Modified Gravity, Vector Fields, and Galileons

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Based on works with:

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Dark energy

- **Current acceleration** is compatible with **positive cosmological constant**
  - ▷ Impressive **fine-tuning** is required

- **Idea**: use **new fields** besides Einstein gravity to **drive acceleration**
  - ▷ Quintessence
    `Scalar field with appropriate interactions`
  - ▷ Modified gravity
    `New gravitational d.o.f.'s control the cosmological dynamics at large scales`

Dark energy

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  - ▷ Modified gravity
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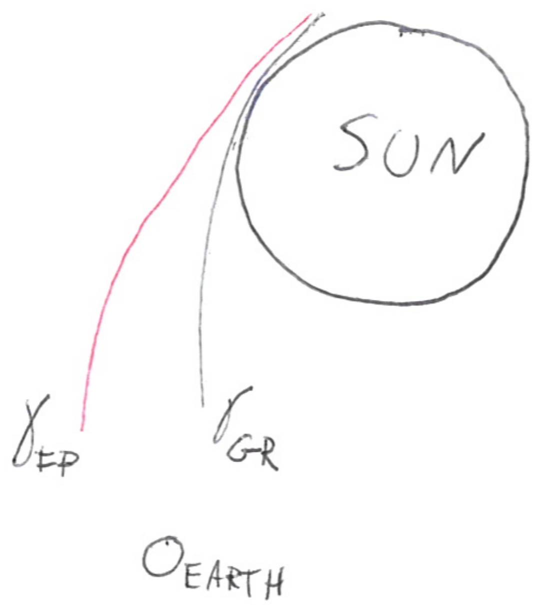
Problems:

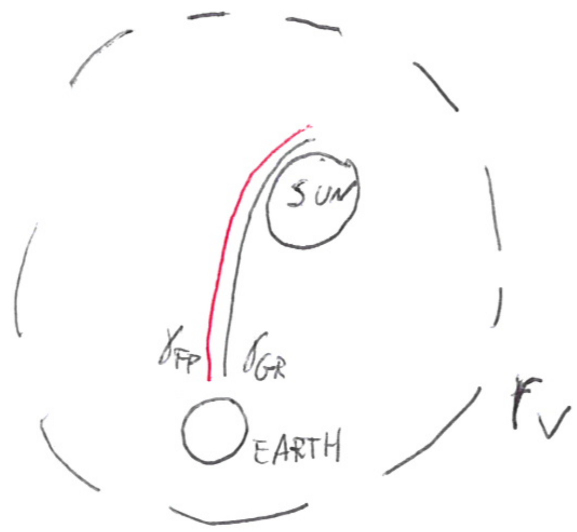
- We need **very light fields** to drive dark energy ($m \simeq H$):
- Why don't we see them with observations at **solar system scales**?
- For scalars: What's keeping their mass **small**?
  (scalar masses receive large corrections)

Dark energy

Cosmic acceleration and screening mechanisms:

- Chamaleon, **Vainshtein mechanism**





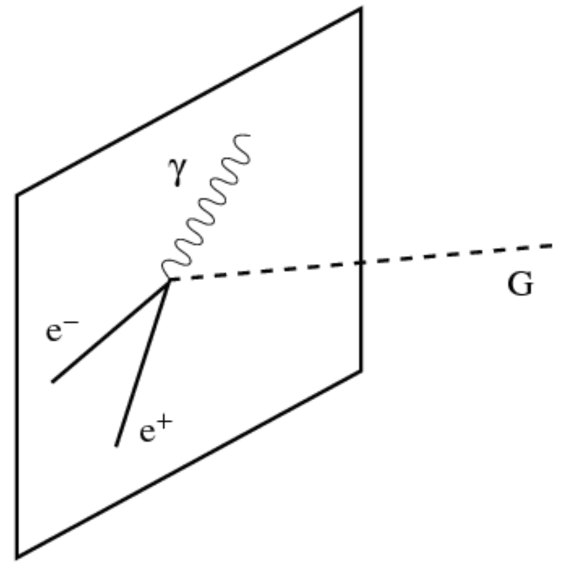
- Non-linear dynamics at scales below a **radius** r_V:
  Strong coupling effects suppress extra forces ($\Rightarrow$ GR results)

