Planck cosmology and more than you *ever* wanted to know about beam window functions

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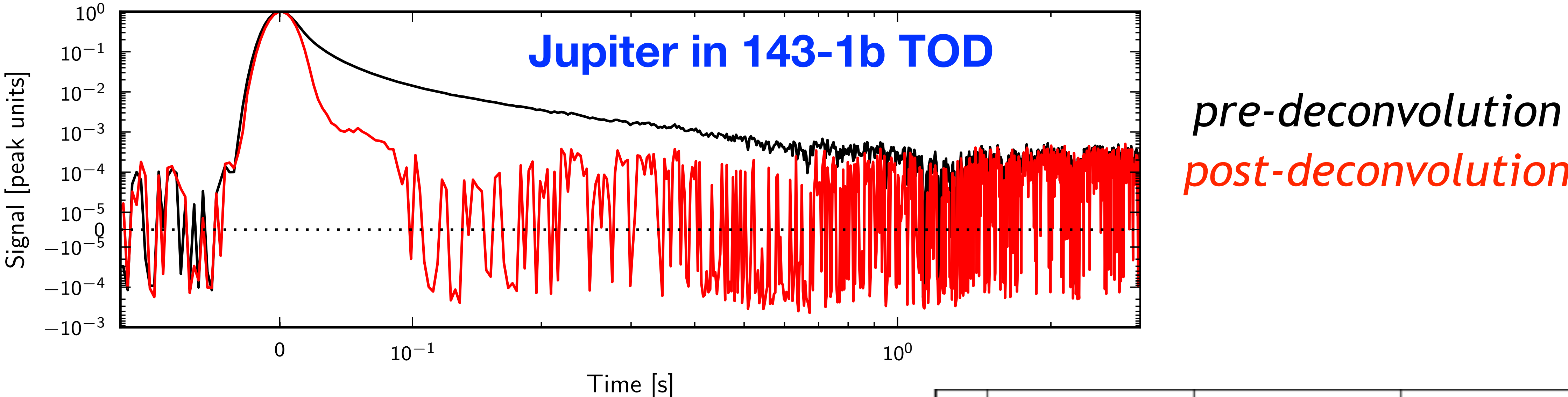
on behalf of the *Planck* collaboration

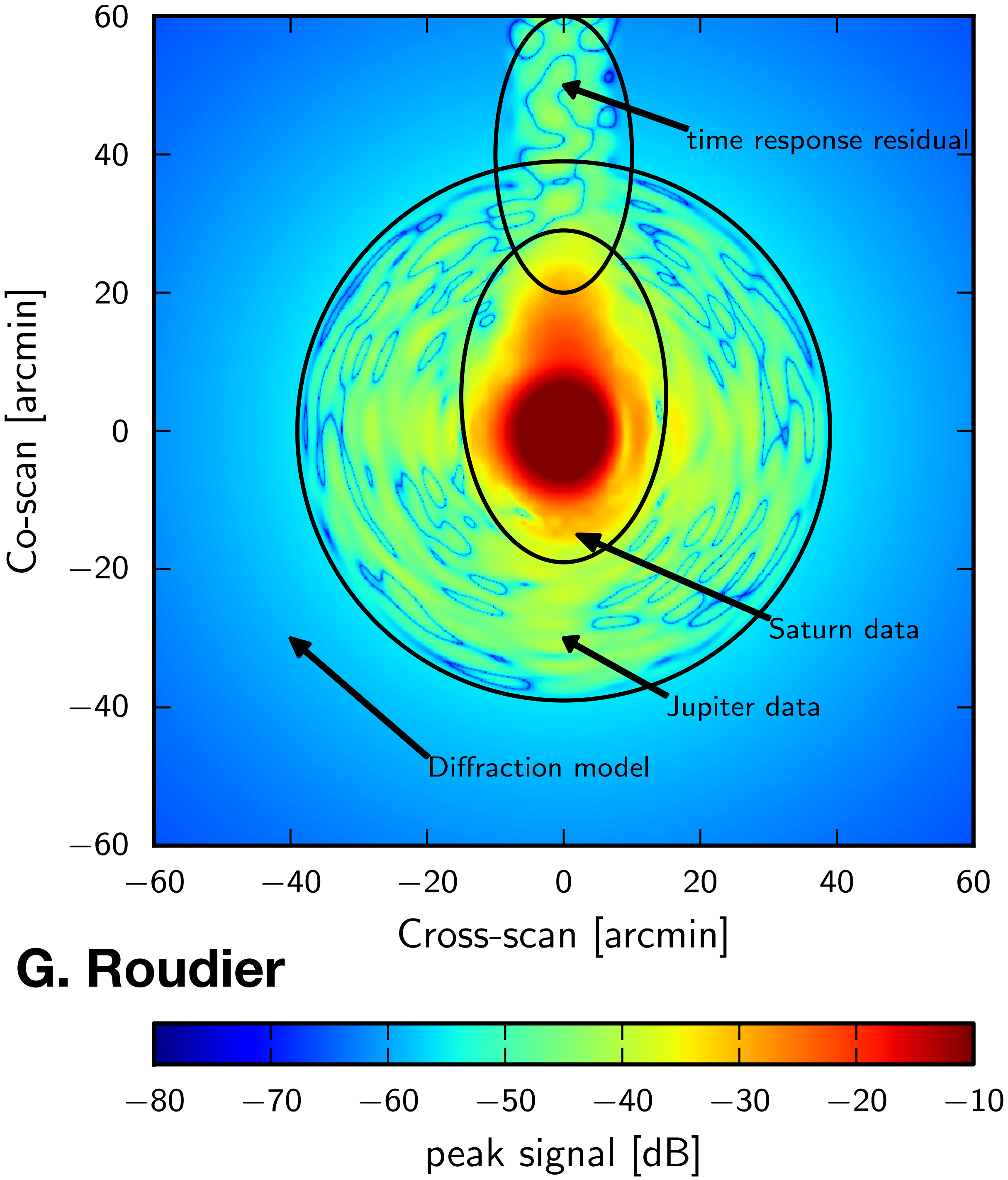
At a time t, the polarised CMB detector j sees

