Telescopes and Coupling

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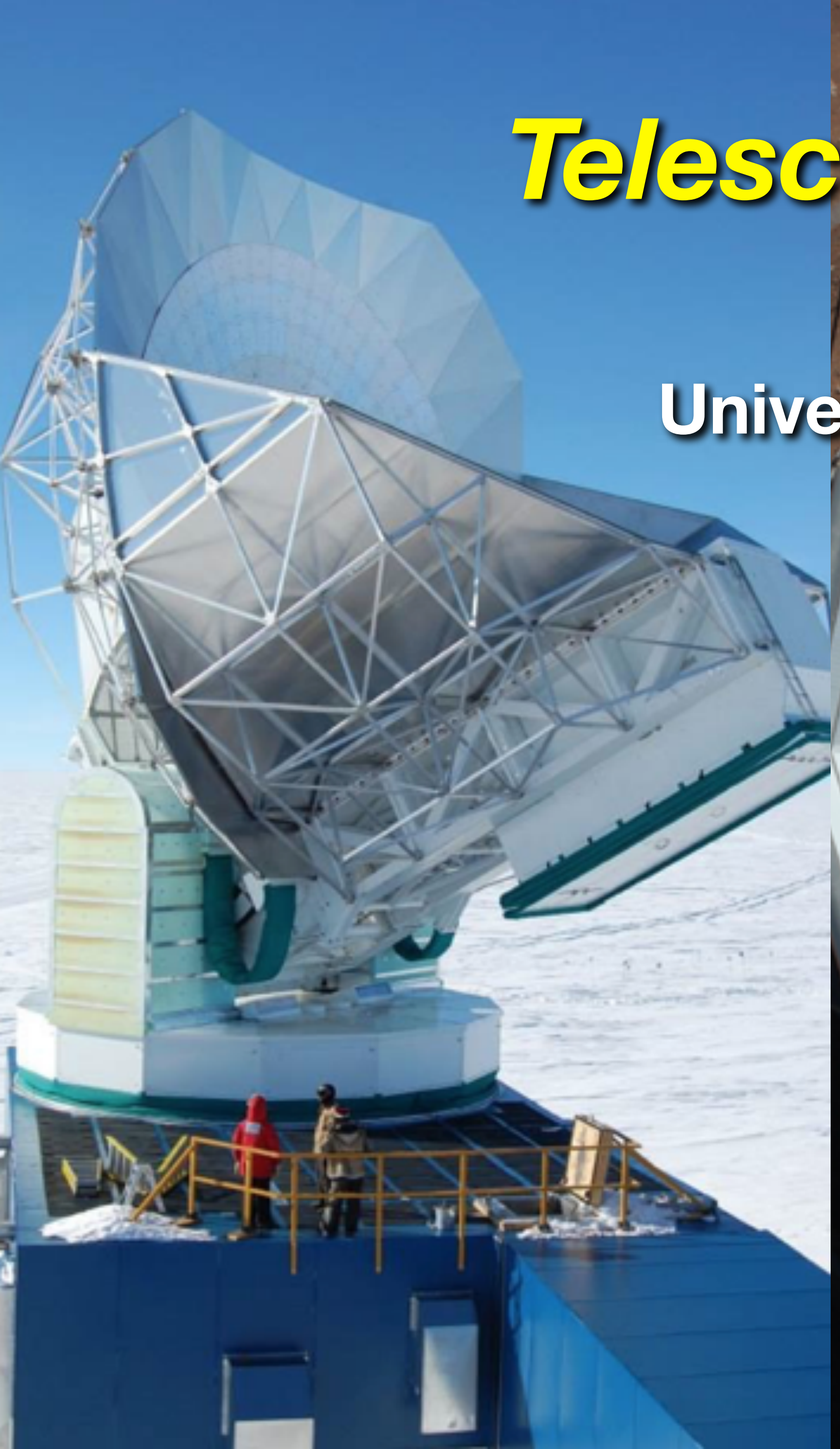
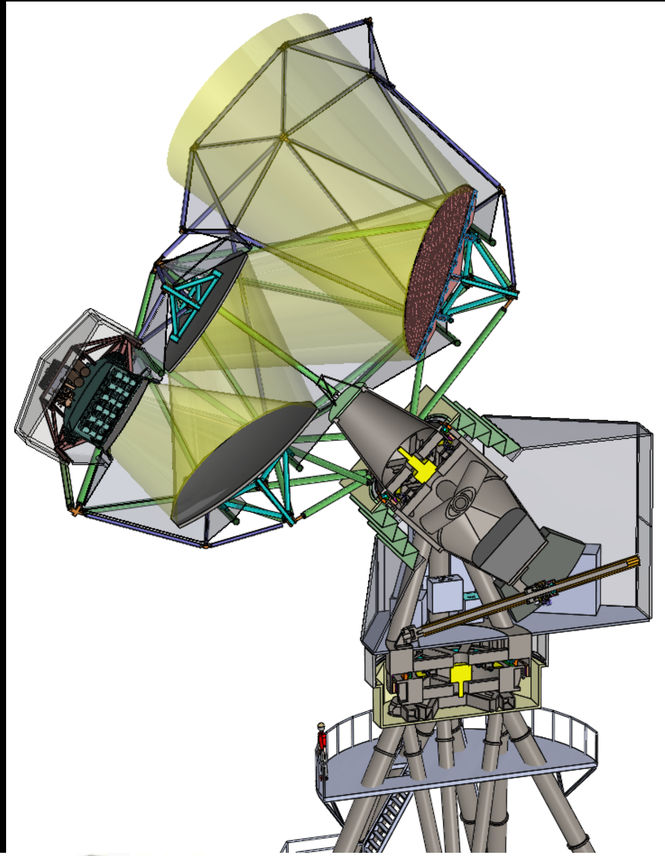
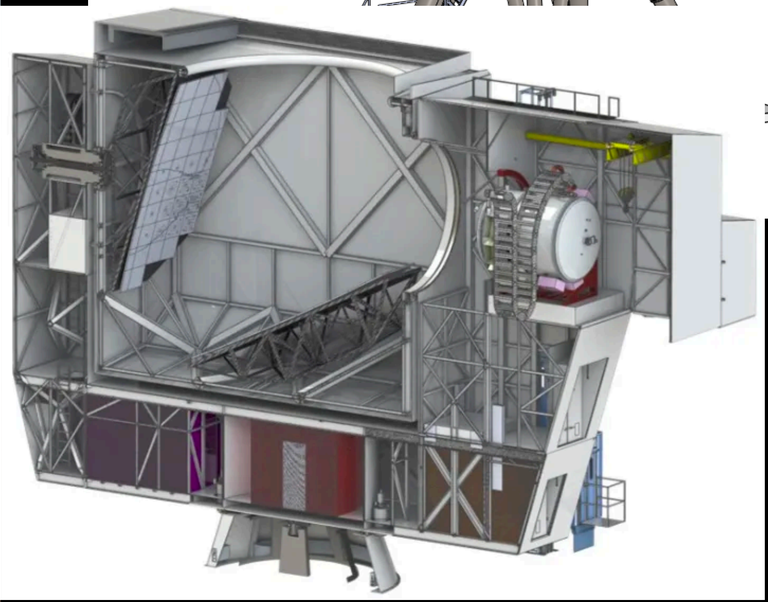


