Ultralight Scalar DM with Repulsive Self-Interaction

JiJi Fan

Brown University

Chicago kicp workshop, April 11,2018

I will talk about scalar dark matter with mass between 10^{-22} eV and eV and with the leading self-interaction being repulsive.

In the literature, ultralight dark matter is sometimes used for fuzzy dark matter with mass 10^{-22} eV (Hu, Barkana, Gruzinov 2000; more recently, Hui, Ostriker, Tremaine, Witten 2016)

Based on on-going work with Djuna Croon (Dartmouth), Chen Sun (CAS/Dartmouth); work done by myself in Phys. Dark Univ. 14 (2016);

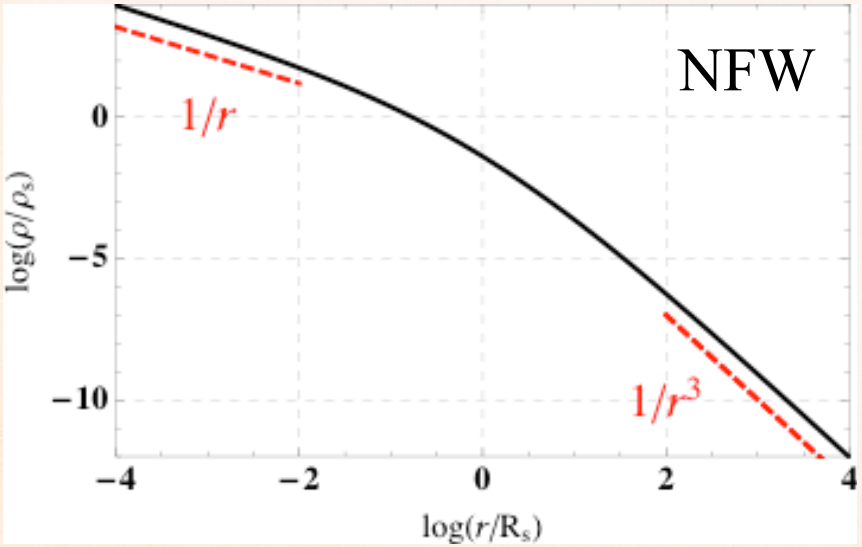
OUTLINE

- Motivation
- Possible particle physics models
- Cosmology and astrophysics: dark stars

Galactic scale dark matter puzzles: cusp-core problem

N-body dark matter-only simulations predict a ***cuspy*** profile

$$\rho_{\rm DM}(r) = \frac{\rho_s}{r/r_s\,(1+r/r_s)^2}$$



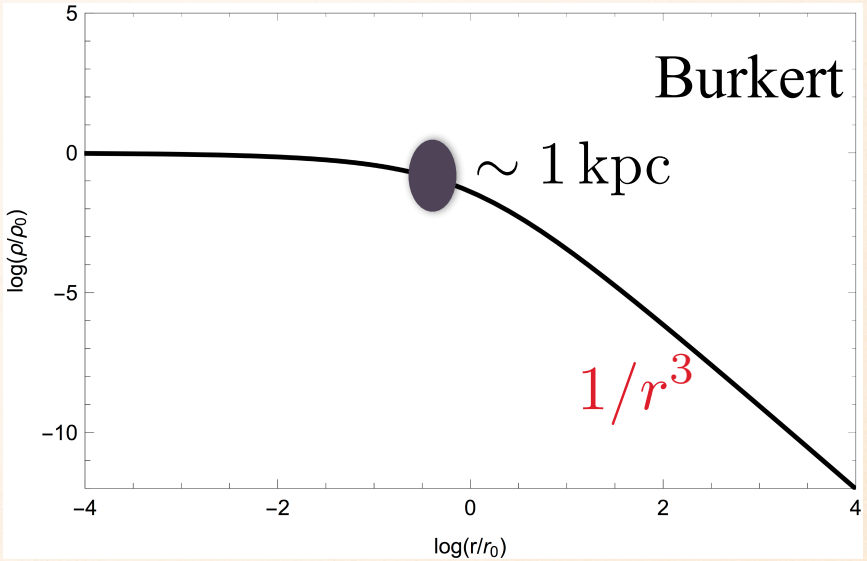
DM local density

radius

Navarro, Frenk, White 1996

The rotational curves of dwarf galaxies can be fitted better by a ***cored*** profile

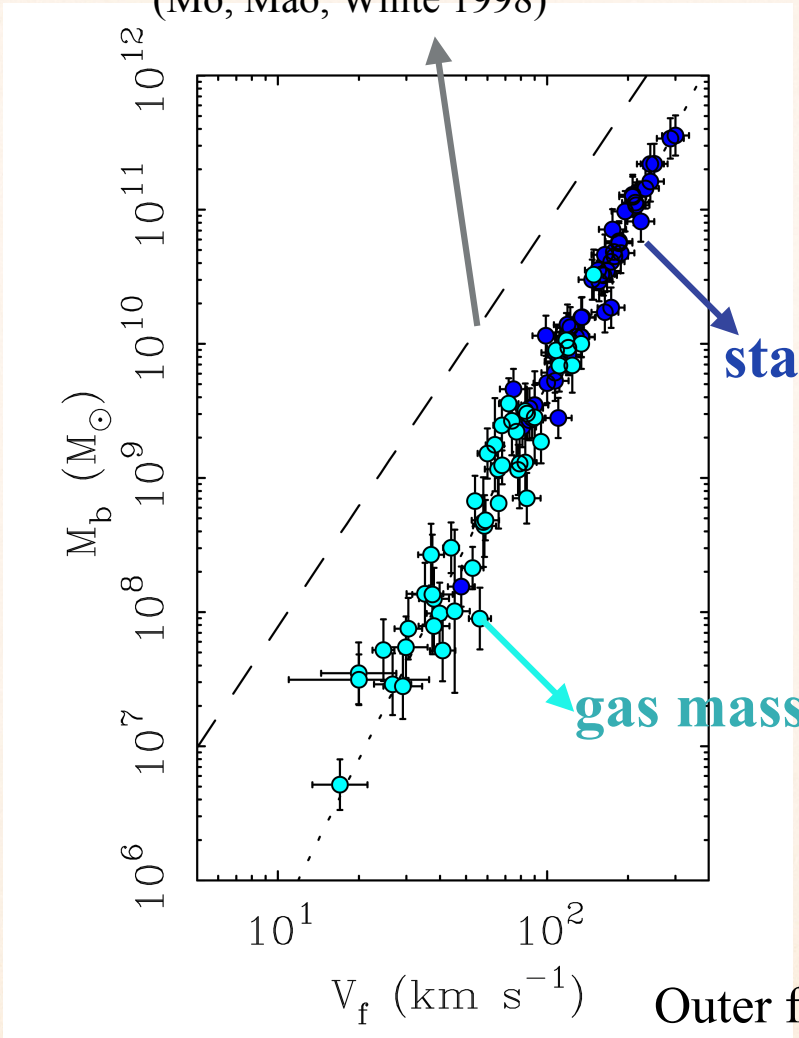
$$\rho_{\rm DM}(r) = \frac{\rho_0 r_0^3}{(r+r_0)(r^2+r_0^2)}$$



Burkert 1995

Galactic scale dark matter puzzles: Baryonic Tully-Fisher relation

"predication" of CDM based on a number of assumptions $M_b \propto v_f^3$
(Mo, Mao, White 1998)



Baryonic mass: sum of stars and gas

Famaey and McGaugh 2011

Outer flat rotational velocity

Puzzle:

Outer flat rotational velocity is mostly determined by DM.

