On the Vainshtein Mechanism

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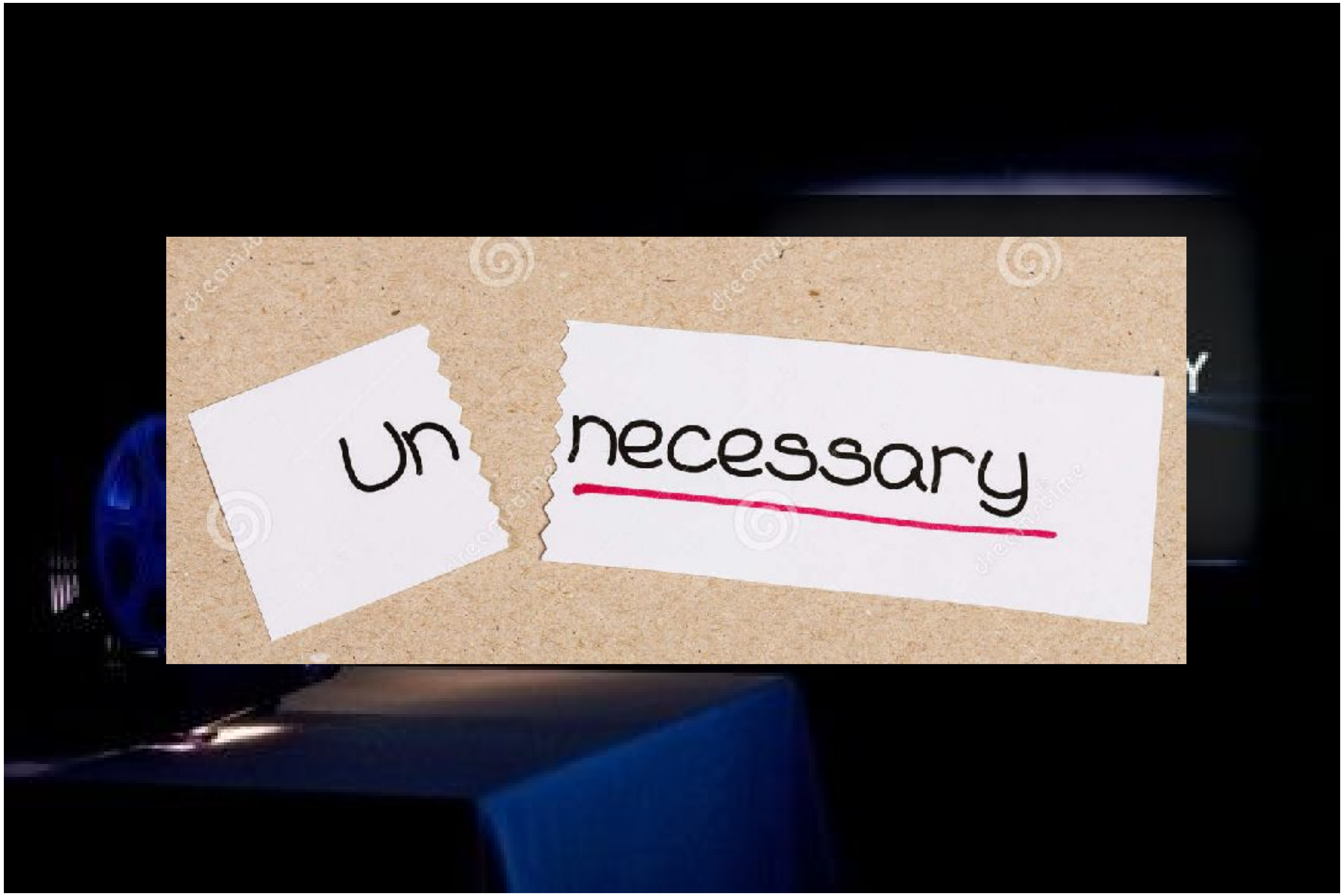
Content

- Motivation
- The Vainshtein Mechanism
  - Basics
  - Exploring solutions and stability (perts)
  - Relaxing symmetry and strong gravity
- Toy model (without gravity)
- Conclusions

Why?



Why?

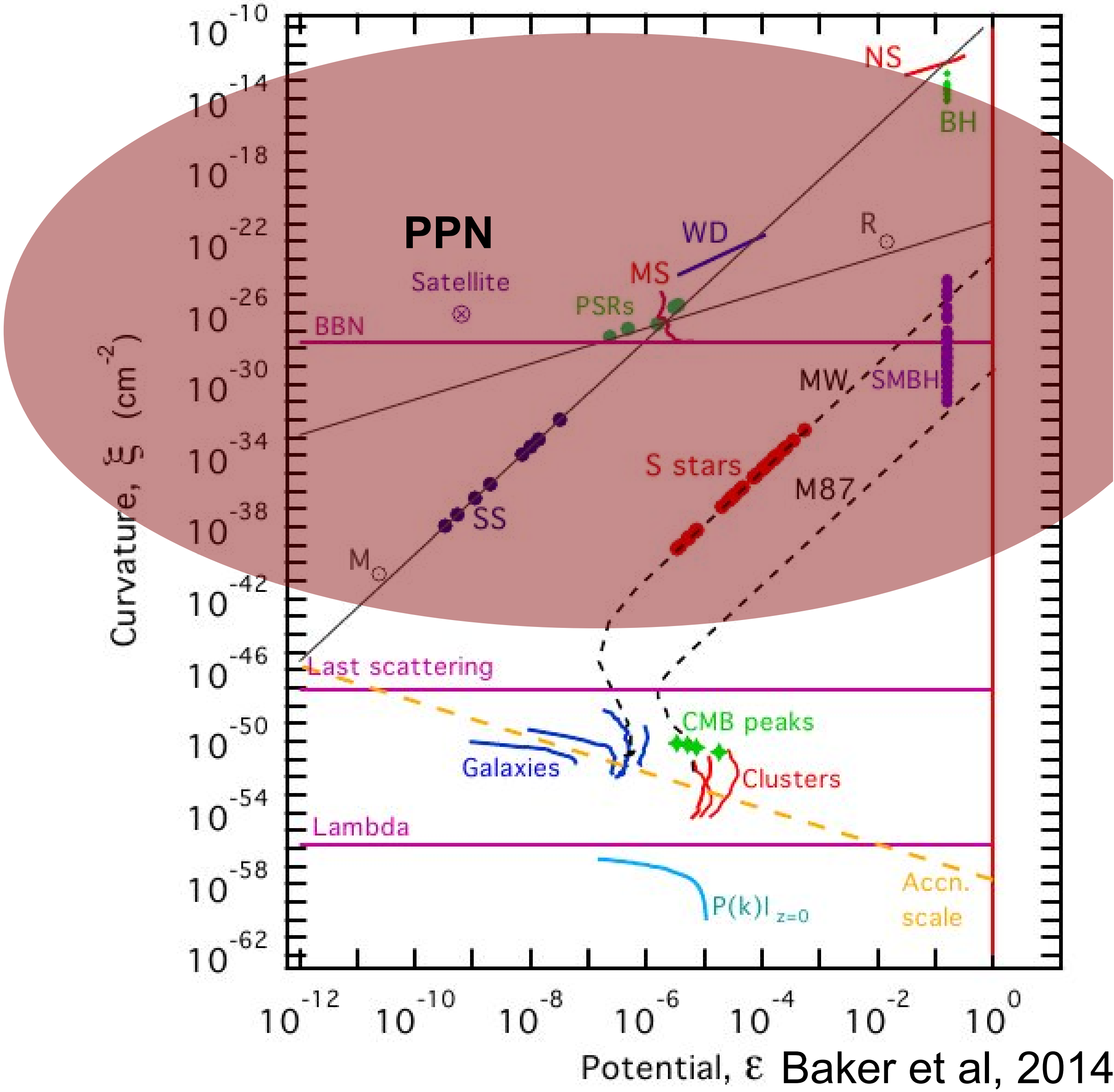
Why?

