

Extending the QCD axion

Prateek Agrawal



KICP Workshop, April 2018
Towards Dark Matter Discovery

[1710.04213] Kiel Howe

[1709.06085] JiJi Fan, Matt Reece, LianTao Wang

[1708.05008] Gustavo Marques-Tavares, Wei Xue

The strong CP problem

$$\mathcal{L}_{QCD} = -\frac{1}{4}G_{\mu\nu}^a G^{a,\mu\nu} - \frac{\alpha_s \theta}{8\pi} G_{\mu\nu}^a \tilde{G}^{a\mu\nu} + y_u Q H u^c + y_d Q \tilde{H} d^c$$

Two sources of CP violation

$$\delta_{\text{CKM}} = \arg \det [y_u y_u^\dagger, y_d y_d^\dagger] \simeq \mathcal{O}(1)$$

$$\bar{\theta} = \arg \det (e^{i\theta} y_u^\dagger y_d^\dagger) \lesssim 10^{-10}$$

[neutron EDM]

Sequestered from the CKM phase

[Ellis, Gaillard, 1979]

No anthropic selection

An unobserved renormalizable term in the SM

The QCD Axion

A Peccei-Quinn U(1) global symmetry, anomalous with QCD

U(1) spontaneously broken at some high scale f_a

[Peccei, Quinn 1977]

Chiral phase now a dynamical field: a (pseudo)-Nambu Goldstone Boson

[Weinberg 1978]

[Wilczek 1978]

$$\mathcal{L}_{QCD} = -\frac{1}{4} G_{\mu\nu}^a G^{a,\mu\nu} + \frac{\theta + a/f_a}{8\pi} \alpha_s G_{\mu\nu}^a \tilde{G}^{a,\mu\nu} + y_u Q H u^c + y_d Q \tilde{H} d^c$$

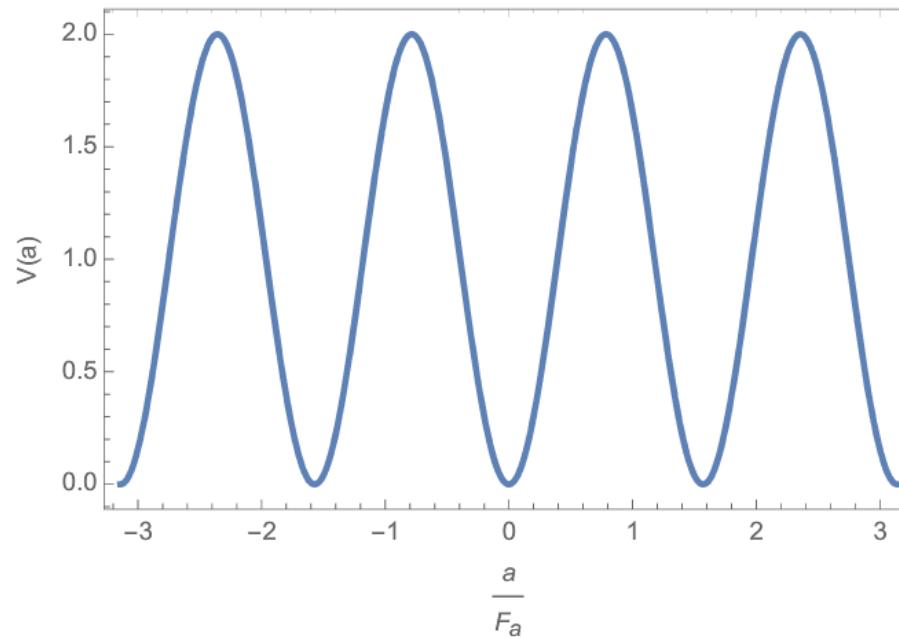
The coupling to QCD breaks the shift symmetry to a discrete shift symmetry

The QCD Axion

The axion potential

$$V(a) \simeq -f_\pi^2 m_\pi^2 \cos\left(\theta + \frac{a}{f_a}\right)$$

[Vafa, Witten 1984]



Dynamically solves the strong CP problem

[Abbott, Sikivie 1983]

Coherent oscillations of the axion make cold dark matter

[Dine, Fischler 1983]

[Preskill, Wise, Wilczek 1983]

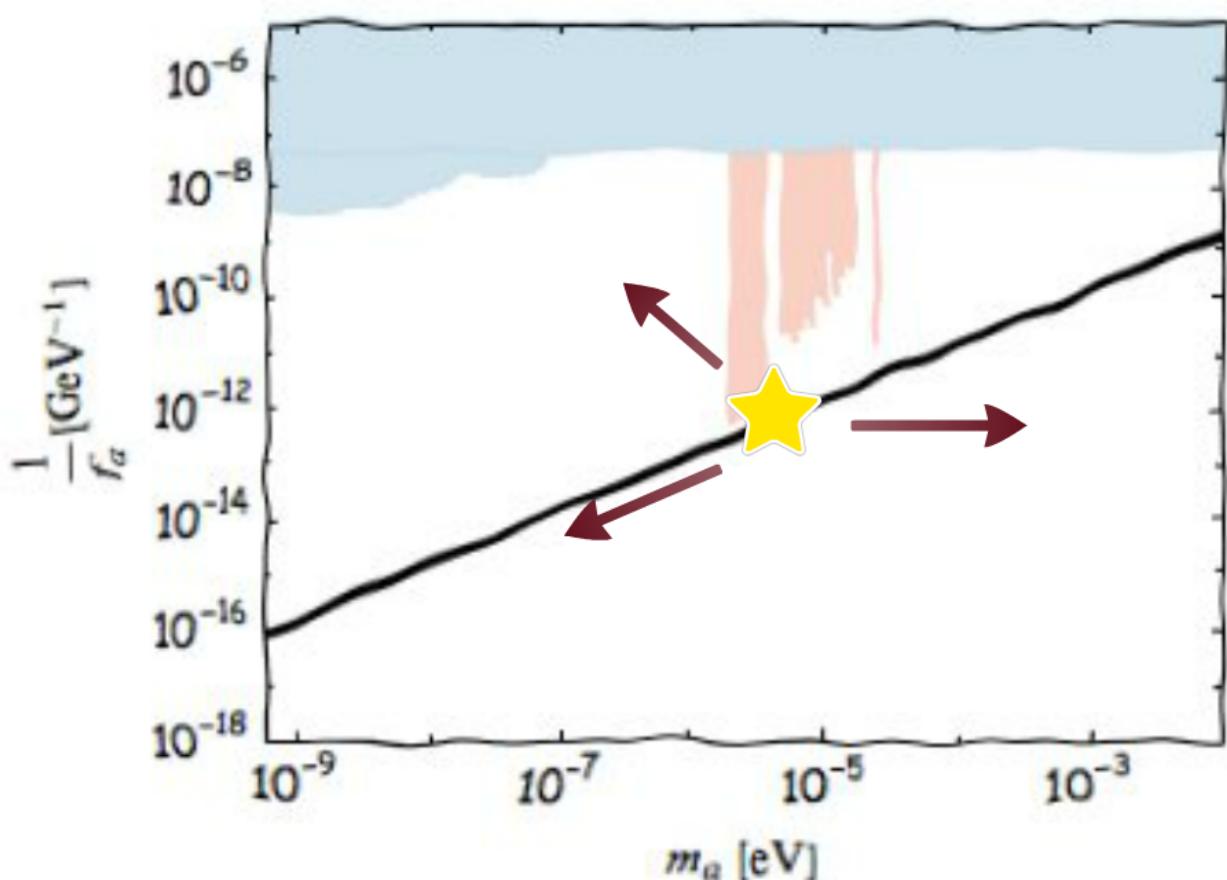
Target

Relic abundance

$$\Omega_a h^2 \simeq 0.1 \left(\frac{f_a}{10^{12} \text{ GeV}} \right)^{7/6} \theta_i^2$$

Mass from QCD

$$m_a = \frac{\sqrt{m_u m_d}}{(m_u + m_d)} \frac{m_\pi f_\pi}{f_a} = 5.7 \mu\text{eV} \left(\frac{10^{12} \text{ GeV}}{f_a} \right)$$



QCD axion couplings

Axion is a compact field with (discrete) shift symmetry

$$a = a + 2\pi F_a$$

Couplings to SM gauge bosons reflect the discrete symmetry

$$\mathcal{L} = N \frac{a}{F_a} \frac{\alpha}{8\pi} G_{\mu\nu}^a \tilde{G}^{a\mu\nu} + E \frac{a}{F_a} \frac{\alpha}{8\pi} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

+ mixing with QCD mesons

Usual basis

$$\mathcal{L} = \frac{a}{f_a} \frac{\alpha}{8\pi} G_{\mu\nu}^a \tilde{G}^{a\mu\nu} + g_{a\gamma\gamma} \frac{a}{4f_a} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

$$g_{a\gamma\gamma} = \frac{\alpha}{2\pi} \left[\frac{E}{N} - 1.92(4) \right]$$



QCD axion couplings to photons

$$g_{a\gamma\gamma} = \frac{\alpha}{2\pi} \left[\frac{E}{N} - 1.92(4) \right]$$

Axion photon coupling is interesting experimentally

Without a cancellation, there is a minimum expected strength

Could it be enhanced?