How could it have such a (clean) scaling relation to the baryonic mass for a wide range of galaxies?

While none of these is convincing evidence for beyond ΛCDM, they do suggest that we may not fully understand the dark matter dynamics at the galactic and sub-galactic scale.

A class of model with potentially interesting galactic and sub-galactic pheno:

$$V(\phi) = \frac{1}{2}m^2\phi^2 + \lambda\phi^4 \qquad \lambda > 0$$

$$\lambda_{dB} = \frac{2\pi}{mv} > \sqrt[3]{\frac{m}{\rho}} \Rightarrow m < 1\,\mathrm{eV}\left(\frac{10^{-3}}{v}\right)^{3/4}\left(\frac{\rho}{1.3\times 10^{-6}\,\mathrm{GeV/cm^3}}\right)^{1/4},$$

De Broglie wavelength

Inter-particle spacing

average density of dark matter in galaxy

Large occupation number: treat as a field

$$V(\phi) = \frac{1}{2}m^2\phi^2 + \lambda\phi^4 \qquad \lambda > 0$$

$$\lambda_{dB} = \frac{2\pi}{mv} > \sqrt[3]{\frac{m}{\rho}} \Rightarrow m < 1\,\mathrm{eV}\left(\frac{10^{-3}}{v}\right)^{3/4}\left(\frac{\rho}{1.3\times10^{-6}\,\mathrm{GeV/cm^3}}\right)^{1/4},$$

De Broglie wavelength

Inter-particle spacing

Large occupation number: treat as a field

$$\lambda_{dB} = \frac{2\pi}{mv} \lesssim 1\,\mathrm{kpc} \Rightarrow m \gtrsim 5\times10^{-23}\,\mathrm{eV}\left(\frac{10^{-3}}{v}\right).$$

avoid modification of large scale structure

A class of model with potentially interesting galactic and sub-galactic pheno:

$$V(\phi) = \frac{1}{2} m^2 \phi^2 + \lambda \phi^4 \qquad \lambda > 0$$

Large ***repulsion*** provides pressure to balance against gravitational attraction and allow for a kpc size dark matter core:
"repulsive dark matter" Goodman and "fluid dark matter" Peebles 2000:

If the leading interaction term is a special type of sextic ***repulsive*** interaction, may explain baryonic Tully-Fisher relation,
"Superfluid dark matter" Berezhiani and Khoury 2015;

$$V(\Phi) = \frac{\Lambda^4}{6(\Lambda_c^2 + |\Phi|^2)^6} \left(|\partial_\mu \Phi|^2 + m^2 |\Phi|^2 \right)^3$$

Explore the scenario with ultralight scalar dark matter and repulsive self-interaction in both particle physics and cosmology/astrophysics.

Models

Effective field theory

Light scalars are dangers in QFT and subject to huge quantum corrections.

The scalar, ϕ, is a pseudo-Nambu-Goldstone boson with an ***approximate shift symmetry***

$$V(\phi) = m^2 \left(\phi^2 + \frac{\lambda_3}{3! f} \phi^3 + \frac{\lambda_4}{4! f^2} \phi^4 + \cdots \right)$$

shift symmetry breaking spurion

decay constant

Expansion parameter: ϕ/f

Non-relativistic effective theory

Another *expansion parameter*: k^2/m^2 or in position space, ∇^2/m^2 Namjoo, Guth, Kaiser 2017;

For virialized system, kinetic energy ~ potential energy

$$\left(\frac{\phi}{f}\right)^2 \sim \frac{\nabla^2}{m^2}$$

No odd power interaction term, leading interaction in the NR limit

$$\frac{\lambda_4^{\rm eff}}{16 f^2}|\psi|^4,$$

$$\lambda_4^{\rm eff} = \lambda_4 - \frac{5}{3}\lambda_3^2$$

How to get repulsion?

Models with attractive self-interaction:

QCD axion $V(\phi) = \Lambda^4 \left(1 - \cos\left(\frac{\phi}{f}\right)\right)$

$$-\frac{m^2}{24 f^2}\phi^4,$$

Difficult to have a BEC with galactic scale correlation length as argued by Sikivie et.al.

Guth, Hertzberg and Prescod-Weinstein; Eby, Suranyi, Vaz and Wijewardhana 2014

How to get repulsion?

Models with attractive self-interaction:

Dilaton: pNGB from breaking of a ***non***-compact global symmetry (spontaneous breaking of conformal symmetry)

$$\mathcal{L}_{\rm CFT} \to \mathcal{L}_{\rm CFT} + \lambda_{\mathcal{O}} \mathcal{O}(x)$$ explicit breaking

Calculable models: small expansion parameter exists

- Nearly marginal operator

$$V(\phi) = \frac{1}{2} m^2 \phi^2 + \frac{5}{3!} \frac{m^2}{f} \phi^3 + \frac{11}{4!} \frac{m^2}{f^2} \phi^4 + \cdots$$

$$\lambda_4^{\rm eff} = \lambda_4 - \frac{5}{3} \lambda_3^2 = -\frac{92}{3}$$

How to get repulsion?

Models with attractive self-interaction:

Dilaton: breaking of a ***non***-compact global symmetry (conformal symmetry)

$$\mathcal{L}_{\rm CFT} \to \mathcal{L}_{\rm CFT} + \lambda_{\mathcal{O}} \mathcal{O}(x)$$ explicit breaking

Calculable models: small expansion parameter exist

- Relevant operator (dimension Δ_o) with small dimensionful coefficient

$$V(\phi) = \frac{1}{2} m^2 \phi^2 + \frac{\Delta_{\mathcal{O}} + 1}{3!} \frac{m^2}{f} \phi^3 + \frac{3 - 2\Delta_{\mathcal{O}} + \Delta_{\mathcal{O}}^2}{4!} \frac{m^2}{f^2} \phi^4 + \cdots$$

$$\lambda_4^{\rm eff} = -\frac{2}{3}\Delta_{\mathcal{O}}^2 - \frac{16}{3}\Delta_{\mathcal{O}} + \frac{4}{3} < 0$$

How to get repulsion?

Multiple (at least three) cosine terms in potential

$$V(\phi) = -\cos\left(a_1 \frac{\phi}{f}\right) + \cos\left(a_2 \frac{\phi}{f}\right) + \cos\left(a_3 \frac{\phi}{f}\right)$$

Positive mass: $a_1^2 - a_2^2 - a_3^2 > 0$

Repulsion: $-a_1^4 + a_2^4 + a_3^4 > 0$

Could be realized in the model with an axion-like particle from a 5D gauged U(1): fifth dimension compactified on a circle and one-loop effective potential generated by 5D matter (boson or fermion) charged under the U(1).