Screening

Mechanism

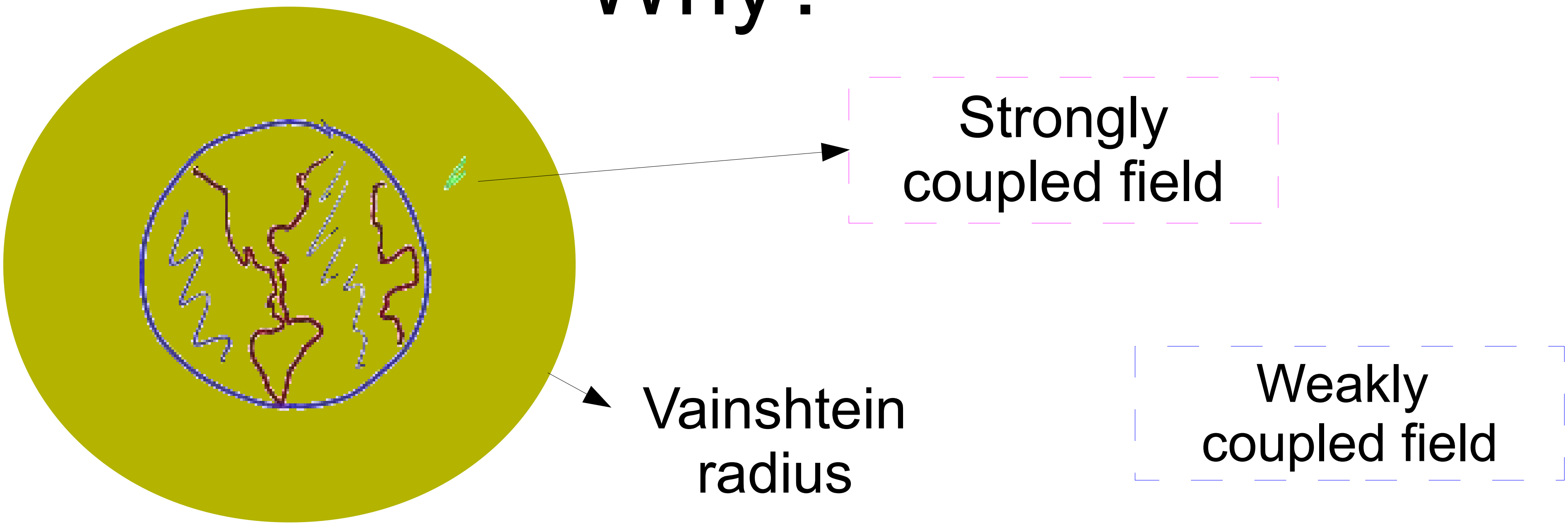
*Cosmological modifications of order 1

*Strong gravity regime may also show something



Baker et al, 2014

Why?



2nd. Laboratory

- Derivative interactions arise from UV physics
- Strongly coupled regime
- Dualities & UV completion? (e.g. Keltner & Tolley, 1502.05706)
- Is it helpful for condensed matter or particle physics?

Vainshtein Mechanism (Review)

In massive gravity by Vainshtein, 1972 (see Babichev and Deffayet, 2013)

Start with

$$S = \int d^4x\sqrt{-g}\left(M_p^2 R - \frac{1}{2}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - \frac{\alpha^2}{\Lambda^3}\Box\Phi g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi\right)$$

Spherically symmetric ansatz

$$\begin{aligned} ds^2 &= -e^{2\nu(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \\ \Phi &= \Phi(r). \end{aligned}$$

Vainshtein Mechanism (Review)

Scalar equation could be integrated once (Nicolis et al)

$$e^{\nu-\lambda}r^2\Lambda^3\Phi' + 2\alpha^2 r e^{\nu-3\lambda}(2+r\nu')\Phi'^2 = \Lambda^3\zeta$$

with solution

$$\Phi' = \frac{-e^{\nu-\lambda}r^2\Lambda^3 \pm \sqrt{e^{2(\nu-\lambda)}r^4\Lambda^6 + 8\alpha^2\Lambda^3\zeta r e^{\nu-3\lambda}(2+r\nu')}}{4\alpha^2 r e^{\nu-3\lambda}(2+r\nu')}$$

~ Mass

Around flat space (test field) for large α

$$\Phi'_{\rm flat} = \left(\frac{\Lambda^3\zeta}{4r\alpha^2}\right)^{1/2} - \frac{\Lambda^3 r}{8\alpha^2} + \left(\frac{\Lambda^9 r^5}{2^{12}\zeta\alpha^6}\right)^{1/2} + O\left(\alpha^{-4}\right)$$

From Galileon **From Kinetic**

$$r^3 \sim \frac{\alpha^2\xi}{\Lambda^3}$$

Vainshtein Mechanism (Review)

$$\Phi'_{\text{flat}} = \left(\frac{\Lambda^3 \zeta}{4r\alpha^2}\right)^{1/2} - \frac{\Lambda^3 r}{8\alpha^2} + \left(\frac{\Lambda^9 r^5}{2^{12}\zeta\alpha^6}\right)^{1/2} + O\left(\alpha^{-4}\right)$$

1) Suggests 1/sqrt(coupling) expansion (Gabadadze el al, Padilla et al)

2) Should recover GR (Schwarzschild) at short scales.

3) Consistent construction with this expansion

New Vainshtein radius $\frac{r - r_s}{r - \frac{3}{4}r_s} r^3 \sim \frac{\alpha^2 \xi_0}{\Lambda^3}$

(larger than in Minkowski)

"Vainshtein" effective theory

Description that captures the Vainshtein effect from Horndeski's Lagriangian

(Koyama, Niz, Tasinato)

Truncate the theory so that

1) It is an expansion around flat space

$$\phi = \phi_0 + \pi, \qquad g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

2) Leading order in $h_{\mu\nu}$ and $\pi, \partial\pi$

3) Keep all second order derivatives of π, $(i.e.\ \partial^2\pi)$

with an associated scale Λ

"Vainshtein" effective theory

Remarks

- It is equivalent to demand the Galilean symmetry (c.f. Nicolis et al)

$$\pi \to \pi + c + b_\mu x^\mu$$

- Reduces to (limits of) well know theories:

  dRGT massive gravity, DGP , (restricted) Galileons, etc.