- At scales $r \gg r_V$ new dofs drive acceleration

Dark energy

Possible realizations of such scenarios:

– **Extradimensions**, brane-world scenarios



– **Break symmetries** (massive gravity): new dofs are 'longitudinal polarizations'

Realize Galileons as Goldstones of broken symmetries (see e.g. [Goon et al])

– **Put by hand** new dof

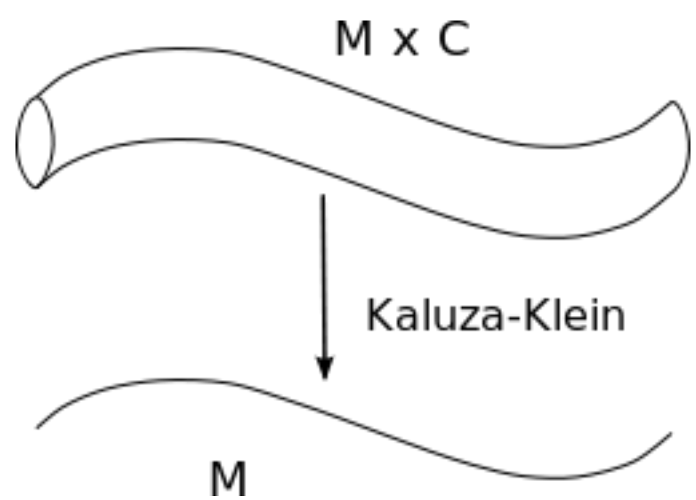
Derivative interactions and symmetries are important

What's the role of vector fields for modified gravity ?

`Can they have anything to do with cosmic acceleration and screening mechanisms?`

- Vectors play role in different scenarios

  – Original **Kaluza-Klein idea**: get electromagnetism from 5d Einstein gravity



  – Brane-world models: DBI action contains world-volume vector fields

  – Other well-known models: TeVeS, Einstein Ether etc

  – Vectors and dark energy: [Koivisto, Mota; Uzan et al,..]

What's the role of vector fields for modified gravity ?

Can they have anything to do with cosmic acceleration and screening mechanisms?

- Vectors **less explored** in dark energy models exploiting derivative self-interactions
  - Sake of simplicity (scalar-tensor theories are already complicated enough)
  - They don't couple to sources
  - They appear at least quadratically in the action

- **But they can have cosmological consequences in modified gravity**

  Strongly coupled around self-accelerating solutions in massive gravity

- **Vector Horndeski** is simple:

  Most general theory **with gauge symmetry**, coupling **vector to gravity**, with **second order** eqs of motion

$$\Delta\mathcal{L} = \sqrt{-g}\,\epsilon^{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\nu_1\nu_2\nu_3\nu_4}F_{\mu_1\mu_2}F^{\nu_1\nu_2}R_{\mu_3\mu_4}^{\ \ \ \ \ \nu_3\nu_4}$$

Galileons from broken gauge invariance

Galileons are interesting for

Screening effects

$\Leftrightarrow$ $\Updownarrow$

Acceleration of the universe

$\Leftrightarrow$

Non-renormalization theorems

$$
\begin{aligned}
\mathcal{L}_2 &= -\frac{1}{2}\left(\partial\pi\right)^2 \\
\mathcal{L}_3 &= \left(\partial\pi\right)^2 \Box\pi \\
\mathcal{L}_4 &= \left(\partial\pi\right)^2 \left[\left(\Box\pi\right)^2 - \left(\partial_\mu\partial_\nu\pi\right)^2\right] \\
\mathcal{L}_5 &= \left(\partial\pi\right)^2 \left[\left(\Box\pi\right)^3 + 2\left(\partial_\mu\partial_\nu\pi\right)^3 - 3\Box\pi\left(\partial_\mu\partial_\nu\pi\right)^2\right]
\end{aligned}
$$