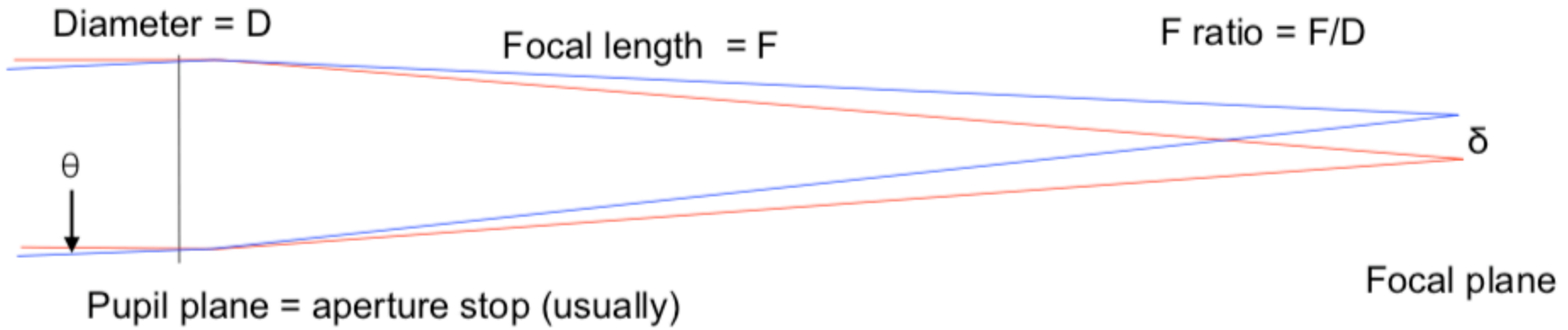




Purpose of a Telescope

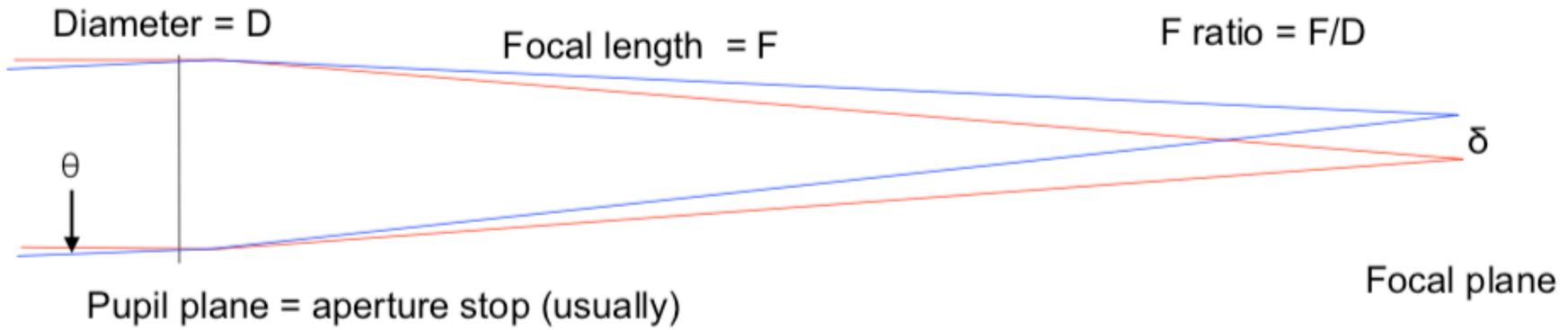
- Form an image of an object an infinity
- Resolve closely spaced objects
  - Large “baselines” needed for this
- Detect faint objects
  - Larger collecting area needed for this
- Form images of many objects at once
  - Multiplexing advantange
- Be a platform for a broad range of instrumentation