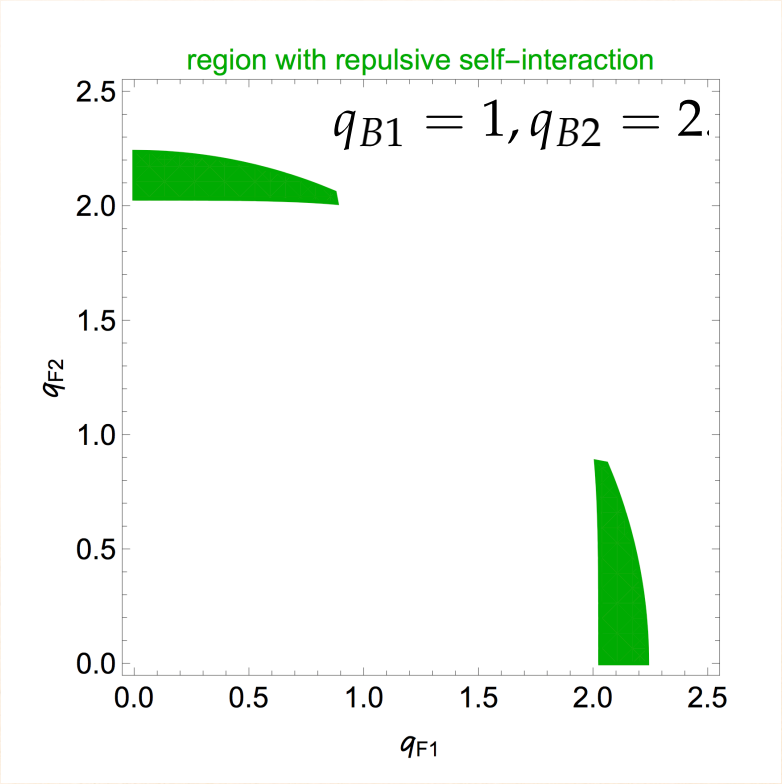
Say n bosons and m fermions,

$$V(\phi) \supset -\Lambda^4 \left(\sum_{i=1}^{n} \cos\left(\frac{q_{Bi}\phi}{f}\right) - \sum_{i=1}^{m} \cos\left(\frac{q_{Fi}\phi}{f}\right) \right)$$

bosons fermions

$$\sum_{i=1}^{n} q_{Bi}^2 > \sum_{i=1}^{m} q_{Fi}^2$$

$$\sum_{i=1}^{n} q_{Bi}^4 < \sum_{i=1}^{m} q_{Fi}^4.$$



Cosmological and Astrophysical Implications

Cosmological constraint

Linearized dark matter density perturbation $\delta\rho/\bar{\rho} \propto r$

$$\ddot{r} + 2H\dot{r} + \left(\frac{k^2}{4m^2a^2} + \frac{\lambda_4^{\rm eff}\bar{\rho}}{4m^2f^2} \right) \frac{k^2 r}{a^2} = 4\pi G\bar{\rho} r,$$

Hubble friction

quantum pressure

pressure from repulsion; absent in the equation for ordinary cold dark matter

gravity source

Cosmological constraint

Linearized dark matter density perturbation $\delta\rho/\bar{\rho} \propto r$

$$\ddot{r} + 2H\dot{r} + \left(\frac{k^2}{4m^2a^2} + \frac{\lambda_4^{\rm eff}\bar{\rho}}{4m^2f^2} \right) \frac{k^2 r}{a^2} = 4\pi G\bar{\rho} r,$$

Hubble friction

quantum pressure

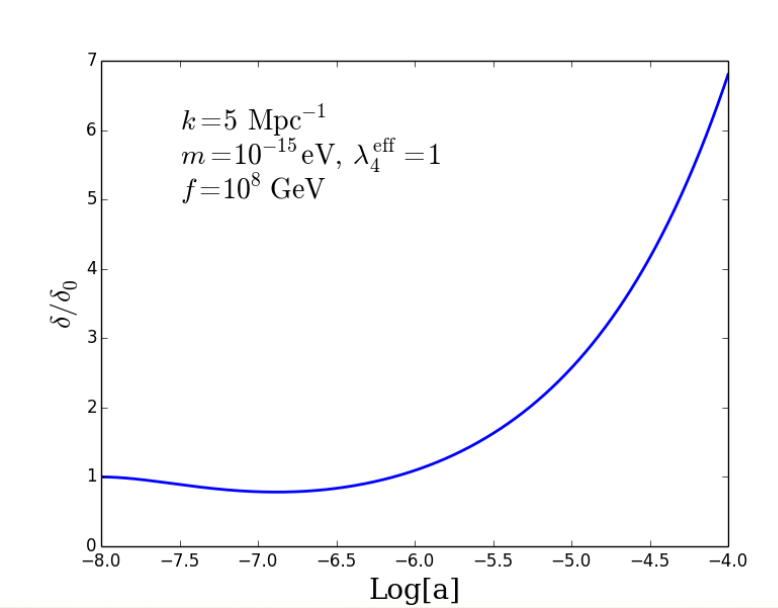
pressure from repulsion; absent in the equation for ordinary cold dark matter

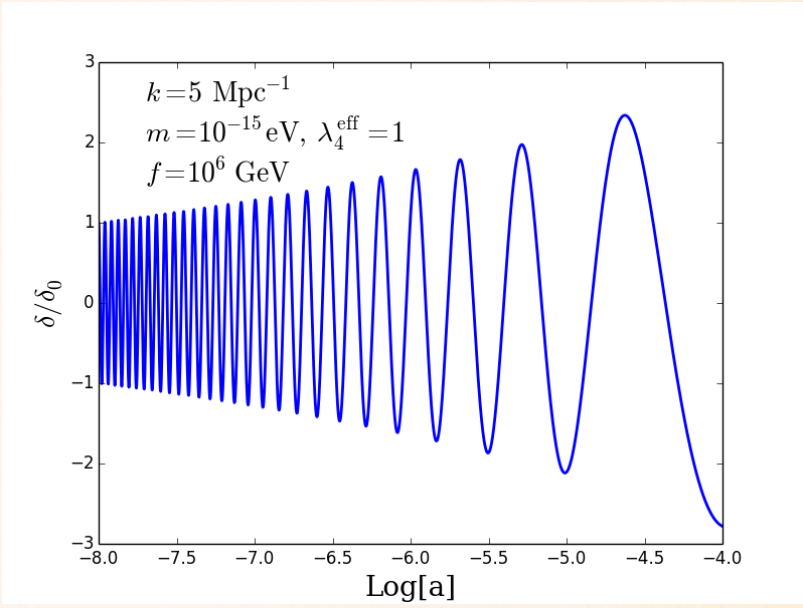
gravity source

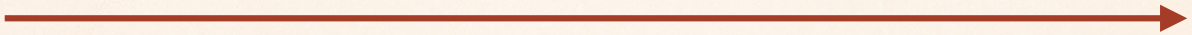
New Jeans scale: below it, density pert. suppressed

$$k_\lambda = \sqrt{\frac{2}{\lambda_4^{\rm eff}}} \frac{mfa}{M_p} = 2.7 \left(\frac{m}{10^{-20}\,{\rm eV}} \right) \left(\frac{f}{10^{13}\,{\rm GeV}} \right) \sqrt{\frac{1}{\lambda_4^{\rm eff}}} \left(\frac{a}{a_{\rm eq}} \right) {\rm Mpc}^{-1}.$$

