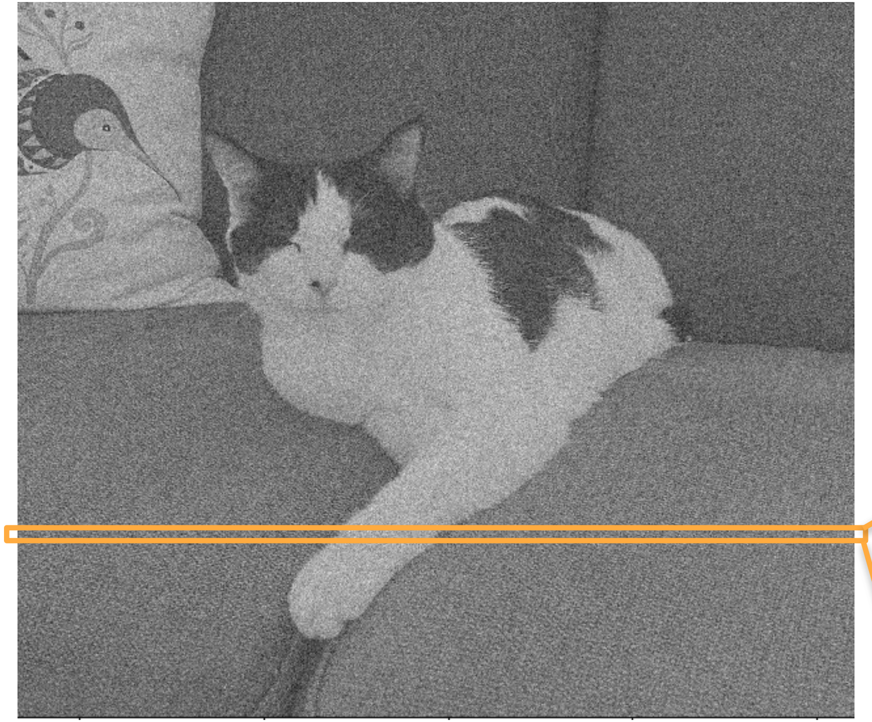
Noise, NEP, NET

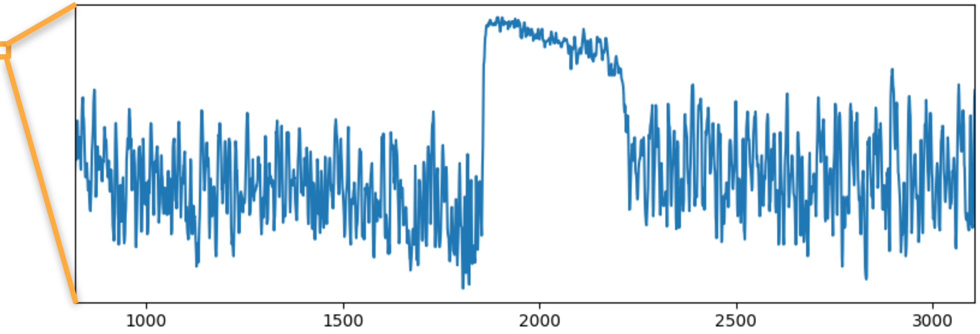
2022-08-09

What is noise?