  (de Rham et al) (Dvali et al) (Berezhiani et al, Nicolis et al)

- Does not include other models from Horndeski which (may) present the Vainshtein mechanism,

  eg. the Fab Four (Charmousis al, Kaloper et al)
  or k-mouflage (Babichev et al)

"Vainshtein" effective theory

$$\mathcal{L}_2 = K(\phi, X) \qquad \mathcal{L}_3 = -G_3(\phi, X)\Box\phi$$

$$\mathcal{L}_4 = G_4(\phi, X)R + G_{4X}(\phi, X)\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]$$

$$\mathcal{L}_5 = G_5(\phi, X)G^{\mu\nu}\nabla_\mu\nabla_\nu\phi - \frac{1}{6}G_{5X}(\phi, X)\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]$$

$$X := -(\partial\phi)^2/2$$

Under this decoupling limit

$$\mathcal{L}^{\rm eff} = -\frac{1}{4}\bar{h}^{\mu\nu}\mathcal{E}^{\alpha\beta}{}_{\mu\nu}\bar{h}_{\alpha\beta} + \frac{\eta}{2}\pi\Box\pi + \frac{1}{2M_{pl}}\bar{h}^{\mu\nu}T_{\mu\nu}$$
$$+\frac{\mu}{\Lambda^3}\mathcal{L}_3^{\rm gal} + \frac{\nu}{\Lambda^6}\mathcal{L}_4^{\rm gal} + \frac{\varpi}{\Lambda^9}\mathcal{L}_5^{\rm gal}$$
$$-\xi\bar{h}^{\mu\nu}X^{(1)}_{\mu\nu} - \frac{1}{\Lambda^3}\alpha\bar{h}^{\mu\nu}X^{(2)}_{\mu\nu} + \frac{1}{2\Lambda^6}\beta\bar{h}^{\mu\nu}X^{(3)}_{\mu\nu}$$

“Vainshtein” effective theory

$$\mathcal{L}^{\text{eff}} = -\frac{1}{4}\bar{h}^{\mu\nu}\mathcal{E}^{\alpha\beta}{}_{\mu\nu}\bar{h}_{\alpha\beta} + \frac{\eta}{2}\pi\Box\pi + \frac{1}{2M_{pl}}\bar{h}^{\mu\nu}T_{\mu\nu}$$
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$$-\xi\bar{h}^{\mu\nu}X^{(1)}_{\mu\nu} - \frac{1}{\Lambda^3}\alpha\bar{h}^{\mu\nu}X^{(2)}_{\mu\nu} + \frac{1}{2\Lambda^6}\beta\bar{h}^{\mu\nu}X^{(3)}_{\mu\nu}$$

$$\Pi_{\mu\nu} = \partial_\mu\partial_\nu\pi$$

$$X^{(1)}_{\mu\nu} = \eta_{\mu\nu}[\Pi] - \Pi_{\mu\nu}$$

$$X^{(2)}_{\mu\nu} = \Pi^2_{\mu\nu} - [\Pi]\Pi_{\mu\nu} + \frac{1}{2}\eta_{\mu\nu}([\Pi]^2 - [\Pi^2]),$$

$$X^{(3)}_{\mu\nu} = 6\Pi^3_{\mu\nu} - 6\Pi^2_{\mu\nu}[\Pi] + 3\Pi_{\mu\nu}([\Pi]^2 - [\Pi^2]) - \eta_{\mu\nu}([\Pi]^3 - 3[\Pi][\Pi^2] + 2[\Pi^3])$$

“Vainshtein” effective theory

$$\mathcal{L}^{\text{eff}} = -\frac{1}{4}\bar{h}^{\mu\nu}\mathcal{E}^{\alpha\beta}{}_{\mu\nu}\bar{h}_{\alpha\beta} + \frac{\eta}{2}\pi\Box\pi + \frac{1}{2M_{pl}}\bar{h}^{\mu\nu}T_{\mu\nu}$$
$$+\frac{\mu}{\Lambda^3}\mathcal{L}_3^{\text{gal}} + \frac{\nu}{\Lambda^6}\mathcal{L}_4^{\text{gal}} + \frac{\varpi}{\Lambda^9}\mathcal{L}_5^{\text{gal}}$$

$$-\xi\bar{h}^{\mu\nu}X^{(1)}_{\mu\nu} - \frac{1}{\Lambda^3}\alpha\bar{h}^{\mu\nu}X^{(2)}_{\mu\nu} + \frac{1}{2\Lambda^6}\beta\bar{h}^{\mu\nu}X^{(3)}_{\mu\nu}$$

Field redefinition

$$\bar{h}_{\mu\nu} = \hat{h}_{\mu\nu} - 2\xi\pi\eta_{\mu\nu} + \frac{2\alpha}{\Lambda^3}\partial_\mu\pi\partial_\nu\pi$$

New couplings

$$-\frac{2\xi}{M_{pl}}\pi T + \frac{2\alpha}{M_{pl}\Lambda^3}\partial_\mu\pi\partial_\nu\pi T^{\mu\nu}$$

Spherically symmetric solutions

Spherically symmetric ansatz imply the equations:

Metric perts

1 $$\frac{M_{pl}}{\Lambda^3}\frac{\Phi'}{r} = \frac{M_{pl}}{\Lambda^3}\frac{\Psi'}{r} = \beta x^3 + A(r)$$ → "Mass" inside a given radius