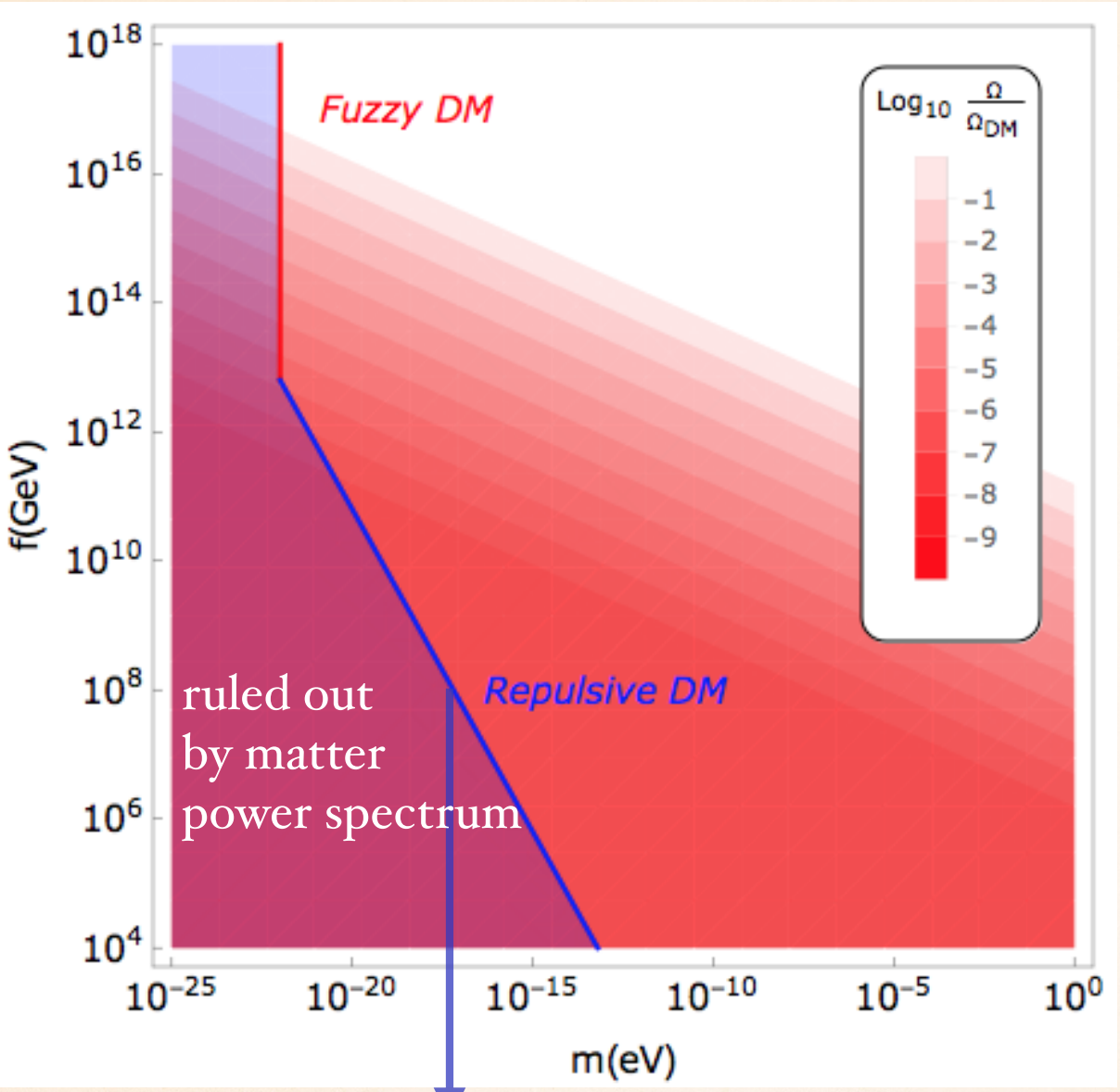
Numerical solution:





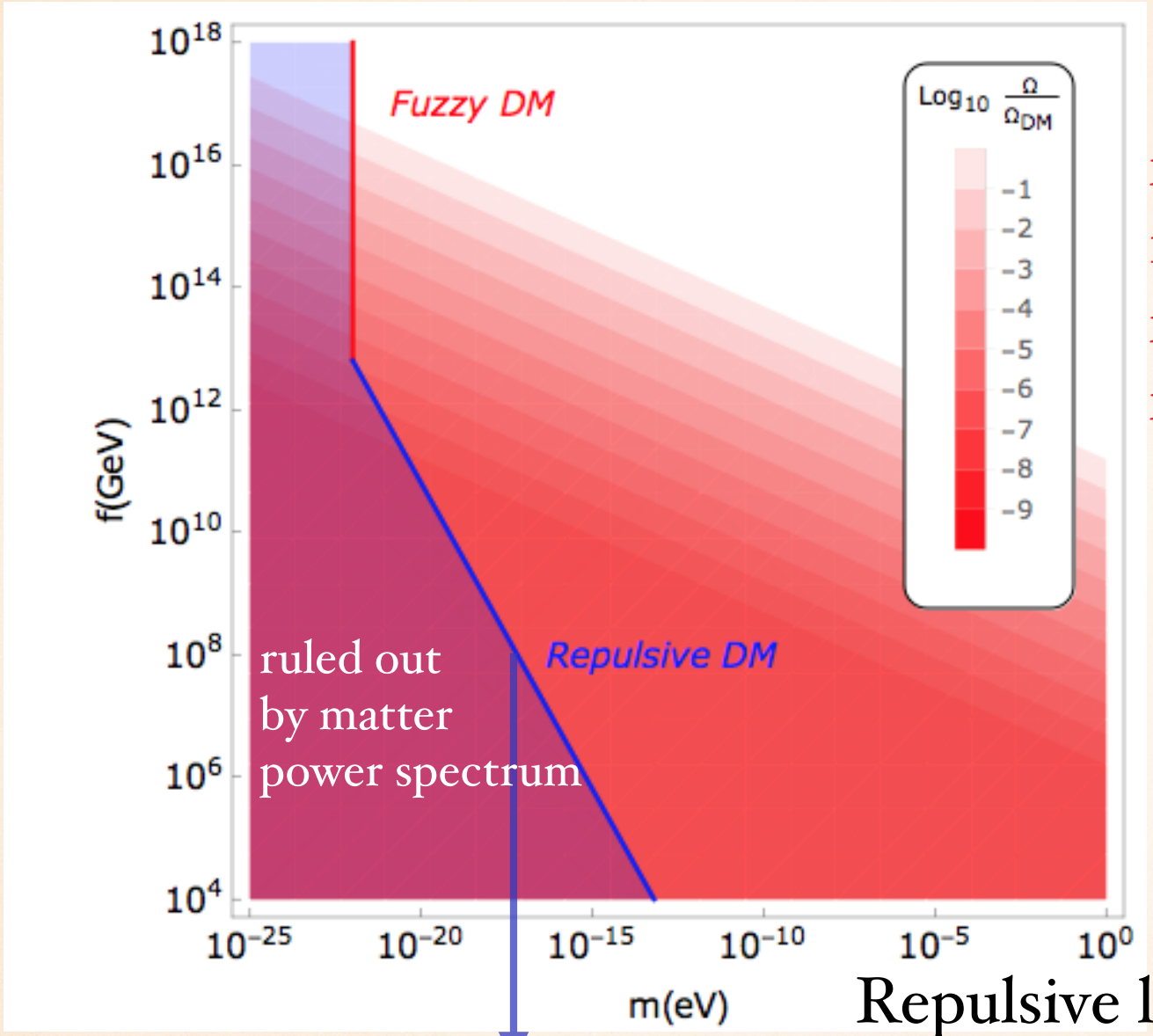


increase repulsion pressure



relic abundance
from
mis-alignment
mechanism

$$k_\lambda^{\mathrm{eq}} = 1\,\mathrm{Mpc}^{-1}$$



relic abundance from mis-alignment mechanism

$$k_\lambda^{\mathrm{eq}} = 1\,\mathrm{Mpc}^{-1}$$

Repulsive large enough to modify galactic scale dynamics $\Longrightarrow$ small relic abundance from mis-alignment

Sub-galactic: dark stars

Now move on to discuss the possibility of smaller objects: dark stars.

Will not discuss the formation mechanisms: some simulations yet still many open questions, e.g., formation through gravitational cooling, Seidel and Suen 1993; for QCD axion, axion minicluster Kolb and Tkachev from 1993;

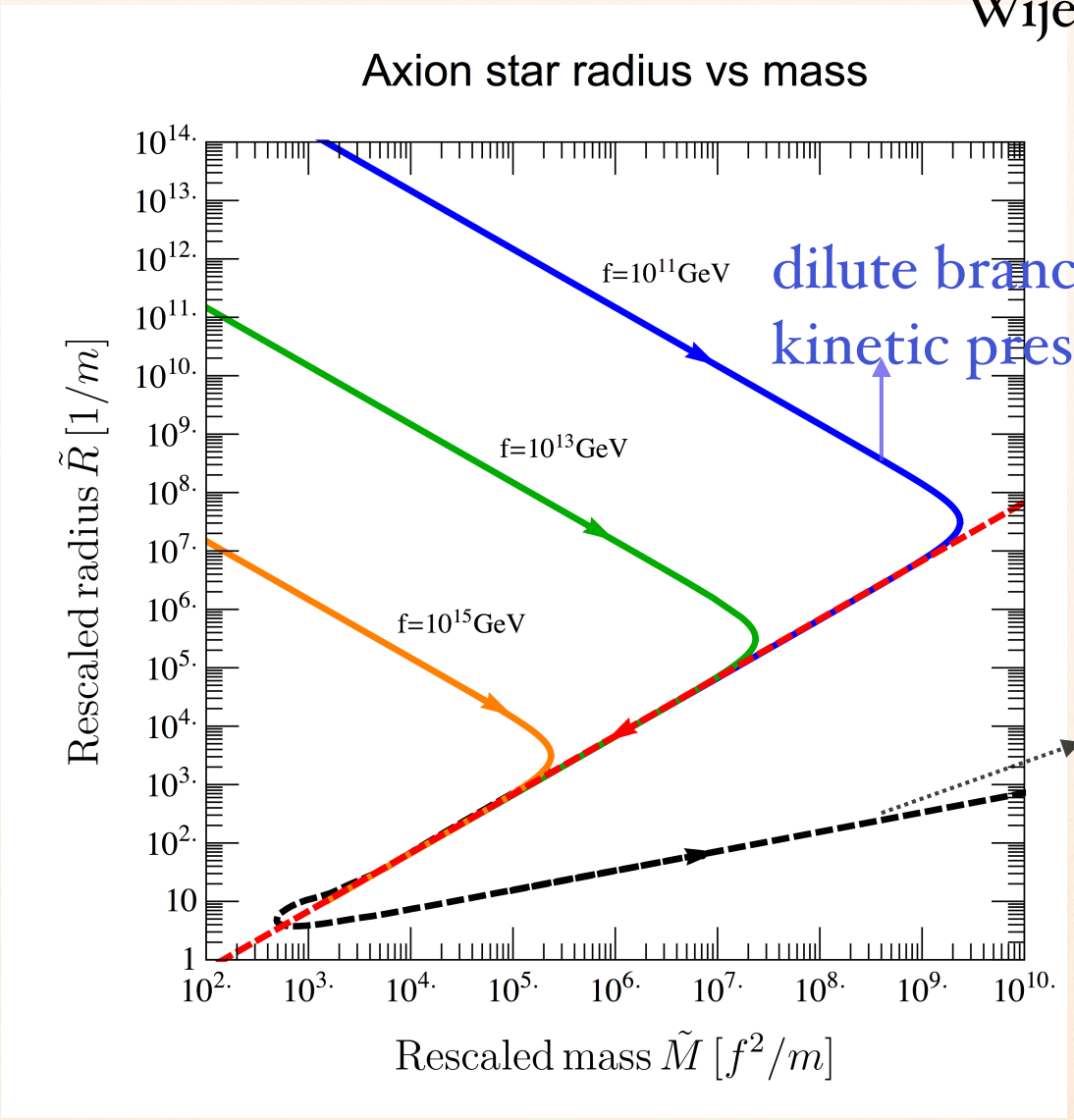
Will focus on solutions to equations of motion.

Sub-galactic: dark stars

A lot of recent work on ***QCD axion*** stars

$10^{-11} M_\odot$

Chavanis; Visinelli, Baum, Redondo, Freese, Wilczek; Schiappacasse, Hertzberg; Eby, Suranyi, Wijewardhana 2017

