

A unifying description of dark energy

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Based on:

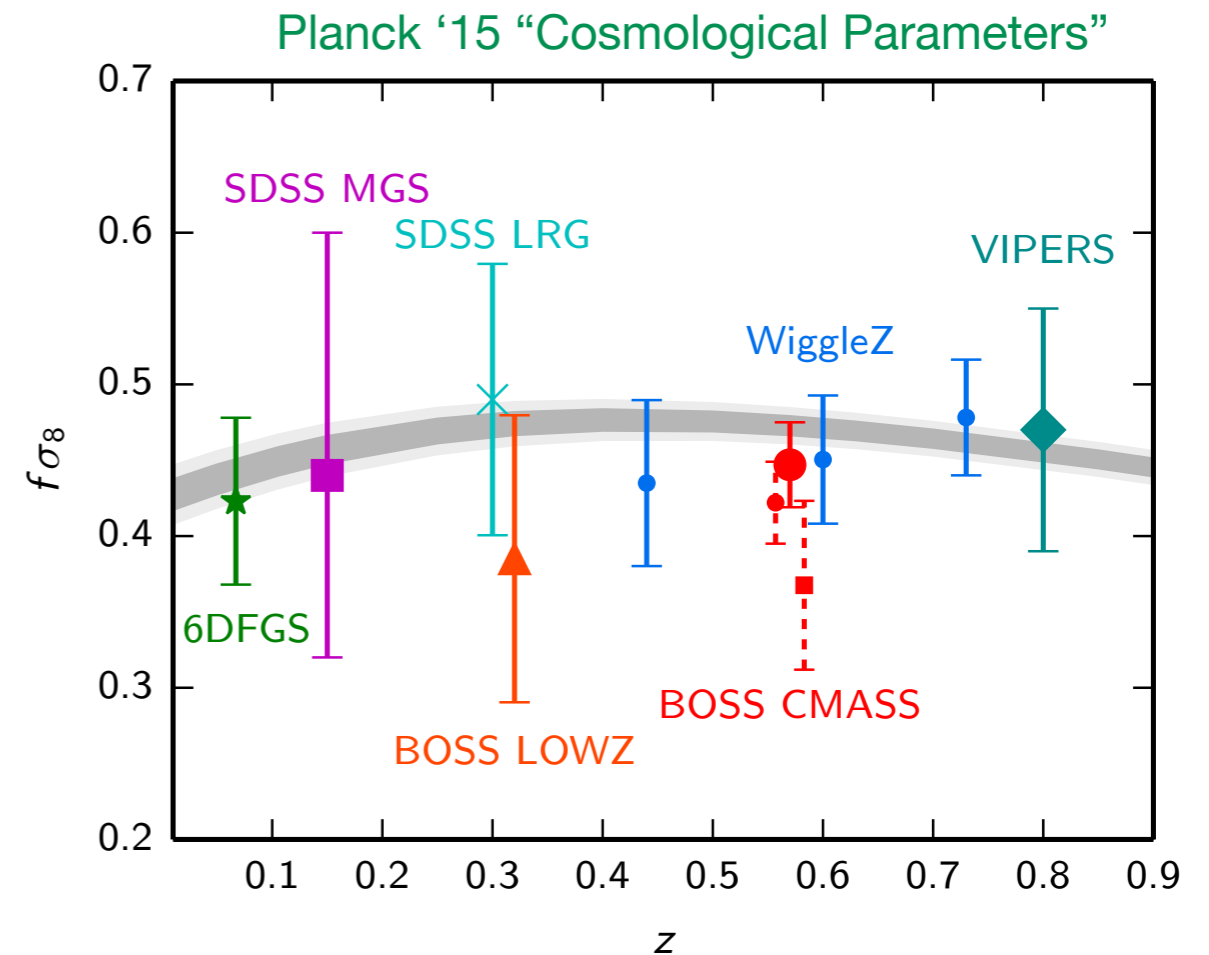
- 1504.05481 and 1509.02191 with J. Gleyzes, D. Langlois and M. Mancarella

(1411.3712 with J. Gleyzes and D. Langlois,
1304.4840 with J. Gleyzes, D. Langlois and F. Piazza,
1210.0201 with G. Gubitosi and F. Piazza)