Signal
+ Noise

Signal

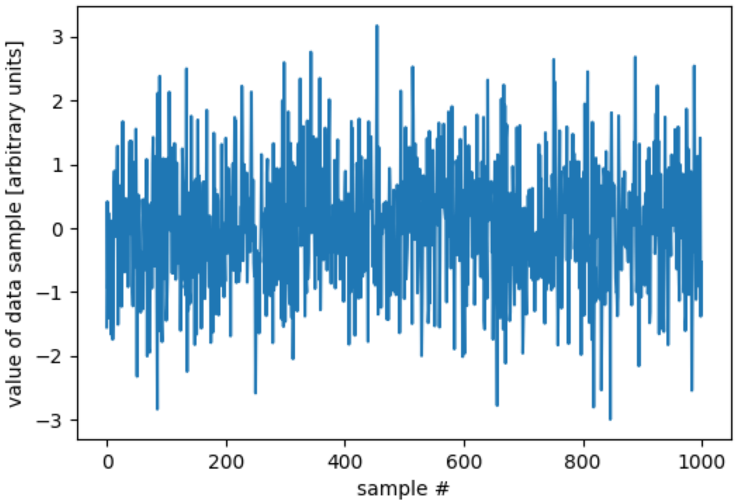


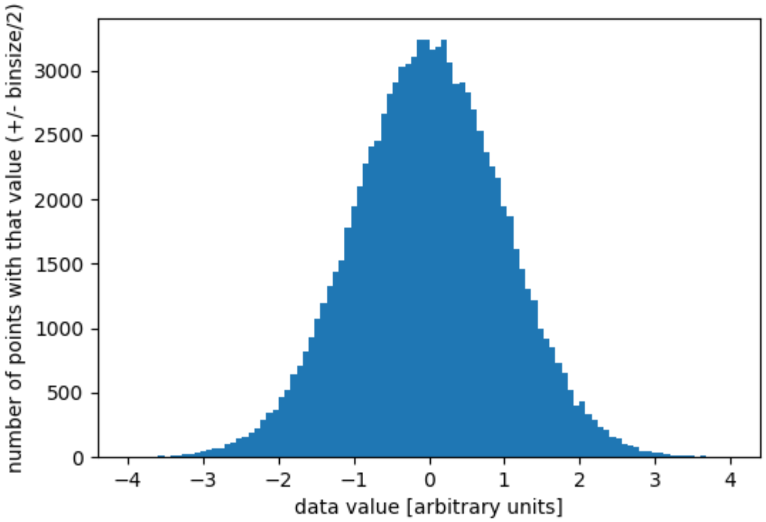
What is noise?

Three broad categories of “stuff in your data that isn’t your signal”:

1. Truly random fluctuations, usually from quantum-level processes.
    - Not coherent with anything, so it always ***integrates*** down. Really, truly noise (in the physics sense).
2. Non-random fluctuations with a basis in some macroscopic phenomenon (often related to something human-made), but which do not live in the exact same basis as your signal.
    - Can be made to behave like noise and integrate down, but depends on how lucky and/or clever you are. Trying to keep signal in a basis that is fully separable from this stuff is a big part of experiment and observing design.
3. Non-random fluctuations that live in the exact same basis as your signal. 😱

Aside #1: “Gaussian”