$$x(r) = \frac{1}{\Lambda^3}\frac{\pi'}{r}$$

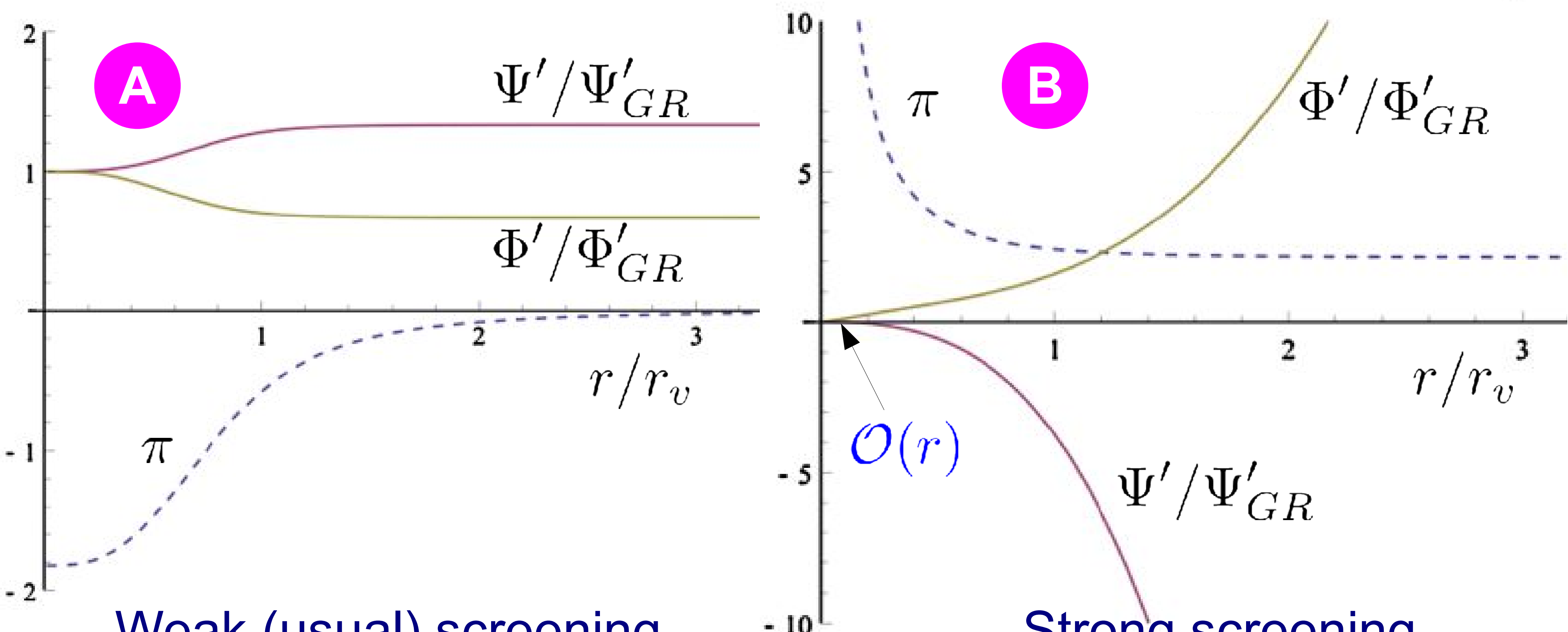
2 $$P(x, A) \equiv \xi A(r) + \left(\frac{\eta}{2} + 3\xi^2\right) x + \left(\mu + 6\alpha\xi - 3\beta A(r)\right) x^2 + \left(\nu + 2\alpha^2 + 4\beta\xi\right) x^3 - 3\beta^2 x^5 = 0$$

Expect many more solutions than before!

Spherically symmetric solutions

Koyama et al, Sbisa et al, Chkareuli et al, Sjors et al, Kaloper et al, Kimura et al, Berezhiani et al, Babichev et al, Li et al, Narikawa et al, Hiramatsu et al, de Rham et al, Renaux-Petel, ...

Two important behaviours within the Vainshtein radius

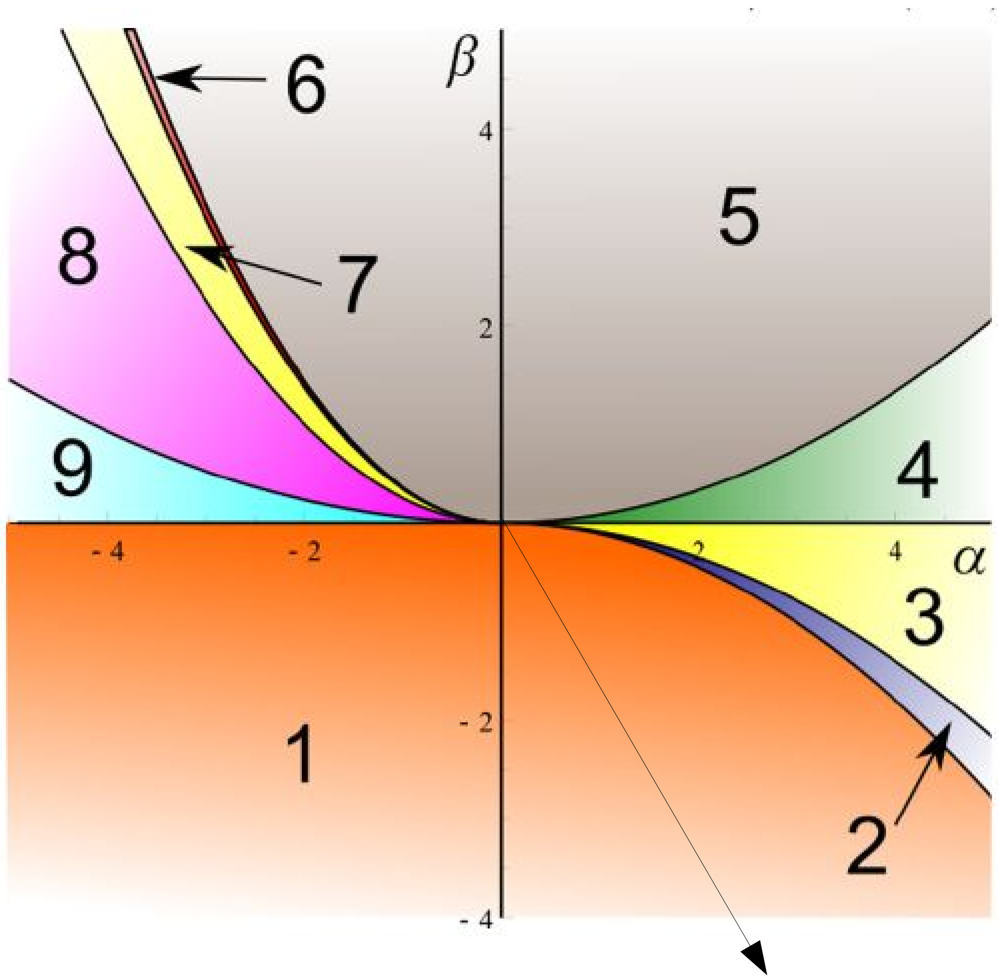


Weak (usual) screening

Strong screening

Spherically symmetric solutions

Construct phase space of how inner-outer solns connect



For dRGT

(Sbisa et al)

How A and B connect to asymptotic solutions

$$\mathcal{L} \sim \sqrt{-g}R + 2m^2\sqrt{-\det(\partial_\mu \phi^\alpha \partial_\nu \phi^\beta \eta_{\alpha\beta})}$$

Stability

Koyama et al

Scalar perturbations around weak screening soln. A

$$S_\varphi = \frac{1}{2}\int d^4x\Big[K_t(r)(\partial_t\varphi)^2 - K_r(r)(\partial_r\varphi)^2 - K_\Omega(r)(\partial_\Omega\varphi)^2\Big]$$

Where all the $K's$ are functions of the effective theory parameters, particularly of $\alpha,\ \beta$

which tune the terms $\alpha\ \partial_\mu\pi\partial_\nu\pi T^{\mu\nu}, \quad \beta\ \bar{h}^{\mu\nu}X^{(3)}_{\mu\nu}$

Stability

1. $\alpha = \beta = 0$ Original Galileon, studied by Nicolis et al

Superluminal propagation in radial modes

Extremely subluminal in radial piece!

