Axial-vector contributions to CEvNS rates

J.L.Newstead
Arizona State University

Predicting CEvNS rates

- We want a consistent formalism for calculating the event rates of CEvNS experiments across different source (nu nubar)/detector configurations (isotopes)
- I will revise the formalism of semi-leptonic electroweak nuclear scattering developed by Donnelly & Peccei (1978) and Haxton and Donnelly (1979)
- This talk will draw heavily from the derivations in Walecka's "Theoretical nuclear and subnuclear physics" (1995) and Kim & Giunti "Fundamentals of neutrino physics and astrophysics" (2007)

The fundamentals

- The effective NC interaction lagrangian, where the currents sum over all fermions with the V-A structure:

$$\mathscr{L}_{\text{eff}}^{(\text{NC})} = -\frac{G_{\text{F}}}{\sqrt{2}}\, j_Z^{\mu} j_{Z\mu}$$

- We want the matrix elements for leptonic-hadronic currents:

$$\langle f|\hat{H}_W|i\rangle = \frac{G_F}{\sqrt{2}} \int d^3x \langle f| j_{\mu}^{lep} \hat{\mathcal{J}}^{\mu}(\vec{x})|i\rangle$$

Nucleon currents and their form factors

- It is useful to work with proton and neutron currents, higher order effects can still be included but likely wont be necessary
- We want to write the hadronic currents using the quark currents:

$$J_Z^\mu = v_Z^\mu - a_Z^\mu$$

$$v_Z^\mu = \bar{Q}_L \gamma^\mu \tau_3 Q_L - 2\sin^2(\theta_w) j_\gamma^\mu - \frac{1}{2}\sum_{q=s,c,b,t} \bar{q}\gamma^\mu q$$

$$a_Z^\mu = \bar{Q}_L \gamma^\mu \gamma^5 \tau_3 Q_L - \frac{1}{2}\sum_{q=s,c,b,t} \bar{q}\gamma^\mu \gamma^5$$

- The nucleon current is then:

$$\begin{aligned}\langle N|J_Z^\mu|N\rangle &= \langle N|v_Z^\mu - a_Z^\mu|N\rangle \\ &= \bar{u}_N \left[\gamma^\mu F_1^{Z(N)}(q^2) - \gamma^\mu\gamma^5 G_A^{Z(N)}(q^2)\right] u_N\end{aligned}$$

- With form factors:

$$F_1^{Z(N)}(q^2) = I_3^N(F_1^p - F_1^n) - 2\sin^2(\theta_w)F_1^N - \frac{1}{2}F_1^{s(N)}$$

$$G_A^{Z(N)}(q^2) = I_3^N(G_A^p - G_A^n) - \frac{1}{2}G_A^{s(N)}$$

Nucleon currents and their form factors

- In the low-q limit $F_1^{(N)}$ is electric charge (no isoscalar contributions)

$$F_1^{Z(N)}(q^2) = I_3^N(F_1^p - F_1^n) - 2\sin^2(\theta_w)F_1^N - \frac{1}{2}F_1^{s(N)}$$
$$G_A^{Z(N)}(q^2) = I_3^N(G_A^p - G_A^n) - \frac{1}{2}G_A^{s(N)}$$

- The form factors become:

$$F_1^{Z(N)}(q^2 \to 0) = I_3^N - 2\sin^2(\theta_w)Q^N \equiv g_V^N$$
$$G_A^{Z(N)}(q^2 \to 0) = I_3^N g_A - \frac{1}{2}g_A^{s(N)} \equiv g_A^N$$

- Our nucleon currents are thus (in low-q limit):

$$\mathcal{J}_Z^\mu = \bar{N}\gamma^\mu\left(g_V^N - g_A^N\gamma^5\right)N$$

- Where the charges are:

$$g_V^p = 0.015 \quad g_V^n = -0.51$$
$$g_A^p = 0.63 \quad g_A^n = -0.59$$

Hadronic matrix element

$$\langle f|\hat{H}_W|i\rangle = \frac{G_F}{\sqrt{2}} \int d^3x \langle f| j_\mu^{lep} \hat{\mathcal{J}}^\mu(\vec{x})|i\rangle$$