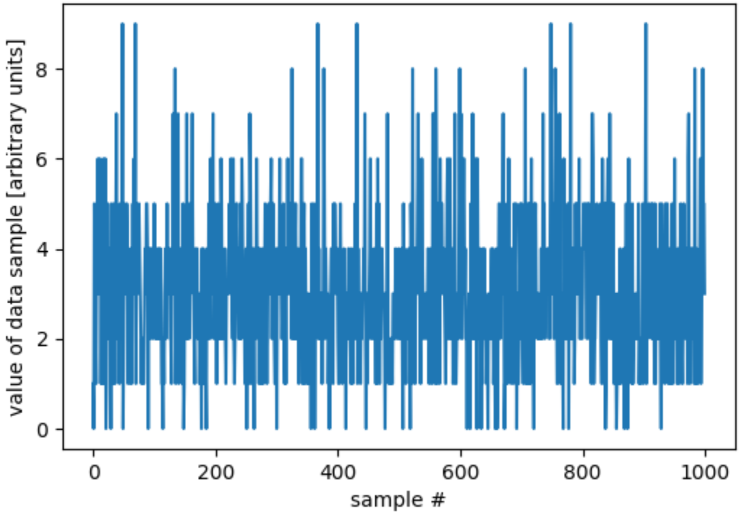


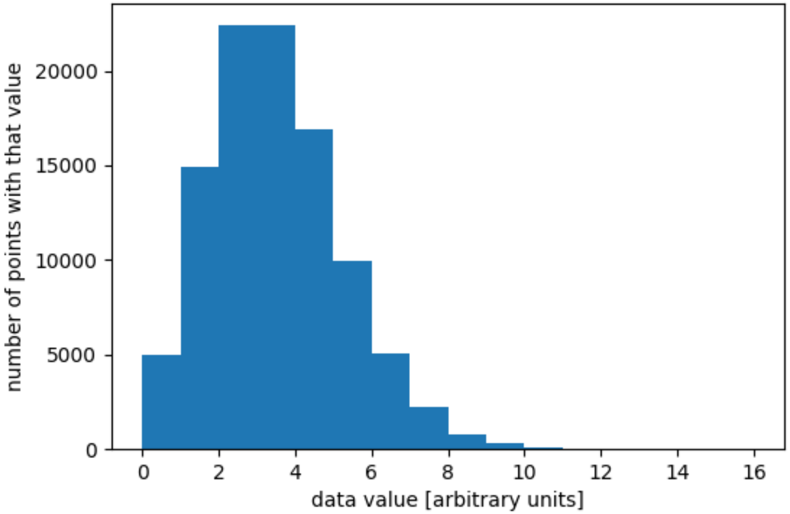


fakedata = np.random.randn(100000)

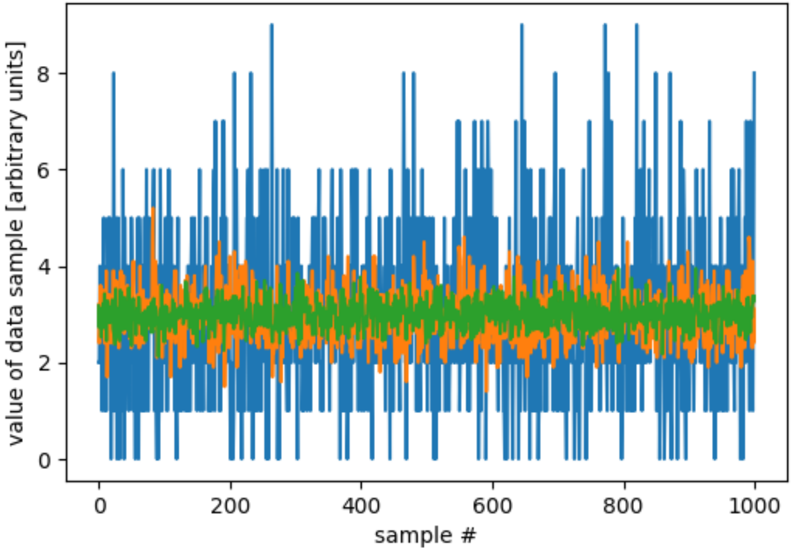
h/t T. Crawford

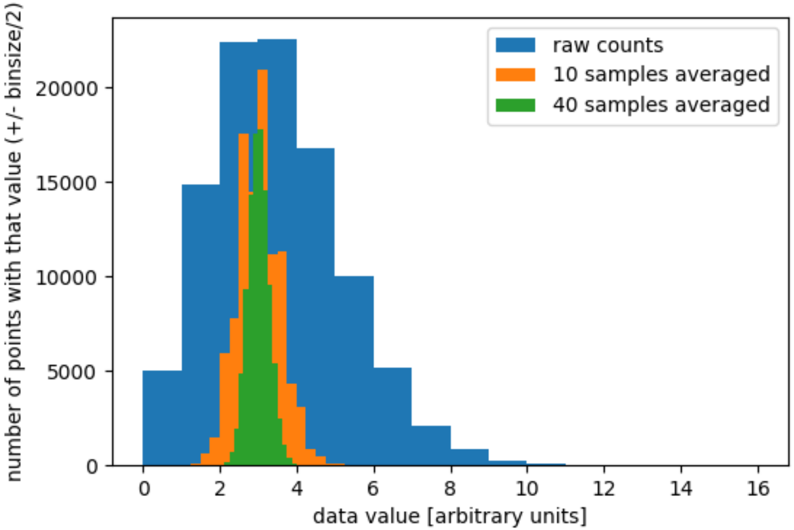
Also “Gaussian”





fakedata2 = np.random.poisson(3, size=100000)






```
fakedata5 = np.random.poisson(3, size=4000000)
fakedata6 = np.array([np.sum(fakedata5[np.int(40*i):np.int(40*i+40)]) for i in np.arange(len(fakedata5)/40)])

# i.e., average over 40 samples
```