KICP, Chicago - October 13, 2015

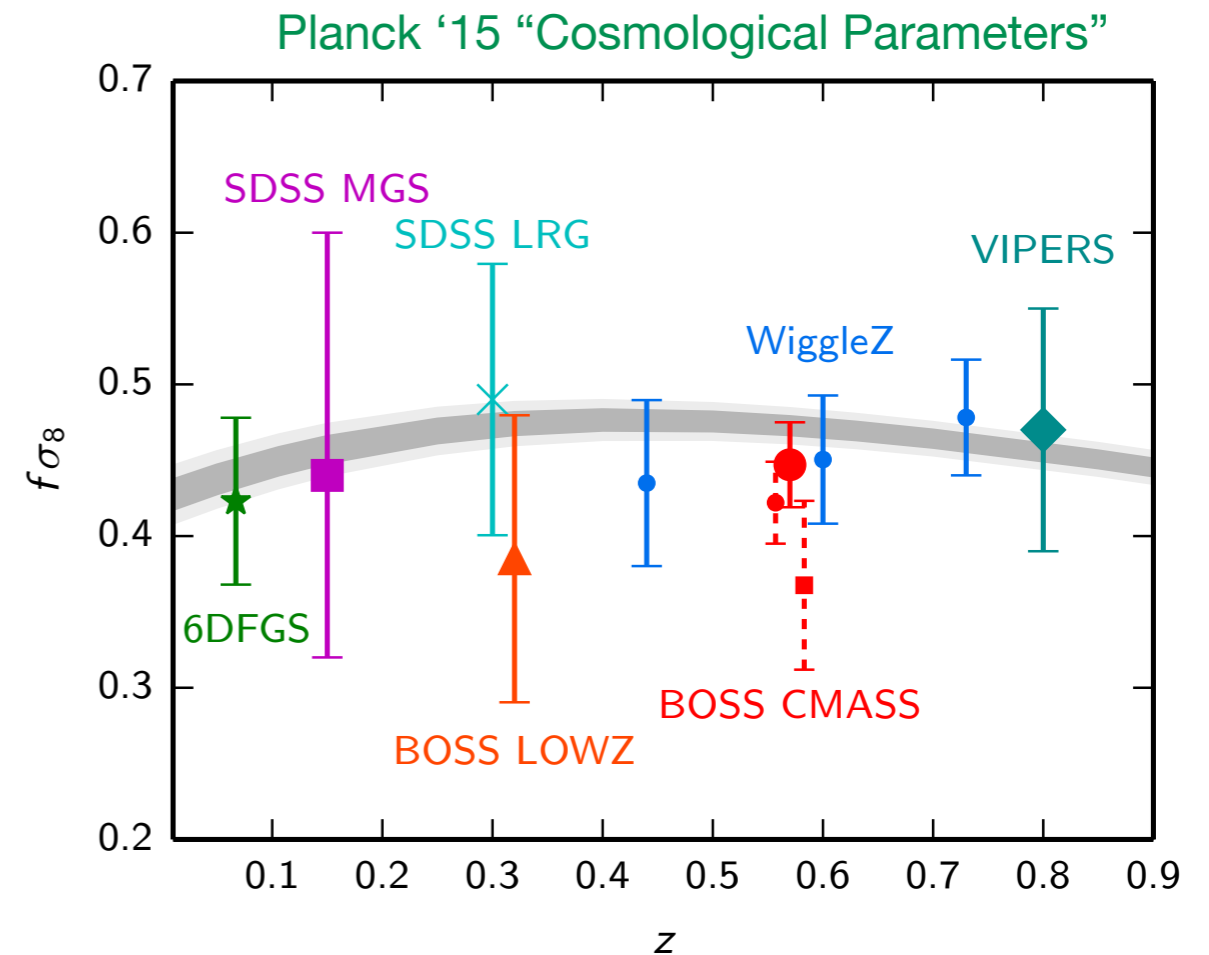
Motivations

- Observed expansion history consistent with Λ CDM ($w=-1$)
- Λ CDM expansion predicts a **unique** growth of structures, consistent with data. Any dynamics beyond Λ CDM implies **time/space deviations**
- Expected 1-2 order-of-magnitude improvement in measurement of the **growth** history of structures, over large redshift range (Euclid, LSST, etc.).



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- Given the many DE and MG models, we need a simple way to bridge theoretical modelling with observations: **unifying and effective treatment**.
- Focus on **single field** models and **large cosmological scales** (linear perturbation theory is applicable). Minimal non-redundant action.
- Analogous approach (EOM) is the Parameterized Post-Friedmann framework

e.g. Baker, Ferreira, Skordis '12

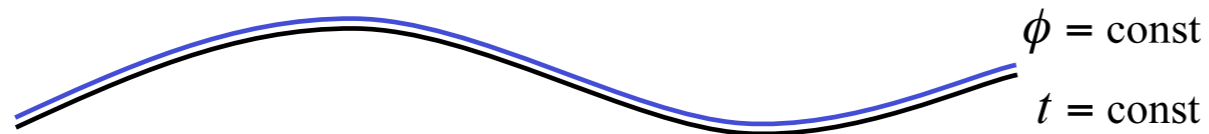
Constructing the action

1. Scalar field breaks time diffs; gravitational action preserves spatial diffs

► ADM (3+1) decomposition in unitary gauge:

Creminelli et al. '06; Cheung et al. '07

$$ds^2 = -N^2 dt^2 + h_{ij}(N^i dt + dx^i)(N^j dt + dx^j)$$



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2. Action: all terms that respect spatial diffs in the action (Jordan frame)

$$S = \int d^4x \sqrt{-g} L[t; N, K_j^i, {}^{(3)}R_j^i, \dots]$$

▶ Lapse	N	<i>time kinetic energy of scalar</i>	$\sim \dot{\phi}$
▶ Extrinsic curvature	K_{ij}	<i>time kinetic energy of metric</i>	$\sim \partial_t g_{ij}$
▶ Intrinsic 3d curvature	${}^{(3)}R_{ij}$	<i>spatial kinetic energy of metric</i>	$\sim \partial_k g_{ij}$

$$K_{ij} = \frac{1}{2N} (\dot{h}_{ij} - \nabla_i N_j - \nabla_j N_i)$$

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3. Expand at quadratic order (i.e. linear theory)

► 3-d tensors: $\delta N \equiv N - 1$, $\delta K_{ij} \equiv K_{ij} - H h_{ij}$, ${}^{(3)}R_{ij}$

$$L(N, K_j^i, R_j^i, \dots) = \bar{L} + L_N \delta N + \frac{\partial L}{\partial K_j^i} \delta K_j^i + \frac{\partial L}{\partial R_j^i} \delta R_j^i + L^{(2)} + \dots$$

$$\begin{aligned} L^{(2)} = & \frac{1}{2} L_{NN} \delta N^2 + \frac{1}{2} \frac{\partial^2 L}{\partial K_j^i \partial K_l^k} \delta K_j^i \delta K_l^k + \frac{1}{2} \frac{\partial^2 L}{\partial R_j^i \partial R_l^k} \delta R_j^i \delta R_l^k + \\ & + \frac{\partial^2 L}{\partial K_j^i \partial R_l^k} \delta K_j^i \delta R_l^k + \frac{\partial^2 L}{\partial N \partial K_j^i} \delta N \delta K_j^i + \frac{\partial^2 L}{\partial N \partial R_j^i} \delta N \delta R_j^i + \dots \end{aligned}$$

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4. Remove higher time and space derivatives and define convenient coefficients

1304.4840 with Gleyzes, Langlois, Piazza

Notation from 1404.3713 Bellini & Sawicki

$$S^{(2)} = \int d^4x a^3 \frac{M^2(t)}{2} \left[\delta K_{ij} \delta K^{ij} - \delta K^2 + \delta_2(\sqrt{h}/a^3 {}^{(3)}R) + \delta N {}^{(3)}R \right. \\ \left. + \alpha_K(t) H^2(t) \delta N^2 + 4\alpha_B(t) H(t) \delta N \delta K + \alpha_T(t) \delta_2(\sqrt{h}/a^3 R) + \alpha_H(t) \delta N {}^{(3)}R \right]$$

