pi] \Pi_{\mu\nu} + \frac{1}{2} \eta_{\mu\nu} ([\Pi]^2 - [\Pi^2]) , \]

\[ X_{\mu\nu}^{(3)} = 6\Pi_{\mu\nu}^3 - 6\Pi_{\mu\nu}^2 [\Pi] + 3\Pi_{\mu\nu} ([\Pi]^2 - [\Pi^2]) \]

\[ -\eta_{\mu\nu} ([\Pi]^3 - 3[\Pi][\Pi^2] + 2[\Pi^3]) \]
"Vainshtein" effective theory

\[ \mathcal{L}^{\text{eff}} = -\frac{1}{4} \bar{h}^{\mu \nu} \mathcal{E}^{\alpha \beta}_{\mu \nu} \bar{h}_{\alpha \beta} + \frac{\eta}{2} \pi \Box \pi + \frac{1}{2M_{pl}} \bar{h}^{\mu \nu} T_{\mu \nu} \]

\[ + \frac{\mu}{\Lambda^3} \mathcal{L}^{\text{gal}}_3 + \frac{\nu}{\Lambda^6} \mathcal{L}^{\text{gal}}_4 + \frac{\omega}{\Lambda^9} \mathcal{L}^{\text{gal}}_5 \]

\[ -\xi \bar{h}^{\mu \nu} X^{(1)}_{\mu \nu} - \frac{1}{\Lambda^3} \alpha \bar{h}^{\mu \nu} X^{(2)}_{\mu \nu} + \frac{1}{2\Lambda^6} \beta \bar{h}^{\mu \nu} X^{(3)}_{\mu \nu} \]

Field redefinition

\[ \bar{h}_{\mu \nu} = \hat{h}_{\mu \nu} - 2\xi \pi \eta_{\mu \nu} + \frac{2\alpha}{\Lambda^3} \partial_\mu \pi \partial_\nu \pi \]

New couplings

\[ -\frac{2\xi}{M_{pl}} \pi T + \frac{2\alpha}{M_{pl} \Lambda^3} \partial_\mu \pi \partial_\nu \pi T^{\mu \nu} \]
Spherically symmetric solutions

Spherically symmetric ansatz imply the equations:

1. \[
\frac{M_{pl}}{\Lambda^3} \frac{\Phi'}{r} = \frac{M_{pl}}{\Lambda^3} \frac{\Psi'}{r} = \beta x^3 + A(r) \rightarrow \text{“Mass” inside a given radius}
\]

\[
x(r) = \frac{1}{\Lambda^3} \frac{\pi'}{r}
\]

2. \[
P(x, A) \equiv \xi A(r) + \left(\frac{\eta}{2} + 3\xi^2\right) x + (\mu + 6\alpha\xi - 3\beta A(r)) x^2
\]

\[
+ (\nu + 2\alpha^2 + 4\beta\xi) x^3 - 3\beta^2 x^5 = 0
\]

Expect many more solutions than before!
Spherically symmetric solutions

Koyama et al, Sbisa et al, Chkareuli et al, Sjors et al, Kaloper et al, Kimura et al, Berezhiani et al, Babichev et al, Li et al, Narikawa et al, Hiramatsu et al, de Rham et al, Renaux-Petel, ...

Two important behaviours within the Vainshtein radius

Weak (usual) screening

Strong screening
Spherically symmetric solutions

Construct phase space of how inner-outer solns connect

For dRGT

(Sbisa et al)

How A and B connect to asymptotic solutions

\[ \mathcal{L} \sim \sqrt{-g} R + 2m^2 \sqrt{- \det(\partial_\mu \phi^\alpha \partial_\nu \phi^\beta \eta_{\alpha\beta})} \]
Stability

Koyama et al

Scalar perturbations around weak screening soln. A

\[ S_\varphi = \frac{1}{2} \int d^4 x \left[ K_t(r)(\partial_t \varphi)^2 - K_r(r)(\partial_r \varphi)^2 - K_\Omega(r)(\partial_\Omega \varphi)^2 \right] \]

Where all the \( K' \)s are functions of the effective theory parameters, particularly of \( \alpha, \beta \)

which tune the terms \( \alpha \partial_\mu \pi \partial_\nu \pi T^{\mu\nu}, \beta \bar{h}^{\mu\nu} X^{(3)}_{\mu\nu} \)
Stability

1. \( \alpha = \beta = 0 \) Original Galileon, studied by Nicolis et al
   Superluminal propagation in radial modes
   Extremely subluminal in radial piece!