Galileons from broken gauge invariance

Take $U(1)$ vector gauge theory;

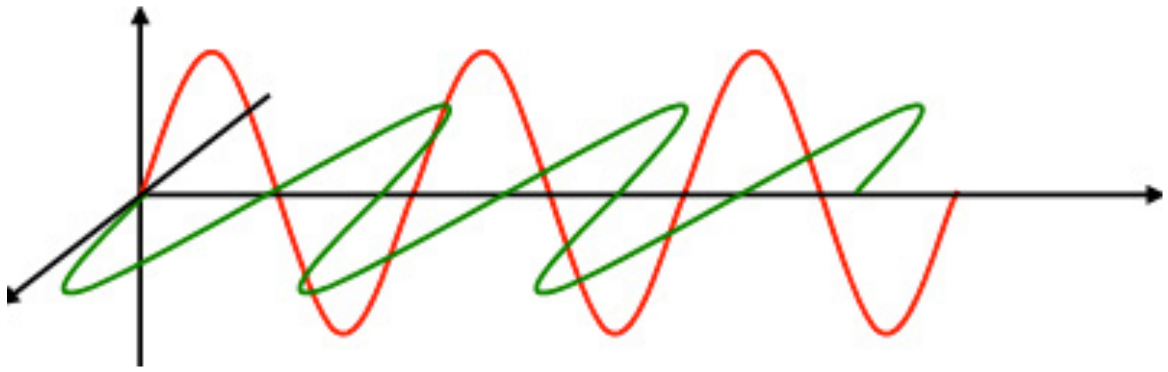
break symmetry through interactions including derivatives

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \text{symmetry breaking part}$$

Task

Realise Galileons as 'Goldstone bosons' of broken gauge symmetry

(in appropriate decoupling limit)



Vector Galileons

[GT, Heisenberg]

$$\begin{aligned}
\mathcal{L}_{(0)} &= -m^2\, A_\mu A^\mu\,, \\
\mathcal{L}_{(1)} &= -\beta_2\, A_\mu A^\mu\, \left(\partial_\rho A^\rho\right)\,, \\
\mathcal{L}_{(2)} &= -\frac{\beta_3}{m^2}\, A_\mu A^\mu\, \left[\left(\partial_\rho A^\rho\right)\left(\partial_\nu A^\nu\right) - \left(\partial_\rho A^\nu\right)\left(\partial^\rho A_\nu\right)\right]\,, \\
\mathcal{L}_{(3)} &= -\frac{\beta_4}{m^4}\, A_\mu A^\mu \Big[-2\left(\partial_\mu\, A^\mu\right)^3 + 3\,\left(\partial_\mu\, A^\mu\right)\,\left(\partial_\rho\, A^\sigma \partial^\rho A_\sigma\right) + 3\,\left(\partial_\mu\, A^\mu\right)\,\left(\partial_\rho\, A^\sigma \partial_\sigma A^\rho\right) \\
&\quad -\partial_\mu\, A^\nu\, \partial_\nu\, A^\rho\, \partial_\rho\, A^\mu - 3\, \partial_\mu\, A^\nu\, \partial_\nu\, A^\rho\, \partial^\mu\, A_\rho \Big]\,,
\end{aligned}$$

$$\text{Vector } A_\mu = (A_0,\, A_i) \text{ with } A_i = A_i^T + \partial_i \pi$$

Criteria to follow

- A_0 vector component keeps **not** dynamical **(no ghost)**
- The vector longitudinal polarization π **is dynamical**
- Decoupling limit give Galileon selfinteractions for π

$$m \to 0 \quad , \quad \beta \to 0 \quad , \quad \frac{m^3}{\beta} = \Lambda^3 = \text{finite}$$