- The hadronic current can be written as a sum over nucleons:

$$\hat{\mathcal{J}}^\mu(\vec{x}) = \sum_{i=1}^{A} \mathcal{J}_Z^\mu(i)\, \delta^3(\vec{x} - \vec{x}_i)$$

- For $q << m_N$ we can write this as:

$$\mathcal{J}_Z^0 \approx g_V^N \xi^\dagger \xi$$
$$\mathcal{J}_Z^i \approx -g_A^N \xi^\dagger \sigma^i \xi$$

$$\hat{\mathcal{J}}^0(\vec{x}) \approx \sum_{i=1}^{A} g_V^N(i)\, \delta^3(\vec{x} - \vec{x}_i)$$

$$\hat{\boldsymbol{\mathcal{J}}}(\vec{x}) \approx \sum_{i=1}^{A} -g_A^N(i) \boldsymbol{\sigma}(i)\, \delta^3(\vec{x} - \vec{x}_i)$$

Expanding the lepton currents

- We are only interested in the neutrino part of the leptonic current

$$\langle f|\hat{H}_W|i\rangle = \frac{G_F}{\sqrt{2}}\int d^3x \langle f|j_\mu^{lep}\hat{\mathcal{J}}^\mu(\vec{x})|i\rangle$$
$$= \frac{G_F}{\sqrt{2}}\int d^3x\, e^{-i\vec{q}\cdot\vec{x}}\left(l_0\mathcal{J}^0(\vec{x}) - \vec{l}\cdot\boldsymbol{\mathcal{J}}(\vec{x})\right)$$

Where:

$$j_\mu^{lep}(\vec{x}) = e^{i\vec{q}.\vec{x}}l_\mu$$

$$l_\mu = i\bar{\nu}\gamma_\mu(1-\gamma^5)\frac{\tau_3}{2}\nu$$

- To evaluate these matrix elements we will perform a spherical decomposition and multipole expansion

Spherical decomposition

$$= \frac{G_F}{\sqrt{2}} \int d^3x \, e^{-i\vec{q}\cdot\vec{x}} \, \left(l_0 \mathcal{J}^0(\vec{x}) - \vec{l} \cdot \boldsymbol{\mathcal{J}}(\vec{x}) \right)$$

$$\vec{l} \, e^{i\vec{q}.\vec{x}} = \sum_{\lambda} (-1)^{\lambda} \, l_{-\lambda} \hat{e}_{\lambda} \, e^{-i\vec{q}.\vec{x}}$$

$$= \frac{G_F}{\sqrt{2}} \int d^3x \, \left[e^{-i\vec{q}\cdot\vec{x}} l_0 \mathcal{J}^0(\vec{x}) - e^{i\vec{q}\cdot\vec{x}} \left(l_z \hat{e}_z - \sum_{\lambda=\pm 1} l_{-\lambda} \hat{e}_{\lambda} \right) \cdot \boldsymbol{\mathcal{J}}(\vec{x}) \right]$$

Multipole expansion

Define the irruducible tensor operators:

$$\hat{\mathcal{M}}_{JM}(q) \equiv \int d^3x \, [j_J(qx)Y_{JM}(\Omega_x)] \, \hat{\mathcal{J}}^0(\vec{x})$$

$$\hat{\mathcal{T}}^{el}_{JM}(q) \equiv \frac{1}{|\vec{q}|} \int d^3x \left[\nabla \times (j_J(qx)\vec{Y}^M_{JJ}(\Omega_x) \right] \cdot \hat{\boldsymbol{\mathcal{J}}}(\vec{x})$$

The matrix element is then:

$$\langle f|\hat{H}_Z|i\rangle = \frac{G_F}{\sqrt{2}} \langle f| \left(\sum_{J\geq 0} \sqrt{4\pi(2J+1)}(-i^J) \, l_0 \hat{\mathcal{M}}_{J0}(q) + \sum_{J\geq 1} \sqrt{2\pi(2J+1)}(-i^J) \sum_{\lambda\pm 1} l_\lambda \, \lambda \hat{\mathcal{T}}^{el}_{J-\lambda}(q) \right) |i\rangle$$