Building blocks of dark energy

$$S^{(2)} = \int d^4x a^3 \frac{M^2(t)}{2} \left[\delta K_{ij} \delta K^{ij} - \delta K^2 + \delta_2(\sqrt{h}/a^3 {}^{(3)}R) + \delta N {}^{(3)}R \right. \\ \left. + \alpha_K(t) H^2(t) \delta N^2 + 4\alpha_B(t) H(t) \delta N \delta K + \alpha_T(t) \delta_2(\sqrt{h}/a^3 R) + \alpha_H(t) \delta N {}^{(3)}R \right]$$

* General Relativity (LCDM)

	α_K	α_B	α_M	α_T	α_H

Building blocks of dark energy

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* Standard kinetic term: quintessence, *k*-essence $P(\phi, (\partial\phi)^2)$

α_K parametrizes **kineticity** of dark energy $\sim (1+w) \Omega_{DE} / c_s^2$

kineticity					
	α_K	α_B	α_M	α_T	α_H
quintessence, k-essence	✓				

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* Kinetic braiding: DGP, KGB $K(\phi, (\partial\phi)^2) \square \phi$

α_B parametrizes kinetic mixing with gravity (braiding)

	kineticity α_K	kinetic braiding α_B	α_M	α_T	α_H
quintessence, k-essence	✓				
DGP, kinetic braiding	✓	✓			

Building blocks of dark energy

$$S^{(2)} = \int d^4x a^3 \frac{M^2(t)}{2} \left[\delta K_{ij} \delta K^{ij} - \delta K^2 + \delta_2(\sqrt{h}/a^3 {}^{(3)}R) + \delta N {}^{(3)}R \right. \\ \left. + \alpha_K(t) H^2(t) \delta N^2 + 4\alpha_B(t) H(t) \delta N \delta K + \alpha_T(t) \delta_2(\sqrt{h}/a^3 R) + \alpha_H(t) \delta N {}^{(3)}R \right]$$

* **Non-minimal couplings: Brans-Dicke, $f(R)$** $f(\phi)R$, $f(R)$, $f(G)$

$\alpha_M = \frac{d \ln M^2}{H dt}$ parametrizes **non-minimal coupling** to R (ex: $\alpha_M = -2\alpha_B$ in $f(R)$)

	kineticity α_K	kinetic braiding α_B	non-minimal coupling α_M	α_T	α_H
quintessence, k-essence	✓				
DGP, kinetic braiding	✓	✓			
Brans-Dicke, $f(R)$	✓	✓	✓		

Building blocks of dark energy

$$S^{(2)} = \int d^4x a^3 \frac{M^2(t)}{2} \left[\delta K_{ij} \delta K^{ij} - \delta K^2 + \delta_2(\sqrt{h}/a^3 {}^{(3)}R) + \delta N {}^{(3)}R \right. \\ \left. + \alpha_K(t) H^2(t) \delta N^2 + 4\alpha_B(t) H(t) \delta N \delta K + \alpha_T(t) \delta_2(\sqrt{h}/a^3 R) + \alpha_H(t) \delta N {}^{(3)}R \right]$$

* Enhanced tensor sound speed: all Horndeski theories

α_T parametrizes deviation from **tensor sound-speed** = c

	kineticity α_K	kinetic braiding α_B	non-minimal coupling α_M	tensor sound- speed α_T	α_H
quintessence, k-essence	✓				
DGP, kinetic braiding	✓	✓			
Brans-Dicke, f(R)	✓	✓	✓		
Horndeski	✓	✓	✓	✓	

Building blocks of dark energy

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* Kinetic mixing with matter: beyond Horndeski theories

α_H parametrizes **extensions of Horndeski theories**

	kineticity α_K	kinetic braiding α_B	non-minimal coupling α_M	tensor sound-speed α_T	kinetic mixing with matter α_H
quintessence, k-essence	✓				
DGP, kinetic braiding	✓	✓			
Brans-Dicke, f(R)	✓	✓	✓		
Horndeski	✓	✓	✓	✓	
Beyond Horndeski	✓	✓	✓	✓	✓

Building blocks of dark energy

$$S^{(2)} = \int d^4x a^3 \frac{M^2(t)}{2} \left[\delta K_{ij} \delta K^{ij} - \delta K^2 + \delta_2(\sqrt{h}/a^3 ({}^3)R) + \delta N ({}^3)R \right. \\ \left. + \alpha_K(t) H^2(t) \delta N^2 + 4\alpha_B(t) H(t) \delta N \delta K + \alpha_T(t) \delta_2(\sqrt{h}/a^3 R) + \alpha_H(t) \delta N ({}^3)R \right]$$

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α_H parametrizes **extensions of Horndeski theories**

► Stability conditions:

	Scalar	Tensor
No ghosts	$\alpha_K + 6\alpha_B^2 > 0$	$M^2 > 0$
No gradient instability	$c_s^2(\alpha_i) \geq 0$	$\alpha_T \geq -1$

Theoretical restriction on the parameter space

Universal couplings

► Horndeski case ($\alpha_H = 0$):

$$S_{\text{gravity}} = \int d^4x \mathcal{L}_g(g_{\mu\nu}; \alpha_K, \alpha_B, \alpha_M, \alpha_T)$$

► **Equivalence Principle.** All species are coupled to the same metric:

For each species:
$$S_m = \int d^4x \sqrt{-g} \mathcal{L}(g_{\mu\nu}, \psi_m)$$

Non-universal couplings

1504.05481 with Gleyzes, Langlois, Mancarella

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► **Equivalence Principle.** Species are coupled to different metrics:

For each species:

$$S_m = \int d^4x \sqrt{-g} \mathcal{L}(\tilde{g}_{\mu\nu}, \psi_m)$$

$$\tilde{g}_{\mu\nu} = C(\phi)g_{\mu\nu} + D(\phi)\partial_\mu\phi\partial_\nu\phi$$

Two new parameters
per species:

$$\alpha_C \equiv \frac{1}{2} \frac{d \ln C}{d \ln a} \qquad \alpha_D \equiv \frac{D}{C - D}$$

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▶ Structure of Horndeski invariant under the above metric transformation

