Modified Gravity, Vector Fields, and Galileons

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Based on works with:

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Dark energy

- **Current acceleration** is compatible with **positive cosmological constant**
  - Impressive **fine-tuning** is required

- **Idea:** use **new fields** besides Einstein gravity to **drive acceleration**
  - Quintessence
    - Scalar field with appropriate interactions
  - Modified gravity
    - New gravitational d.o.f.’s control the cosmological dynamics
      at large scales
Dark energy

• **Current acceleration** is compatible with **positive cosmological constant**
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• **Idea**: use **new fields** besides Einstein gravity to **drive acceleration**
  - Quintessence
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- We need **very light fields** to drive dark energy \((m \simeq H)\):

**Problems:**
- Why don’t we see them with observations at **solar system scales**?
- For scalars: What’s keeping their mass **small**? (scalar masses receive large corrections)
Dark energy

Cosmic acceleration and screening mechanisms:

- Chamaleon, Vainshtein mechanism

- Non-linear dynamics at scales below a radius $r_V$:
  Strong coupling effects suppress extra forces ($\Rightarrow$ GR results)

- At scales $r \gg r_V$ new dofs drive acceleration
Dark energy

Possible realizations of such scenarios:

– **Extradimensions**, brane-world scenarios

– **Break symmetries** (massive gravity): new dofs are ‘longitudinal polarizations’

  Realize Galileons as Goldstones of broken symmetries (see e.g. [Goon et al])

– **Put by hand** new dof

*Derivative interactions and symmetries are important*
What’s the role of vector fields for modified gravity?

Can they have anything to do with cosmic acceleration and screening mechanisms?

- Vectors play role in different scenarios
  - Original Kaluza-Klein idea: get electromagnetism from 5d Einstein gravity
  - Brane-world models: DBI action contains world-volume vector fields
  - Other well-known models: TeVeS, Einstein Ether etc
  - Vectors and dark energy: [Koivisto, Mota; Uzan et al,..]
What’s the role of vector fields for modified gravity?

Can they have anything to do with cosmic acceleration and screening mechanisms?

- Vectors less explored in dark energy models exploiting derivative self-interactions
  - Sake of simplicity (scalar-tensor theories are already complicated enough)
  - They don’t couple to sources
  - They appear at least quadratically in the action

- But they can have cosmological consequences in modified gravity
  Strongly coupled around self-accelerating solutions in massive gravity

- Vector Horndeski is simple:
  Most general theory with gauge symmetry, coupling vector to gravity, with second order eqs of motion

\[
\Delta \mathcal{L} = \sqrt{-g} \epsilon^{\mu_1 \mu_2 \mu_3 \mu_4} \epsilon_{\nu_1 \nu_2 \nu_3 \nu_4} F_{\mu_1 \mu_2} F^{\nu_1 \nu_2} R_{\nu_3 \nu_4}^{\mu_3 \mu_4}
\]
Galileons from broken gauge invariance

Galileons are interesting for

Screening effects

Acceleration of the universe

Non-renormalization theorems

\[ \mathcal{L}_2 = -\frac{1}{2} (\partial \pi)^2 \]
\[ \mathcal{L}_3 = (\partial \pi)^2 \Box \pi \]
\[ \mathcal{L}_4 = (\partial \pi)^2 \left[ (\Box \pi)^2 - (\partial_\mu \partial_\nu \pi)^2 \right] \]
\[ \mathcal{L}_5 = (\partial \pi)^2 \left[ (\Box \pi)^3 + 2 (\partial_\mu \partial_\nu \pi)^3 - 3 \Box \pi (\partial_\mu \partial_\nu \pi)^2 \right] \]
Galileons from broken gauge invariance

Take $U(1)$ vector gauge theory;
break symmetry through interactions including derivatives

\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \text{symmetry breaking part} \]

Task

Realise Galileons as ‘Goldstone bosons’ of broken gauge symmetry

(in appropriate decoupling limit)
Vector Galileons

\[ \mathcal{L}_{(0)} = -m^2 A_\mu A^\mu, \]
\[ \mathcal{L}_{(1)} = -\beta_2 A_\mu A^\mu (\partial_\rho A^\rho), \]
\[ \mathcal{L}_{(2)} = -\frac{\beta_3}{m^2} A_\mu A^\mu [(\partial_\rho A^\rho)(\partial_\nu A^\nu) - (\partial_\rho A^\nu)(\partial_\nu A^\rho)], \]
\[ \mathcal{L}_{(3)} = -\frac{\beta_4}{m^4} A_\mu A^\mu \left[ -2 (\partial_\mu A^\mu)^3 + 3 (\partial_\mu A^\mu)(\partial_\rho A^\sigma \partial^\rho A_\sigma) + 3 (\partial_\mu A^\mu)(\partial_\rho A^\sigma \partial_\sigma A^\rho) - \partial_\mu A^\nu \partial_\nu A^\rho \partial_\rho A^\mu - 3 \partial_\mu A^\nu \partial_\nu A^\rho \partial_\rho A^\mu \right], \]