- Large charges

- Alignment / Clockwork

- Kinetic mixing

Representations and charges

Example: KSVZ models integrating out
heavy "quarks"

$$\mathcal{L} = m Q \bar{Q}^c$$

$$\mathcal{L} = 2\mu(\mathbf{R}) \arg(m) \frac{\alpha}{8\pi} F_{\mu\nu}^a \tilde{F}^{a\mu\nu}$$

For U(1)
 $\mu(\mathbf{R}) \rightarrow q^2$

Guiding principles

Simple examples

[hep-ph/9802220] Kim

(No) Landau poles

[1204.5465]

Unification

Giudice, Rattazzi, Strumia

[1610.07593], [1705.05370]
Luzio, Mescia, Nardi

Leads to a limited range of couplings

KNP Alignment

In "aligned" multi-axion models, the light mode can inherit parametrically different couplings to different gauge groups

Two axion model

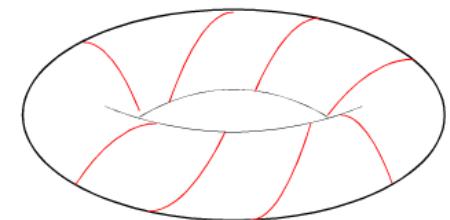
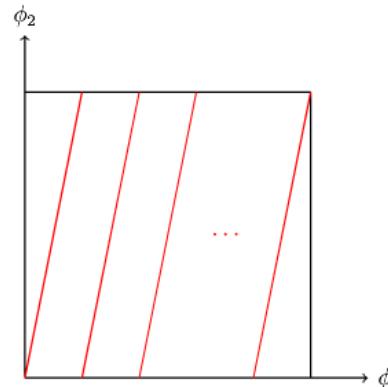
[hep-ph/0409138]
Kim, Nilles, Peloso

$$\frac{a}{f_a} \frac{\alpha_a}{8\pi} F_a \tilde{F}_a + \frac{b}{f_b} \frac{\alpha_b}{8\pi} F_b \tilde{F}_b + \cos \left(\frac{a}{f_a} + \frac{Nb}{f_b} \right)$$

The massless mode $\phi = \frac{-Q f_a a + f_b b}{\sqrt{f_b^2 + Q^2 f_a^2}}$

$$\frac{Q\phi}{f_{\text{eff}}} \frac{\alpha_a}{8\pi} F_a \tilde{F}_a + \frac{\phi}{f_{\text{eff}}} \frac{\alpha_b}{8\pi} F_b \tilde{F}_b$$

$$f_{\text{eff}} = \sqrt{f_b^2 + Q^2 f_a^2}$$



[1404.6209]
Choi, Kim, Yun

Clockwork: Rinse, repeat

Clockwork

Extend KNP alignment to multiple axions

Example: A confinement tower

| | $SU(M)_{h;1}$ | $SU(N)_{h;2}$ | \cdots | $SU(N)_{h;n-1}$ | $SU(3)_C$ | $U(1)_Y$ |
|--------------------------|-------------------|-------------------|----------|-------------------|-----------------|----------|
| ϕ_1 | 1 | 1 | 1 | 1 | 1 | 0 |
| $Q_{1a}(\tilde{Q}_{1a})$ | Adj | 1 | 1 | 1 | 1 | 0 |
| $Q_{1b}(\tilde{Q}_{1b})$ | 1 | 1 | 1 | 1 | 3 ($\bar{3}$) | 0 |
| ϕ_2 | 1 | 1 | 1 | 1 | 1 | 0 |
| $Q_{2a}(\tilde{Q}_{2a})$ | M (\bar{M}) | 1 | 1 | 1 | 1 | 0 |
| $Q_{2b}(\tilde{Q}_{2b})$ | 1 | Adj | 1 | 1 | 1 | 0 |
| ϕ_3 | 1 | 1 | 1 | 1 | 1 | 0 |
| $Q_{3a}(\tilde{Q}_{3a})$ | 1 | M (\bar{M}) | 1 | 1 | 1 | 0 |
| $Q_{3b}(\tilde{Q}_{3b})$ | 1 | 1 | Adj | 1 | 1 | 0 |
| \cdots | \cdots | \cdots | \cdots | \cdots | \cdots | \cdots |
| ϕ_n | 1 | 1 | 1 | 1 | 1 | 0 |
| $Q_{na}(\tilde{Q}_{na})$ | 1 | 1 | 1 | M (\bar{M}) | 1 | 0 |
| $Q_{nb}(\tilde{Q}_{nb})$ | 1 | 1 | 1 | 1 | 1 | 1 (-1) |



[arXiv:1511.00132]

Choi, Im

[arXiv:1511.01827]

Kaplan, Rattazzi

[arXiv:1611.0985]

Farina, Pappadopulo, Rompineve, Tesi

[1709.06085]

PA, Fan, Reece, Wang

$$\mathcal{L} = \frac{1}{8\pi F_0} \left[\left(\sum_{i=1}^{n-1} (Ma_i + a_{i+1}) \alpha_i H_i \tilde{H}_i \right) + a_1 \alpha_s G \tilde{G} + a_n \alpha_{\text{em}} F \tilde{F} \right]$$

Confinement Tower

$$\mathcal{L} = \frac{1}{8\pi F_0} \left[\left(\sum_{i=1}^{n-1} (M a_i + a_{i+1}) \alpha_i H_i \tilde{H}_i \right) + a_1 \alpha_s G \tilde{G} + a_n \alpha_{\text{em}} F \tilde{F} \right]$$

Below the hidden confinement scale(s)

$$V = \Lambda_{\text{QCD}}^4 \cos \left(\frac{a_1}{F_0} \right) + \sum_{i=1}^{n-1} \Lambda_i^4 \cos \left(\frac{M a_i + a_{i+1}}{F_0} \right)$$

$$a_1 \approx \frac{a_n}{M^{n-1}} \quad a_n \text{ light eigenvector}$$

Exponential enhancement of the couplings to photons

$$\mathcal{L} = \frac{a_n}{8\pi M^{n-1} F_0} \left[\alpha_s G \tilde{G} + M^{n-1} \alpha_{\text{em}} F \tilde{F} \right]$$

Kinetic mixing

Large number of axions in string constructions

If axions arise from higher dimensional gauge fields, can get large kinetic mixing from topology of compactification