Parameter-space rotation

1504.05481 with Gleyzes, Langlois, Mancarella

► Total of $4 + 2 N_S$ parameters:

$$S_{\text{gravity}} = \int d^4x \mathcal{L}_g(g_{\mu\nu}; \alpha_K, \alpha_B, \alpha_M, \alpha_T)$$

$$S_{\text{matter}} = \sum_I^{N_S} \int d^4x \sqrt{-g} \mathcal{L}_I(g_{\mu\nu}; \alpha_{C,I}, \alpha_{D,I}; \psi_I)$$

► With a rotation in parameter space, $\tilde{\alpha}_i = \mathcal{F}_i(\alpha_j)$, we can choose a base where one of the species is minimally coupled: $4 + 2 N_S - 2 = 2(N_S + 1)$

Parameter-space rotation

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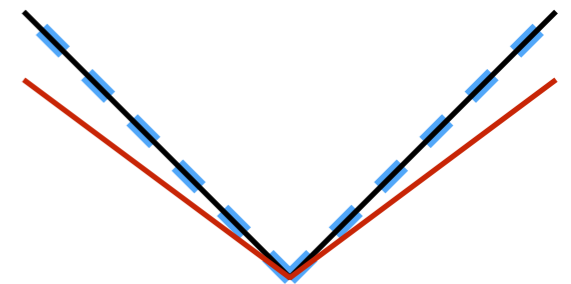
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- ▶ Ghost and gradient stability conditions are invariant under rotation in par. space

- ▶ Observables invariant. Example:

$$\frac{\tilde{c}_I^2}{\tilde{c}_J^2} = \frac{c_I^2}{c_J^2}$$



- ▶ Inflation: no matter ($N_s = 0$). We have 2 independent parameters, ex. α_K and α_B

$$\delta N^2, \quad \delta N \delta K$$

1407.8439 with Creminelli, Gleyzes, Noreña

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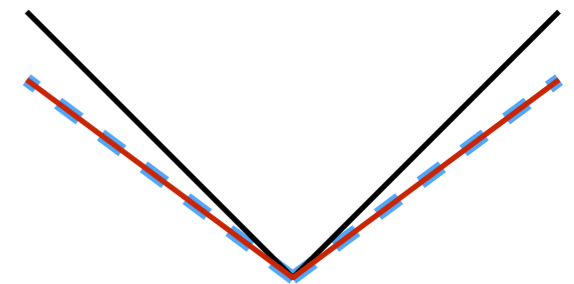
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1407.8439 with Creminelli, Gleyzes, Noreña

Constraining dark energy

- Can we constrain these parameters?

Constraining dark energy

1509.02191 with Gleyzes, Langlois, Mancarella
(see also Baker et al. '13 & Leonard et al. '15,
Piazza et al. '13, Perenon et al. '15)

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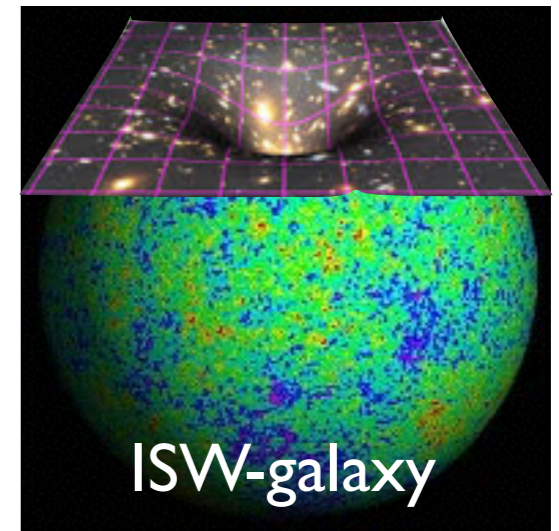
- Undo unitary gauge: $t \rightarrow t + \pi(t, \vec{x})$

- Newtonian gauge (scalar flucts): $dt^2 = -(1 + 2\Phi)dt^2 + a^2(t)(1 - 2\Psi)d\vec{x}^2$

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- Fisher matrix forecasts, Euclid-like specifications.



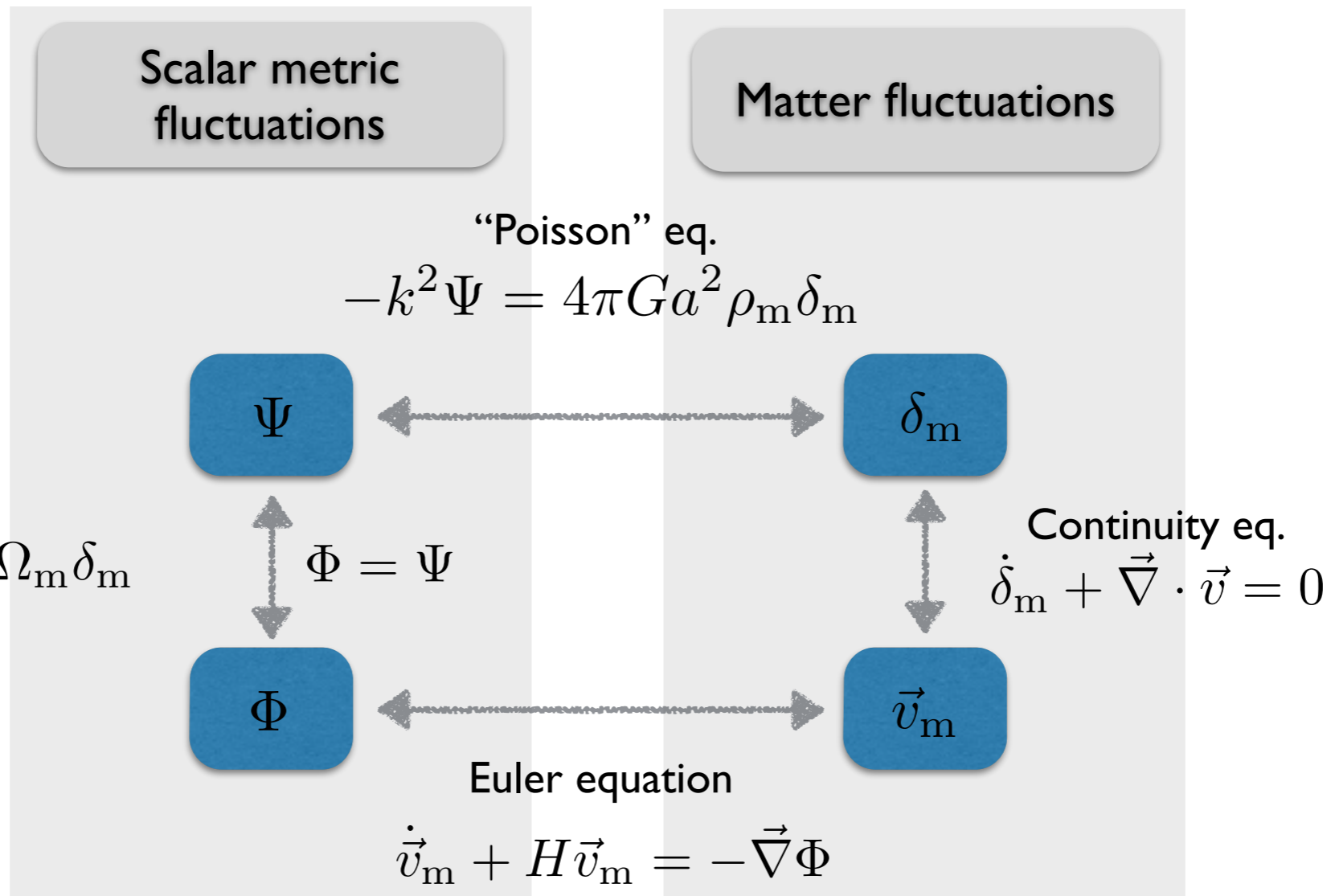
$$\delta_{g,s}(\vec{k}) = (b_g^2 + \mu^2 f_{\text{eff}})\delta_m(t, k)$$