Vector \( A_\mu = (A_0, A_i) \) with \( A_i = A_i^T + \partial_i \pi \)

Criteria to follow

- \( A_0 \) vector component keeps not dynamical (no ghost)
- The vector longitudinal polarization \( \pi \) is dynamical
- Decoupling limit give Galileon selfinteractions for \( \pi \)

\[ m \to 0 \quad , \quad \beta \to 0 \quad , \quad \frac{m^3}{\beta} = \Lambda^3 = \text{finite} \]
Vector Galileons

Motivations

► Relation with Galileons: possibly share some of the nice features mentioned above

► Galileons from broken symmetries: analogy with massive gravity.

  - Opportunity to understand (some of) its features in a simpler set-up

► Moreover

  - Easier to study dynamics of perturbations around cosmological space-times, outside decoupling limit

  - Higgs mechanism: spontaneous symmetry breaking leads to vector galileons
Screening with vector galileons

Analogue of Vainsthein mechanism

Example: Electric field produced by point charge

$$A_\mu = (A_0, A_1, A_2, A_3)$$

With $$A_i = A_i^T + \partial_i \chi$$ and $$\partial_i A_i^T = 0$$

$$A_0 \simeq -\frac{Q}{r}$$

$$\chi \simeq -\frac{\beta Q}{m^2 r^2}$$
Coupling to gravity

• Same technical issues one meets coupling to gravity scalar Galileons

Must ensure that EOMs remain second order

\[ \mathcal{L}_{(0)}^{\text{cov}} = -m^2 A_\mu A^\mu \]
\[ \mathcal{L}_{(1)}^{\text{cov}} = -\beta_1 A_\mu A^\mu (\nabla_\rho A^\rho), \]
\[ \mathcal{L}_{(2)}^{\text{cov}} = -\frac{\beta_2}{m^2} A_\mu A^\mu \left[ (\nabla_\rho A^\rho)(\nabla_\nu A^\nu) - (\nabla_\rho A^\nu)(\nabla^\rho A_\nu) - \frac{1}{4} R A_\sigma A^\sigma \right] \]

Non-minimal coupling with Ricci scalar

The theory propagates five degrees of freedom around Minkowski
• Look for homogeneous cosmological expansion driven by vectors

▶ Metric Ansatz

\[ ds^2 = -dt^2 + a^2(t) \delta^{ij} \, dx_i \, dx_j \]

\[ A_\mu = (A_0(t), 0, 0, 0) \]

Simplest possibility: time-like vector background
Dark energy from vector galileons

- Look for homogeneous cosmological expansion driven by vectors

\[ A_\mu = (A_0(t), 0, 0, 0) \]

Vector equation is algebraic

\[ A_0 \left( m^2 - 3 \beta_1 A_0 H + 9 \frac{\beta_2}{m^2} A_0^2 H^2 \right) = 0 \]

With \( m \) the vector mass, \( \beta_i \) dimensionless vector galileon coupling constants
Dark energy from vector galileons

- Look for homogeneous cosmological expansion driven by vectors

\[ A_\mu = (A_0(t), 0, 0, 0) \]

Vector solution:

\[ A_0^\pm(t) = \frac{\beta_1 \pm \sqrt{\beta_1^2 - 4\beta_2}}{6 \beta_2} \frac{m^2}{H(t)}, \]

\[ = \frac{c_{\pm} m^2}{H(t)}. \]

de Sitter solution with

\[ H^2 \approx \beta \frac{m^3}{M_{Pl}} \]

Size of acceleration proportional to the symmetry breaking parameters
Linearized perturbations around de Sitter solution

Equations for perturbations can be studied analytically

- **Tensor Fluctuations**

  The non-minimal coupling of the vector to gravity induces a renormalization of the Planck mass

- **Vector Fluctuations**

  **Interesting:** vectors have zero mass around de Sitter solution

- **Scalar Fluctuations**

  **Strong coupling issue** scalar perturbations have **no kinetic terms**
  (analog of massive gravity)

  Probably signal instability
again, similar problems as in other models

Generalizations: going beyond time-like vector background

... but then one has to be careful of not inducing spatial anisotropies, so we need to keep the theory simple ...
Vectors coupled to gravity: Magnetogenesis and Vector Inflation

Couple gauge potential $A_\mu$ to curvature $R_{\mu\nu}$

$$S = \int d^4 x \sqrt{-g} \left[ \frac{M_{Pl}^2}{2} R - \frac{1}{4} F^2 - \frac{1}{2} (m^2 + \xi R) A_\mu A^\mu \right]$$