Cicoli, Goodwill and Ringwald '12

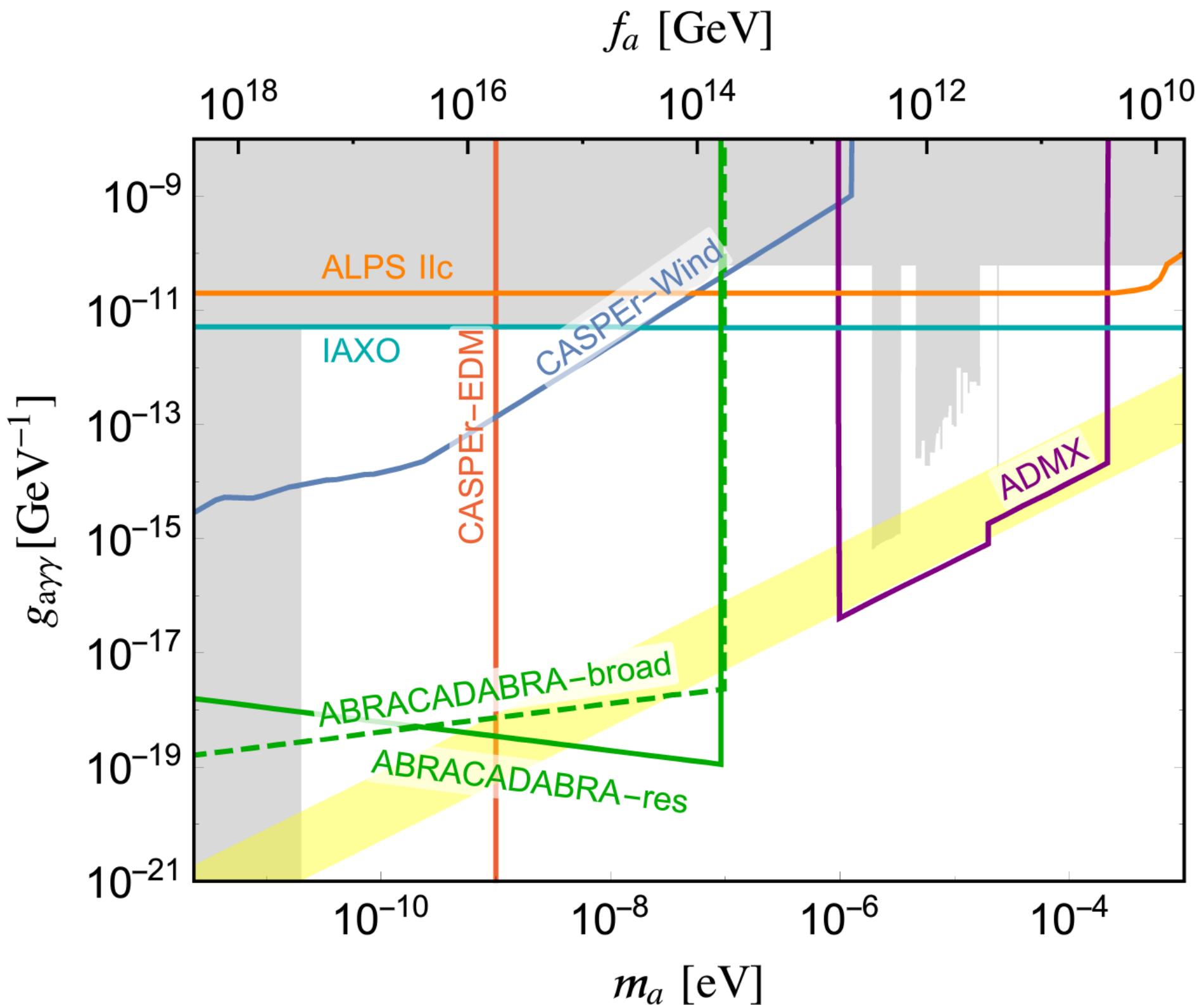
Two axion example

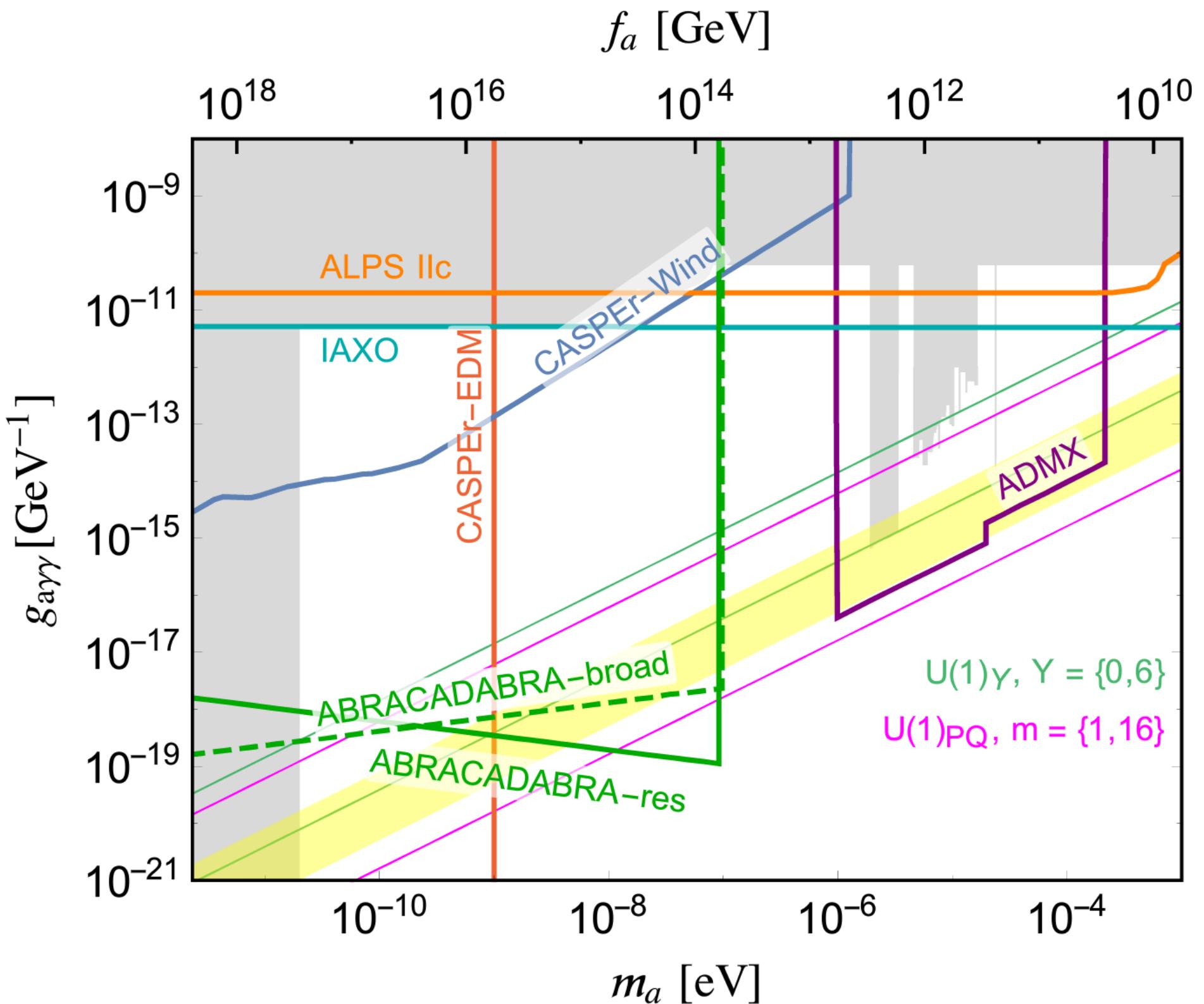
$$\frac{1}{2}\partial_\mu a \partial^\mu a + \frac{1}{2}\partial_\mu b \partial^\mu b + \epsilon \partial_\mu a \partial^\mu b + c_b \frac{\alpha}{8\pi} \frac{b}{F_b} F_{\mu\nu} \tilde{F}^{\mu\nu} - V_G(a) - V_H(b)$$