2. $\alpha \neq 0, \ \beta = 0$ Disformal coupling $\alpha \, \partial_\mu \pi \partial_\nu \pi T^{\mu\nu}$

Leads to Restricted Galileon (Berezhiani et al)

$\alpha < 0$ has a ghost mode

There are no asymptotically flat Vainshtein solutions

Stability

Koyama et al

❸ $\alpha \neq 0, \ \beta \neq 0$

$$S_\varphi \sim \int d^4x \Big[K_t (\partial_t \varphi)^2 - K_r (\partial_r \varphi)^2 - K_\Omega (\partial_\Omega \varphi)^2 \Big]$$

Inside the Vainshtein radius and outside the source, the leading terms are

$$K_t \sim 0, \quad K_r \sim -12\beta A(r) x_0, \quad K_\Omega \sim 6\beta A(r) x_0,$$

Speed of fluctuations is always superluminal, and **unstable**.

- The instability can be avoided inside the source
- Need to look beyond static & spherical symmetry!

Beyond Vanilla...

Play with the matter sector

Chagoya, Koyama, Niz & Tasinato

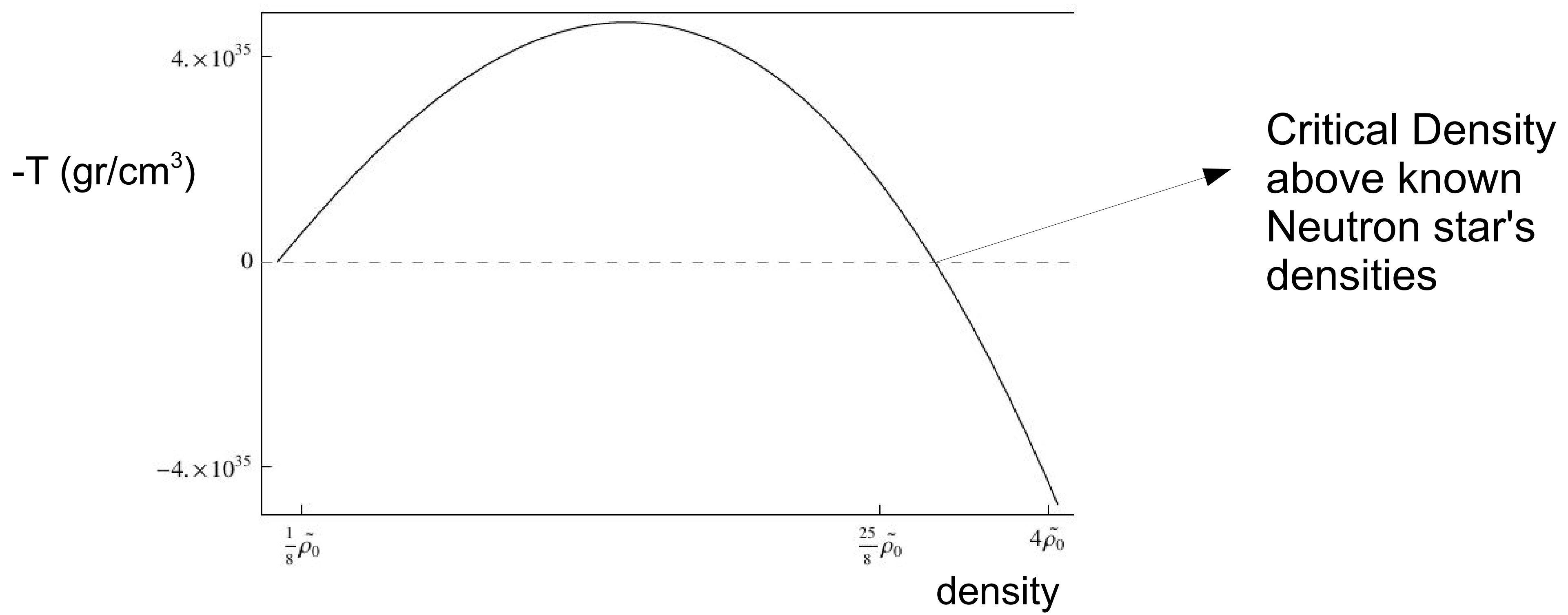
TOV	Found that: small corrections	✓
Scalarisation (see Teruaki's talk)	avoids it	✓
Polytropic EOS	mechanism works better for higher densities	✓

Beyond Vanilla...

Play with the matter sector

Chagoya, Koyama, Niz & Tasinato

Polytropic EOS. Only issue in strong gravity regime



Beyond Vanilla...

Away from spherical symmetry

Slow rotation shows no big corrections

Chagoya, Koyama, Niz & Tasinato

Filaments (cylindrical) solutions present suppressed Vainshtein effect

Gabadadze et al, Bloomfield et al, Falck et al.

Pancakes (planar) solutions do not exhibit screening

Brax et al, Bloomfield et al, Falck et al.

Are these solutions unique to gravitational systems?

Let us rewind a bit and consider a simple toy model

Toy Model

Simple Bi-galileon

Chagoya, Niz, Tasinato
(preliminary)

$$S=\int d^4x\left[\partial_\mu\phi\partial^\mu\phi+\partial_\mu\pi\partial^\mu\pi+\alpha\partial^\mu\phi\partial^\nu\phi X^{(1)}_{\mu\nu}(\pi)\right],$$

$$X^{(1)}_{\mu\nu}=\eta_{\mu\nu}[\Pi]-\Pi_{\mu\nu}$$

EOM (two integration constants)