Vector Galileons

Motivations

- Relation with Galileons: possibly share some of the nice features mentioned above
- Galileons from broken symmetries : **analogy with massive gravity**.
  - `Opportunity to understand (some of) its features in a simpler set-up`

- **Moreover**

- Easier to study **dynamics of perturbations around cosmological space-times**, outside decoupling limit
- **Higgs mechanism**: spontaneous symmetry breaking leads to vector galileons

Screening with vector galileons

Analogue of Vainsthein mechanism

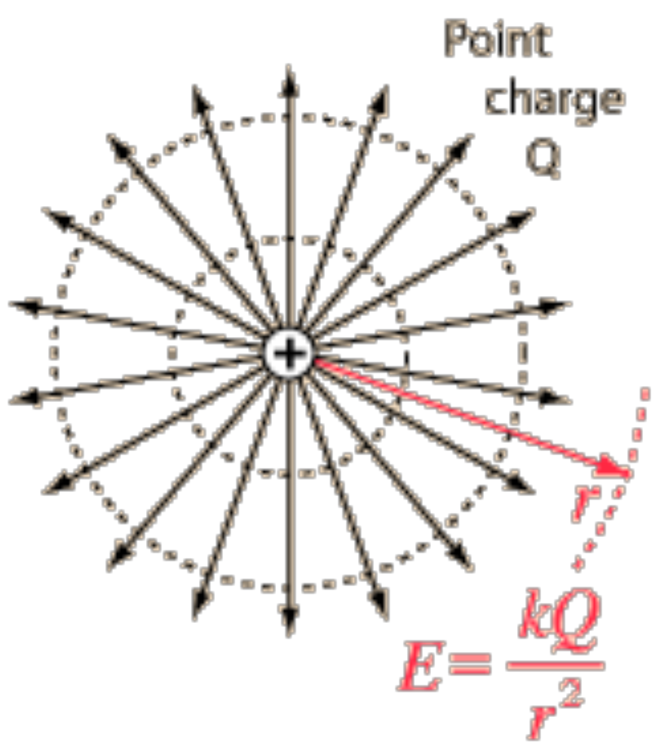
Example: Electric field produced by point charge

$$A_\mu \; = \; (A_0, A_1, A_2, A_3)$$

With $A_i \; = \; A_i^T + \partial_i \, \chi$ and $\partial_i \, A_i^T \; = \; 0$

$$A_0 \;\simeq\; -\frac{Q}{r}$$

$$\chi \;\simeq\; -\frac{\beta \, Q}{m^2 \, r^2}$$



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Coupling to gravity

- Same **technical issues** one meets coupling to gravity scalar Galileons

`Must ensure that EOMs remain second order`

$$
\begin{aligned}
\mathcal{L}^{cov}_{(0)} &= -m^2 A_\mu A^\mu \\
\mathcal{L}^{cov}_{(1)} &= -\beta_1\, A_\mu A^\mu \,\left(\nabla_\rho A^\rho\right)\,, \\
\mathcal{L}^{cov}_{(2)} &= -\frac{\beta_2}{m^2}\, A_\mu A^\mu \left[\left(\nabla_\rho A^\rho\right)\left(\nabla_\nu A^\nu\right) - \left(\nabla_\rho A^\nu\right)\left(\nabla^\rho A_\nu\right) - \frac{1}{4}\, R\, A_\sigma A^\sigma\right]
\end{aligned}
$$

$\Updownarrow$

`Non-minimal coupling with Ricci scalar`

`The theory propagates five degrees of freedom around Minkowski`

Dark energy from vector galileons

- Look for homogeneous cosmological expansion driven by vectors

  ▷ Metric Ansatz
$$ds^2 \;=\; -dt^2 + a^2(t)\,\delta^{ij}\,dx_i\,dx_j$$

$$A_\mu \;=\; (A_0(t)\,,0\,,0\,,0)$$

$$\Updownarrow$$

`Simplest possibility:  time-like vector background`

Dark energy from vector galileons

- Look for homogeneous cosmological expansion driven by vectors

$$A_\mu \;=\; \left(A_0(t)\,,0\,,0\,,0\right)$$

Vector equation is algebraic $$A_0\left(m^2-3\,\beta_1\,A_0\,H+9\,\frac{\beta_2}{m^2}\,A_0^2\,H^2\right)\;=\;0$$