As you average samples in a non-Gaussian distribution, the noise gets smaller AND more Gaussian —> Central Limit Theorem

Noise Equivalent Power (NEP)

- **NEP: the incident signal power required to obtain a signal equal to the noise in a one Hz bandwidth**
  - Describes S/N of a detector, not just the ‘noise’
  - Lower NEP = more sensitive detector
- Measure NEP as a function of frequency to fully capture behavior
  - Often quoted as a single number that describes the ‘white noise regime’
- Sources of white noise in a CMB experiment
  - Photon noise (Poisson + Bunching)
  - Detector noise
    - TES: Phonon noise
    - MKID: Generation-Recombination noise
  - Readout electronics noise

Photon Statistics

- Number of photons emitted from a source as a function of time intrinsically fluctuates
  - Photon noise is this variance in the arrival of photons at a detector
  - CMB experiments are typically ‘photon-noise-dominated’

For a Blackbody source measured by with efficiency η:

$$n = \frac{\eta}{exp(h\nu/kT) - 1} = \left[\frac{photons}{s\ Hz}\right]$$

number of photons (occupation number)

frequency of measurement

temperature of source

Photon Statistics: Average Distribution

- Number of photons emitted from a source as a function of time intrinsically fluctuates
  - Photon noise is this variance in the arrival of photons at a detector
  - CMB experiments are typically 'photon-noise-dominated'

For a Blackbody source measured by with efficiency η, average number of photons measured

$$n = \frac{\eta}{exp(h\nu/kT) - 1} = \left[\frac{photons}{s\ Hz}\right]$$

Example:
T = 20 K source
η =0.3
v = 150 GHz
Rayleigh-Jeans Limit

$$\frac{h\nu}{k_B T} = \frac{6.626\,10^{-34}\,(150\,10^9\,Hz)}{1.38\,10^{-23}(20K)} \sim 0.3$$

$$n \approx \frac{0.3}{1 + 0.3 - 1} \sim 1\,\frac{photons}{s\,Hz}$$ per mode

For 30 GHz bandwidth centered @ 150 GHz

~1 * 30 x 10^9 photons/s

Lots of photons!

Photon Noise

Variance in the number of photons

$$< \Delta n^2 >= n + n^2$$

Energy/power in photons, $E = nh\nu$, so can write variance in energy:

$$(h\nu)^2 < \Delta n^2 >$$

Assume detectors are not polarization sensitive, then we get variance in the power from ***2 polarization modes***

$$(2h\nu)^2 n + (2h\nu)^2 n^2$$

Integrate over frequency to get variance in power in that band!

$$\int 4(h\nu)^2 n d\nu + \int 4(h\nu)^2 n^2 d\nu$$

Photon Noise NEP

Previous equation represents a ***power variance*** in incoming photons … i.e., NEP!

$$NEP_\gamma^2 = \int 4(h\nu)^2 n d\nu + \int 4(h\nu)^2 n^2 d\nu$$

Rewrite in terms of photon power and assume spectrum is constant over the frequency band.

$$NEP_\gamma^2 = \int 2h\nu P_\nu d\nu + \int P_\nu^2 d\nu$$

$$P_{opt} \approx P_{\nu_0} \Delta\nu$$

$$NEP_\gamma^2 \approx 2h\nu_0 P_{opt} + \frac{P_{opt}^2}{\Delta\nu}$$

Poisson/shot noise term

Bunching term

Caveat: typically in front of bunching term there is a dimensionless coherence parameter (ξ) that is neglected here.