Telescope Principles



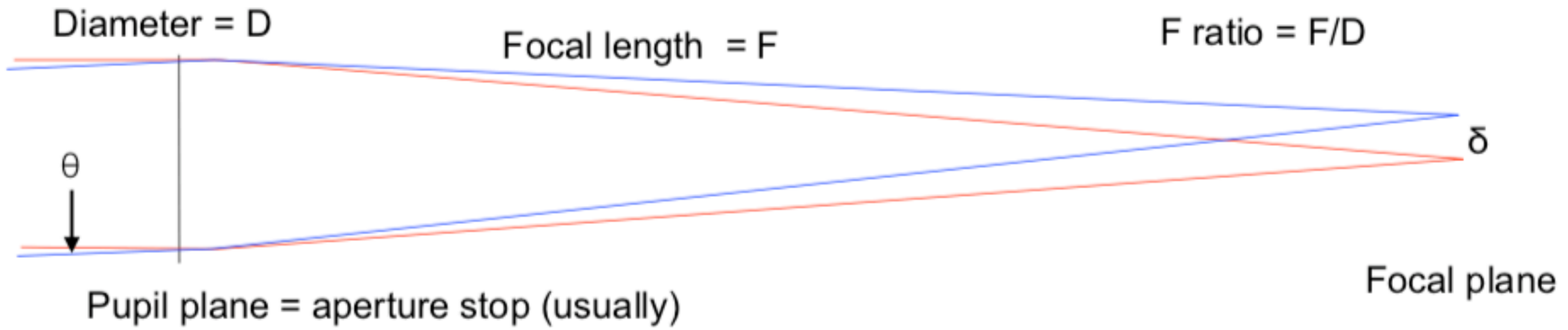
- The Focal length (F) determines the pixel (or plate) scale at the focal plane, the ratio between the angle on the sky and physical dimension of the detector.
  - Angular scale on-sky to physical separation on focal plane: **$\delta = \theta \times F$**
  - **F/D = the "f-number" (or f/#)**. So can re-express as: **$\delta = \theta \times D \times (f/\#)$**
  - If you are at the diffraction limit, $\theta \sim 1.2\ \lambda/D$
    - Implying: $\delta = \theta \times F \rightarrow \delta = (1.2\ \lambda/D) \times F \rightarrow \delta = 1.2\ \lambda\ (F/D) \rightarrow$ **$\delta = 1.2\ (f/\#)\ \lambda$**
  - For CMB experiments, we typically don't want horn/pixel-spacing less than the diffraction limit (<1.2 (f/#) λ), i.e., otherwise we are "over-sampling" the focal plane and photon noise will be correlated between detectors because they are measuring the same diffraction beam.

Telescope Principles



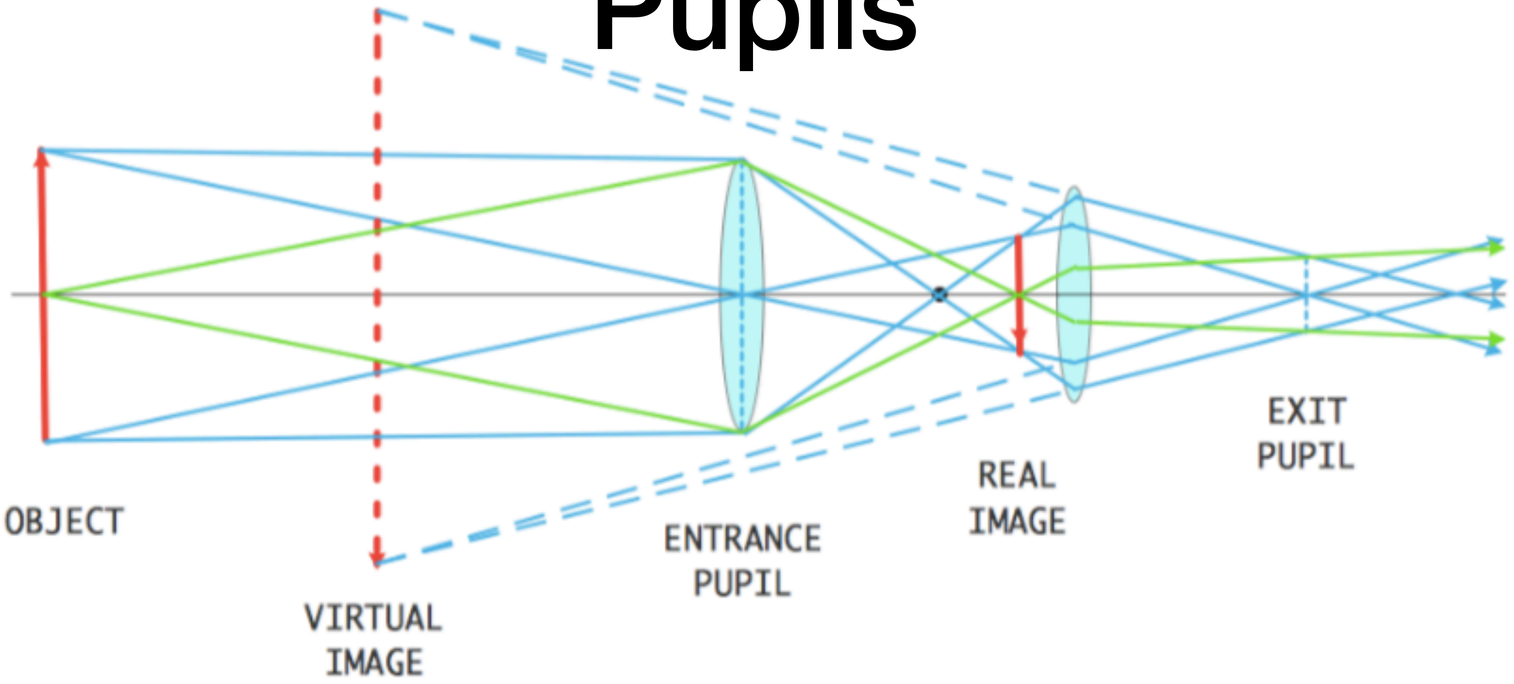
- Examples:
  - SPT:
    - 10-m telescope (only 8-m illuminated) with f# = 1.4 at the focal plane for SPT-3G
    - Plate-scale: δ = (1') (1.4x8) ~ (2.9e-4 rad x 11.2-m) = 3.2 mm (per arcmin)
    - Diffraction-spot size at (λ=2-mm): 1.2 (f/#) λ = (1.2) (1.4) * (2-mm) = 3.4-mm
    - Lenslet (or pixel)-size = 5-mm
      - Horn/lenslet/pixel size often discussed in units of "F"-lambda.
      - Divide lenslet-size by (f/#) λ
      - Pixel-size: 5-mm/((f/#) λ) = 1.8 "F" λ
      - So pixel size, is "***1.8 times "F" lambda***" , or roughly 1.5x diffraction limit at 2-mm wavelength