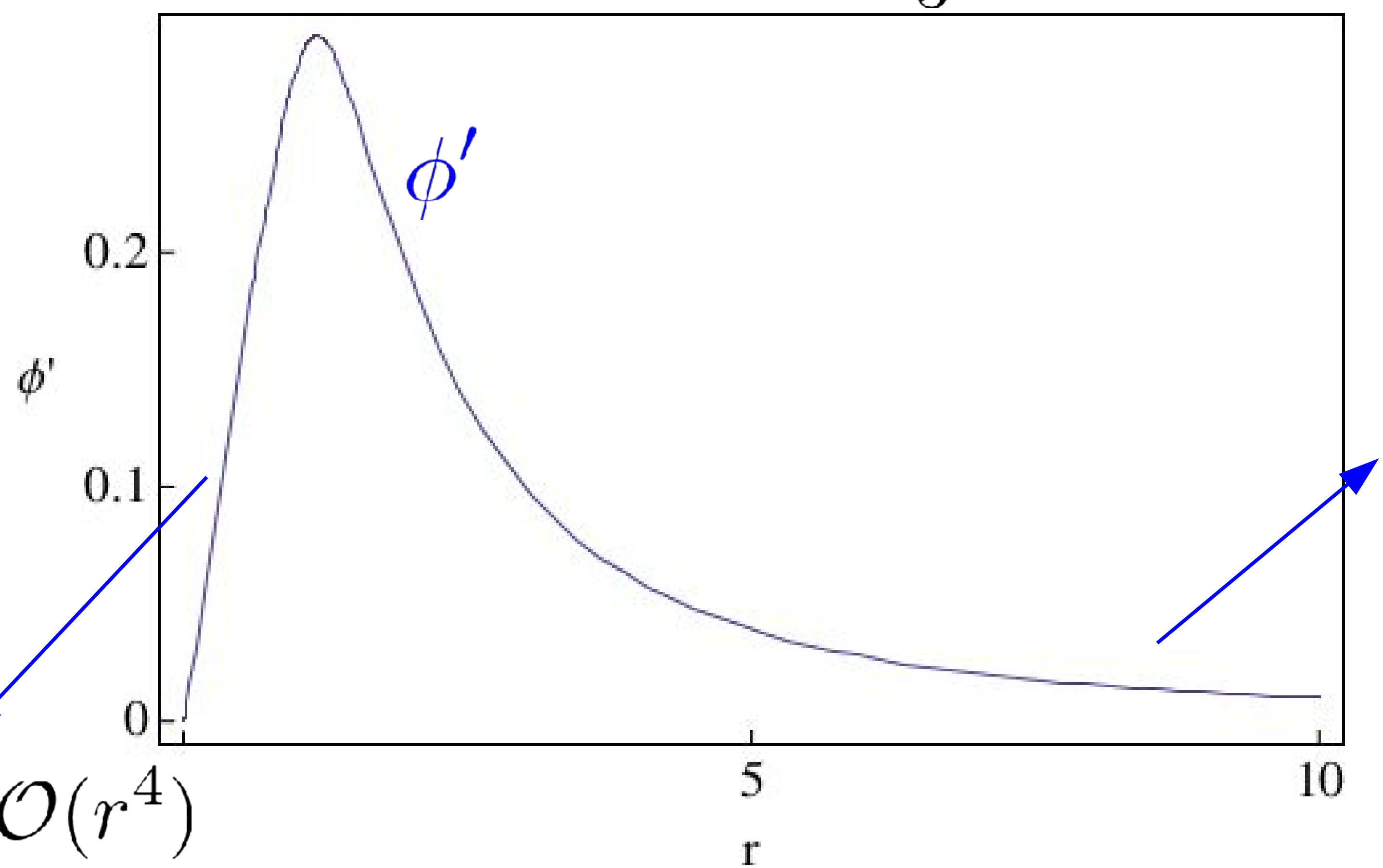
$$-r\zeta_\phi-\left(r^3+4\alpha\zeta_\pi\right)\phi'+8r\alpha^2\left(\phi'\right)^3=0$$

Toy Model

1

1/r singularity removal (strong screening)

Existence cond. $|\zeta_\pi| > \frac{2}{3}|\zeta_\phi|$



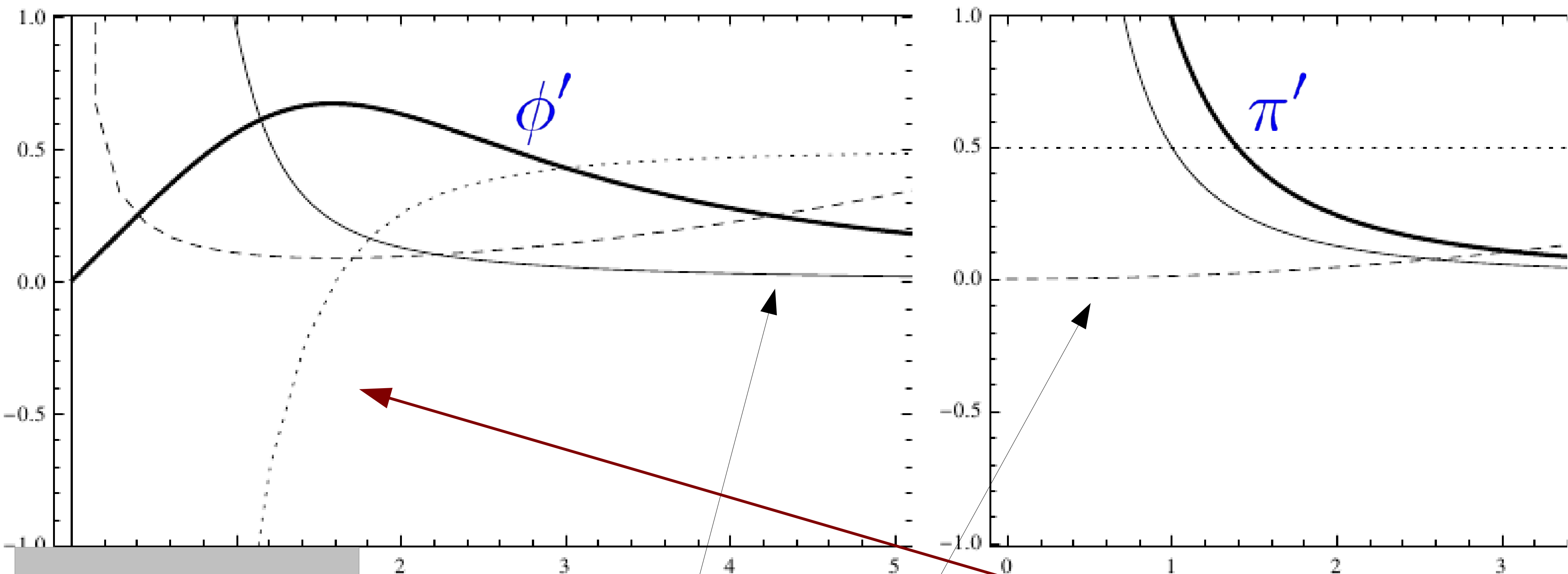
$\sim \frac{\zeta_\phi}{4\zeta_\pi} r + \mathcal{O}(r^4)$

$\sim \frac{\zeta_\phi}{r^2}$

Toy Model

1

Other field & Perturbations



No Ghost

$$|\zeta_\pi| > \sqrt{3}|\zeta_\phi|$$

$$S_\varphi \sim \sum_i \int d^4x \left[K_t(\partial_t \varphi_i)^2 - K_r(\partial_r \varphi_i)^2 - K_\Omega(\partial_\Omega \varphi_i)^2 \right] + \text{ints.}$$

Toy Model

1

Superluminality inside the Vainshtein radius?