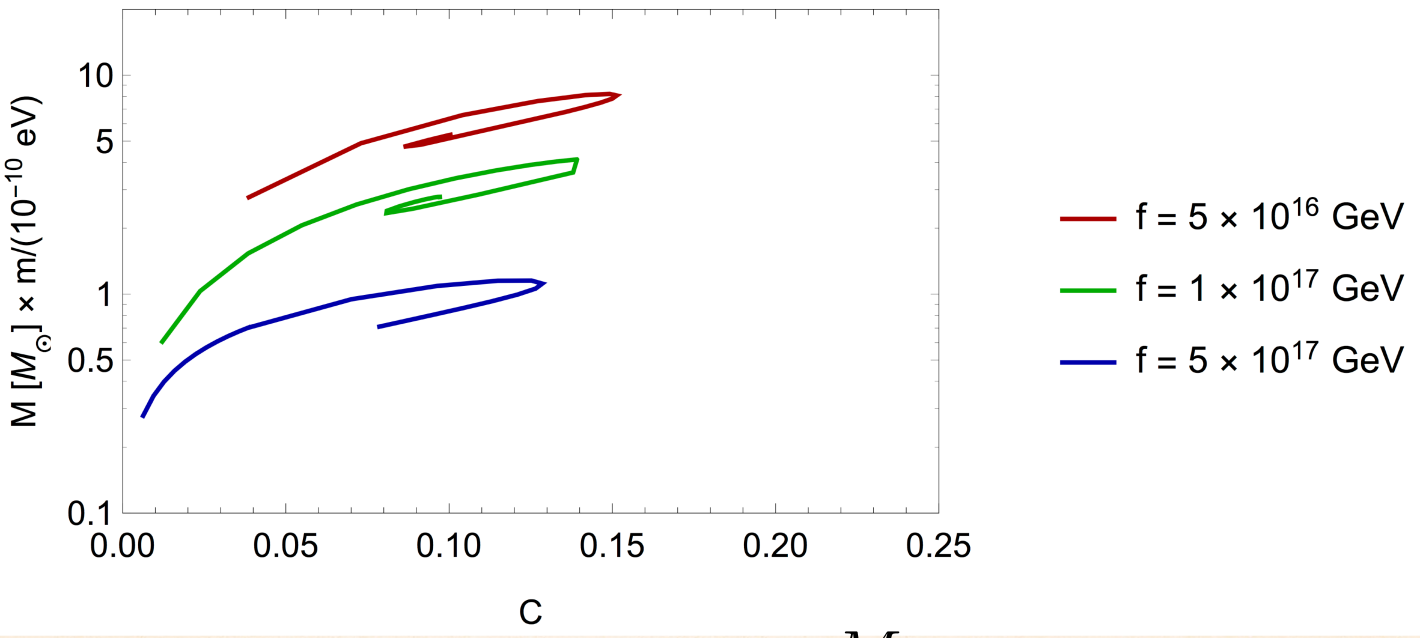


Spherically symmetric boson star from repulsion:
Colpi, Shapiro, Wassermann 1986

$$V(\phi) = \frac{1}{2}m^2\phi^2 + \lambda\phi^4 \qquad \lambda > 0$$

Solve coupled Einstein-Klein-Gordan equations of motion

mass



Compactness $C = \frac{M}{R}$

Spherically symmetric star solution from repulsion:
Colpi, Shapiro, Wassermann 1986

$$V(\phi) = \frac{1}{2}m^2\phi^2 + \lambda\phi^4 \qquad \lambda > 0$$

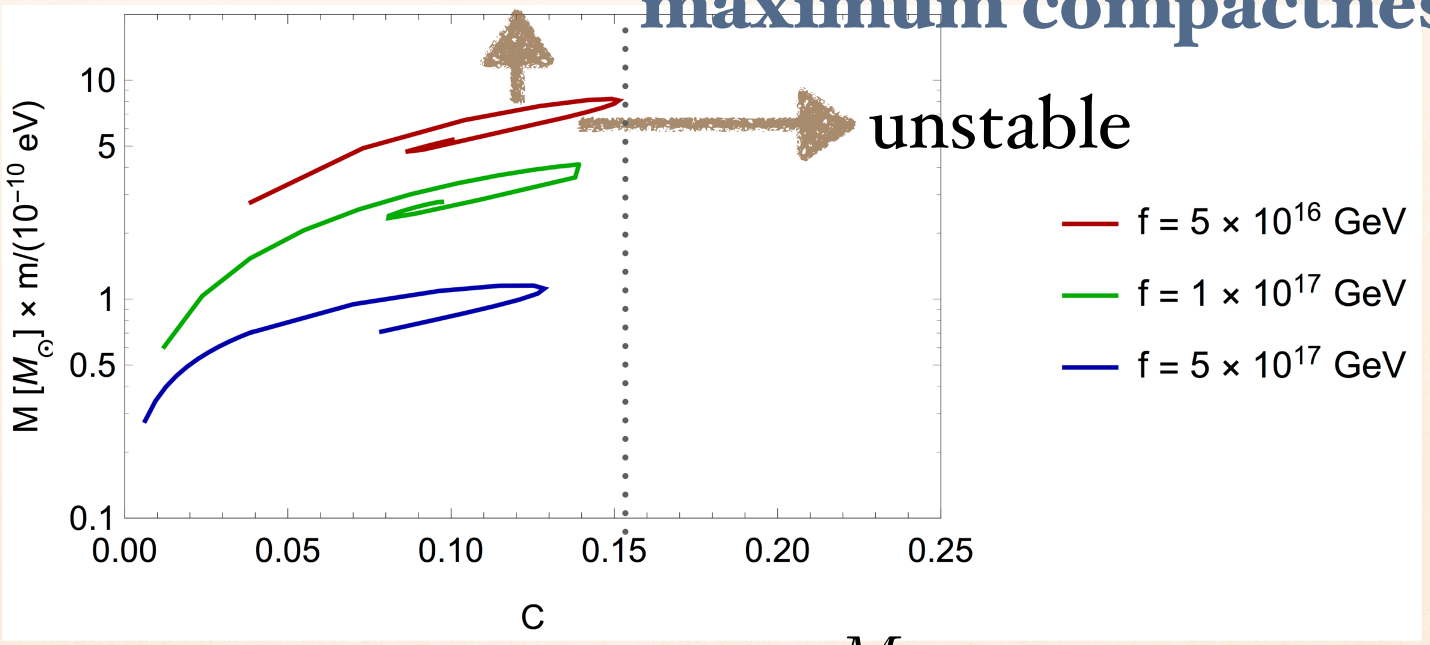
Solve coupled Einstein-Klein-Gordan equations of motion

stable

maximum compactness ~ 0.15

unstable

mass

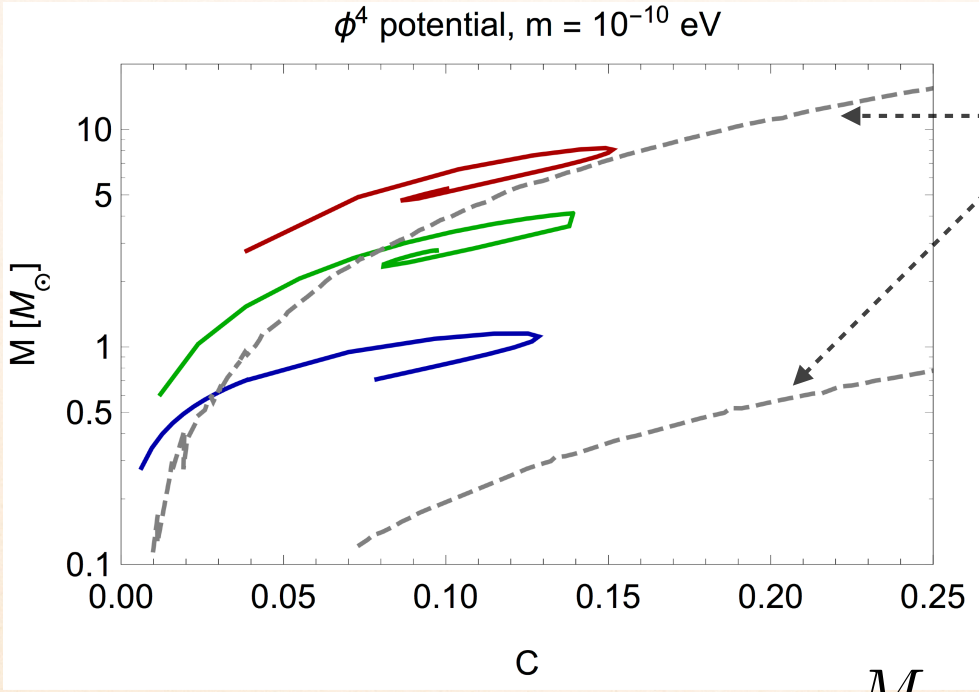


Compactness $C = \frac{M}{R}$

Spherically symmetric star solution from repulsion:
Colpi, Shapiro, Wassermann 1986