Telescope Principles



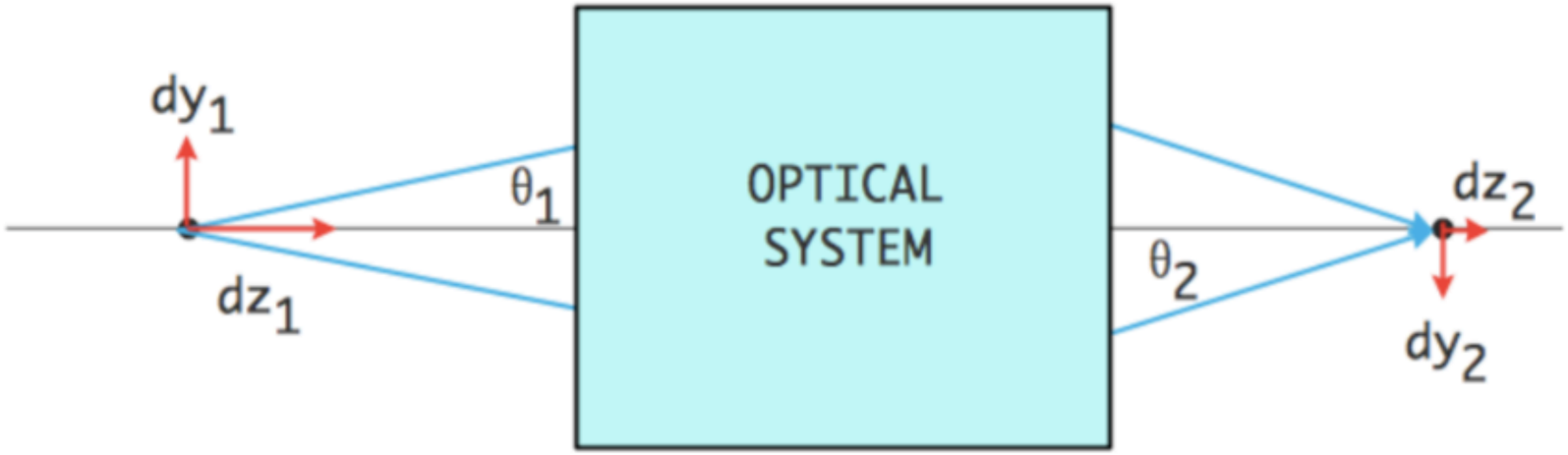
- Examples:
  - DES (Optical):
    - 4-m Blanco telescope with f/# = 2.7 -> F ~ 2.7 x D = 11-m
    - δ = (1”) (F) ~ (4.85e-6 radians) (11-m) ~ 53 um (per arcsec)
    - DES CCD’s are 15 x 15 um pixels, or 15/53 = 0.28” square pixels
    - Best point-spread function (PSF) at Blanco is ~0.5”. Optical telescopes are not at “diffraction-limit” typically limited by atmospheric “seeing”, most optical systems over-sample the PSF.
  - LSST (Optical):
    - D=8.4-m with f/#=1.2 -> F = (8.4-m)(1.2) = 10-m
    - δ = (1”) (10-m) ~ 48 um (per arcsec)
    - LSST CCDs are 10 x 10 um pixels, so 0.2” / pixel

Pupils



- An aperture or “stop”, and all images of it are called **pupils**
- The first pupil encountered by rays entering the optical system is called the **entrance pupil.**
- The last pupil encourantered by rays exiting the optical system is called the **exit pupil.**
- A ray passing through the center of a pupil is called a **chief ray.**
- A ray passing through the edge of a pupil is called the **marginal ray.**