The cross section

Summing over the spins and averaging over nuclear spins (only):

$$\frac{d\sigma}{d\Omega} = \frac{G_F^2 E_\nu^2}{4\pi^2} \frac{4\pi}{2J_i + 1} \left(\langle l_0 \rangle \langle l_0 \rangle^* \sum_{J=0}^{\infty} |\langle J_i || \hat{\mathcal{M}}_J || J_i \rangle|^2 + \frac{1}{2} \langle \vec{l} \rangle \cdot \langle \vec{l} \rangle^* \sum_{J=1}^{\infty} |\langle J_i || \hat{\mathcal{T}}_J^{\mathrm{el}} || J_i \rangle|^2 \right)$$

Evaluating the neutrino traces and putting this in terms of the recoil energy:

$$\frac{d\sigma}{dE_R} = \frac{G_F^2 m_T}{\pi} \frac{4\pi}{2j+1} \left[\left(1 - \frac{m_T E_R}{2E_\nu^2}\right) \sum_{J=0,2..}^{\infty} |\langle j || \hat{\mathcal{M}}_J || j \rangle|^2 + \frac{1}{2} \left(1 + \frac{E_R m_T}{2E_\nu^2}\right) \sum_{J=1,3..}^{\infty} |\langle j || \hat{\mathcal{T}}_J^{\mathrm{el}} || j \rangle|^2 \right]$$

(leading order in Er/Enu)

Form factors

Define form factors as:

$$F_M^{(N,N')}(q^2) = \frac{4\pi}{2j+1} \sum_{J=0,2,\ldots} \langle j||M_J^{(N)}||j\rangle\langle j||M_J^{(N')}||j\rangle$$

$$F_{\Sigma'}^{(N,N')}(q^2) = \frac{4\pi}{2j+1} \sum_{J=1,3,\ldots} \langle j||\Sigma_J'^{(N)}||j\rangle\langle j||\Sigma_J'^{(N')}||j\rangle$$

Then the cross section can be written:

$$\frac{d\sigma}{dE_R} = \frac{G_F^2 m_T}{\pi}\left[\left(1 - \frac{m_T E_R}{2E_\nu^2}\right)\left({g_V^n}^2 F_M^{nn}(q^2) + 2g_V^p g_V^n F_M^{pn}(q^2) + {g_V^p}^2 F_M^{pp}(q^2)\right)\right.$$
$$\left. + \frac{1}{2}\left(1 + \frac{m_T E_R}{2E_\nu^2}\right)\left({g_A^n}^2 F_{\Sigma'}^{nn}(q^2) + 2g_A^p g_A^n F_{\Sigma'}^{pn}(q^2) + {g_A^p}^2 F_{\Sigma'}^{pp}(q^2)\right)\right]$$

Calculating the form factor

- To calculate a form factor you have to choose a basis
- Here I use the results of Haxton et al. who evaluate them in the harmonic oscillator basis
- Defining the single particle operators as:

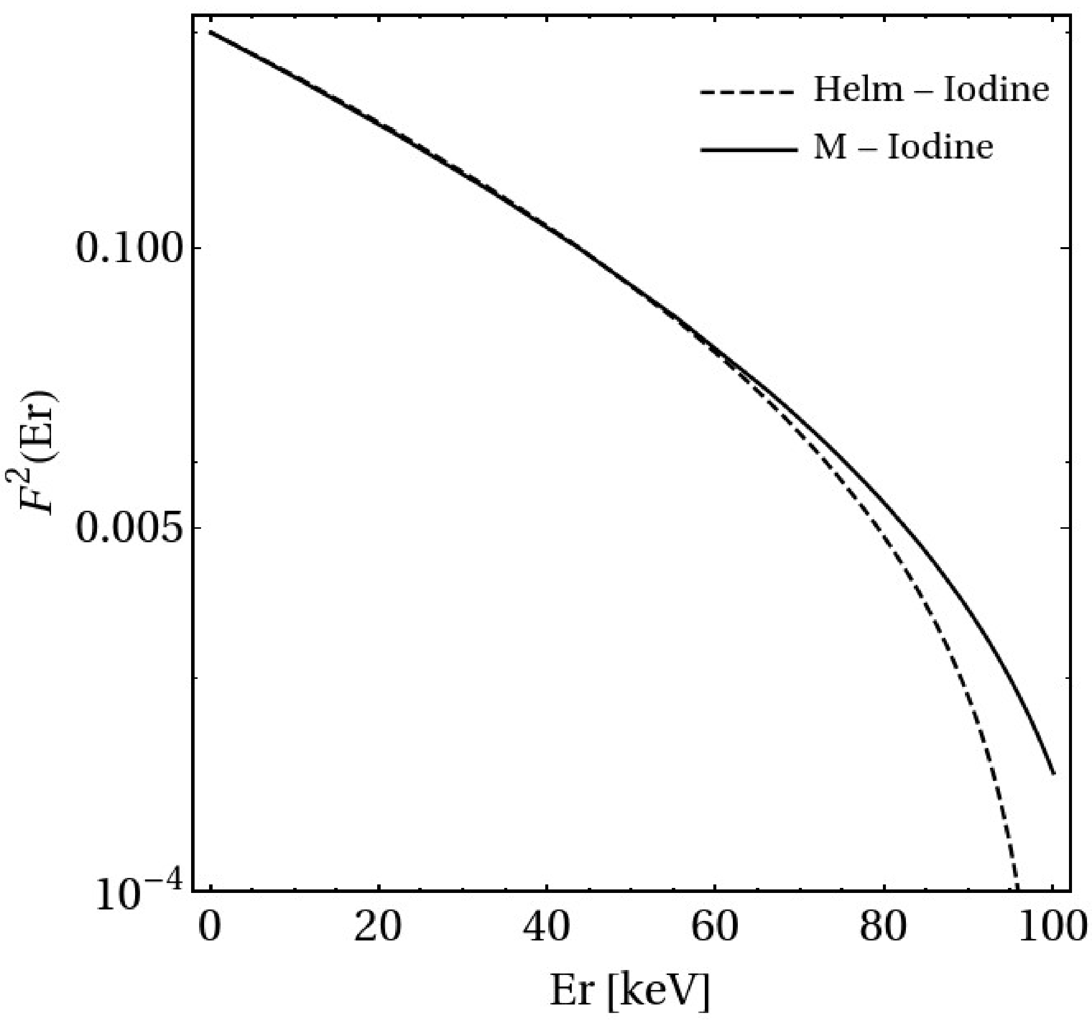
$$M_{JM}(q\vec{x}_i) \equiv j_J(qx_i)Y_{JM}(\Omega_{x_i})$$