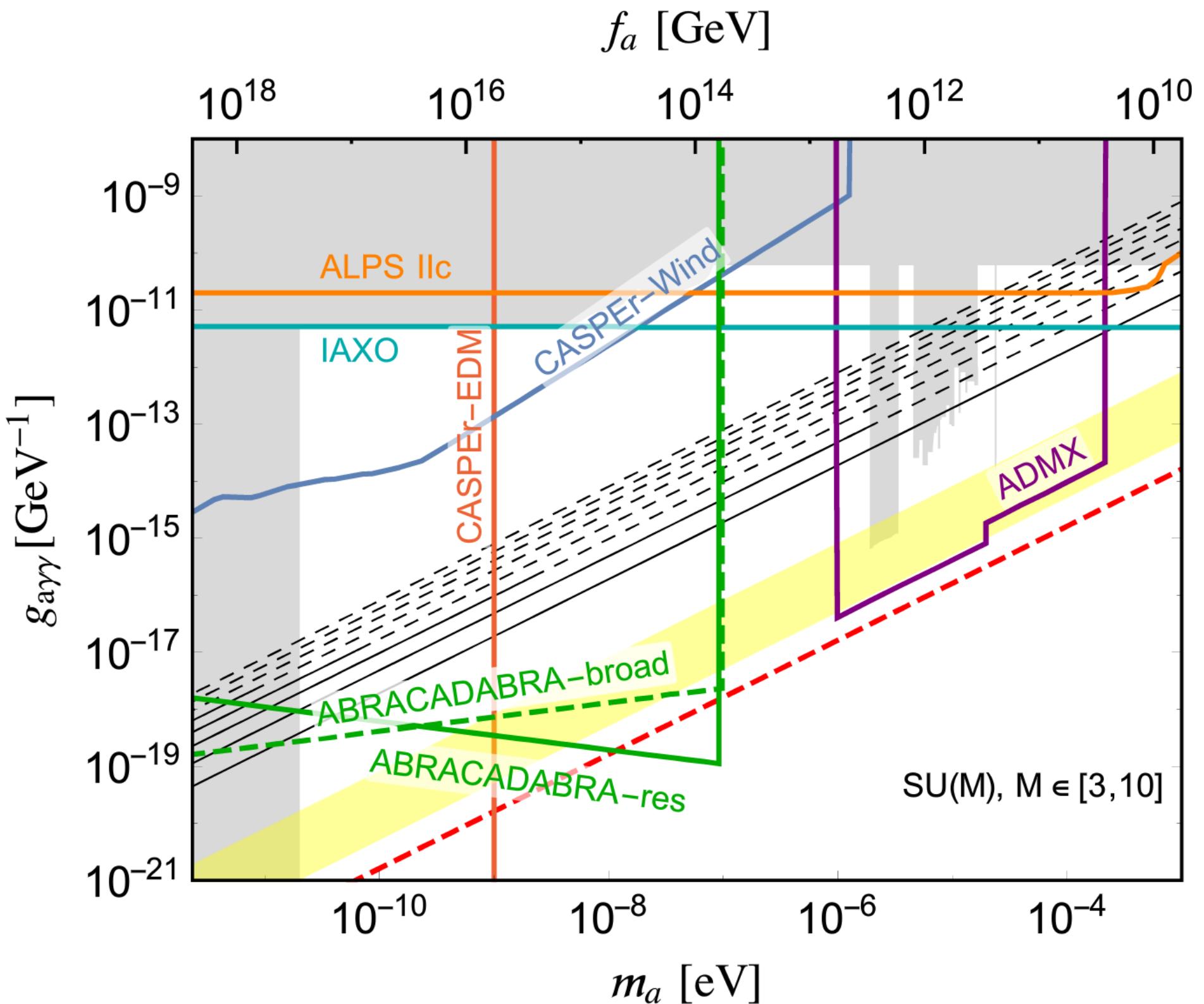
When $m_b \ll m_a$

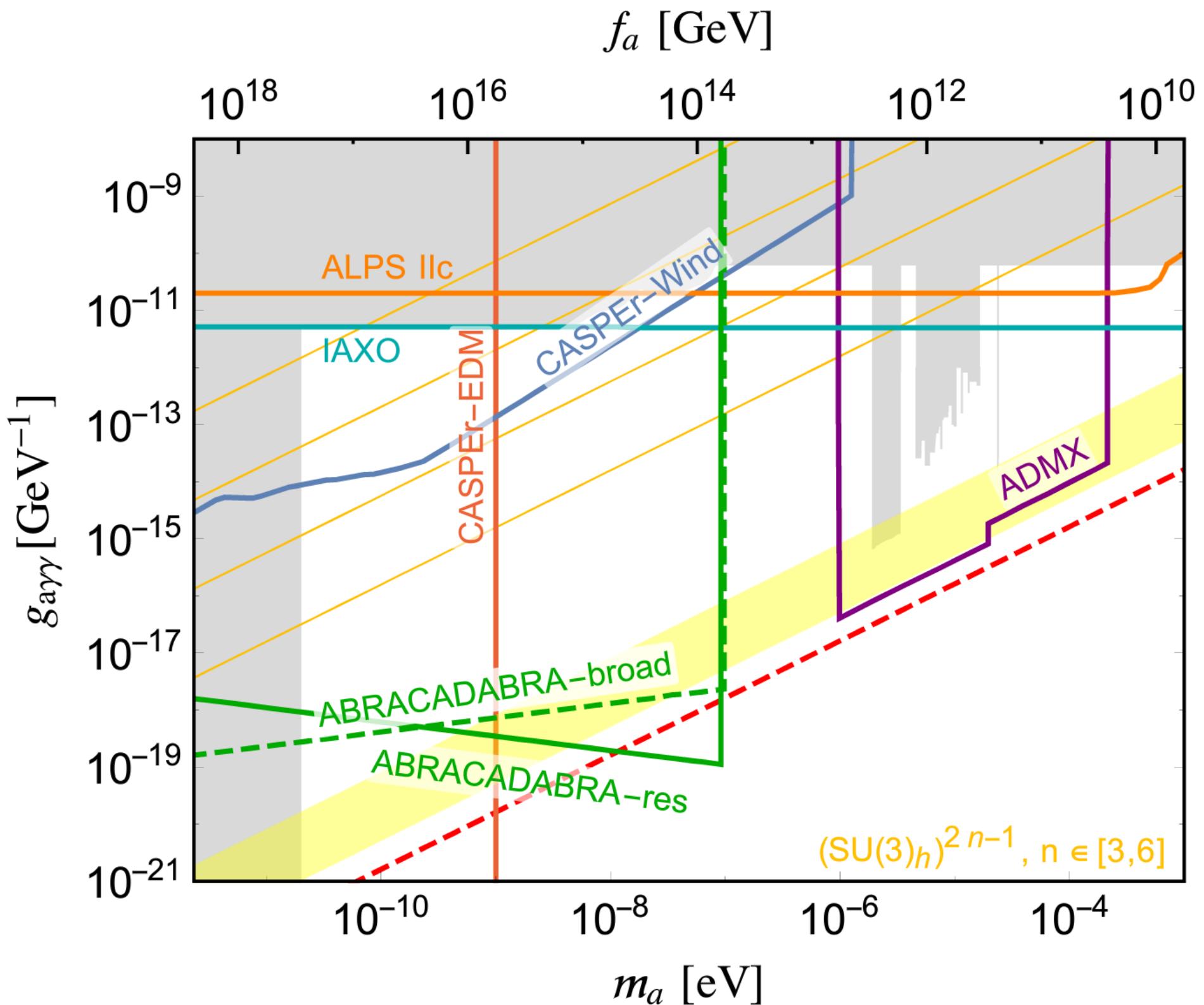
$$\mathcal{L}_{\text{diag}} \supset \left(\frac{c_b \epsilon F_a}{F_b} + \mathcal{O}(\epsilon^2) \right) \frac{\alpha}{8\pi} \frac{a}{F_a} F_{\mu\nu} \tilde{F}^{\mu\nu}.$$

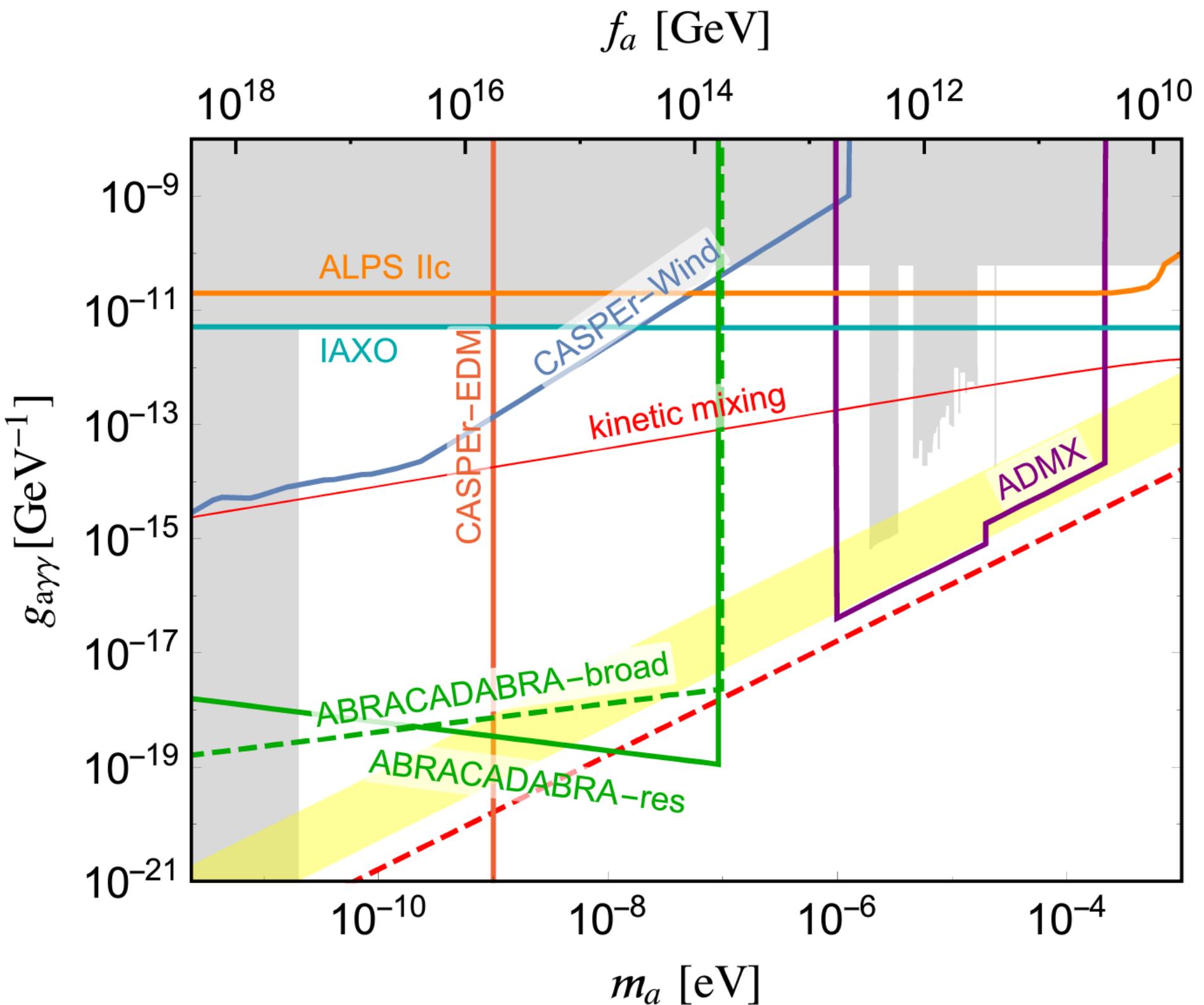
We can get an *irrational* value for enhancement