$$\Phi + \Psi$$

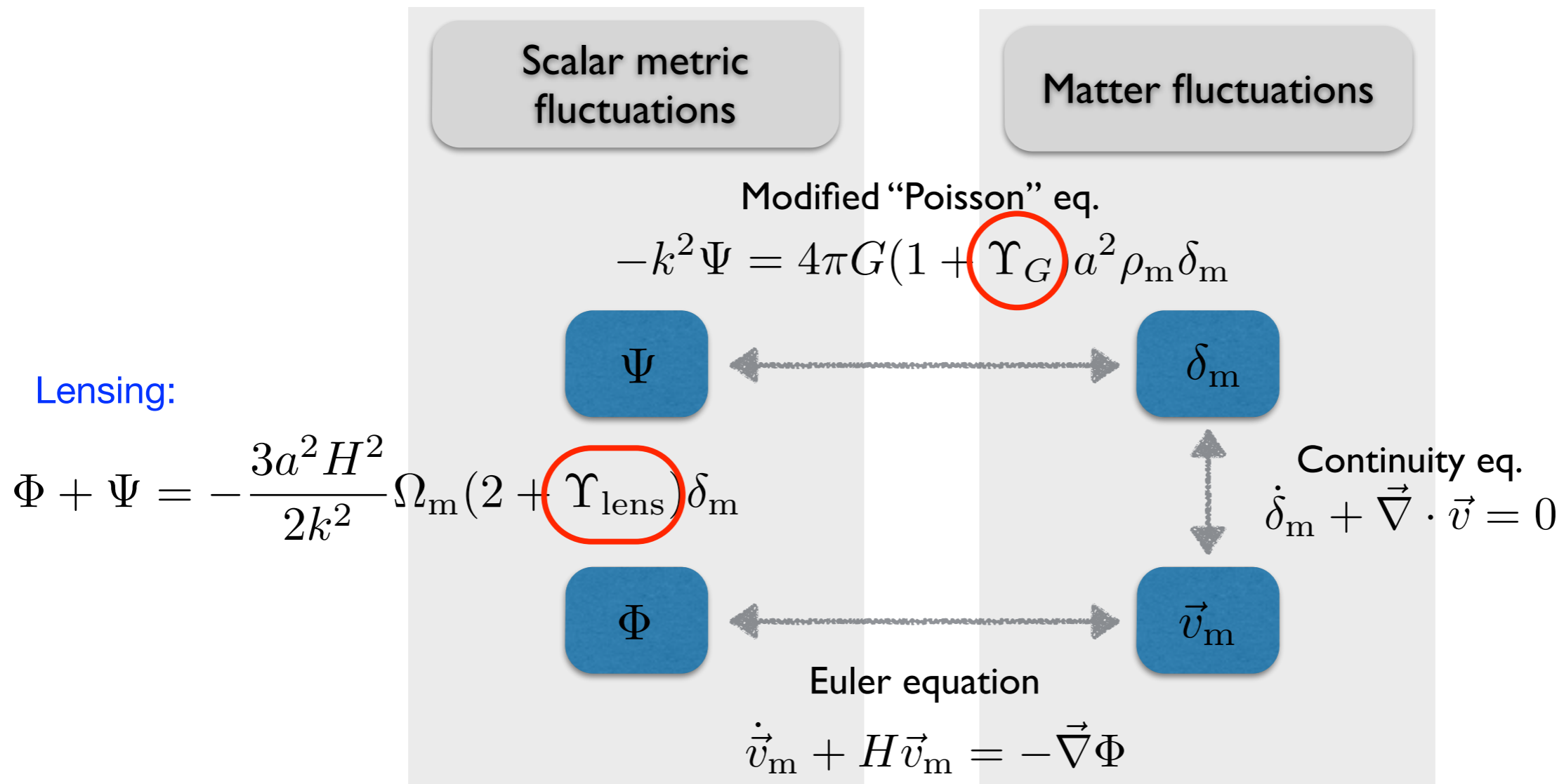
$$\dot{\Phi} + \dot{\Psi}$$

- Quasi-static approximations — valid on scales $k \gg aHc_s^{-1}$. Sawicki, Bellini '15
E.g., for surveys such as Euclid $c_s \gtrsim 0.1$.