With m the vector mass, β_i dimensionless vector galileon coupling constants

Dark energy from vector galileons

- Look for homogeneous cosmological expansion driven by vectors

$$A_\mu = (A_0(t)\,,0\,,0\,,0)$$

Vector solution:

$$A_0^{\pm}(t) = \frac{\beta_1 \pm \sqrt{\beta_1^2 - 4\beta_2}}{6\,\beta_2}\,\frac{m^2}{H(t)}\,, \\ = \frac{c_\pm\, m^2}{H(t)}\,.$$

de Sitter solution with

$$H^2 \simeq \beta\,\frac{m^3}{M_{Pl}}$$

`Size of acceleration proportional to the symmetry breaking parameters`

Linearized perturbations around de Sitter solution

Equations for perturbations can be studied analytically

- Tensor Fluctuations

The non-minimal coupling of the vector to gravity induces a renormalization of the Planck mass

- Vector Fluctuations

Interesting: vectors have zero mass around de Sitter solution

- Scalar Fluctuations

Strong coupling issue scalar perturbations have **no kinetic terms**

(analog of massive gravity)

Probably signal instability

`again, similar problems as in other models`

Generalizations: going beyond time-like vector background

... but then one has to be careful of not inducing spatial anisotropies, so we need to keep the theory simple ...

Vectors coupled to gravity: Magnetogenesis and Vector Inflation

Couple gauge potential A_μ to curvature $R_{\mu\nu}$

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{Pl}^2}{2} R - \frac{1}{4} F^2 - \frac{1}{2} \left(m^2 + \xi R \right) A_\mu A^\mu \right]$$

Break the conformal invariance of Maxwell eqs to get interesting cosmology:

- Magnetogenesis [Turner, Widrow]

Magnetic field magnitude: $|\vec{B}|^2 \propto a^{-5+\sqrt{1-48\,\xi}}$

Vectors coupled to gravity: Magnetogenesis and Vector Inflation

Couple `gauge potential` A_μ to `curvature` $R_{\mu\nu}$

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{Pl}^2}{2} R - \frac{1}{4} F^2 - \frac{1}{2} \left(m^2 + \xi R \right) A_\mu A^\mu \right]$$

- <u>`Vector inflation`</u> [Golovnev, Mukhanov, Vanchurin; Armendariz-Picon, ...],

  FRW cosmology from **triplet of vectors** each pointing in **different spatial direction**

  $A_\mu^{(1)} = (0,\, a(t)\, \mathcal{B}(t),\, 0,\, 0) \quad ; \quad A_\mu^{(2)} = (0,\, 0,\, a(t)\, \mathcal{B}(t),\, 0) \quad ; \quad A_\mu^{(3)} = (0,\, 0,\, 0,\, a(t)\mathcal{B}(t))$

...or large number of vectors with random directions...

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$$S = \int d^4x \sqrt{-g} \left[\frac{M_{Pl}^2}{2} R - \frac{1}{4} F^2 - \frac{1}{2} \left(m^2 + \xi R \right) A_\mu A^\mu \right]$$

- <u>Vector inflation</u>

$$A_\mu^{(1)} = (0,\, a(t)\, \mathcal{B}(t),\, 0,\, 0) \quad ; \quad A_\mu^{(2)} = (0,\, 0,\, a(t)\, \mathcal{B}(t),\, 0) \quad ; \quad A_\mu^{(3)} = (0,\, 0,\, 0,\, a(t) \mathcal{B}(t))$$

Choose $\xi = -1/6$.