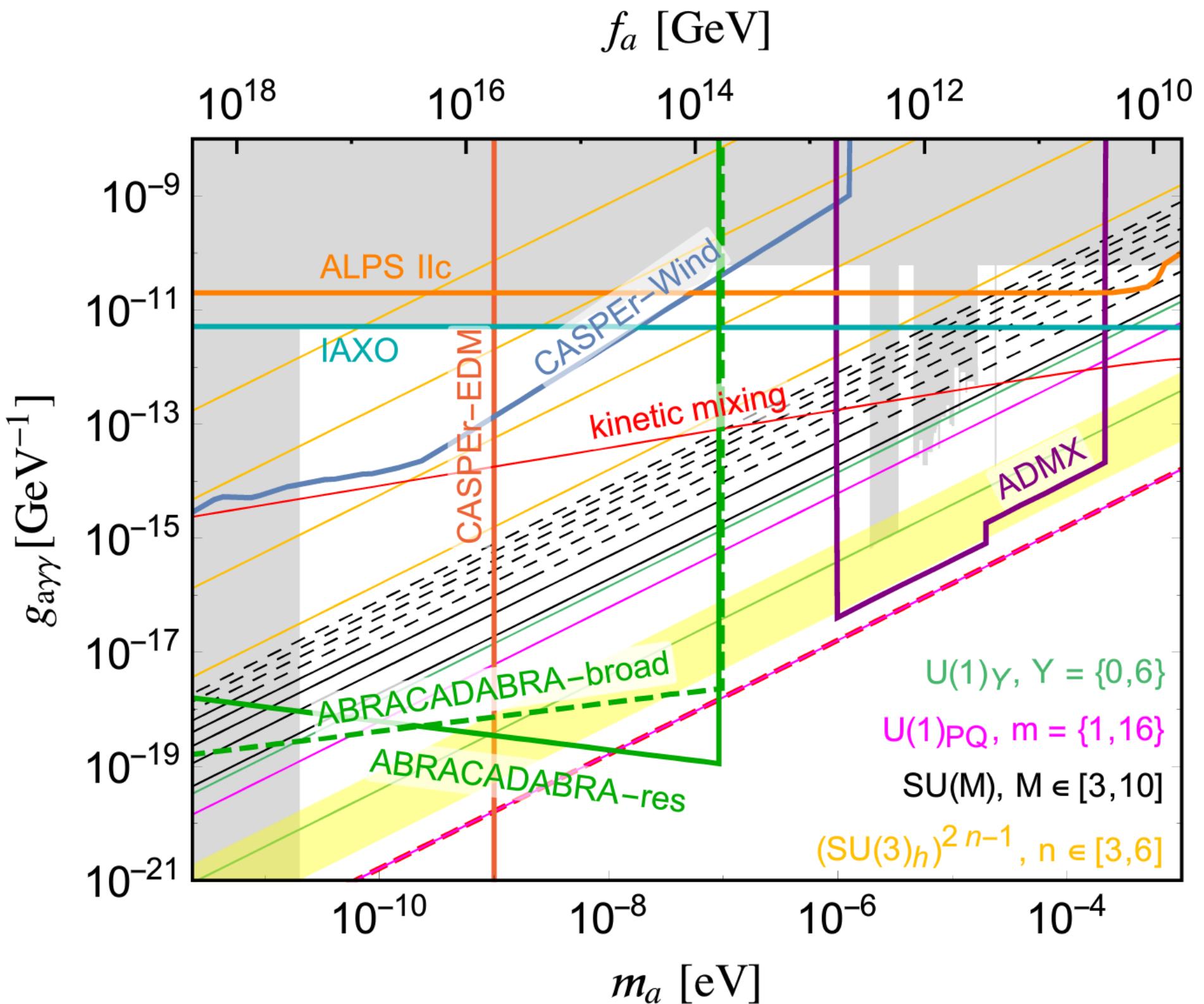












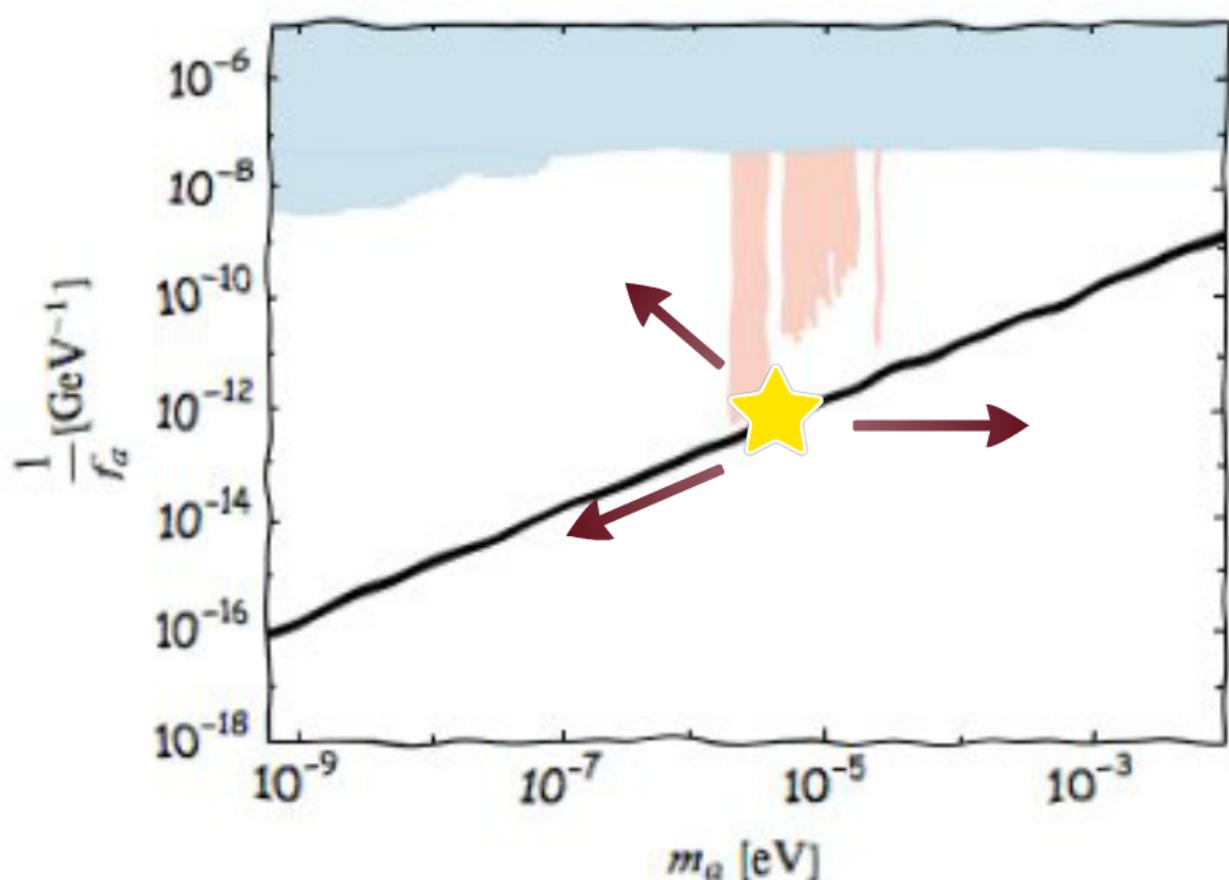
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Mass from QCD

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Axion cosmology

Assume PQ symmetry is broken before inflation

Inflation ensures entire hubble patch has the same θ
 Ω_a set by the misalignment mechanism

Axion starts oscillating when $H \simeq m_a$

$$\rho_a(T_{osc}) = m_a^2 f_a^2 \theta_i^2$$

Subsequent evolution conserves axion number
(adiabatic evolution)

$$\theta(t) = \frac{1}{a^{3/2}} \theta_i \cos(m_a t)$$

Large f_a

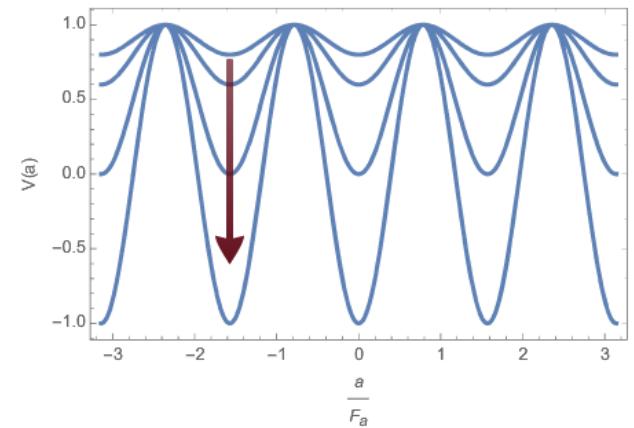
Above QCD phase transition axion mass depends on temperature

$$m_a(T) = \begin{cases} m_a(0) , & T < 200 \text{ MeV} \\ b m_a(0) \left(\frac{200 \text{ MeV}}{T}\right)^4 , & T \geq 200 \text{ MeV} \end{cases}$$

$b = 0.018$

QCD axion abundance set by f_a

$$\Omega_a h^2 \sim \begin{cases} 2 \times 10^4 \left(\frac{f_a}{10^{16} \text{ GeV}}\right)^{7/6} \theta_i^2 , & f_a < 2 \times 10^{15} \text{ GeV} \\ 5 \times 10^3 \left(\frac{f_a}{10^{16} \text{ GeV}}\right)^{3/2} \theta_i^2 , & f_a > 2 \times 10^{17} \text{ GeV} \end{cases}$$



Model independent string axion $f_a = 10^{16} \text{ GeV}$

[hep-th/0605206]
Svrcek, Witten

Initial misalignment may be tuned

(anthropic axion)

Particle production

Depleting axion energy into dark radiation

$$\mathcal{L} = \frac{a}{f_a} \frac{\alpha}{8\pi} G_{\mu\nu}^a \tilde{G}^{a\mu\nu} + \frac{\beta a}{f_a} \frac{\alpha_D}{8\pi} (F_D)_{\mu\nu} \tilde{F}_D^{\mu\nu}$$

Equation of motion for the gauge field

$$A''_{\pm} + \left(k^2 \mp \frac{\beta k \phi'}{f_a} \right) A_{\pm} = 0$$

(in conformal time)