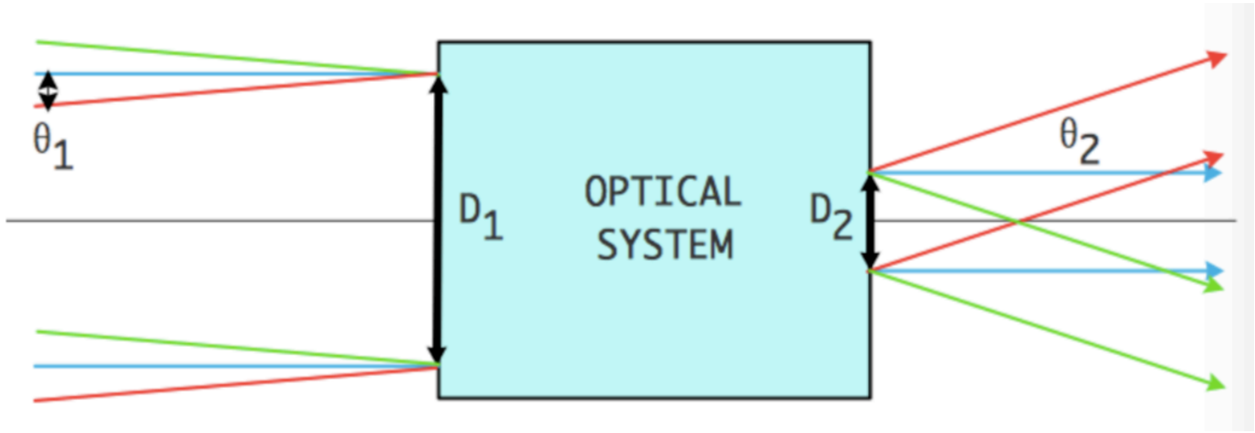
Magnification



- For a general optical system, the magnification is just given by:

  $M = dy_2/dy_1 = \theta_1/\theta_2$

Lagrange Invariant: Throughput



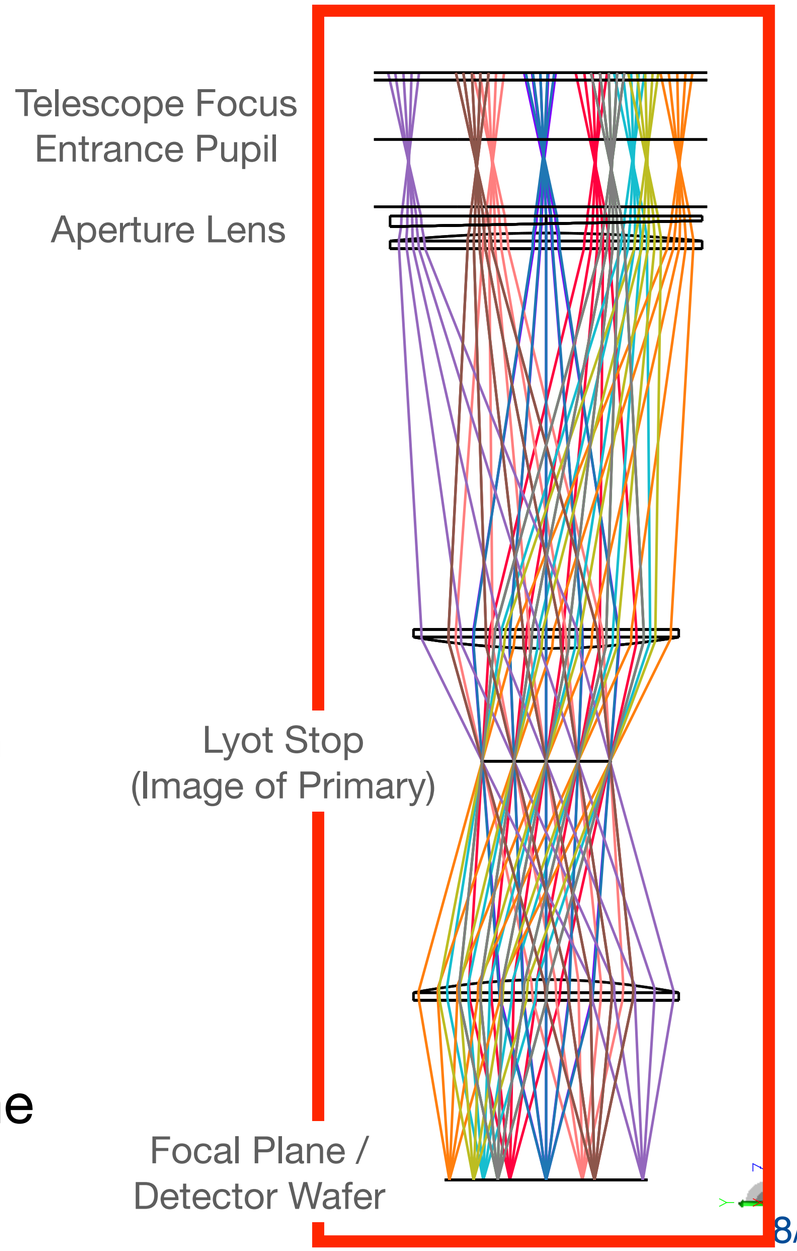
- At any pupil, the angle subtended by the rays passing through the pupil is inversely proportional to the pupil diameter