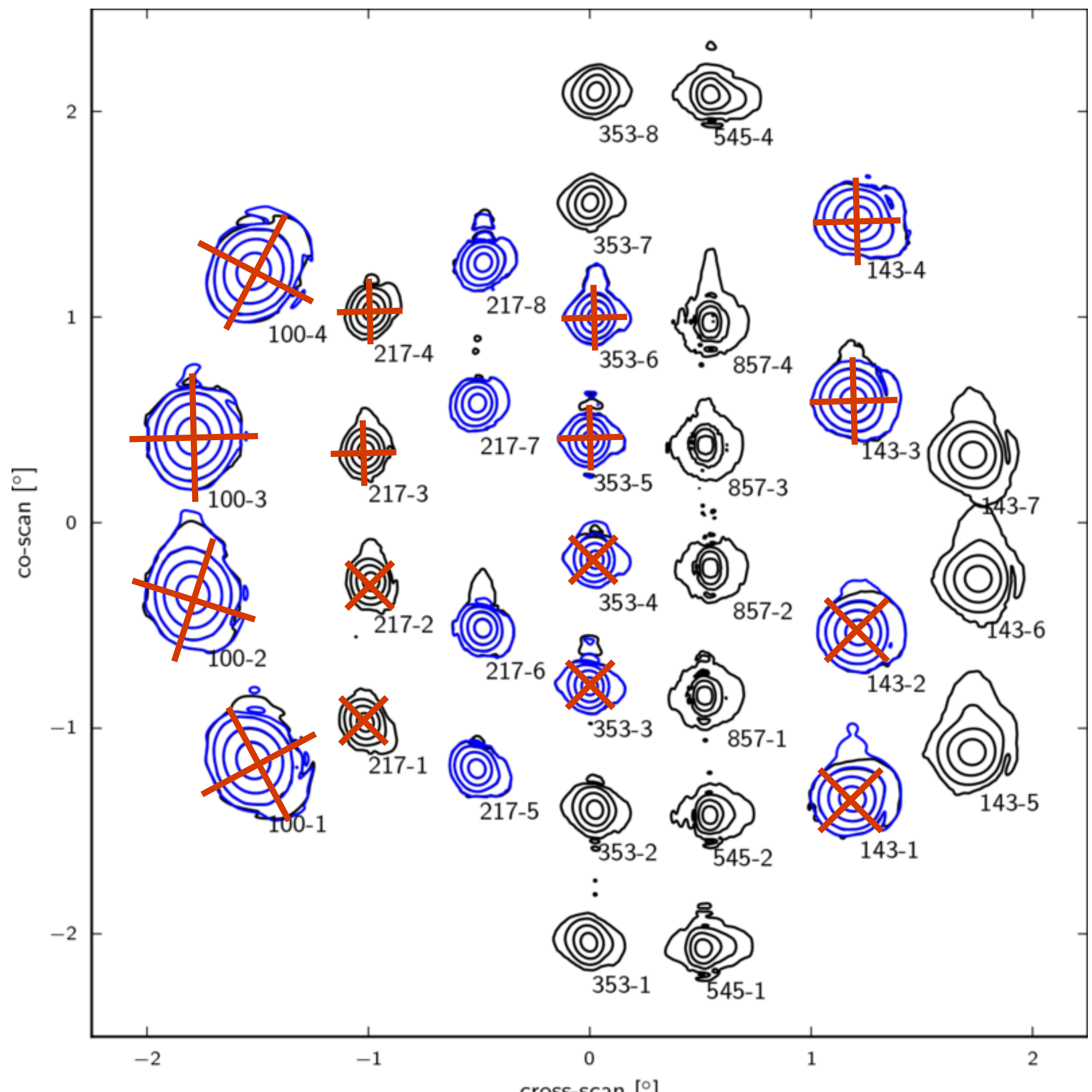
$$d_{j,t} = B_j(\psi_{j,t})^* g_j \, [I + \rho_j \, Q \cos(2\psi_{j,t}) + \rho_j \, U \sin(2\psi_{j,t})] + n_{j,t}$$

where B_j, g_j, ρ_j and ψ_j are poorly known or unknown before flight

- determination of beam (B_j)
- mismatched or non-circular beam → T to P leakage
- error on beam (B_j)
- error on gain (g_j), polar efficiency (ρ_j), polar angles (ψ_j)
- are the beams copolar ?


Jupiter in 143-1b TOD
Signal [peak units]
Time [s]
pre-deconvolution
post-deconvolution


time response residual
Saturn data
Jupiter data
Diffraction model
Co-scan [arcmin]
Cross-scan [arcmin]
peak signal [dB]

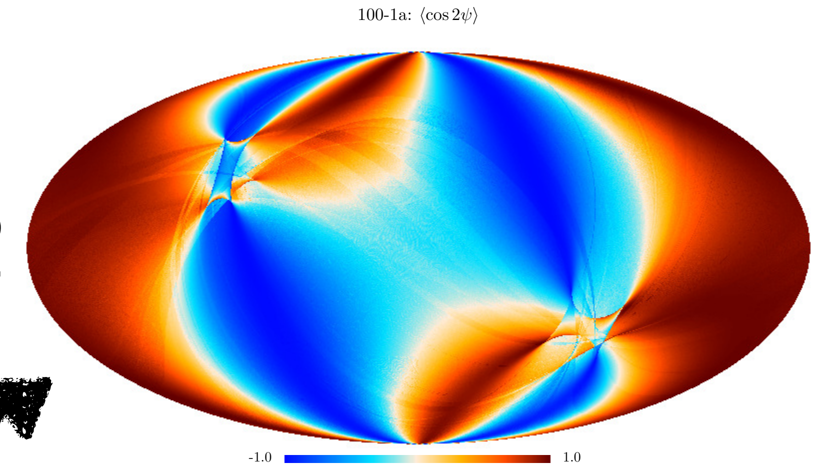

353-8
545-4
353-7
143-4
100-4
217-8
217-4
353-6
857-4
100-3
217-7
217-3
353-5
857-3
143-3
143-7
100-2
217-2
353-4
857-2
143-2
143-6
217-6
100-1
217-1
353-3
857-1
143-5
217-5
353-2
545-2
143-1
353-1
545-1
co-scan [°]
cross-scan [°]

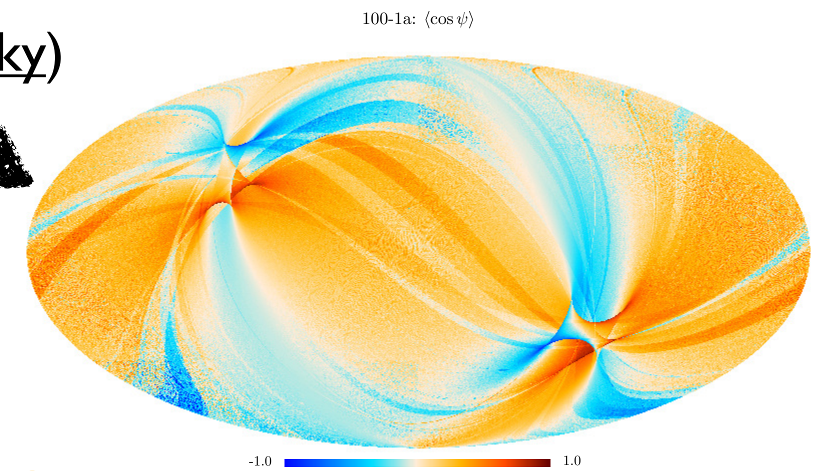
G. Roudier

QuickPol

- **Temperature QuickBeam** (used in Planck DR1 and DR2):
  - ✦ $C'^{TT}_\ell = \Sigma_s\ \omega_s^2\ \ b^*_{\ell s}\ \ b_{\ell s}\ \ \ C^{TT}_\ell$
    - $b_{\ell s}$: weighted combination of scanning beams in DetSet,
    - ω_s^2: encodes scanning strategy (<u>assumed to vary slowly across the sky</u>)