Not straightforward to calculate velocities because fields are coupled, but found solutions, for $r \to 0$

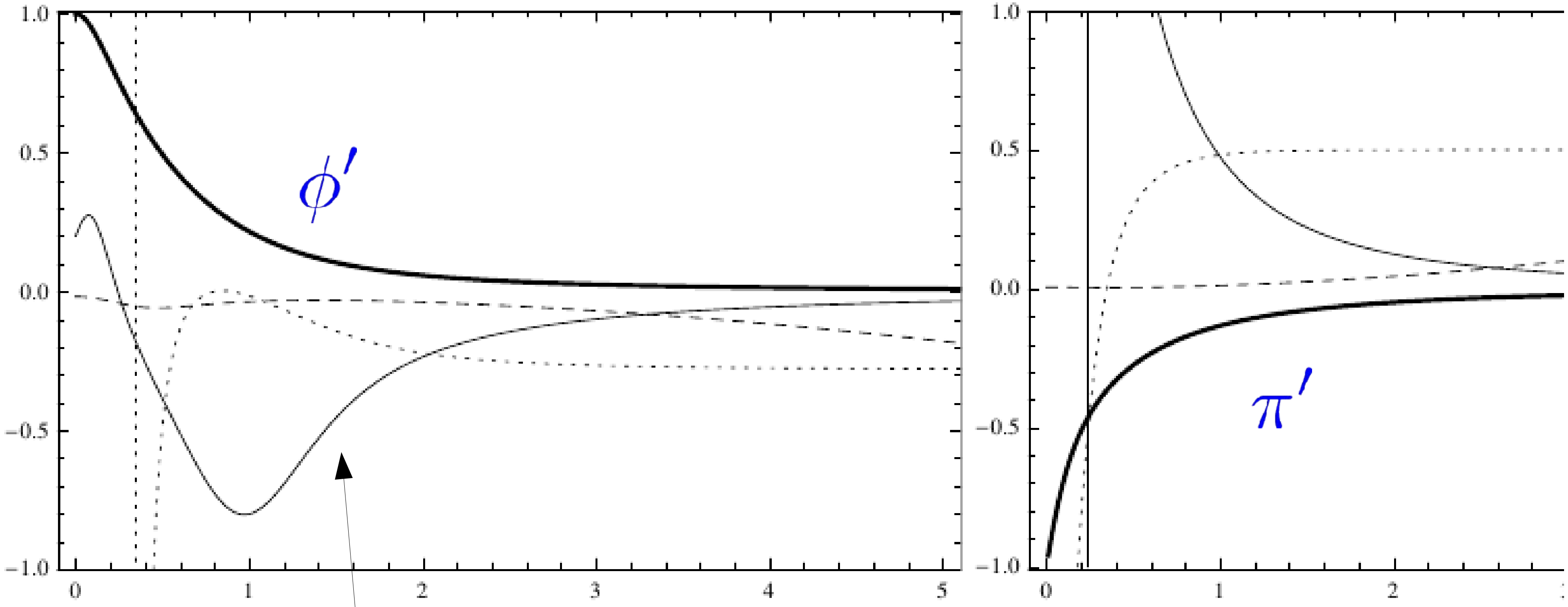
$$\delta\phi = -\frac{\zeta_\phi w^2}{120 r \alpha \zeta_\pi^2}\left(\kappa_1 e^{-i(r+t)w} + \kappa_2 e^{i(r-t)w}\right).$$

$$\delta\pi = A\delta\phi + \left(\kappa_1 e^{-i(r+t)w} + \kappa_2 e^{i(r-t)w}\right).$$

Toy Model

1

With $(\partial\phi)^2 X_2(\pi)$ there is one further solution where both fields are regular near zero.



Always a Ghost

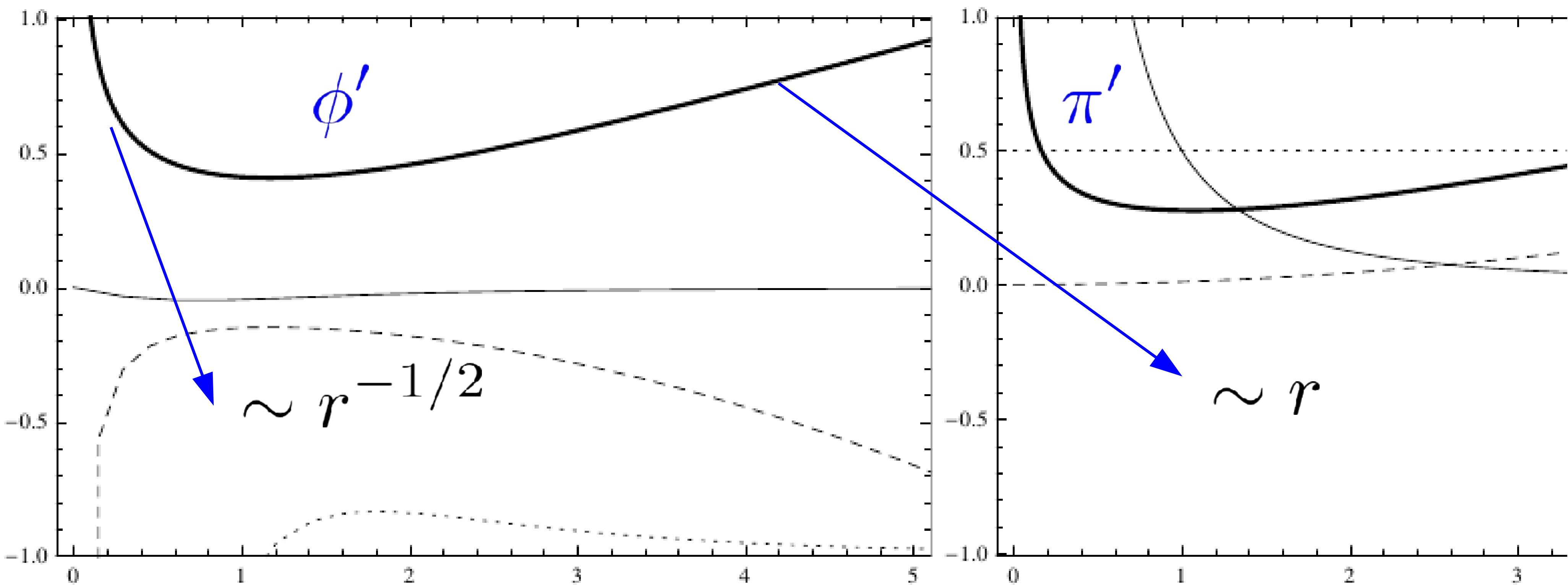
Other operators may avoid Derrick's theorem