$$\omega^2(k)$$

Tachyonic instability

Conditions for efficient depletion

$$A''_{\pm} + \left(k^2 \mp \frac{\beta k \phi'}{f_a} \right) A_{\pm} = 0$$

$\beta \gg 1$

$$k \sim \frac{\beta \phi'}{f_a} \sim \theta_i \beta m_a$$

Growth rate of the tachyonic modes set by k

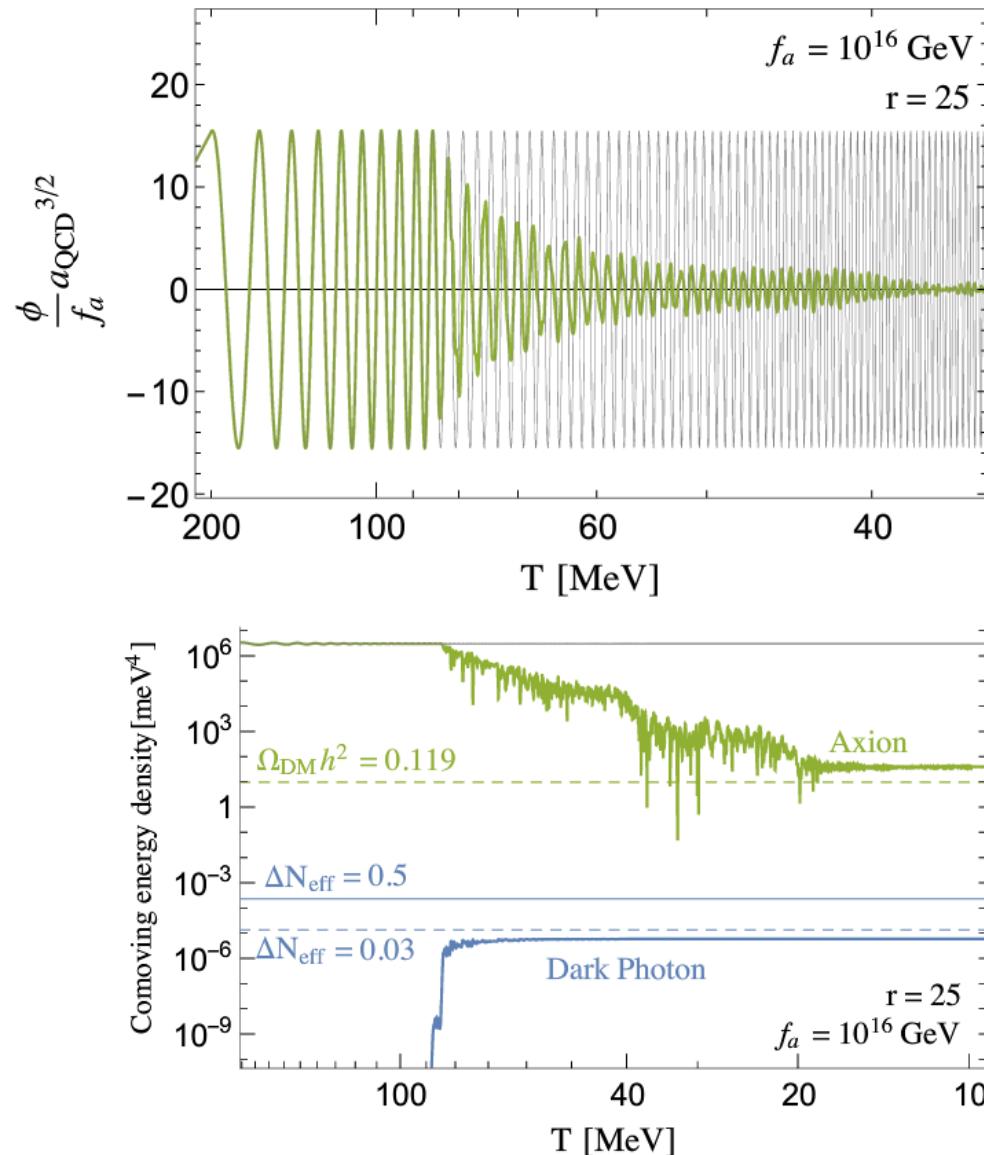
Small (no) thermal mass

Cannot use SM photon

Particles with dark charge should be absent

Results

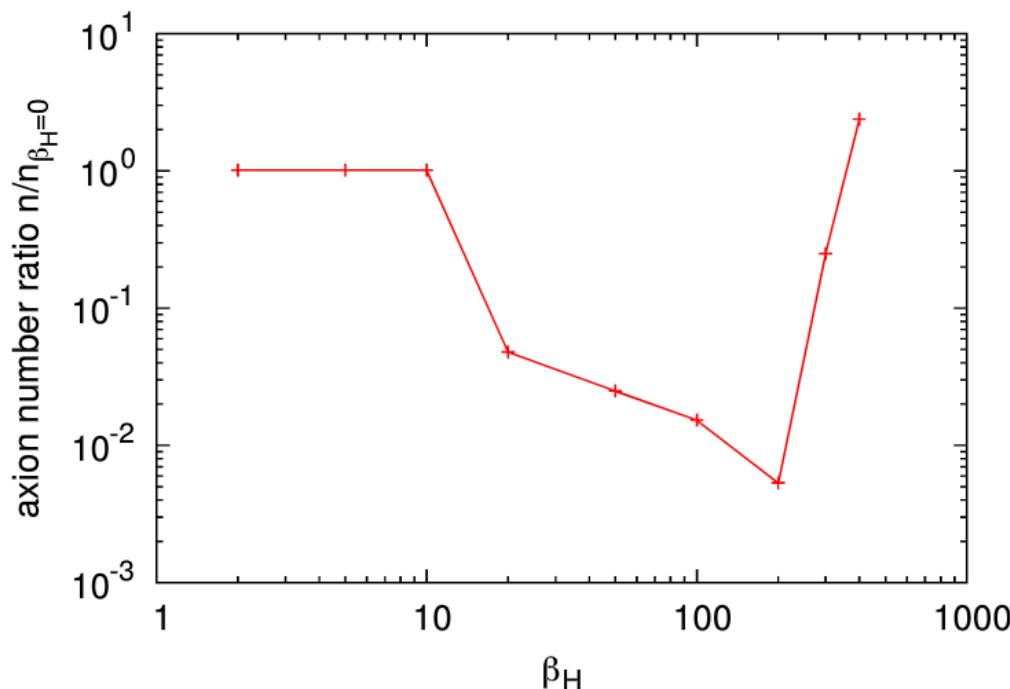
Neglecting backscattering into axion k-modes



Impact of backscattering

High number of dark photons scatter back into axions

Need lattice simulation for k-modes of dark photons and axions



[1802.00444]
Kitajima, Sekiguchi, Takahashi

$f_a \sim 10^{15}$ GeV
may be viable

Effect on matter power spectrum?

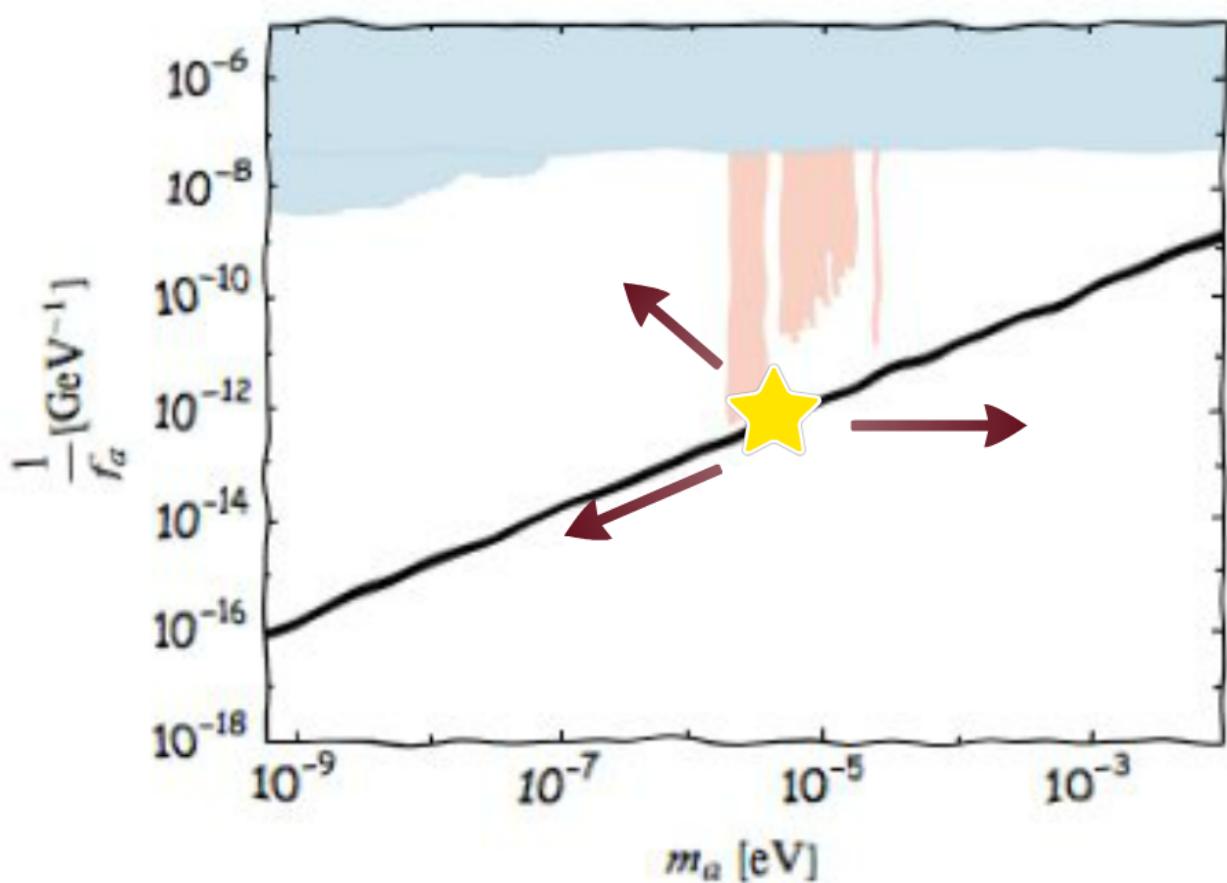
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Heavy QCD axion

Mass of the axion is a robust prediction

$$m_a = \frac{\sqrt{m_u m_d}}{(m_u + m_d)} \frac{m_\pi f_\pi}{f_a} = 5.7 \mu\text{eV} \left(\frac{10^{12} \text{ GeV}}{f_a} \right)$$