Standard case



Modified gravity

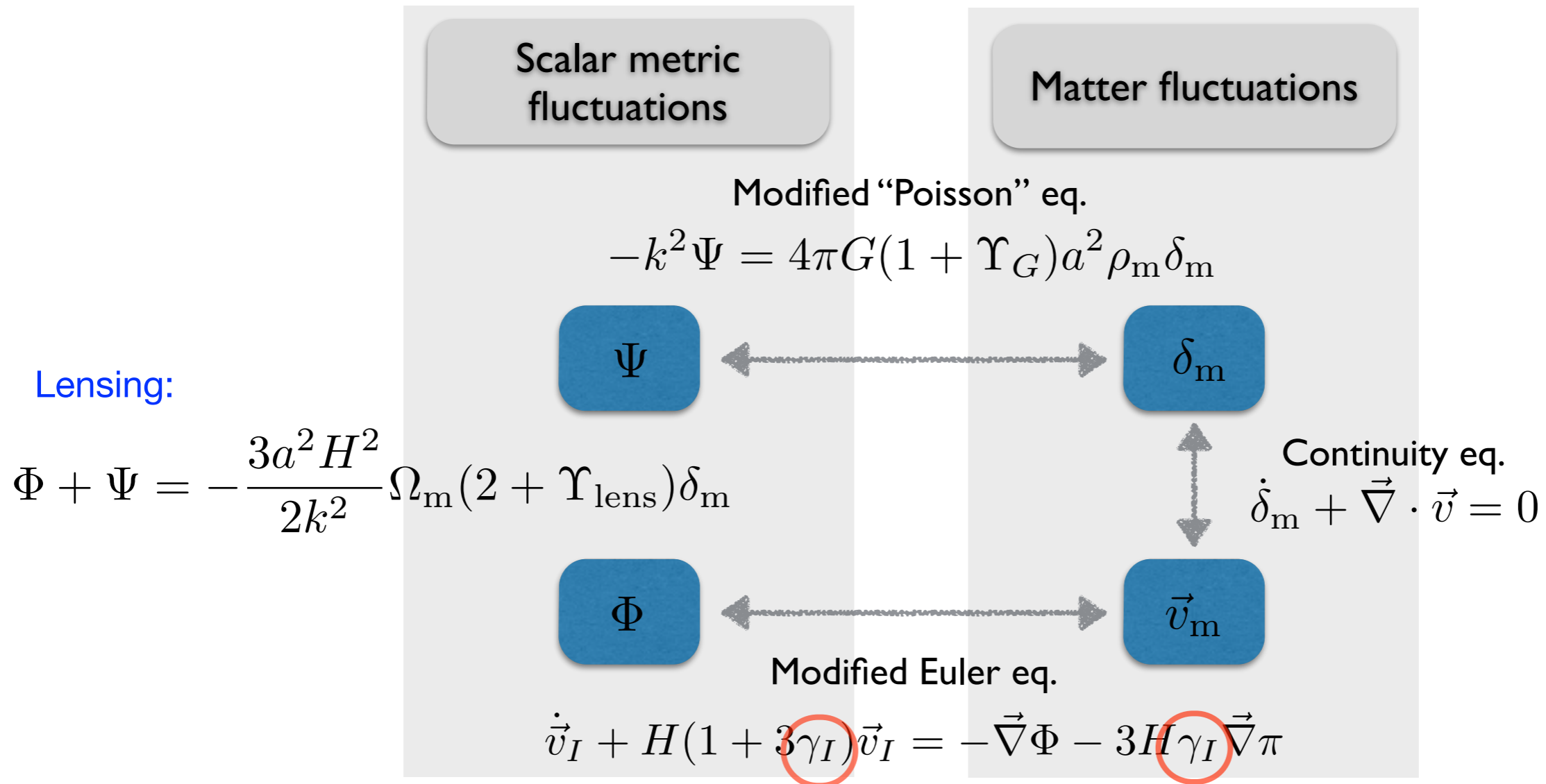


$$\Upsilon_G = \Upsilon_G(\alpha_B, \alpha_M, \alpha_T), \quad \Upsilon_{\text{lens}} = \Upsilon_{\text{lens}}(\alpha_B, \alpha_M, \alpha_T)$$

Modifications of gravity

► Time dependence of coefficients: $\alpha_I(t) = \alpha_{I,0} \frac{1 - \Omega_m(t)}{1 - \Omega_{m,0}}$

+ Nonminimal coupling



$$\Upsilon_G = \Upsilon_G(\alpha_B, \alpha_M, \alpha_T), \quad \Upsilon_{\text{lens}} = \Upsilon_{\text{lens}}(\alpha_B, \alpha_M, \alpha_T), \quad \beta_{\gamma, I} \propto \gamma_I(\alpha_{C_I}, \alpha_{D_I})$$