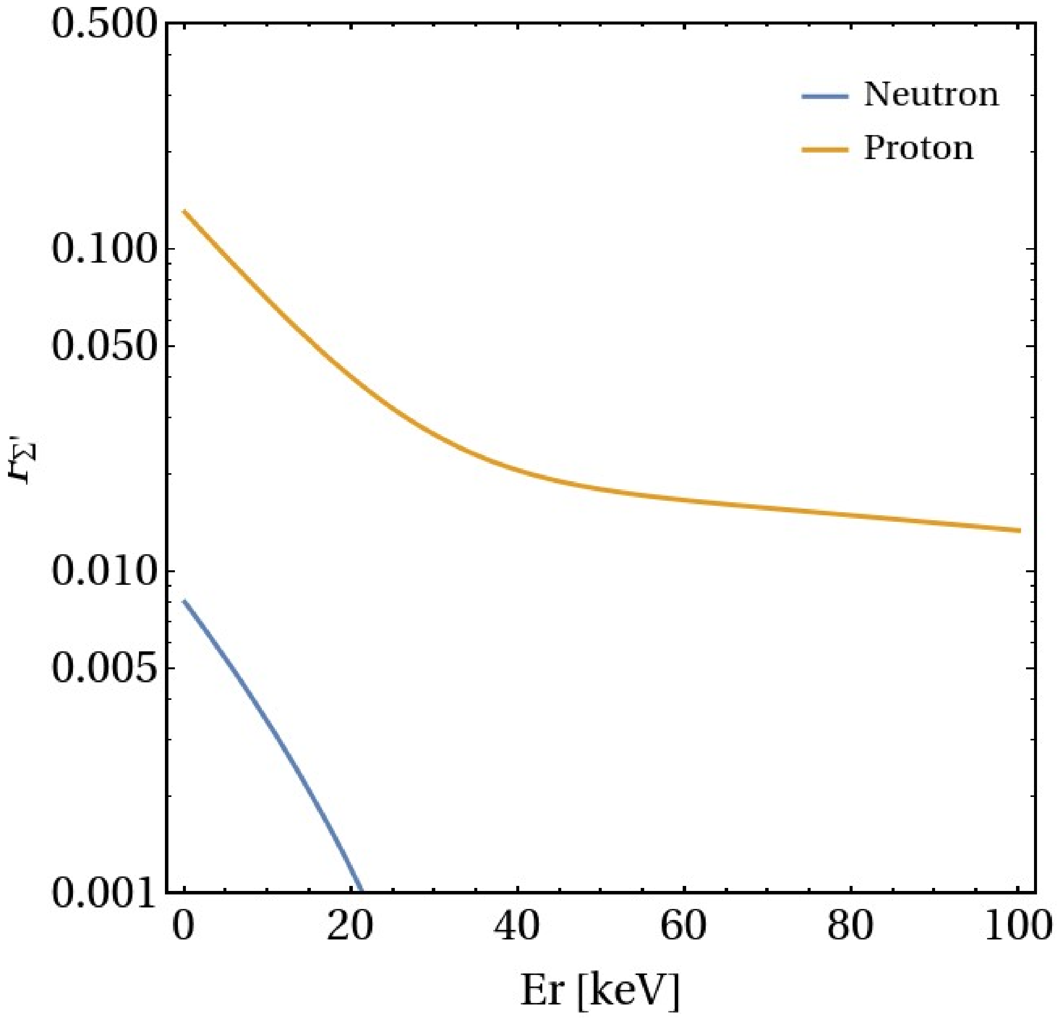
$$\Sigma'_{JM}(q\vec{x}_i) \equiv -i\sum_{i=1}^{A}\left\{\frac{1}{q}\vec{\nabla}_i \times \vec{M}^{M}_{JJ}(q\vec{x}_i)\right\}\cdot\vec{\sigma}(i)$$

$$\hat{\mathcal{M}}_{JM}(q) = \sum_{i=1}^{A} g_V^N(i)\, M_{JM}(q\vec{x}_i) \qquad \hat{\mathcal{T}}^{el}_{JM}(q) = i\sum_{i=1}^{A} g_A^N(i)\, \Sigma'_{JM}(q\vec{x}_i)$$

$$\langle f|M_{JM}(q\vec{x}_i)|i\rangle = \int d^3x_i \psi_i^*(\vec{x}_i)\psi_i(\vec{x}_i)\, j_J(qx)Y_{JM}(\Omega_{x_i})$$