$s=2$





$s=1$

- **Temperature + Polarisation QuickPol** (New in DR3!):
  - ✦ $\mathbf{C'}_\ell = \Sigma_{sij}\ \ \mathbf{\Omega}_{sij}\ \circledast\ \ \mathbf{B}^{*t}_{\ell si}\ .\ \mathbf{C}_\ell\ .\ \mathbf{B}_{\ell sj}$
    - **C** : 3x3 *C(l)* matrix
    - **B** : weighted scanning polarised beams in DetSet
    - **Ω** : encodes scanning strategy weighted by map-making IQU inverse covariance matrix
      can be based on a subset of pixels !
  - ✦ provides effective beam window matrix $\mathbf{W}_\ell$ describing C_ℓ coupling
  - ✦ extended to gain and polar efficiency uncertainty
  - ✦ Backward *C(l)* fitting can then still be used as a rain check to detect/catch remaining systematics

Map(s) Power Spectra

Sky Power Spectra

$$\begin{pmatrix} \tilde{C}^{TT}_\ell \\ \tilde{C}^{EE}_\ell \\ \tilde{C}^{BB}_\ell \\ \tilde{C}^{TE}_\ell \\ \tilde{C}^{TB}_\ell \\ \tilde{C}^{EB}_\ell \\ \tilde{C}^{ET}_\ell \\ \tilde{C}^{BT}_\ell \\ \tilde{C}^{BE}_\ell \end{pmatrix} = \mathbf{W}_\ell \cdot \begin{pmatrix} C^{TT}_\ell \\ C^{EE}_\ell \\ C^{BB}_\ell \\ C^{TE}_\ell \\ C^{TB}_\ell \\ C^{EB}_\ell \end{pmatrix}$$

For each ℓ, $\mathbf{W}_\ell$ is a 9x6 (diagonal dominated) matrix

Hivon, Mottet & Ponthieu, 2017

$$W_l^{XY,TT} = \sum_s \sum_{j_1 j_2} \begin{pmatrix} \hat{\Omega}^s_{00}\hat{b}^{(j_1)*}_{l,s}\hat{b}^{(j_2)}_{l,s} \\ \hat{b}^{(j_1)*}_{l,s+2}\left(\hat{\Omega}^s_{-2-2}\hat{b}^{(j_2)}_{l,s+2}+\hat{\Omega}^s_{-22}\hat{b}^{(j_2)}_{l,s-2}\right)+\hat{b}^{(j_1)*}_{l,s-2}\left(\hat{\Omega}^s_{2-2}\hat{b}^{(j_2)}_{l,s+2}+\hat{\Omega}^s_{22}\hat{b}^{(j_2)}_{l,s-2}\right) \\ \hat{b}^{(j_1)*}_{l,s+2}\left(\hat{\Omega}^s_{-2-2}\hat{b}^{(j_2)}_{l,s+2}-\hat{\Omega}^s_{-22}\hat{b}^{(j_2)}_{l,s-2}\right)+\hat{b}^{(j_1)*}_{l,s-2}\left(\hat{\Omega}^s_{22}\hat{b}^{(j_2)}_{l,s-2}-\hat{\Omega}^s_{2-2}\hat{b}^{(j_2)}_{l,s+2}\right) \\ -\hat{b}^{(j_1)*}_{l,s}\left(\hat{\Omega}^s_{0-2}\hat{b}^{(j_2)}_{l,s+2}+\hat{\Omega}^s_{02}\hat{b}^{(j_2)}_{l,s-2}\right) \\ -i\hat{b}^{(j_1)*}_{l,s}\left(\hat{\Omega}^s_{02}\hat{b}^{(j_2)}_{l,s-2}-\hat{\Omega}^s_{0-2}\hat{b}^{(j_2)}_{l,s+2}\right) \\ i\hat{b}^{(j_1)*}_{l,s+2}\left(\hat{\Omega}^s_{-22}\hat{b}^{(j_2)}_{l,s-2}-\hat{\Omega}^s_{-2-2}\hat{b}^{(j_2)}_{l,s+2}\right)+i\hat{b}^{(j_1)*}_{l,s-2}\left(\hat{\Omega}^s_{22}\hat{b}^{(j_2)}_{l,s-2}-\hat{\Omega}^s_{2-2}\hat{b}^{(j_2)}_{l,s+2}\right) \\ -\hat{b}^{(j_2)}_{l,s}\left(\hat{\Omega}^s_{-20}\hat{b}^{(j_1)*}_{l,s+2}+\hat{\Omega}^s_{20}\hat{b}^{(j_1)*}_{l,s-2}\right) \\ -i\hat{b}^{(j_2)}_{l,s}\left(\hat{\Omega}^s_{-20}\hat{b}^{(j_1)*}_{l,s+2}-\hat{\Omega}^s_{20}\hat{b}^{(j_1)*}_{l,s-2}\right) \\ i\hat{b}^{(j_1)*}_{l,s+2}\left(\hat{\Omega}^s_{-2-2}\hat{b}^{(j_2)}_{l,s+2}+\hat{\Omega}^s_{-22}\hat{b}^{(j_2)}_{l,s-2}\right)-i\hat{b}^{(j_1)*}_{l,s-2}\left(\hat{\Omega}^s_{2-2}\hat{b}^{(j_2)}_{l,s+2}+\hat{\Omega}^s_{22}\hat{b}^{(j_2)}_{l,s-2}\right) \end{pmatrix}$$