Toy Model

2

Confinement

Existence cond. $\zeta_\phi > \zeta_\pi$

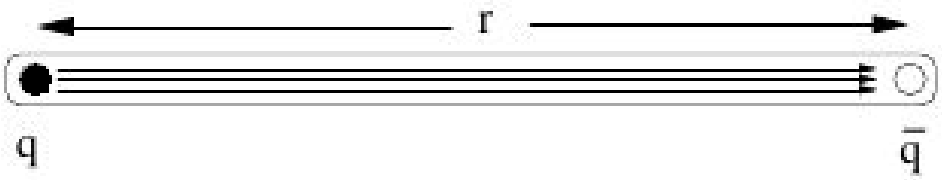


Perturbations are always unstable

Toy Model

2

Other/higher operators



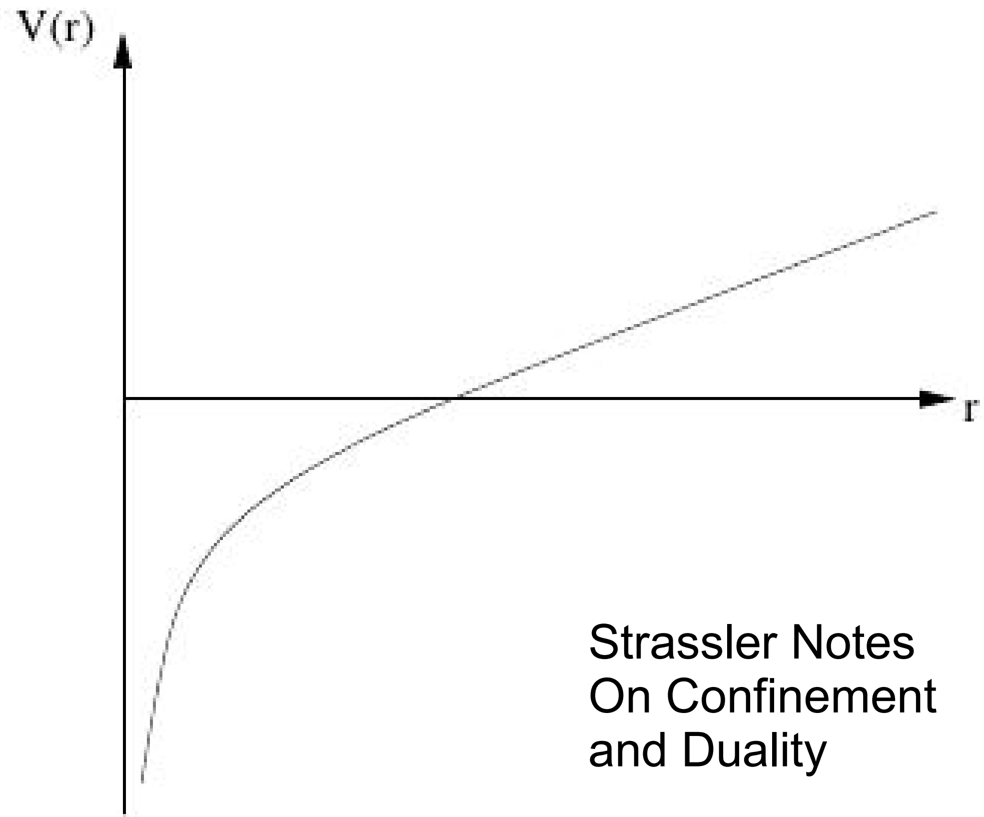
1) Change the exponent

near r=0

$$\sim r^{-n}$$

2) Improve stability

of perturbations



Strassler Notes On Confinement and Duality

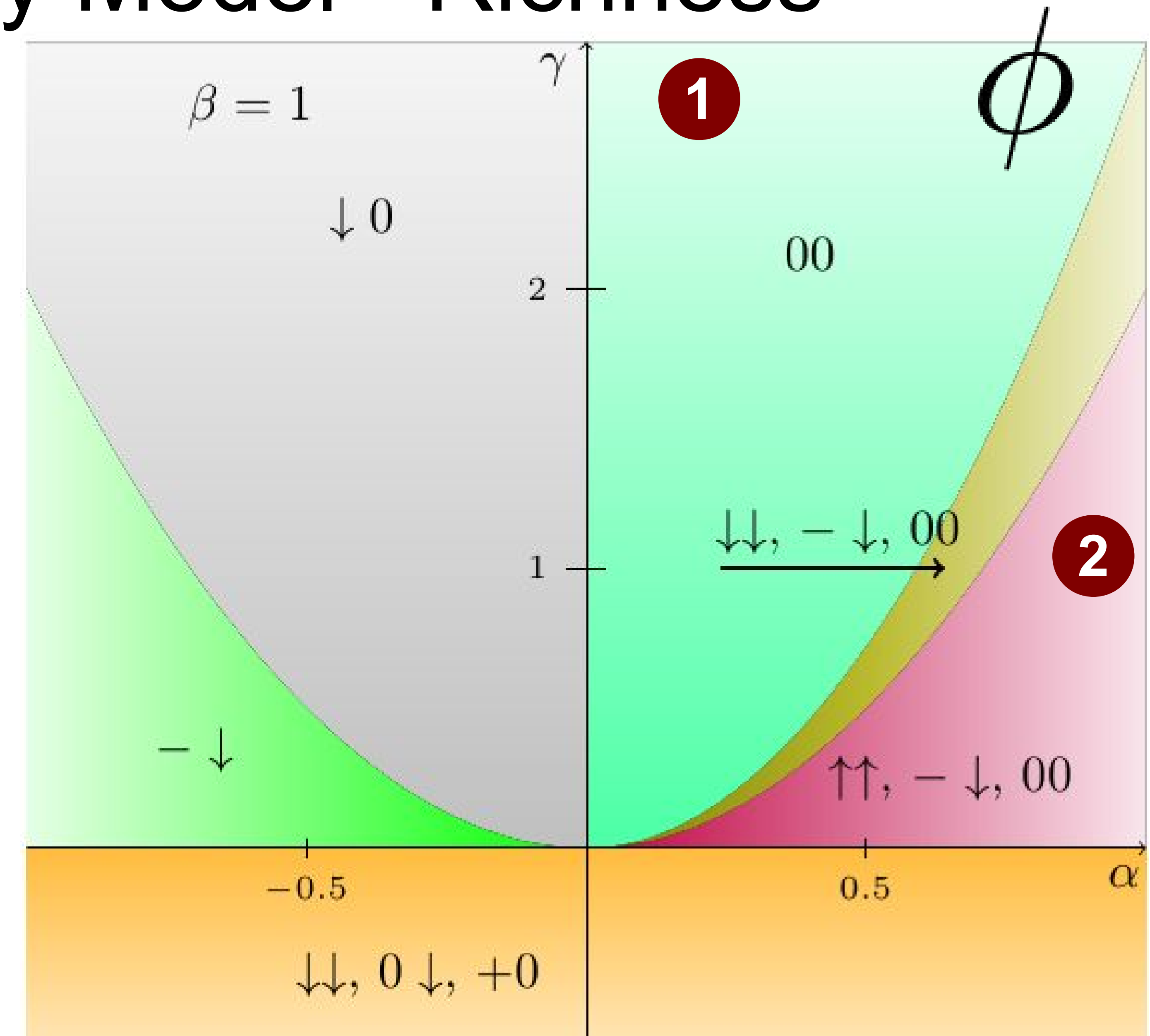
Is this solution relevant for confinement?

Toy Model - Richness

$\alpha(\partial\phi)^2 X_1(\pi)$

$\gamma(\partial\phi)^2 X_2(\pi)$

$\beta(\partial\phi)(\partial\pi)$



$\downarrow 0$

Inner Outer

Conclusions

- Models with derivative interactions (of Galileon type) have rich space of solutions
- Are solutions tied to instabilities?
  - Some may go away by adding other operators or when calculating the relevant observables (group vs front velocity, see Keltner and Tolley)
- May have interesting applications beyond modified gravity