Form factors

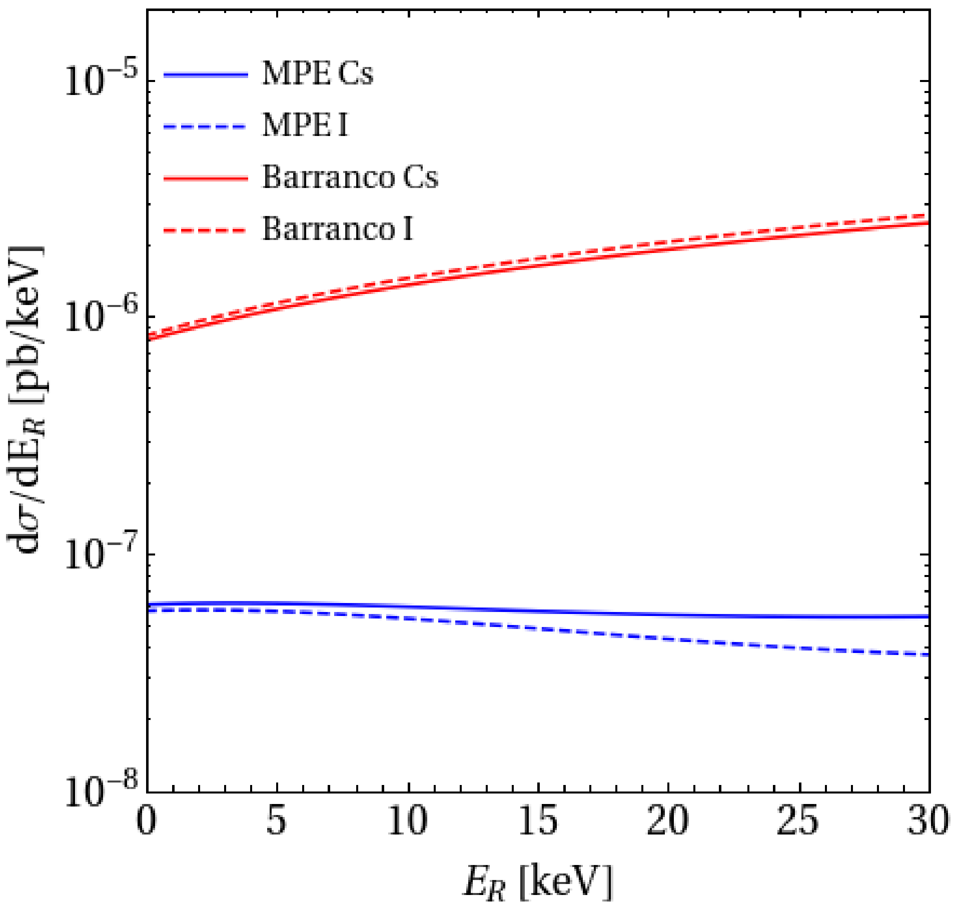




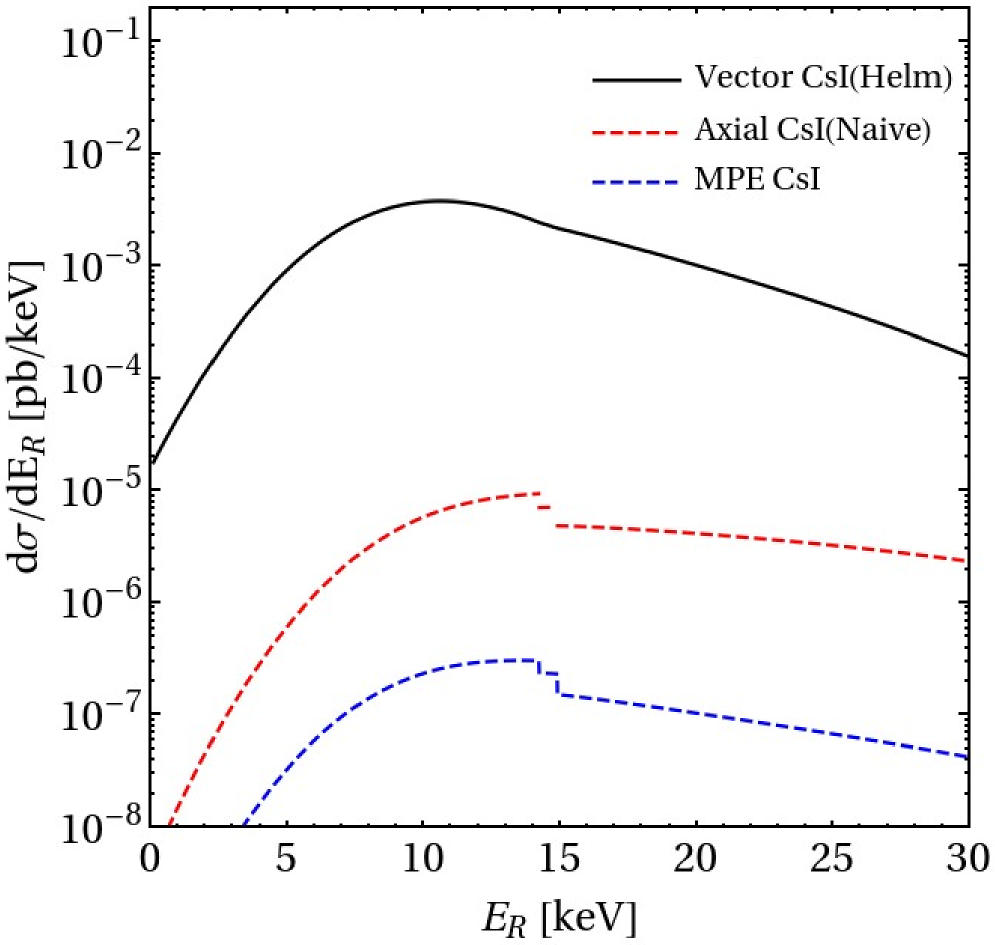
Multipole expansion calculated with Dmformfactor code