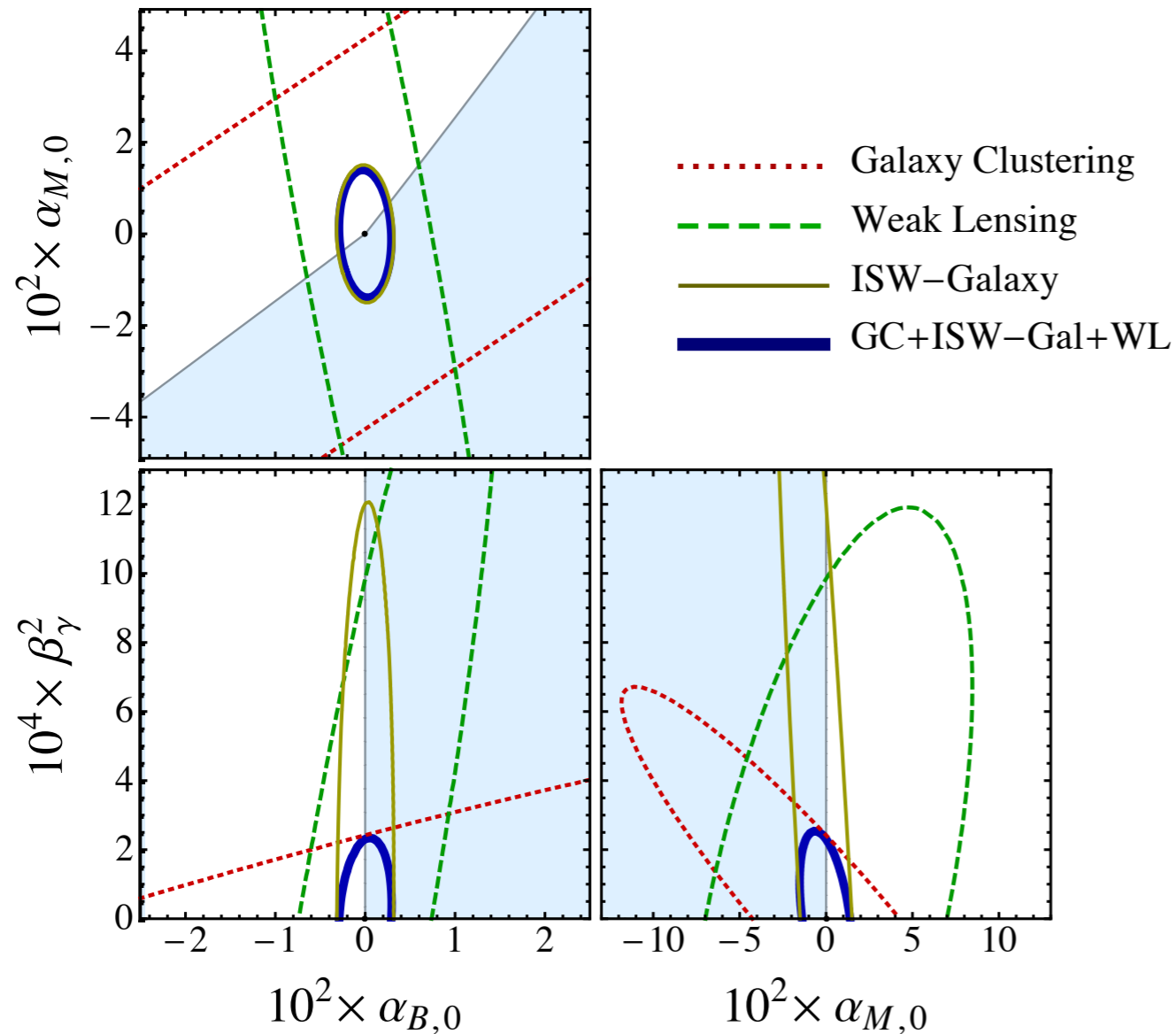
Modifications of gravity

non-minimal coupling

► Time dependence of coefficients: $\alpha_I(t) = \alpha_{I,0} \frac{1 - \Omega_m(t)}{1 - \Omega_{m,0}}$, $\beta_\gamma = \beta_{\gamma,c} = \text{const}$
 $\beta_{\gamma,b} = 0$

Baryons + coupled CDM

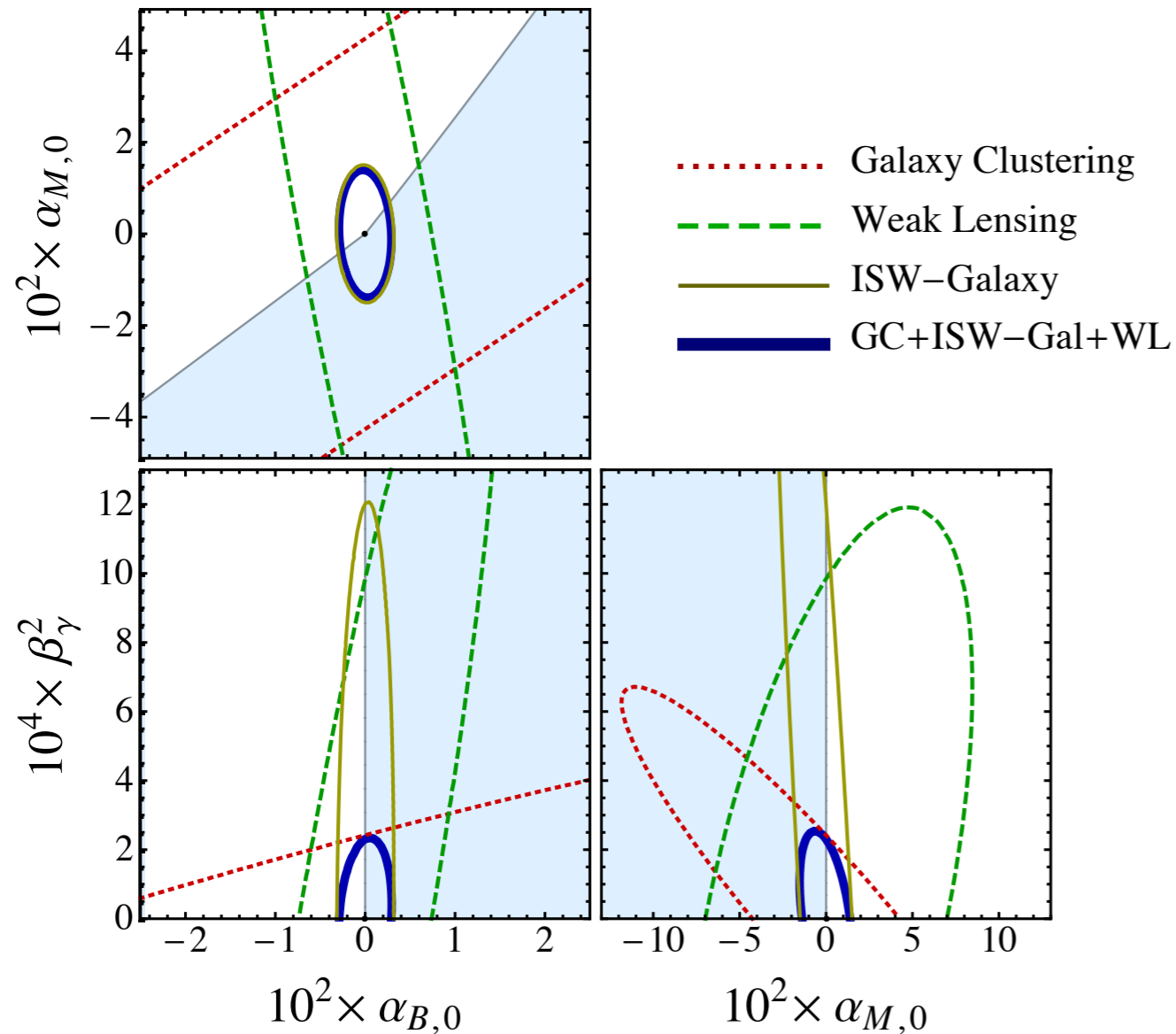
Obs.	$10^3 \times \sigma(\alpha_{B,0})$	$10^3 \times \sigma(\alpha_{M,0})$	$10^3 \times \sigma(\alpha_{T,0})$	$10^4 \times \sigma(\beta_\gamma^2)$
GC	18.6	24.5	–	1.4
WL	4.2	40.1	–	5.7
ISW-g	1.8	8.6	–	6.9
Comb	1.7	8.0	–	1.3