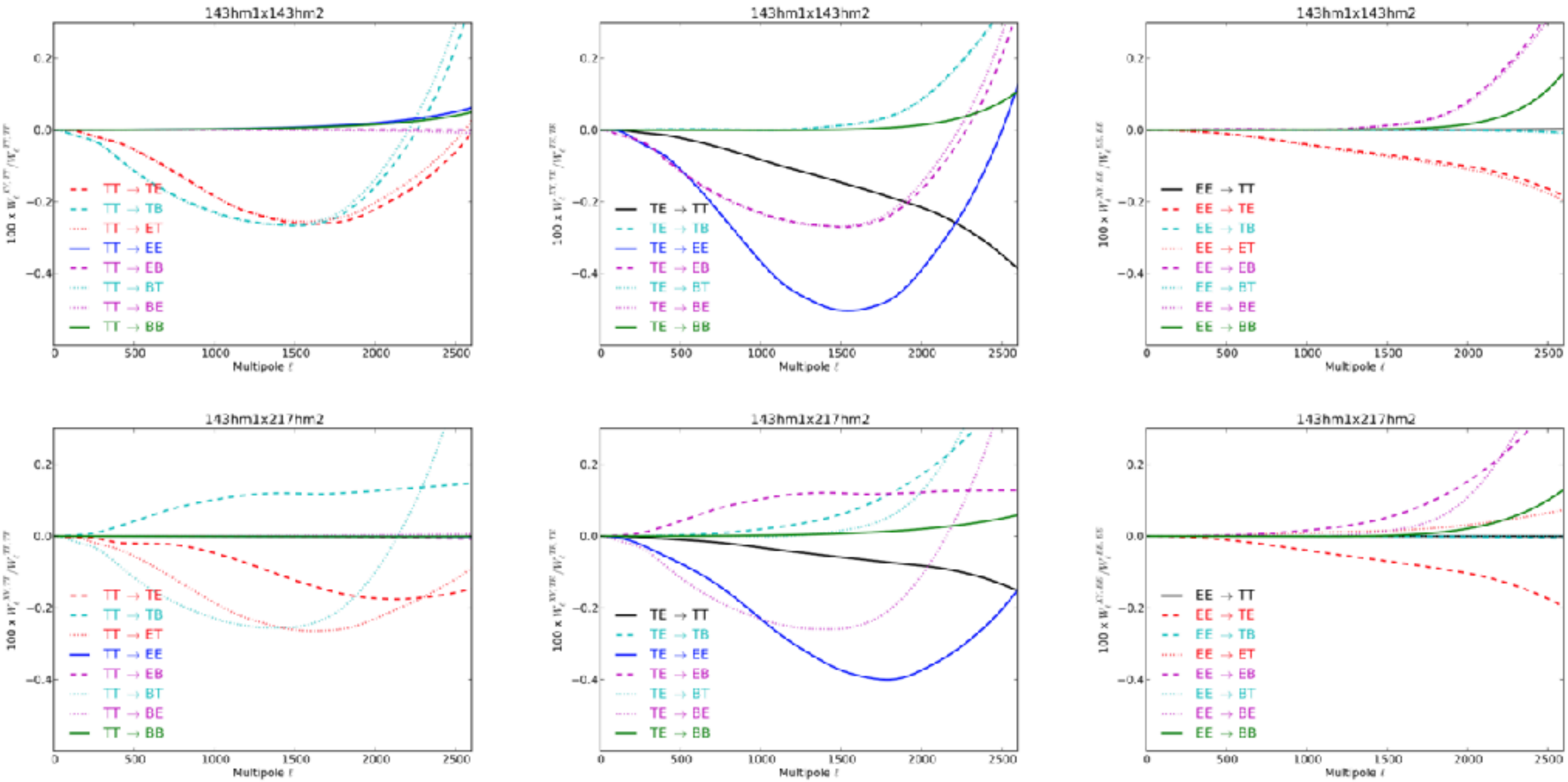
TT column of beam window matrix

EE column

ρ': polar efficiency

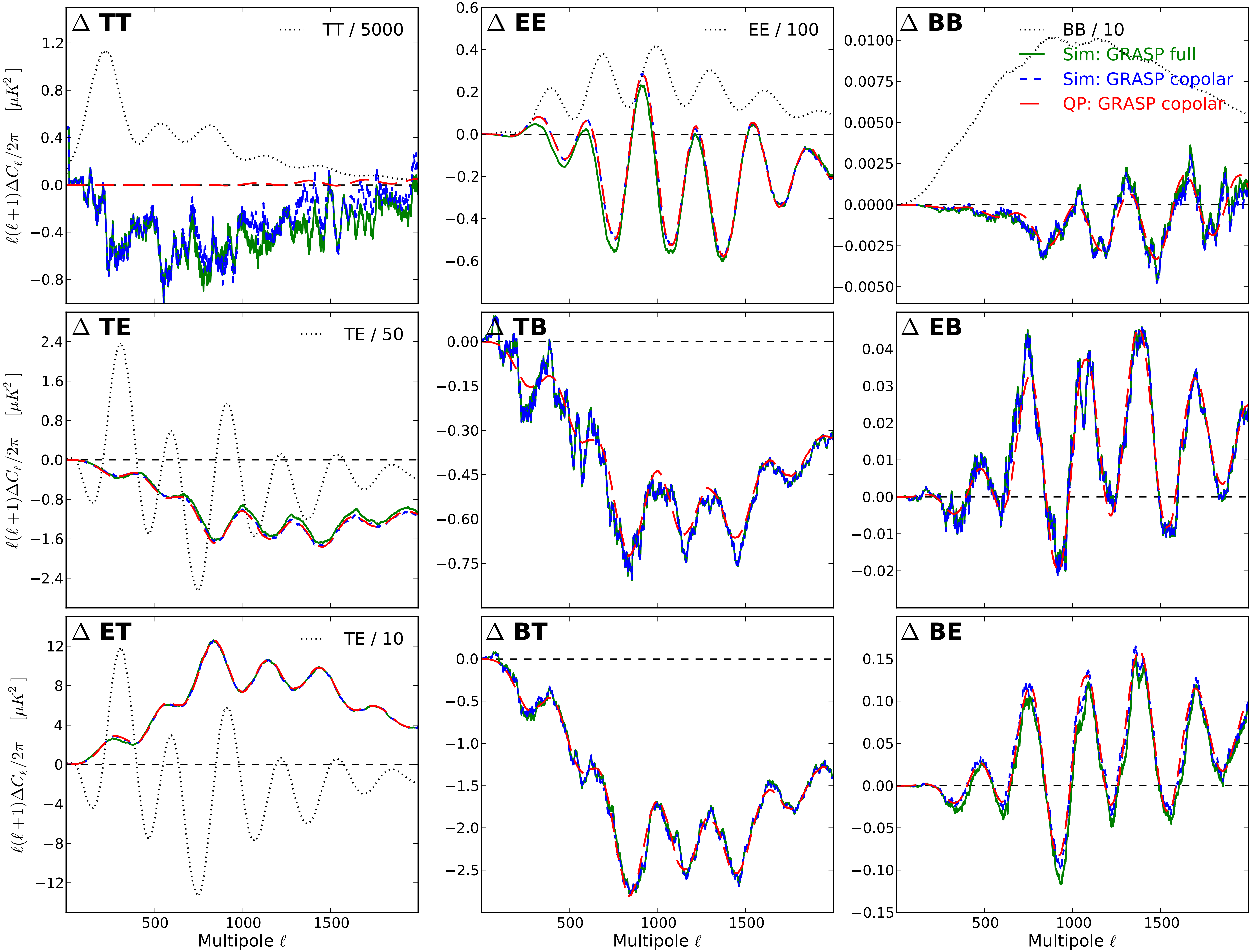
$$W_l^{XY,EE} = \sum_s \sum_{j_1 j_2} \frac{\rho'_{j_1}\rho'_{j_2}}{4} \left(\begin{array}{l} \hat{\Omega}^s_{00}\left(\hat{b}^{(j_1)*}_{l,s-2}+\hat{b}^{(j_1)*}_{l,s+2}\right)\left(\hat{b}^{(j_2)}_{l,s-2}+\hat{b}^{(j_2)}_{l,s+2}\right) \\ \hat{b}^{(j_1)*}_{l,s}\left[\hat{b}^{(j_2)}_{l,s}\left(\hat{\Omega}^s_{-2-2}+\hat{\Omega}^s_{-22}+\hat{\Omega}^s_{2-2}+\hat{\Omega}^s_{22}\right)+\hat{b}^{(j_2)}_{l,s+4}\left(\hat{\Omega}^s_{-2-2}+\hat{\Omega}^s_{2-2}\right)+\hat{b}^{(j_2)}_{l,s-4}\left(\hat{\Omega}^s_{-22}+\hat{\Omega}^s_{22}\right)\right] \\ \quad +\hat{b}^{(j_1)*}_{l,s+4}\left[\hat{b}^{(j_2)}_{l,s}\left(\hat{\Omega}^s_{-2-2}+\hat{\Omega}^s_{-22}\right)+\hat{\Omega}^s_{-2-2}\hat{b}^{(j_2)}_{l,s+4}+\hat{\Omega}^s_{-22}\hat{b}^{(j_2)}_{l,s-4}\right] \\ \quad +\hat{b}^{(j_1)*}_{l,s-4}\left[\hat{b}^{(j_2)}_{l,s}\left(\hat{\Omega}^s_{2-2}+\hat{\Omega}^s_{22}\right)+\hat{\Omega}^s_{2-2}\hat{b}^{(j_2)}_{l,s+4}+\hat{\Omega}^s_{22}\hat{b}^{(j_2)}_{l,s-4}\right] \\ \hat{b}^{(j_1)*}_{l,s}\left[\hat{b}^{(j_2)}_{l,s}\left(\hat{\Omega}^s_{-2-2}-\hat{\Omega}^s_{-22}-\hat{\Omega}^s_{2-2}+\hat{\Omega}^s_{22}\right)+\hat{b}^{(j_2)}_{l,s+4}\left(\hat{\Omega}^s_{-2-2}-\hat{\Omega}^s_{2-2}\right)+\hat{b}^{(j_2)}_{l,s-4}\left(\hat{\Omega}^s_{22}-\hat{\Omega}^s_{-22}\right)\right] \\ \quad +\hat{b}^{(j_1)*}_{l,s+4}\left[\hat{b}^{(j_2)}_{l,s}\left(\hat{\Omega}^s_{-2-2}-\hat{\Omega}^s_{-22}\right)+\hat{\Omega}^s_{-2-2}\hat{b}^{(j_2)}_{l,s+4}-\hat{\Omega}^s_{-22}\hat{b}^{(j_2)}_{l,s-4}\right] \\ \quad +\hat{b}^{(j_1)*}_{l,s-4}\left[\hat{b}^{(j_2)}_{l,s}\left(\hat{\Omega}^s_{22}-\hat{\Omega}^s_{2-2}\right)-\hat{\Omega}^s_{2-2}\hat{b}^{(j_2)}_{l,s+4}+\hat{\Omega}^s_{22}\hat{b}^{(j_2)}_{l,s-4}\right] \\ -\left(\hat{b}^{(j_1)*}_{l,s-2}+\hat{b}^{(j_1)*}_{l,s+2}\right)\left[\hat{b}^{(j_2)}_{l,s}\left(\hat{\Omega}^s_{0-2}+\hat{\Omega}^s_{02}\right)+\hat{\Omega}^s_{0-2}\hat{b}^{(j_2)}_{l,s+4}+\hat{\Omega}^s_{02}\hat{b}^{(j_2)}_{l,s-4}\right] \\ -i\left(\hat{b}^{(j_1)*}_{l,s-2}+\hat{b}^{(j_1)*}_{l,s+2}\right)\left[\hat{b}^{(j_2)}_{l,s}\left(\hat{\Omega}^s_{02}-\hat{\Omega}^s_{0-2}\right)-\hat{\Omega}^s_{0-2}\hat{b}^{(j_2)}_{l,s+4}+\hat{\Omega}^s_{02}\hat{b}^{(j_2)}_{l,s-4}\right] \\ i\hat{b}^{(j_1)*}_{l,s}\left[\hat{b}^{(j_2)}_{l,s}\left(-\hat{\Omega}^s_{-2-2}+\hat{\Omega}^s_{-22}-\hat{\Omega}^s_{2-2}+\hat{\Omega}^s_{22}\right)-\hat{b}^{(j_2)}_{l,s+4}\left(\hat{\Omega}^s_{-2-2}+\hat{\Omega}^s_{2-2}\right)+\hat{b}^{(j_2)}_{l,s-4}\left(\hat{\Omega}^s_{-22}+\hat{\Omega}^s_{22}\right)\right] \\ \quad +i\hat{b}^{(j_1)*}_{l,s+4}\left[\hat{b}^{(j_2)}_{l,s}\left(\hat{\Omega}^s_{-22}-\hat{\Omega}^s_{-2-2}\right)-\hat{\Omega}^s_{-2-2}\hat{b}^{(j_2)}_{l,s+4}+\hat{\Omega}^s_{-22}\hat{b}^{(j_2)}_{l,s-4}\right] \\ \quad +i\hat{b}^{(j_1)*}_{l,s-4}\left[\hat{b}^{(j_2)}_{l,s}\left(\hat{\Omega}^s_{22}-\hat{\Omega}^s_{2-2}\right)-\hat{\Omega}^s_{2-2}\hat{b}^{(j_2)}_{l,s+4}+\hat{\Omega}^s_{22}\hat{b}^{(j_2)}_{l,s-4}\right] \\ -\left(\hat{b}^{(j_2)}_{l,s-2}+\hat{b}^{(j_2)}_{l,s+2}\right)\left[\left(\hat{\Omega}^s_{-20}+\hat{\Omega}^s_{20}\right)\hat{b}^{(j_1)*}_{l,s}+\hat{\Omega}^s_{-20}\hat{b}^{(j_1)*}_{l,s+4}+\hat{\Omega}^s_{20}\hat{b}^{(j_1)*}_{l,s-4}\right] \\ i\left(\hat{b}^{(j_2)}_{l,s-2}+\hat{b}^{(j_2)}_{l,s+2}\right)\left[\left(\hat{\Omega}^s_{20}-\hat{\Omega}^s_{-20}\right)\hat{b}^{(j_1)*}_{l,s}-\hat{\Omega}^s_{-20}\hat{b}^{(j_1)*}_{l,s+4}+\hat{\Omega}^s_{20}\hat{b}^{(j_1)*}_{l,s-4}\right] \\ i\hat{b}^{(j_1)*}_{l,s}\left[\hat{b}^{(j_2)}_{l,s}\left(\hat{\Omega}^s_{-2-2}+\hat{\Omega}^s_{-22}-\hat{\Omega}^s_{2-2}-\hat{\Omega}^s_{22}\right)+\hat{b}^{(j_2)}_{l,s+4}\left(\hat{\Omega}^s_{-2-2}-\hat{\Omega}^s_{2-2}\right)+\hat{b}^{(j_2)}_{l,s-4}\left(\hat{\Omega}^s_{-22}-\hat{\Omega}^s_{22}\right)\right] \\ \quad +i\hat{b}^{(j_1)*}_{l,s+4}\left[\hat{b}^{(j_2)}_{l,s}\left(\hat{\Omega}^s_{-2-2}+\hat{\Omega}^s_{-22}\right)+\hat{\Omega}^s_{-2-2}\hat{b}^{(j_2)}_{l,s+4}+\hat{\Omega}^s_{-22}\hat{b}^{(j_2)}_{l,s-4}\right] \\ \quad -i\hat{b}^{(j_1)*}_{l,s-4}\left[\hat{b}^{(j_2)}_{l,s}\left(\hat{\Omega}^s_{2-2}+\hat{\Omega}^s_{22}\right)+\hat{\Omega}^s_{2-2}\hat{b}^{(j_2)}_{l,s+4}+\hat{\Omega}^s_{22}\hat{b}^{(j_2)}_{l,s-4}\right] \end{array}\right)$$