New contributions to the mass in general not be aligned with $\theta = 0$

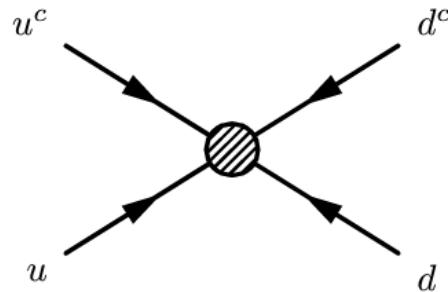
Contribution from small instantons is naturally aligned, but too small in QCD

UV instantons

Two flavor example

$$\mathcal{L}_{QCD} = -\frac{1}{4}G_{\mu\nu}^a G^{a,\mu\nu} + \frac{\theta + a/f_a}{8\pi} \alpha_s G_{\mu\nu}^a \tilde{G}^{a,\mu\nu} + y_u Q H u^c + y_d Q \tilde{H} d^c$$

Contributions above a scale M captured by 't Hooft vertex



$$\mathcal{L}_{\text{eff}} = 2e^{i(\theta+a/f_a)} \int_0^M \frac{d\rho}{\rho^5} \rho^6 \langle uu^c dd^c \rangle e^{-2\pi/\alpha} \left(\frac{2\pi}{\alpha}\right)^6 + \text{h.c.}$$

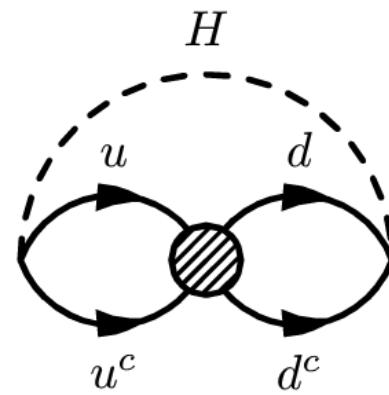
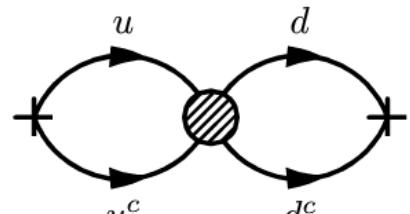
super suppressed in QCD
even more suppressed by
extra yukawas/ loops

Heavy QCD axion

Two flavor example

$$\mathcal{L}_{\text{eff}} = 2e^{i(\theta + a/f_a)} \int_0^M \frac{d\rho}{\rho^5} \rho^6 \langle uu^c dd^c \rangle e^{-2\pi/\alpha} \left(\frac{2\pi}{\alpha} \right)^6 + \text{h.c.}$$

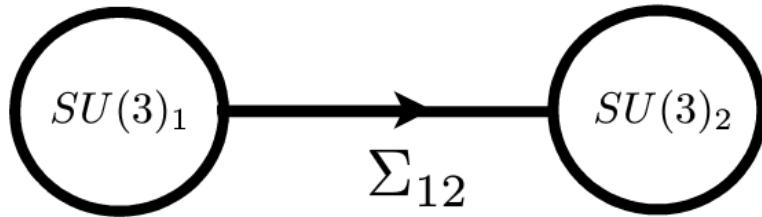
Axion potential



$$V(a) = \Lambda^4 \cos \left(\theta + \frac{a}{f_a} \right)$$

$$\Lambda^4 = M^4 D[\alpha(M)] \left(\prod \frac{m_q}{M} + \prod \frac{y_q}{4\pi} \right) \quad D[\alpha] = D_0 e^{-2\pi/\alpha} \left(\frac{2\pi}{\alpha} \right)^6$$

Two site model



$$Q_i, u_i^c, d_i^c$$

$$f_i > M = \sqrt{g_1^2 + g_2^2} \langle \Sigma_{12} \rangle$$

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4}(G_1)^a_{\mu\nu} (G_1)^{a,\mu\nu} + \frac{g_{s1}^2}{32\pi^2} \left(\frac{a_1}{f_1} - \theta_1 \right) (\tilde{G}_1)^a_{\mu\nu} (G_1)^{a,\mu\nu} \\ & - \frac{1}{4}(G_2)^a_{\mu\nu} (G_2)^{a,\mu\nu} + \frac{g_{s2}^2}{32\pi^2} \left(\frac{a_2}{f_2} - \theta_2 \right) (\tilde{G}_2)^a_{\mu\nu} (G_2)^{a,\mu\nu} \end{aligned}$$

$$V_\Sigma = -m_\Sigma^2 \text{Tr}(\Sigma_{12}\Sigma_{12}^\dagger) + \frac{\lambda}{2} [\text{Tr}(\Sigma_{12}\Sigma_{12}^\dagger)]^2 + \frac{\kappa}{2} \text{Tr}(\Sigma_{12}\Sigma_{12}^\dagger\Sigma_{12}\Sigma_{12}^\dagger)$$

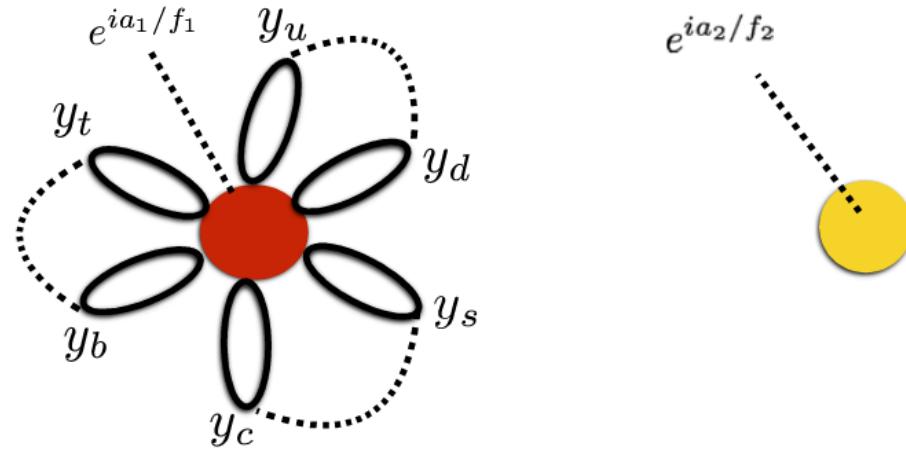
$$SU(3)_1 \times SU(3)_2 \rightarrow SU(3)_c$$

$SU(3) \times SU(3)$

Lagrangian below the scale M

contribution from UV instantons

$$\mathcal{L}_a = \Lambda_1^4 \cos \left(\frac{a_1}{f_1} - \bar{\theta}_1 \right) + \Lambda_2^4 \cos \left(\frac{a_2}{f_2} - \bar{\theta}_2 \right) + \frac{g_s^2}{32\pi^2} \left(\left(\frac{a_1}{f_1} - \bar{\theta}_1 \right) + \left(\frac{a_2}{f_2} - \bar{\theta}_2 \right) \right) G\tilde{G}$$



$$\bar{\theta}_{eff} = \left\langle \left(\frac{a_1}{f_1} + \bar{\theta}_1 \right) + \left(\frac{a_2}{f_2} + \bar{\theta}_2 \right) \right\rangle = 0$$

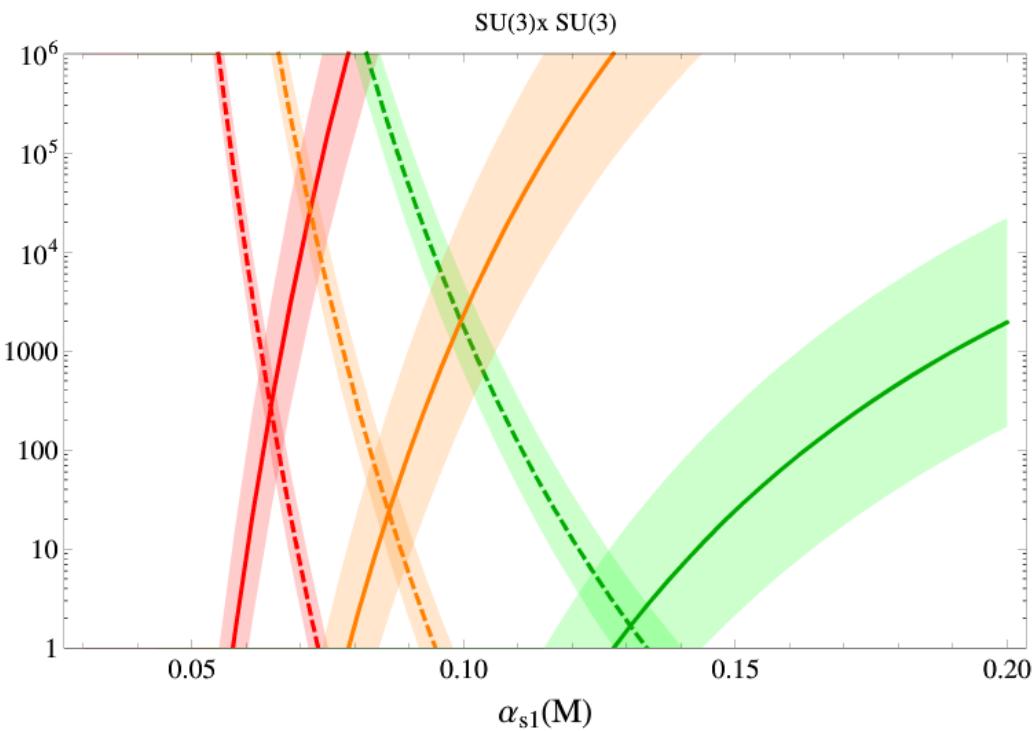
$SU(3) \times SU(3)$

$$\frac{1}{\alpha_s(\mu)} = \frac{1}{\alpha_{s_1}(\mu)} + \frac{1}{\alpha_{s_2}(\mu)}, \quad \mu = M$$