$$D_1\theta_1 = D_2\theta_2$$

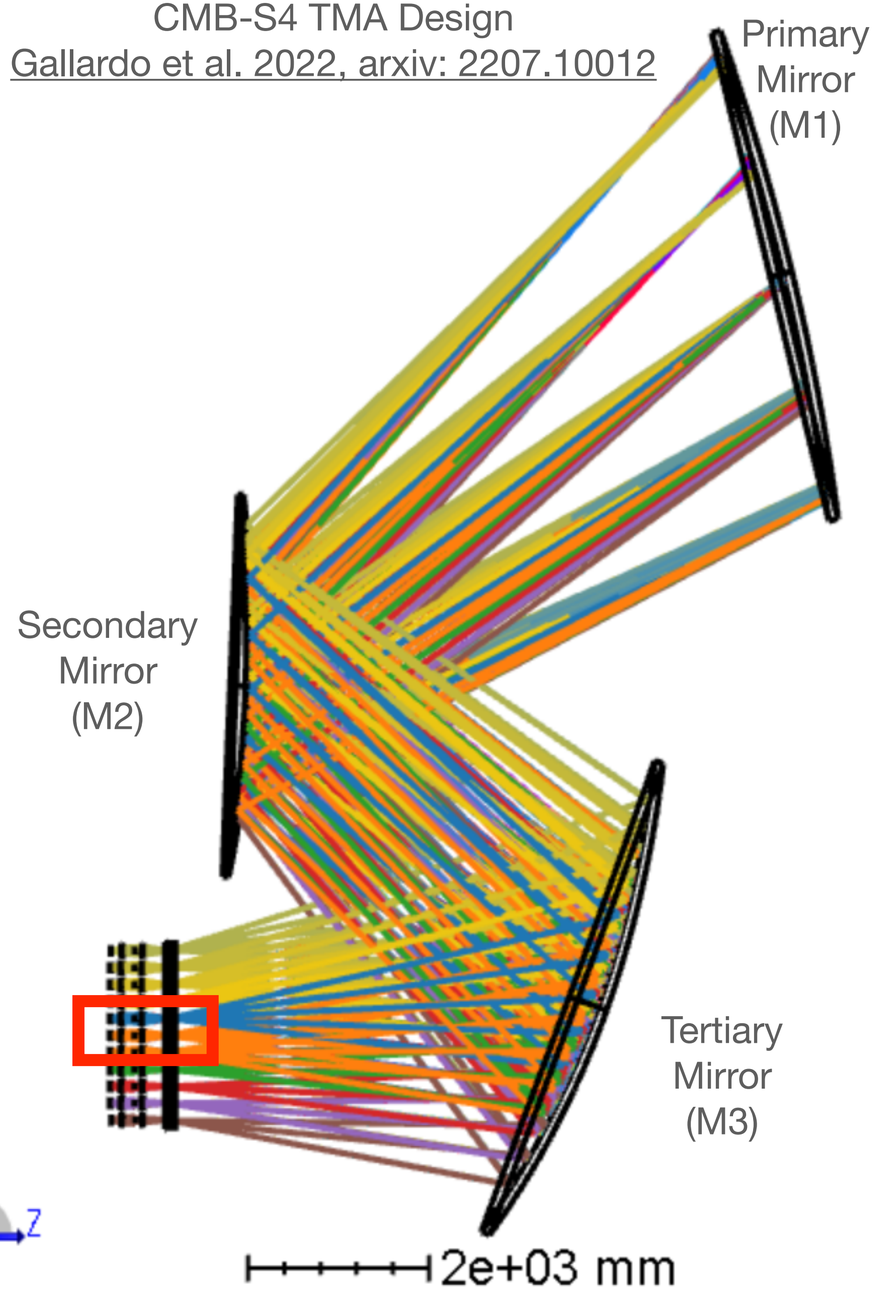
- Throughput (AΩ) and (Dθ) are “Lagrange invariants”, they are quantities that are conserved in an optical system (i.e., we cant increase the throughput with a camera optic downstream)

Optics Tube Example

- Optics tube re-images the telescope focus onto a detector wafer
- Optics tube conserves throughput, but re-images a ~190-mm diameter aperture onto a ~130-mm diameter detector wafers
  - So optics tube has a 190/130 ~1.5x magnification
  - f/# at telescope focus is 2.7, with the optics tube magnification changing this to f/# of 1.8 at the detector focal plane
- Lyot stop is uniformly illuminated by all pixels, because its at an image of the primary / M1 mirror