$$V(\phi) = \frac{1}{2}m^2\phi^2 + \lambda\phi^4 \qquad \lambda > 0$$

Solve coupled Einstein-Klein-Gordan equations of motion



mass

LIGO frequency range for GW events of equal mass exotic compact objects (ECO)

f = 5 × 10^{16} GeV

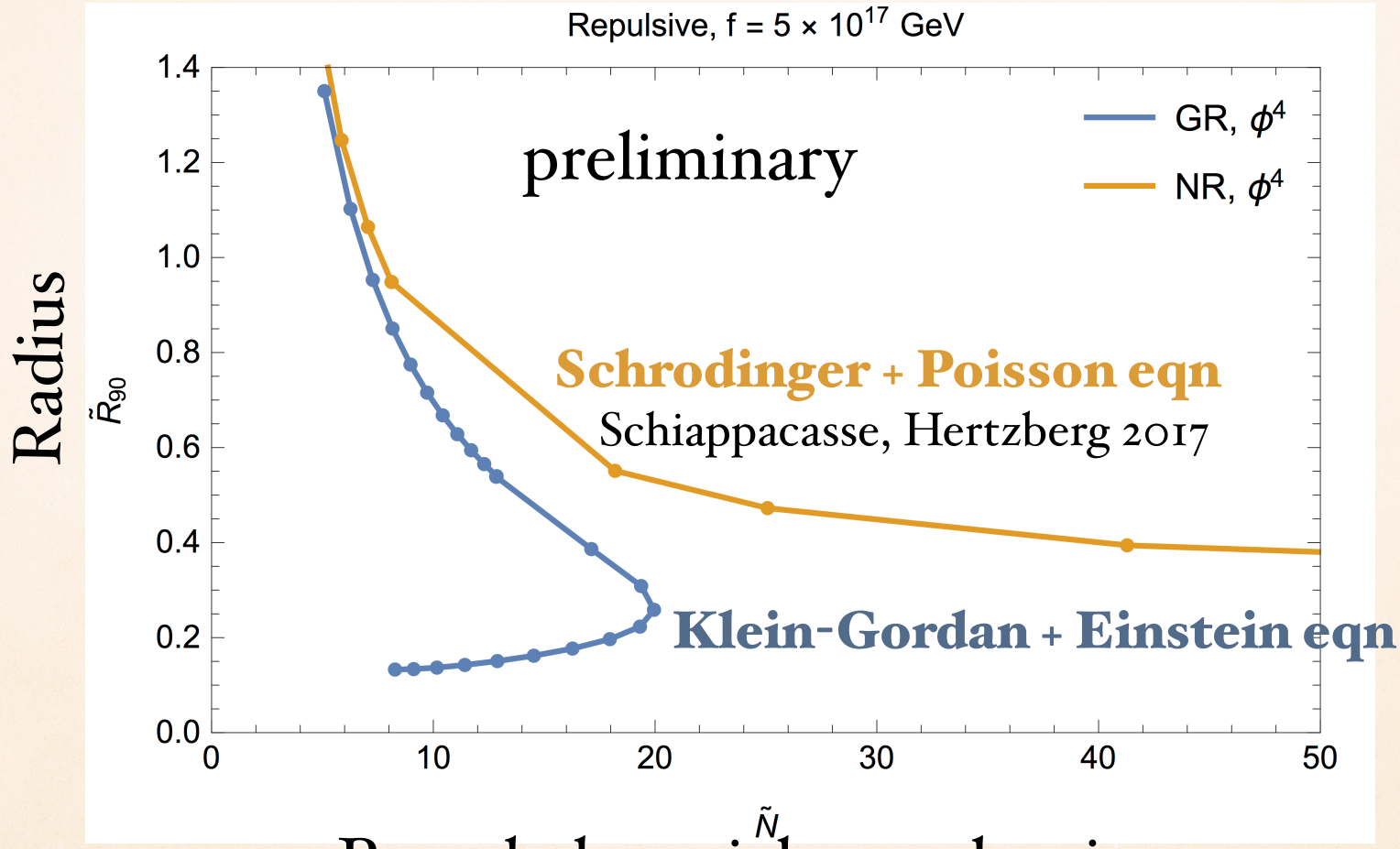
f = 1 × 10^{17} GeV

f = 5 × 10^{17} GeV

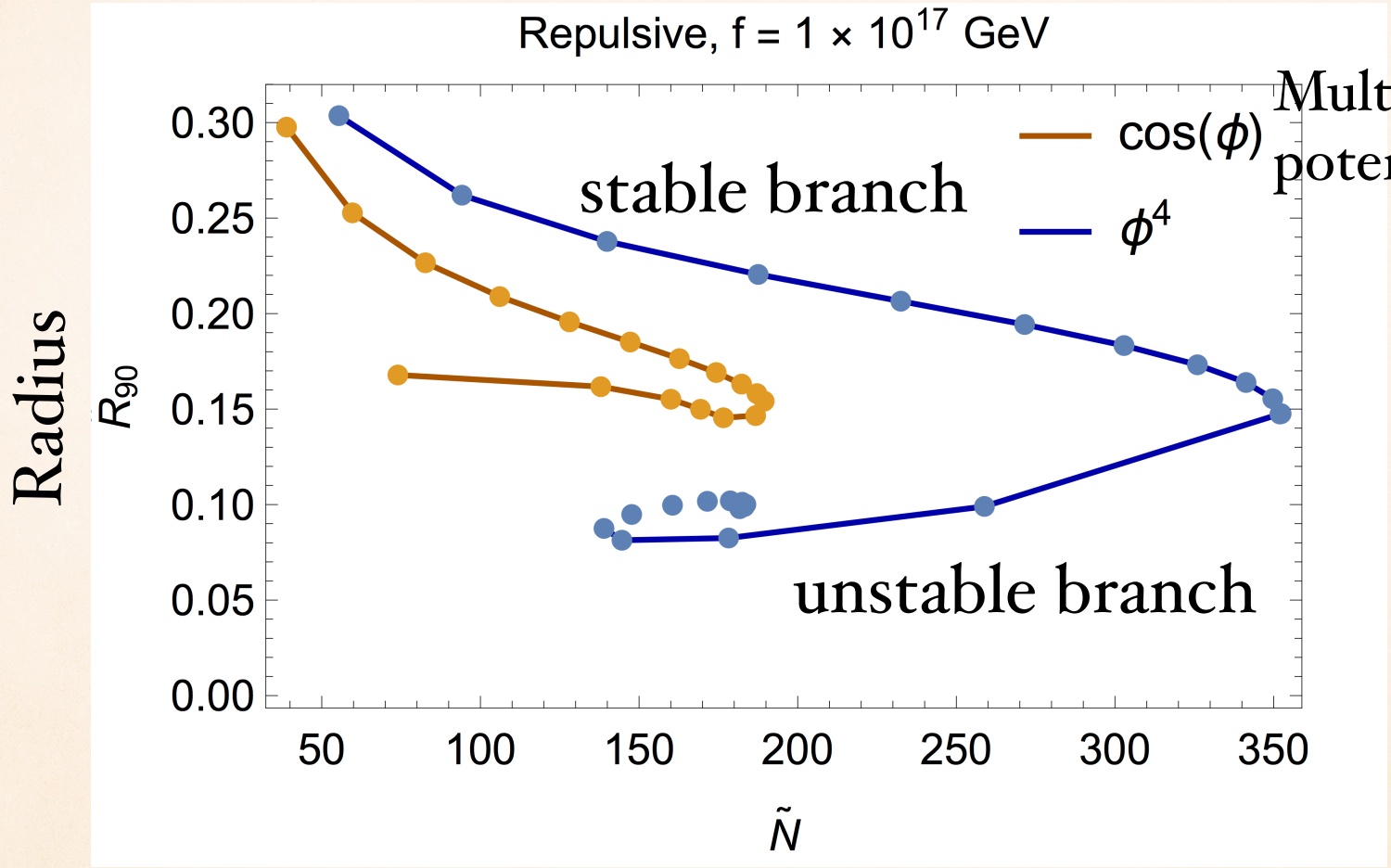
GW detection for ECO formed of heavy bosons with mass keV-GeV:
Giudice et.al, 2016;
Croon et.al, 2018.