$$\Lambda_1^4 \simeq K \frac{4}{5} D[\alpha_{s_1}(M)] M^4$$

$$\Lambda_2^4 = \frac{4}{13} D[\alpha_{s_2}(M)] M^4$$

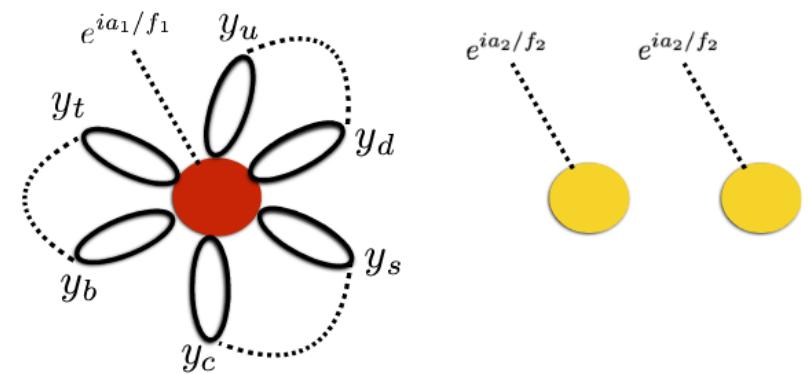
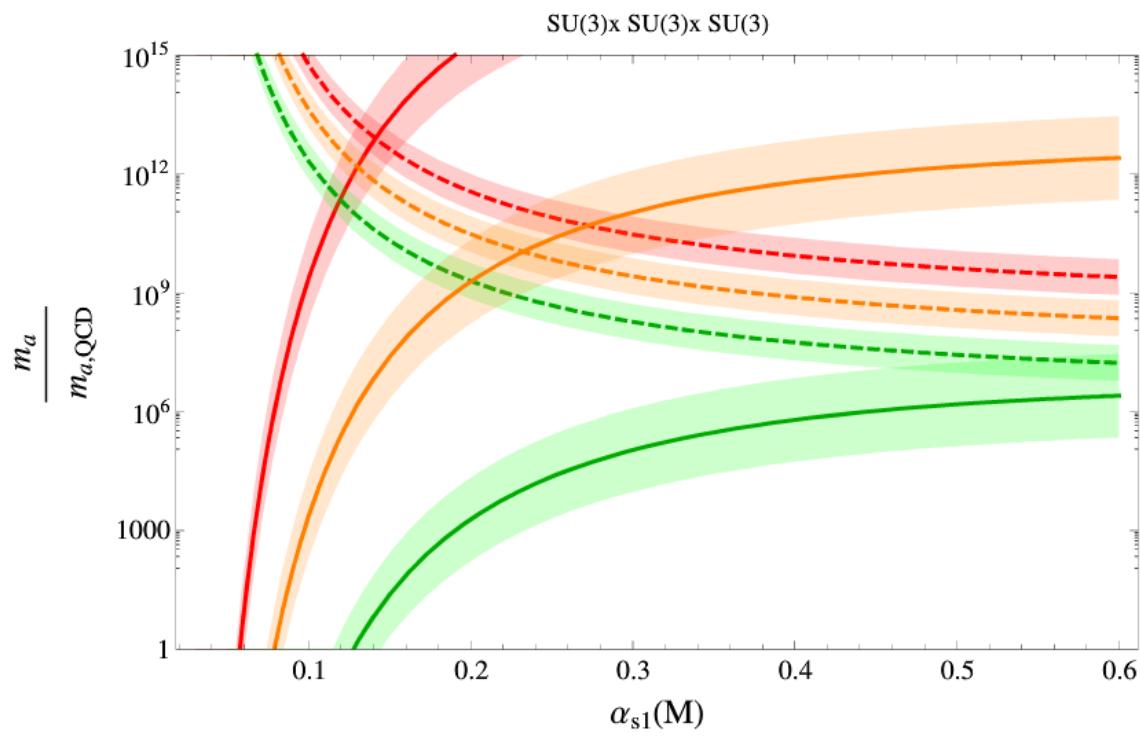
$$m_a \Big| \frac{m_a}{m_{a,\text{QCD}}}$$

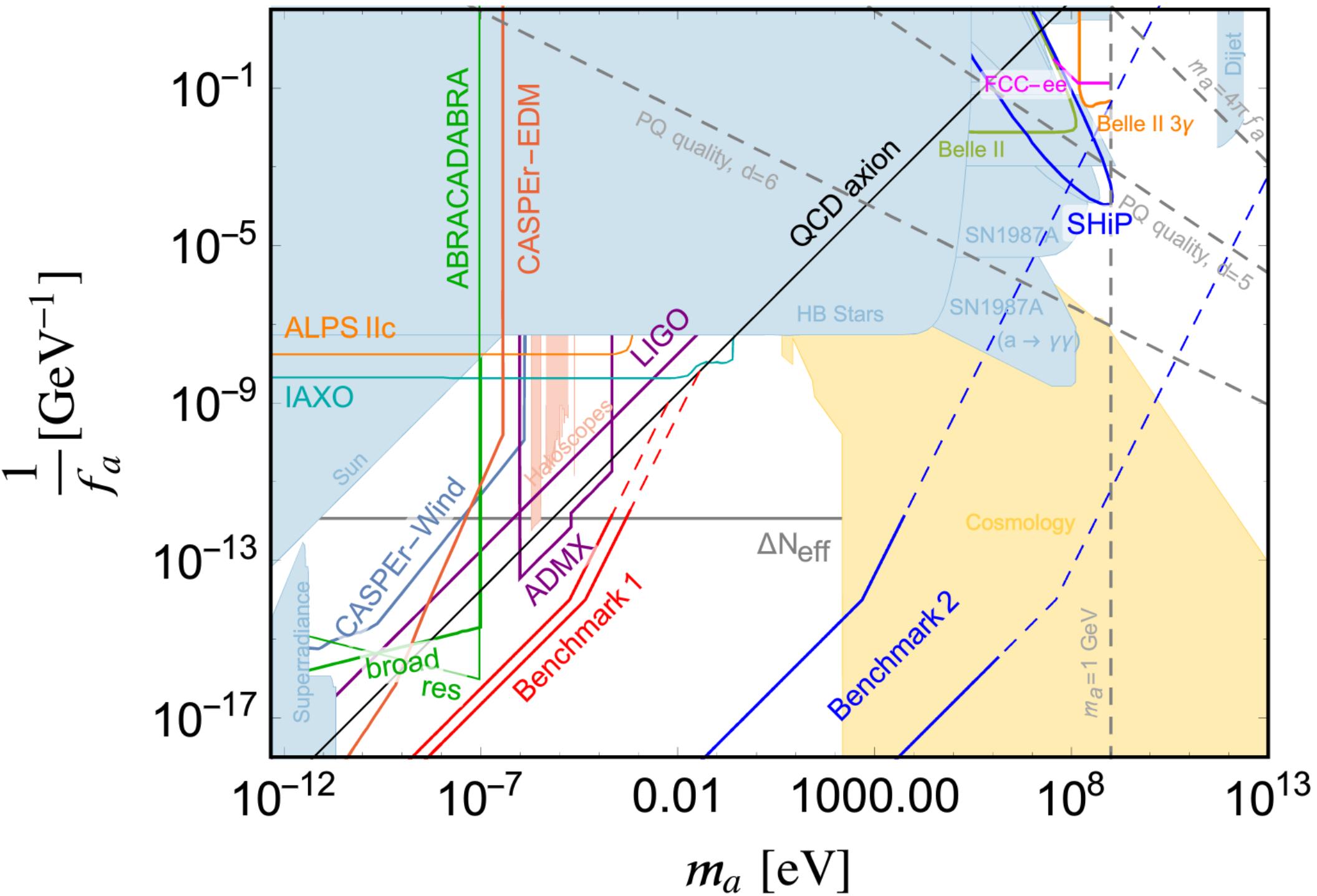


$$K = \left(\frac{y_u}{4\pi}\right) \left(\frac{y_d}{4\pi}\right) \left(\frac{y_c}{4\pi}\right) \left(\frac{y_s}{4\pi}\right) \left(\frac{y_t}{4\pi}\right) \left(\frac{y_b}{4\pi}\right) \approx 10^{-23}.$$

$SU(3) \times SU(3) \times SU(3)$

$$\frac{1}{\alpha_s(\mu)} = \frac{1}{\alpha_{s_1}(\mu)} + \frac{1}{\alpha_{s_2}(\mu)} + \frac{1}{\alpha_{s_3}(\mu)}, \quad \mu = M$$





Conclusion

The QCD axion dark matter model is a very well motivated target for new physics searches

In the simplest models this target is relatively narrow, couplings and cosmological abundance is set by one parameter

It is possible to dramatically enhance the couplings of the axion to photons in extended models

Cosmological mechanisms can be used to increase the range of f_a where axions can be viable dark matter candidates

Axions can be made heavy while preserving the solution to the strong CP problem using small instantons

Motivates casting a wide net in the hunt for axions