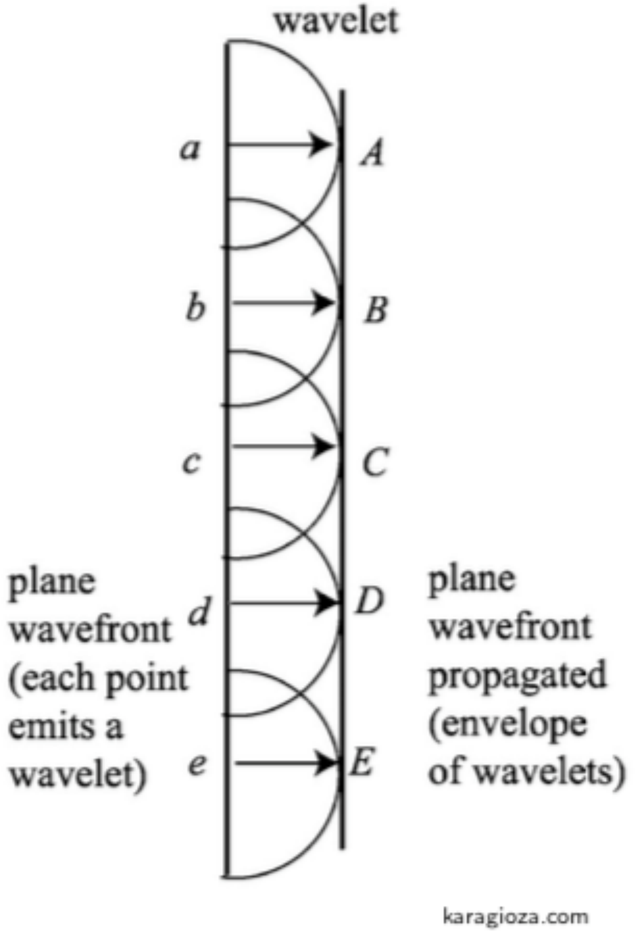


CMB-S4 TMA Design
Gallardo et al. 2022, arxiv: 2207.10012



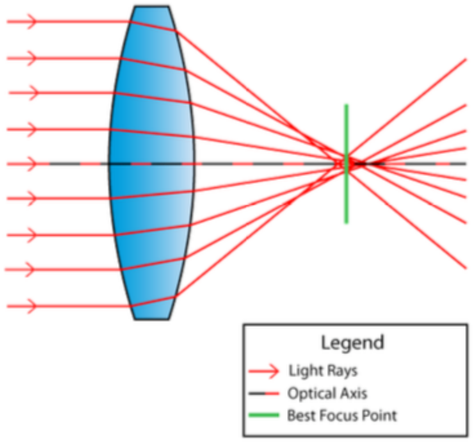
Wavefronts & Aberrations

- Wavefronts are surfaces of constant phase, perpindicular to rays (Hyugen's principle)
- Every point on wavefront becomes source of spherical and secondary waves
- These waves interfere (constructively or destructively) to form new wavefront
- Constructive interference requires the that change in optical path between two wavefronts to be constant

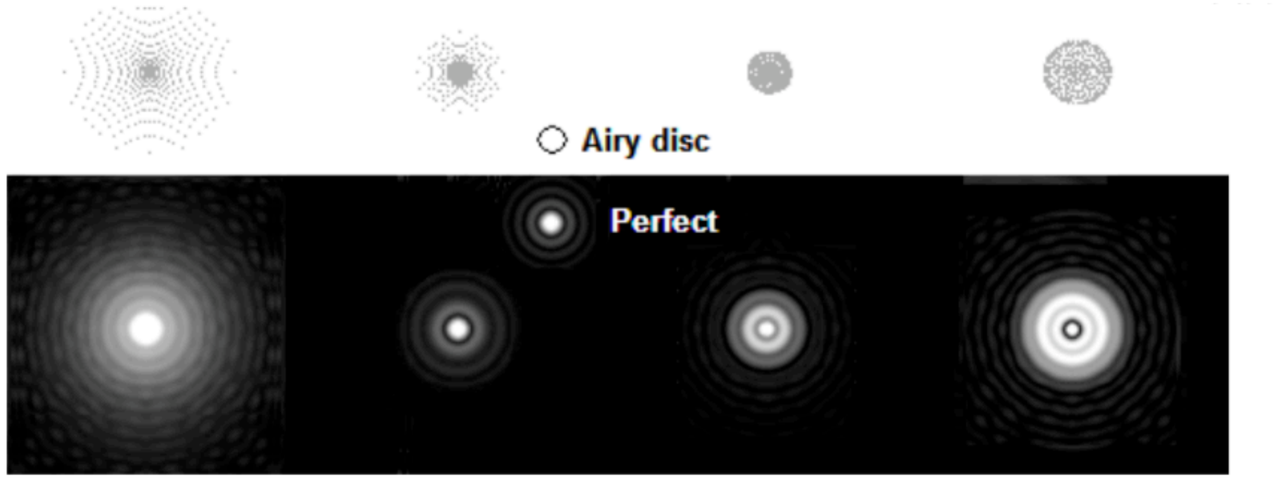


Aberrations

- Three main aberrations: 1) Spherical, 2) Coma, 3) Astigmatism
- **Spherical Aberration**
  - Due to surface difference between a sphere and a parabola
  - Telescope focus is a function of radius in the focal plane
  - Can be removed by an aspheric surface (e.g., a parabola)
  - proportional to $(1/f\#)^4$ -> low f#'s make aberrations more difficult to correct