Compactness $C = \frac{M}{R}$

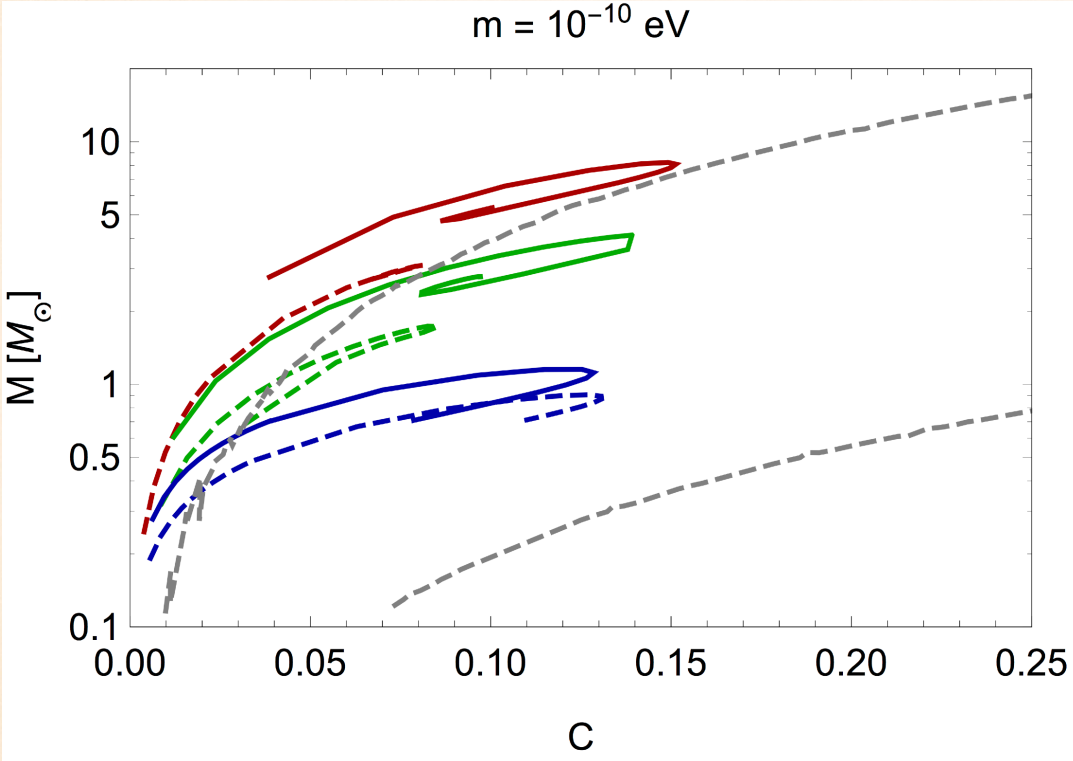
Important to keep the relativistic corrections to find the star solution at large density



Higher-order self-interactions could change the qualitative behavior significantly



Rescaled particle number in star



preliminary

ϕ^4 cos(ϕ), f = 5 × 10^{16} GeV

ϕ^4 cos(ϕ), f = 1 × 10^{17} GeV

ϕ^4 cos(ϕ), f = 5 × 10^{17} GeV

SUMMARY

- Many puzzles of dark matter dynamics at the galactic scale and below;
- For ultralight scalar dark matter, $\lambda\phi^4$ theory is only an approximation to a pNGB model. It is non-trivial to get a pNGB with repulsive self-interaction;
- For repulsion large enough to have dynamics at the galactic scale, the relic abundance from mis-alignment is tiny (need to invoke other mechanisms);
- Even smaller scale compact objects, like dark stars, are possible. Yet the properties (mass-compactness relation) could be very different from the classic results derived in 1986 based on $\lambda\phi^4$.

Thank you!

Back-up

Unsuccessful Attempts 3: little dark matter

Potential from collective symmetry breaking

$$V(\phi,\xi) = \lambda_1 f^2 \left|\xi + \frac{[\phi\phi]}{f} + \cdots\right|^2 + \lambda_2 f^2 \left|\xi - \frac{[\phi\phi]}{f} + \cdots\right|^2,$$

ξ, ϕ : two sets of pNGB

Shift symmetries for each term

$$\phi \to \phi + c,$$
$$\xi \to \xi - \frac{[\phi c] + [c\phi]}{f} \quad \text{or} \quad \xi \to \xi + \frac{[\phi c] + [c\phi]}{f},$$

Integrating out heavy ξ ,

$$V_{\text{eff}}(\phi) = \frac{4\lambda_1\lambda_2}{\lambda_1+\lambda_2}[\phi\phi]^2 + \cdots \qquad m_\phi^2 \sim \frac{\lambda_1^2+\lambda_2^2}{16\pi^2} f^2$$

Naive dimensional analysis: $\left(\frac{1}{2}m^2\phi^2 + \frac{\lambda_3}{3!}\frac{m^2}{f}\phi^3 + \frac{\lambda_4}{4!}\frac{m^2}{f^2}\phi^4\right)$

$$\lambda_3 = 0, \quad \lambda_4 \sim (4\pi)^2, \quad \frac{m}{f} \sim \frac{1}{4\pi}.$$

Bullet Cluster constraint: collisions of two clusters with a large relative velocity, usually larger than the DM velocity dispersion. The self-interaction bound should still apply

$$\sigma/m \lesssim 1\,\mathrm{cm}^2/g \approx 2\,\mathrm{barn/GeV}$$

$$\lambda_4^{\mathrm{eff}} \left(\frac{m}{f}\right)^2 \lesssim 10^{-11} \left(\frac{m}{\mathrm{eV}}\right)^{3/2}$$

For little DM, NDA determines that the bound could only be satisfied for DM with mass at or above 10 MeV; or small λ.