Fiducial: LCDM, no interactions

Baryons + coupled CDM

Obs.	$10^3 \times \sigma(\alpha_{B,0})$	$10^3 \times \sigma(\alpha_{M,0})$	$10^3 \times \sigma(\alpha_{T,0})$	$10^4 \times \sigma(\beta_\gamma^2)$
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Degeneracy:

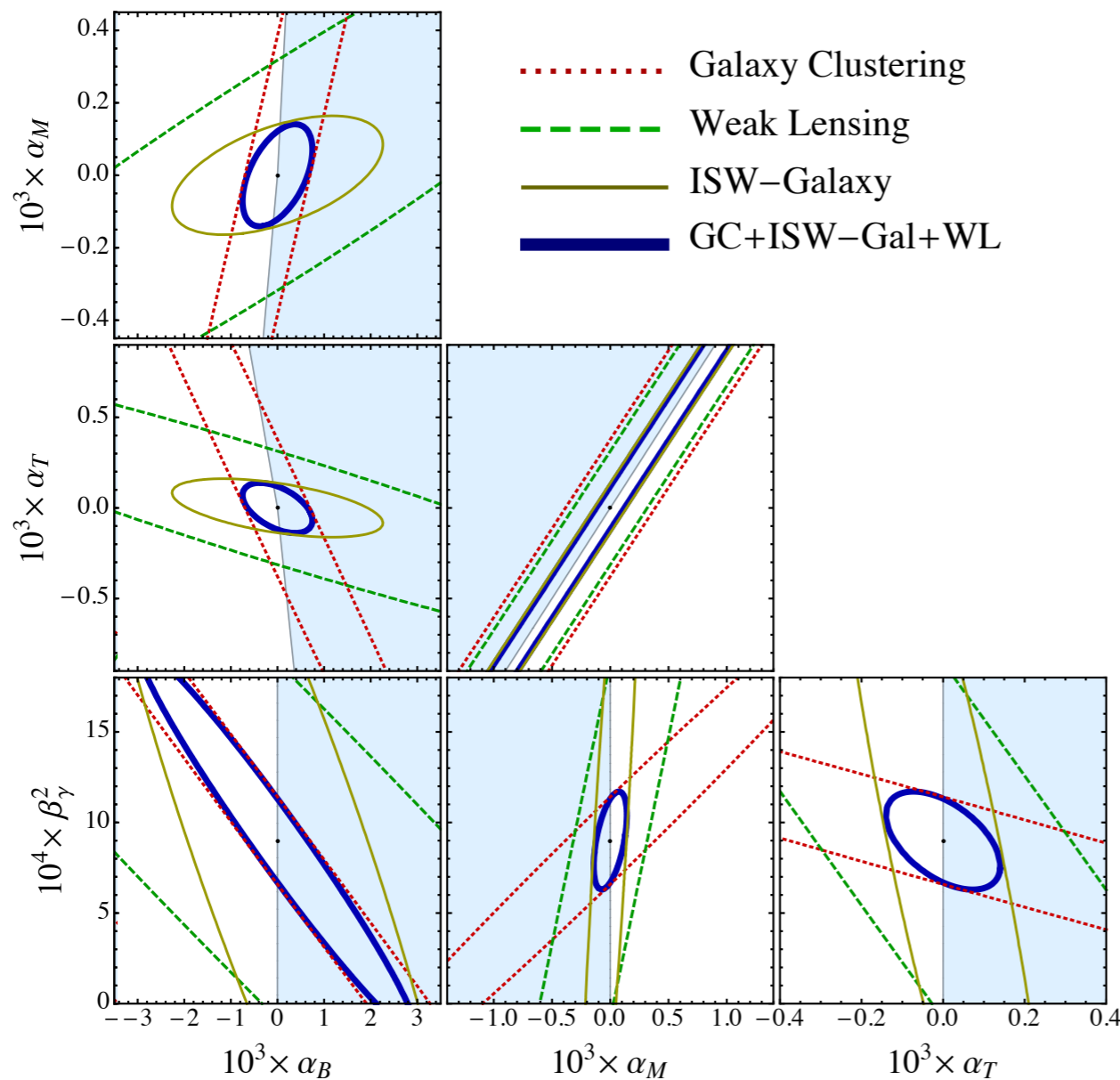
$$\Upsilon_G \simeq \Upsilon_{\text{lens}} \simeq \alpha_M$$

$$\Upsilon_G \simeq \Upsilon_{\text{lens}}/2 \simeq -\frac{2}{2 + 3\Omega_m} \alpha_B$$

lensing degeneracy explained by taking into account of background evolution

Baryons + coupled CDM

Obs.	$10^3 \times \sigma(\alpha_{B,0})$	$10^3 \times \sigma(\alpha_{M,0})$	$10^3 \times \sigma(\alpha_{T,0})$	$10^4 \times \sigma(\beta_\gamma^2)$
GC	0.40	0.22	0.22	1.4
WL	2.14	0.18	0.18	5.6
ISW-g	1.11	0.08	0.08	7.5
Comb	0.37	0.07	0.07	1.3



Fiducial: LCDM, interacting CDM

$$\beta_\gamma = -0.03$$

Conclusions

- * **General** description of linear perturbations in scalar-tensor theories of gravity, with non-universal couplings
- * **Efficient** (minimal) and systematic way to parametrize observations on large scales (linear regime) and address **stability**
- * **Forecasts:** unmarginalized errors $\sim 10^{-3}$ on parameters describing modifications of gravity. Degeneracies and dependence on the fiducial model.
- * **Future:** Relax assumptions (beyond linear regime, more degrees of freedom, etc...), explore phenomenology and forecasts beyond the quasi-static approximation.