Example: COHERENT run 1

Cs and I axial cross sections



CsI axial rates



Conclusions

- The multipole expansion of hadronic currents gives a consistent way to account for the non-point like nature of the nucleus
- Can be extended to include higher order effects (will affect vector rate too), and inelastic (incoherent) scattering
- I find that the axial cross section is smaller than previously suggested, and may be irrelevant for CevENs experiments
- Complete analysis in progress.. stay tuned

Spherical decomposition:

$$\hat{e}_{\pm} = \frac{1}{2}(\hat{x} \mp i\hat{y}), \ \ \hat{e}_0 = \hat{z} \equiv \hat{q}$$

$$\vec{\sigma}\, e^{i\vec{q}.\vec{x}} = \sum_{\lambda} (-1)^{\lambda} \sigma_{-\lambda} \hat{e}_{\lambda} e^{-i\vec{q}.\vec{x}}$$

Multipole expansion:

$$e^{i\vec{q}} = \sum_{J=0} \sqrt{4\pi(2J+1)} i^J j_J(qx) Y_{J0}(\Omega_x)$$

$$\hat{e}_0 e^{i\vec{q}} = \sum_{J=0} \sqrt{4\pi(2J+1)} i^{J-1} \frac{1}{q} \vec{\nabla} j_J(qx) Y_{J0}(\Omega_x)$$

$$\hat{e}_{\pm} e^{i\vec{q}} = \sum_{J \geq 1} \sqrt{2\pi(2J+1)} i^{J-2} \left[\lambda\, j_J(qx) \vec{Y}^{\lambda}_{JJ1}(\Omega_x) + \frac{1}{q} \vec{\nabla} \times \left(j_J(qx) \vec{Y}^{\lambda}_{JJ1}(\Omega_x) \right) \right]$$

Full list of single particle operators:

$$M^{\pm}_{JM_J}(q\vec{x}) = F_1^{(1)}(q_\mu^2)M_J^{M_J}(q\vec{x})\tau_\pm$$

$$T^{el\ \pm}_{JM_J}(q\vec{x}) = \frac{q}{M_N}(F_1^{(1)}(q_\mu^2)\Delta_J^{'M_J}(q\vec{x}) + \tfrac{1}{2}\mu^{(1)}(q_\mu^2)\Sigma_J^{M_J}(q\vec{x}))\tau_\pm$$

$$T^{mag\ \pm}_{JM_J}(q\vec{x}) = -\frac{iq}{M_N}(F_1^{(1)}(q_\mu^2)\Delta_J^{M_J}(q\vec{x}) - \tfrac{1}{2}\mu^{(1)}(q_\mu^2)\Sigma_J^{'M_J}(q\vec{x}))\tau_\pm$$

$$M^{5\ \pm}_{JM_J}(q\vec{x}) = \frac{iq}{M_N}(F_A^{(1)}(q_\mu^2)\Omega_J^{'M_J}(q\vec{x}) + \tfrac{1}{2}\omega F_P^{(1)}(q_\mu^2)\Sigma_J^{''M_J}(q\vec{x}))\tau_\pm$$

$$L^{5\ \pm}_{JM_J}(q\vec{x}) = i\left(F_A^{(1)}(q_\mu^2) - \frac{q^2}{2M_N}F_P^{(1)}(q_\mu^2)\right)\Sigma_J^{''M_J}(q\vec{x})\tau_\pm$$

$$T^{el5\ \pm}_{JM_J}(q\vec{x}) = iF_A^{(1)}(q_\mu^2)\Sigma_J^{'M_J}(q\vec{x})\tau_\pm$$

$$T^{mag5\ \pm}_{JM_J}(q\vec{x}) = F_A^{(1)}(q_\mu^2)\Sigma_J^{M_J}(q\vec{x})\tau_\pm$$