Break the conformal invariance of Maxwell eqs to get interesting cosmology:

**Magnetogenesis** [Turner, Widrow]

Magnetic field magnitude: $|\vec{B}|^2 \propto a^{-5+\sqrt{1-48\xi}}$
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- Vector inflation [Golovnev, Mukhanov, Vanchurin; Armendariz-Picon, ...],
  FRW cosmology from triplet of vectors each pointing in different spatial direction
  $$A^{(1)}_\mu = (0, a(t) B(t), 0, 0) \quad ; \quad A^{(2)}_\mu = (0, 0, a(t) B(t), 0) \quad ; \quad A^{(3)}_\mu = (0, 0, 0, a(t) B(t))$$

  ...or large number of vectors with random directions...
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**Vector inflation**

$$ A^{(1)}_\mu = (0, a(t) B(t), 0, 0) \quad ; \quad A^{(2)}_\mu = (0, 0, a(t) B(t), 0) \quad ; \quad A^{(3)}_\mu = (0, 0, 0, a(t) B(t)) $$

Choose $\xi = -1/6$.

$$ H^2 = \frac{1}{2 M_{Pl}^2} \left[ \dot{B}^2 + m^2 B^2 \right] $$

Equations of motion:

$$ \ddot{B} + 3 H \dot{B} + m^2 B = 0 $$

Very similar to chaotic inflation
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- **Vector inflation**

$$A^{(1)}_\mu = (0, a(t) B(t), 0, 0) ; \quad A^{(2)}_\mu = (0, 0, a(t) B(t), 0) ; \quad A^{(3)}_\mu = (0, 0, 0, a(t) B(t))$$

Interesting:

- Simple alternative to scalar-field inflation!

- Small violation of isotropy can be easily generated, by coupling $N$ vectors

- Cosmological perturbations have specific features:
  - coupling between modes at linearized level, anisotropic stress etc etc

- Can be used for dark energy as well
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- **Vector inflation**

$$A^{(1)}_\mu = (0, a(t) B(t), 0, 0) \quad ; \quad A^{(2)}_\mu = (0, 0, a(t) B(t), 0) \quad ; \quad A^{(3)}_\mu = (0, 0, 0, a(t) B(t))$$

But:

This scenario has ghost instabilities [Himmetoglu, Contaldi, Peloso]

maybe not surprising, since the EOMs are higher order
Vectors coupled to gravity: Magnetogenesis and Vector Inflation

Way out to resurrect the model

Use vector galileons

Add derivative ‘‘counterterms’’ that make the EOMs second order

\[
S = \int d^4x \sqrt{-g} \left[ \frac{M_{Pl}^2}{2} R - \frac{1}{4} F^2 - \frac{1}{2} \left( m^2 + \xi R \right) A_\mu A^\mu + \xi \left[ (\nabla_\mu A^\mu)^2 - \nabla_\mu A_\nu \nabla^\mu A^\nu \right] \right]
\]

Such counterterms are total derivatives in flat space
Vectors coupled to gravity: Magnetogenesis and Vector Inflation

\[ S = \int d^4 x \sqrt{-g} \left[ \frac{M_{Pl}^2}{2} R - \frac{1}{4} F^2 - \frac{1}{2} \left( m^2 + \xi R \right) A_\mu A^\mu + \xi \left[ (\nabla_\mu A^\mu)^2 - \nabla_\mu A_\nu \nabla^\mu A^\nu \right] \right] \]

The theory is still quadratic in the fields
vector inflation can be studied as Mukhanov’s scenario

The background equations are:

\[ H^2 = \frac{1}{2 M_{Pl}^2} \left[ \dot{\mathcal{B}}^2 + (m^2 + (1 + 6\xi) H^2) \mathcal{B}^2 + 2(1 + 4\xi) \dot{\mathcal{B}} \mathcal{B} H \right] \]

\[ \ddot{\mathcal{B}} + 3 H \dot{\mathcal{B}} + \left[ m^2 + H^2 (2 + \xi (6 - 4\epsilon) - \epsilon) \right] \mathcal{B} = 0 \]

Cosmological perturbations don’t have previously found ghosts.
Still checking the details...
Conclusions

- **Vector Galileons** allow to get **galileons** as **Goldstone bosons** of $U(1)$ gauge symmetry breaking operators.

- They represent a **concrete, simple setting** for studying field theory and cosmology set-ups enjoying galileon symmetries in appropriate limits.

- They share some of the **features** (and problems) with systems like massive gravity. Accelerating solutions driven by time-like vectors have strong coupling problems.

- Given the simplicity of the set-up, cosmology can be studied relatively easily, including possibilities so far unexplored:
  - anisotropic field configurations, that leads to (quasi)isotropic FRW metric
  - stability of configurations with vectors non-minimally coupled with curvature