2. \( \alpha \neq 0, \beta = 0 \) Disformal coupling \( \alpha \, \partial_\mu \pi \partial_\nu \pi T^{\mu \nu} \)
   Leads to Restricted Galileon (Berezhiani et al)
   \( \alpha < 0 \) has a ghost mode
   There are no asymptotically flat Vainshtein solutions
Inside the Vainshtein radius and outside the source, the leading terms are

\[ S_\varphi \sim \int d^4x \left[ K_t (\partial_t \varphi)^2 - K_r (\partial_r \varphi)^2 - K_\Omega (\partial_\Omega \varphi)^2 \right] \]

\[ K_t \sim 0, \quad K_r \sim -12\beta A(r)x_0, \quad K_\Omega \sim 6\beta A(r)x_0, \]

Speed of fluctuations is always superluminal, and unstable.

- The instability can be avoided inside the source
- Need to look beyond static & spherical symmetry!
Beyond Vanilla...

Play with the matter sector

TOV

Scalarisation

(see Teruaki's talk)

Polytropic EOS

Chagoya, Koyama, Niz & Tasinato

Found that:
small corrections

avoids it

mechanism works better for higher densities
Beyond Vanilla...

Play with the matter sector

Polytropic EOS. Only issue in strong gravity regime

Critical Density above known Neutron star's densities

\[-T \text{ (gr/cm}^3)\]

\[\frac{1}{8} \rho_0 \quad \frac{25}{8} \rho_0 \quad 4 \rho_0\]

density
Beyond Vanilla...

Away from spherical symmetry

Slow rotation shows no big corrections

Filaments (cylindrical) solutions present suppressed Vainshtein effect

Gabadadze et al, Bloomfield et al, Falck et al.

Pancakes (planar) solutions do not exhibit screening

Brax et al, Bloomfield et al, Falck et al.
Are these solutions unique to gravitational systems?

Let us rewind a bit and consider a simple toy model.

keep it simple
Toy Model

Simple Bi-galileon

\[ S = \int d^4x \left[ \partial_\mu \phi \partial^\mu \phi + \partial_\mu \pi \partial^\mu \pi + \alpha \partial^\mu \phi \partial^\nu \phi X_{\mu\nu}^{(1)}(\pi) \right], \]

\[ X_{\mu\nu}^{(1)} = \eta_{\mu\nu} [\Pi] - \Pi_{\mu\nu} \]

EOM (two integration constants)

\[-r \zeta_\phi - \left( r^3 + 4\alpha \zeta_\pi \right) \phi' + 8r\alpha^2 (\phi')^3 = 0\]
Toy Model

1/r singularity removal (strong screening)

Existence cond. \(|\zeta_\pi| > \frac{2}{3} |\zeta_\phi|\)

\[ \sim \frac{\zeta_\phi}{4\zeta_\pi} r + \mathcal{O}(r^4) \]

\[ \sim \frac{\zeta_\phi}{r^2} \]
No Ghost

\[ |\zeta_\pi| > \sqrt{3} |\zeta_\phi| \]

\[ S_\varphi \sim \sum_i \int d^4x \left[ K_l (\partial_t \varphi_i)^2 - K_r (\partial_r \varphi_i)^2 - K_\Omega (\partial_\Omega \varphi_i)^2 \right] + \text{ints.} \]
Superluminality inside the Vainshtein radius?

Not straightforward to calculate velocities because fields are coupled, but found solutions, for $r \to 0$

$$\delta \phi = -\frac{\zeta \phi w^2}{120 r \alpha \zeta_{\pi}^2} \left( \kappa_1 e^{-i(r+t)w} + \kappa_2 e^{i(r-t)w} \right).$$

$$\delta \pi = A \delta \phi + \left( \kappa_1 e^{-i(r+t)w} + \kappa_2 e^{i(r-t)w} \right).$$
With \((\partial \phi)^2 X_2(\pi)\) there is one further solution where both fields are regular near zero.

Always a Ghost

Other operators may avoid Derrick's theorem
Toy Model

Confinement

Existence cond. \( \zeta_\phi > \zeta_\pi \)

Perturbations are always unstable

\( \sim r^{-1/2} \)

\( \sim r \)
Toy Model

Other/higher operators

1) Change the exponent near $r=0$

$$\sim r^{-n}$$

2) Improve stability of perturbations

Is this solution relevant for confinement?
Toy Model - Richness

\[ \alpha (\partial \phi)^2 X_1(\pi) \]

\[ \gamma (\partial \phi)^2 X_2(\pi) \]

\[ \beta (\partial \phi)(\partial \pi) \]

Inner Outer
Conclusions

• Models with derivative interactions (of Galileon type) have rich space of solutions

• Are solutions tied to instabilities?
  – Some may go away by adding other operators or when calculating the relevant observables (group vs front velocity, see Keltner and Tolley)

• May have interesting applications beyond modified gravity