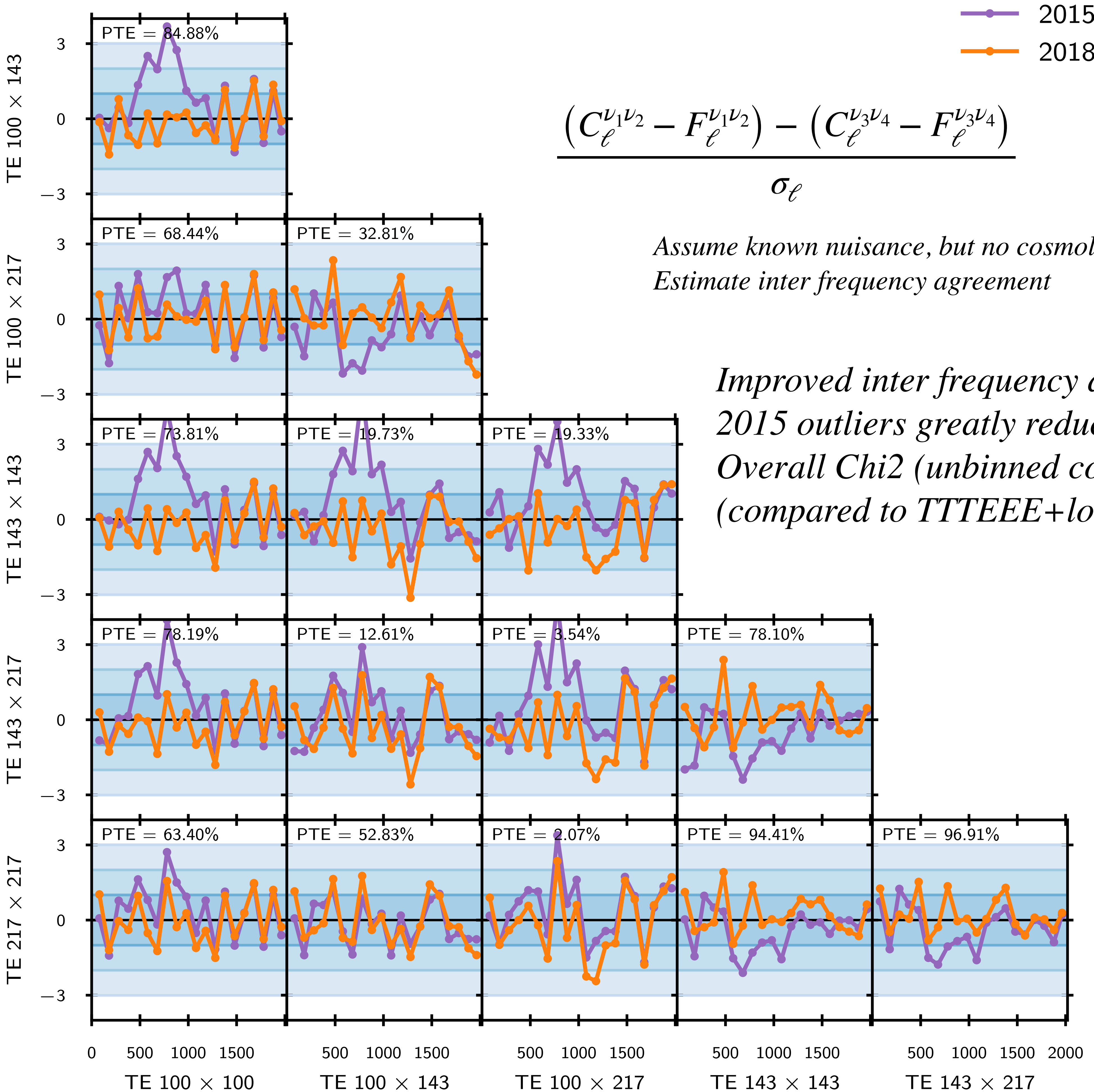
Elements of the beam matrix Wℓ



Comparison to simulations

143ds1x217ds1





$$\frac{\left(C_\ell^{\nu_1\nu_2} - F_\ell^{\nu_1\nu_2}\right) - \left(C_\ell^{\nu_3\nu_4} - F_\ell^{\nu_3\nu_4}\right)}{\sigma_\ell}$$

Assume known nuisance, but no cosmology
Estimate inter frequency agreement

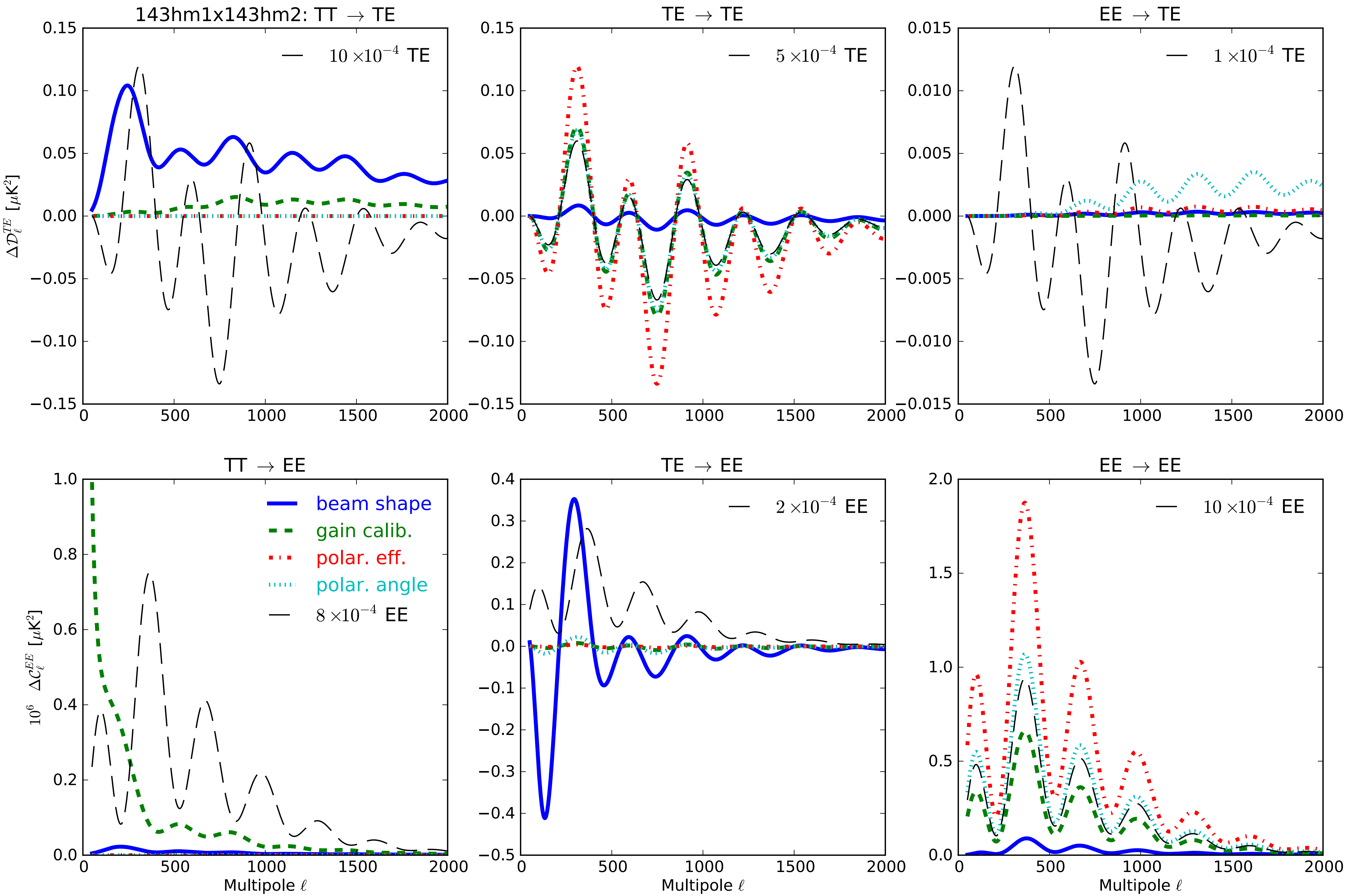
Improved inter frequency agreement
2015 outliers greatly reduced
Overall Chi2 (unbinned coadded) is 1.053 (5% PTE)
(compared to TTTEEE+lowE+lensing best fit)

What if the beam transfer functions are wrong ?