Photon Noise Example

$$NEP_\gamma^2 \approx 2h\nu_0 P_{opt} + \frac{P_{opt}^2}{\Delta\nu}$$

central frequency: 150 GHz
bandwidth: 30 GHz
incident optical power: 6.5 pW

$= 1.29 \times 10^{-33} \quad + \quad 1.41 \times 10^{-33}$

$$NEP_\gamma \approx 52 \times 10^{-18} \frac{W}{\sqrt{Hz}}$$

To be photon-noise-dominated, this term should be the largest source of noise in your system!

Detector Noise

- Depends on what type of detector!

TES Bolometer	MKID
Phonon noise (thermal carrier noise)	Generation-recombination noise
Johnson noise	Two-level system noise

- Focus on TES detectors, since you won’t hear about MKIDs until later today

Phonon Noise

- Thermodynamic fluctuations across the thermal link between TES and the cold thermal bath
- Reduce phonon noise by reducing G and/or T_c, but G must also be chosen for thermal circuit balance

$$NEP_G^2 = \gamma 4 k_B G T_c^2$$

thermal conductance

superconducting transition temperature

Example:
G = 100 pW/K
T_c = 400 mK

$$NEP_G^2 \approx 4 \times 1.38 \times 10^{-23} \times 100 \times 10^{-12} \times 0.4^2$$

$$\approx 37 \times 10^{-18} W/\sqrt{Hz}$$

Johnson Noise

- Thermodynamic fluctuations associated with electrical resistance of the TES
- suppressed as TES enters superconducting transition and loop gain increases

$$NEP_J^2 = 4k_B T_c P_{elec}/\mathcal{L}^2$$

electrical bias power

loopgain of electrothermal feedback

Example:
T_c = 400 mK
P_{elec} = 7 pW
loopgain = 10

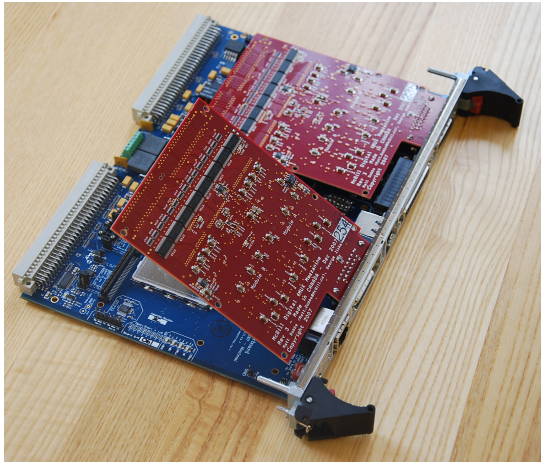
$$NEP_J = 1.2 \times 10^{-18} W/\sqrt{Hz}$$

Readout Noise

- Every resistor, amplifier, DAC, ADC contributes noise to the system
- Typically, electronics noise is specified in current units (Amps/√Hz), must refer to power fluctuation at the TES

$$NEP_{readout} = V_{bias} \times NEI_{readout}$$

- Two ways to reduce readout NEP
  - Reduce current noise from all the electronics
  - Reduce voltage bias by lowering operating resistance of TES



Example:

$NEI_{readout}$ = 20 pA/√Hz P_{elec} = 7 pW

R_{TES} = 1Ω

$$V_{bias} = \sqrt{P_{elec} R_{TES}}$$ = 2.6 μV

$$NEP_{readout} = 53 \times 10^{-18} W/\sqrt{Hz}$$

Total NEP

- Individual terms are added in quadrature to calculate total NEP

$$NEP_{total}^2 = NEP_{\gamma}^2 + NEP_G^2 + NEP_J^2 + NEP_{readout}^2$$

Source	**Noise Level [aW/√Hz]**
Photon Noise	52
Phonon Noise	37
Johnson Noise	1.2
Readout Noise	53
Total	83

😱

Not photon-noise dominated!!!

Total NEP

- Individual terms are added in quadrature to calculate total NEP

$$NEP_{total}^2 = NEP_\gamma^2 + NEP_G^2 + NEP_J^2 + NEP_{readout}^2$$

Source	**Noise Level [aW/√Hz]**
Photon Noise	52
Phonon Noise	37
Johnson Noise	1.2
Readout Noise	26
Total	69

Reduce readout noise to 10 pA/√Hz

Ok! Now this is photon-noise-dominated.