$$H^2 = \frac{1}{2\, M_{Pl}^2} \left[\dot{\mathcal{B}}^2 + m^2\, \mathcal{B}^2 \right]$$

Equations of motion:

$$\ddot{\mathcal{B}} + 3\, H\, \dot{\mathcal{B}} + m^2\, \mathcal{B} = 0$$

Very similar to chaotic inflation

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- Vector inflation

$$A_\mu^{(1)} = (0,\, a(t)\,\mathcal{B}(t),\, 0,\, 0) \quad ; \quad A_\mu^{(2)} = (0,\, 0,\, a(t)\,\mathcal{B}(t),\, 0) \quad ; \quad A_\mu^{(3)} = (0,\, 0,\, 0,\, a(t)\mathcal{B}(t))$$

Interesting:

- Simple alternative to scalar-field inflation!
- Small violation of isotropy can be easily generated, by coupling N vectors
- Cosmological perturbations have specific features:
  coupling between modes at linearized level, anisotropic stress etc etc
- Can be used for dark energy as well

Vectors coupled to gravity: Magnetogenesis and Vector Inflation

Couple gauge potential A_μ to curvature $R_{\mu\nu}$

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{Pl}^2}{2} R - \frac{1}{4} F^2 - \frac{1}{2} \left(m^2 + \xi R \right) A_\mu A^\mu \right]$$

- Vector inflation

$$A_\mu^{(1)} = (0, a(t)\, \mathcal{B}(t), 0, 0) \quad ; \quad A_\mu^{(2)} = (0, 0, a(t)\, \mathcal{B}(t), 0) \quad ; \quad A_\mu^{(3)} = (0, 0, 0, a(t)\mathcal{B}(t))$$

But:

This scenario has ghost instabilities [Himmetoglu, Contaldi, Peloso]

maybe not surprising, since the EOMs are higher order

Way out to resurrect the model

Use vector galileons

Add derivative ``counterterms'' that make the EOMs second order

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{Pl}^2}{2} R - \frac{1}{4} F^2 - \frac{1}{2} \left(m^2 + \xi R \right) A_\mu A^\mu + \xi \left[(\nabla_\mu A^\mu)^2 - \nabla_\mu A_\nu \nabla^\mu A^\nu \right] \right]$$

Such counterterms are total derivatives in flat space

Vectors coupled to gravity: Magnetogenesis and Vector Inflation

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{Pl}^2}{2} R - \frac{1}{4} F^2 - \frac{1}{2} \left(m^2 + \xi R \right) A_\mu A^\mu + \xi \left[(\nabla_\mu A^\mu)^2 - \nabla_\mu A_\nu \nabla^\mu A^\nu \right] \right]$$

The theory is still quadratic in the fields
vector inflation can be studied as Mukhanov's scenario

The background equations are:

$$H^2 = \frac{1}{2M_{Pl}^2} \left[\dot{\mathcal{B}}^2 + \left(m^2 + (1+6\xi) H^2 \right) \mathcal{B}^2 + 2(1+4\xi) \dot{\mathcal{B}} \mathcal{B} H \right]$$

$$\ddot{\mathcal{B}} + 3 H \dot{\mathcal{B}} + \left[m^2 + H^2 (2 + \xi(6-4\epsilon) - \epsilon) \right] \mathcal{B} = 0$$

Cosmological perturbations don't have previously found ghosts.
Still checking the details...

Conclusions

- **Vector Galileons** allow to get **galileons** as **Goldstone bosons** of $U(1)$ gauge symmetry breaking operators.
- They represent a **concrete, simple setting** for studying field theory and cosmology set-ups enjoying galileon symmetries in appropriate limits
- They share some of the **features** (and problems) with systems like massive gravity. Accelerating solutions driven by time-like vectors have strong coupling problems.
- Given the simplicity of the set-up, cosmology can be studied relatively easily, including possibilities so far unexplored:
  - anisotropic field configurations, that leads to (quasi)isotropic FRW metric
  - stability of configurations with vectors non-minimally coupled with curvature