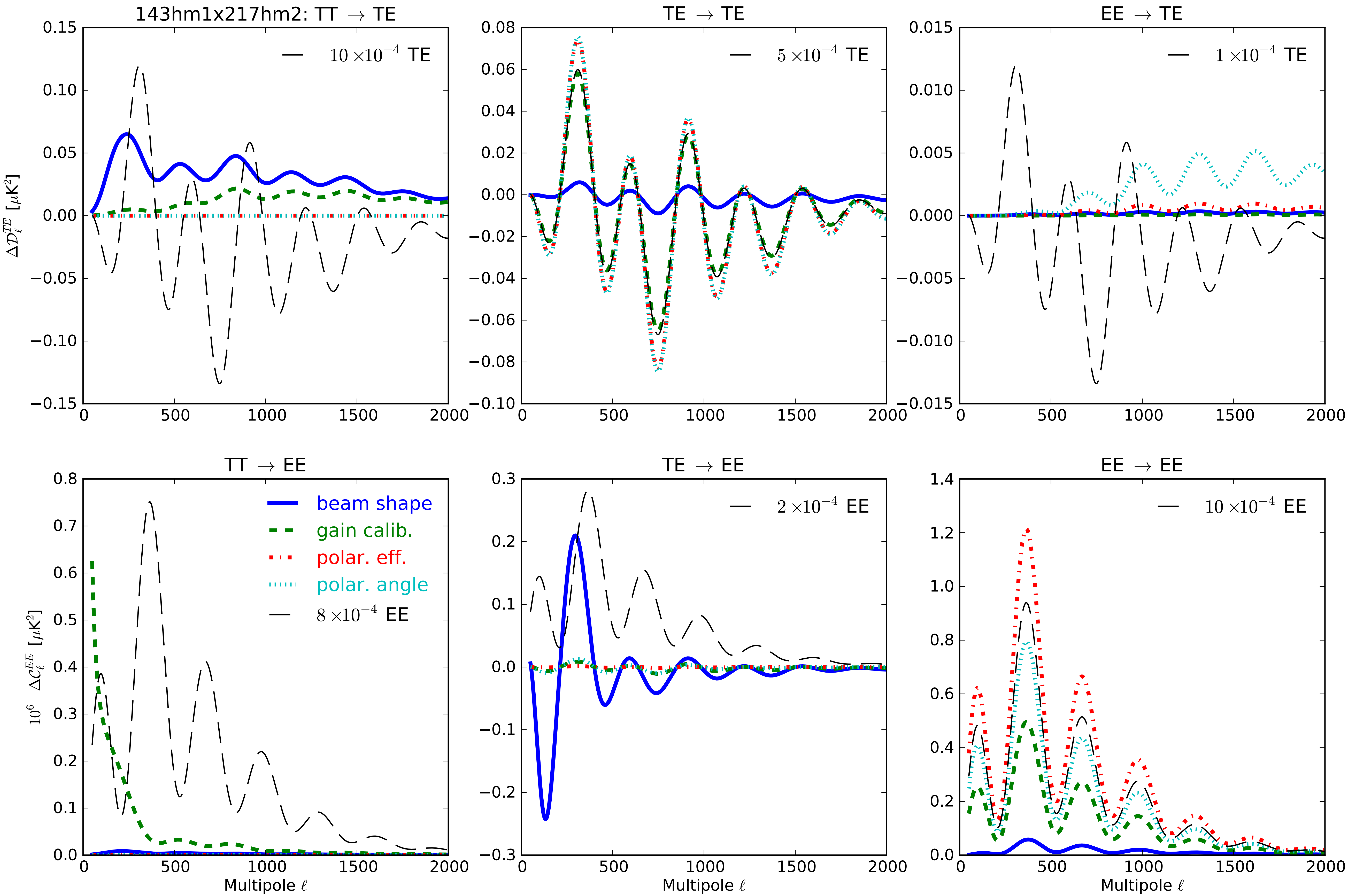
- Allowing flexibility in TT transfer function, with Λ-CDM model
  3 dof (polynomial) at each of the 3 frequencies
  - ✦ if A_{lens}=1 :
    - cosmological parameters:
      1σ change for A_s and n_s,
      slight change (<1σ) for others
    - transfer functions: common modes
  - ✦ if A_{lens} is free :
    - A_{lens} remains at 1.24,
    - cosmological parameters:
      unchanged, with larger errors for A_s and n_s,
    - transfer functions: unchanged,
    - better χ^2

Error propagation in Planck-HFI

- MonteCarlo simulations of QuickPol are run quickly with the following uncertainties on each detector
  - **beam measurements:**
    - ★ detector scanning $b_{\ell m}$ from MC observation of planets,
  - **gain calibration (g):**
    - ★ Gaussian distributed (GD) around nominal value (1.0),
    - ★ $\delta g = 0.1\%$ @ 100-217GHz,
  - **polar efficiency (ρ), $0 < \rho_{SWB} < \rho_{PSB} < 1$**
    - ★ GD around IMO value,
    - ★ $\delta\rho$ = a few 0.1% (read from Rosset+2010),
  - **polarisation orientation (ψ):**
    - ★ GD around IMO value,
    - ★ $\delta\psi$ = 1deg for PSB, 5deg for SWB (adapted from Rosset+2010).



Leading error modes (correlated across all multipoles) are well below ($<10^{-3}$) the TE or EE signals