Note that the other sources still contribute in a meaningful way. System design can further optimize!

Noise Equivalent Temperature

- We measure the CMB in terms of temperature fluctuations on the sky
  - Express sensitivity in the same units
  - Noise equivalent temperature: convert to how big a power fluctuation on the bolometer we would expect for given temperature fluctuation
- Assume in the Rayleigh-Jeans limit
  - 30 GHz bandwidth, 30% efficiency

$$NET = NEP\frac{dT}{dP}$$

$$P_{RJ} = 2k_B T_{RJ}(\eta\Delta\nu)$$

$$\frac{dP_{RJ}}{dT_{RJ}} = 2k_B(\eta\Delta\nu)$$

$$NEP_{total}\times\frac{dT_{RJ}}{dP_{RJ}} = \frac{69aW/\sqrt{Hz}}{2k_B\times 0.3\times 30\times 10^9} = 278\mu K/\sqrt{Hz}$$

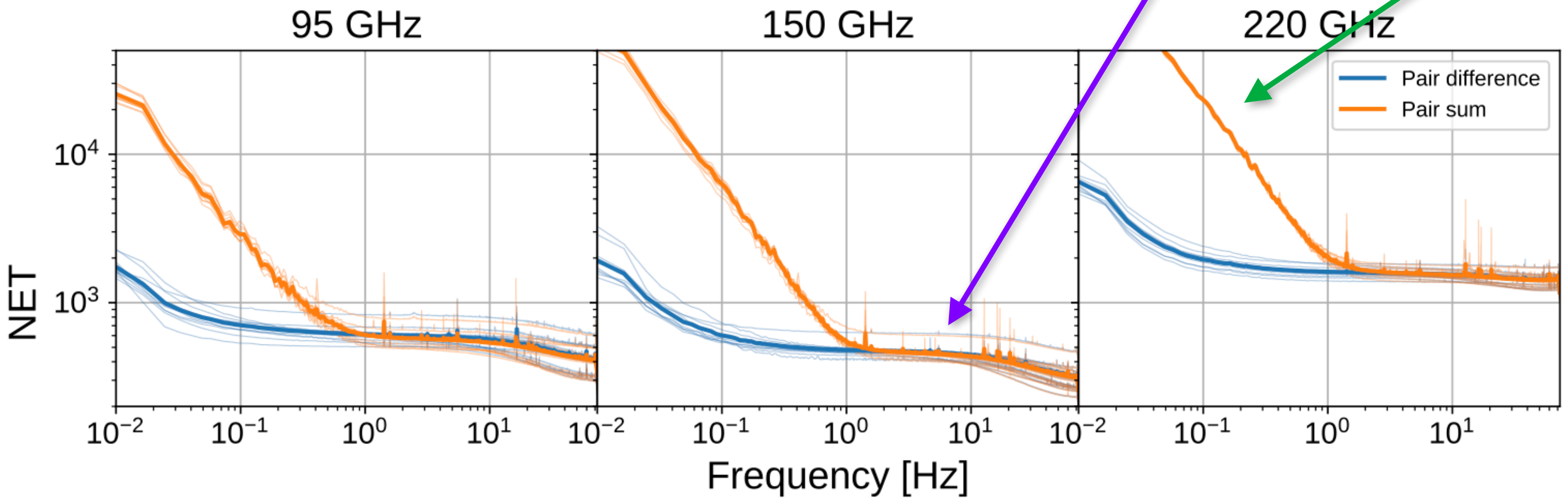
One more thing: non-white noise

- External RF/EMI
- Fluctuations in instrument/readout temperatures
- For ground based experiments…. atmosphere (technically fluctuating photon noise)
- Is it important? Where is your signal in frequency space?

$$NET = \sqrt{B + A \cdot f^{-\alpha}}$$

white noise

1/f noise



Useful References

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