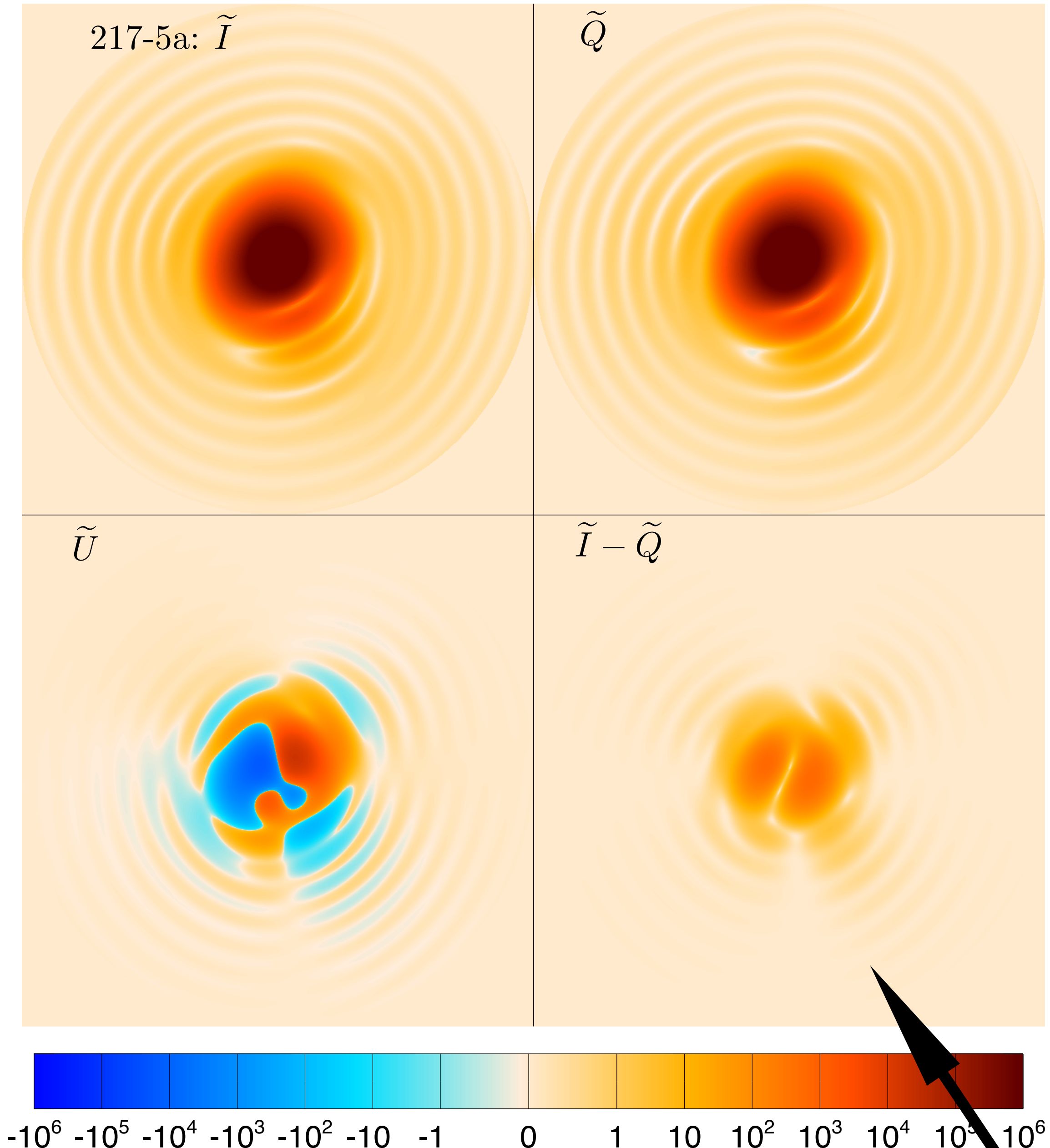


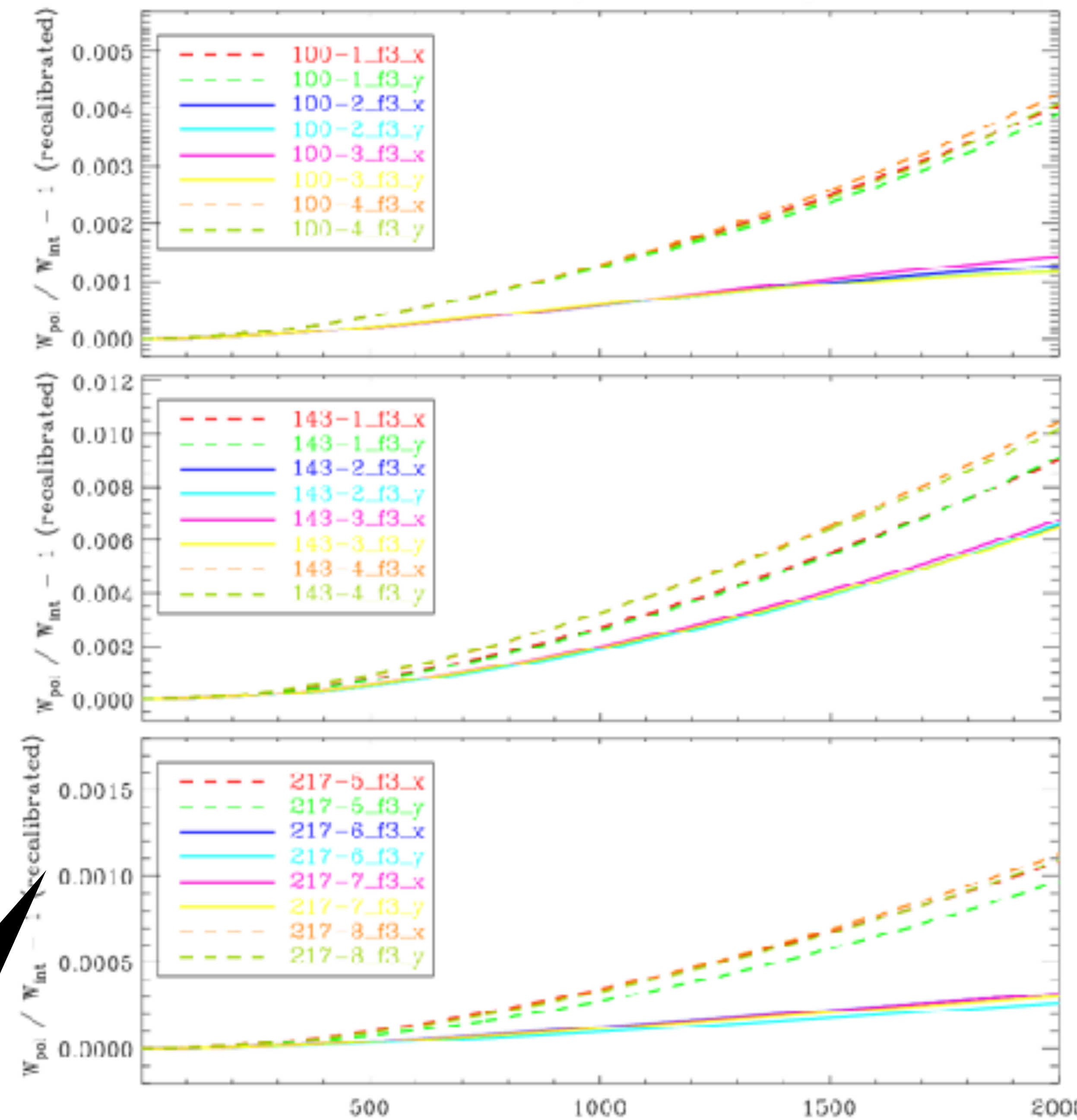
Leading error modes (correlated across all multipoles) are well below (<10^{-3}) the TE or EE signals

Are the beams copolar ?

A copolar beam preserves polarisation $\widetilde{Q} = \widetilde{I}$

$\widetilde{U} = 0$





Optical beams GRASP simulations.

How about actual scanning beams ?

Conclusion

- CMB measurements prone to many sources of systematic errors in beams transfer functions
    - beam shape, gain calibration, polarisation efficiency, polarisation angle …
    - and also: Far Side Lobes, …
- They have been addressed via
    - modelling and correction,
    - template regression,
    - mitigation, or
    - marginalisation.
- cosmological parameters appear stable with respect to these systematics