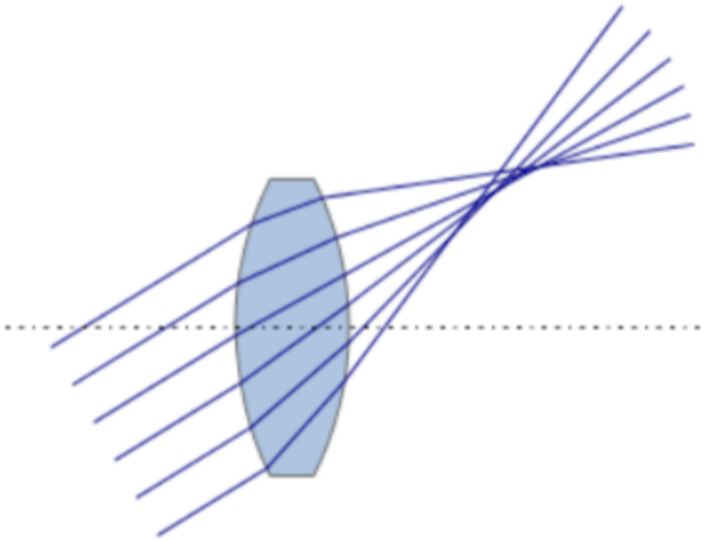
N. Mansurov

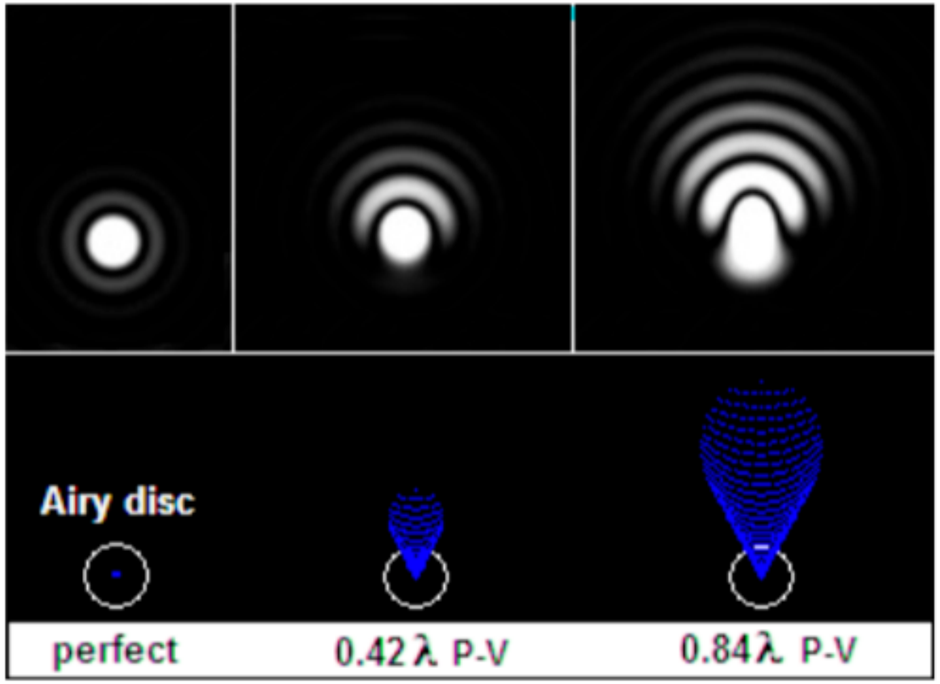


telescope-optics.net

Aberrations

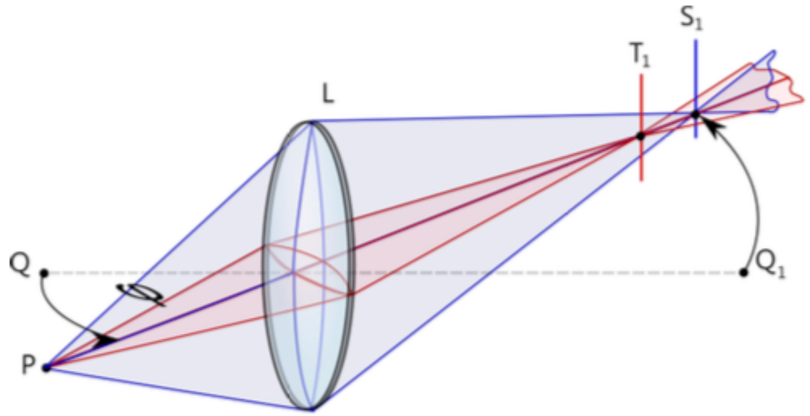
- Three main aberrations: 1) Spherical, 2) Coma, 3) Astigmatism
- **Coma**
  - Increases linearly with field angle (i.e., off-axis pixels)
  - Proportional to $(1/f\#)^3$ -> low f#'s make aberrations more difficult to correct
  - A system will be free from Coma if it obeys Abbe sine condition (see X-ray lecture)

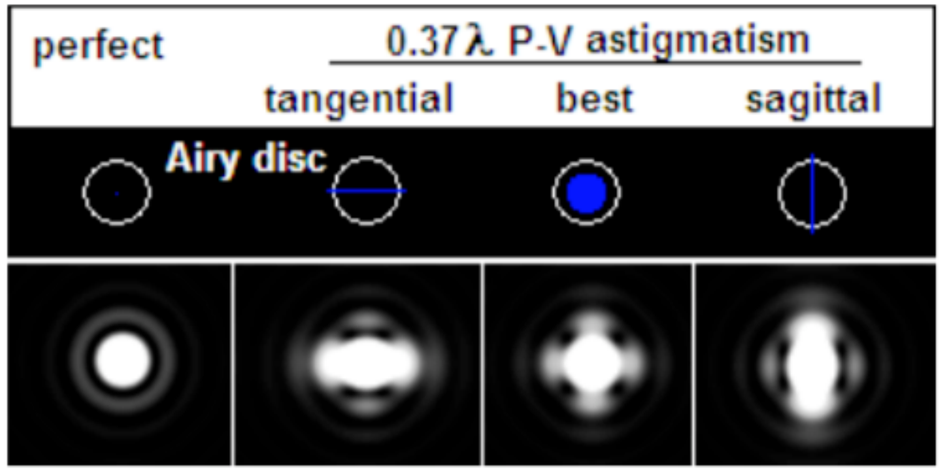




Aberrations

- Three main aberrations: 1) Spherical, 2) Coma, 3) Astigmatism
- **Astigmatism**
  - Focal distance depends on position angle in the pupil
  - Increases like square of the field angle (i.e., even worse for off-axis pixels)
  - Proportional to $(1/f\#)^2$





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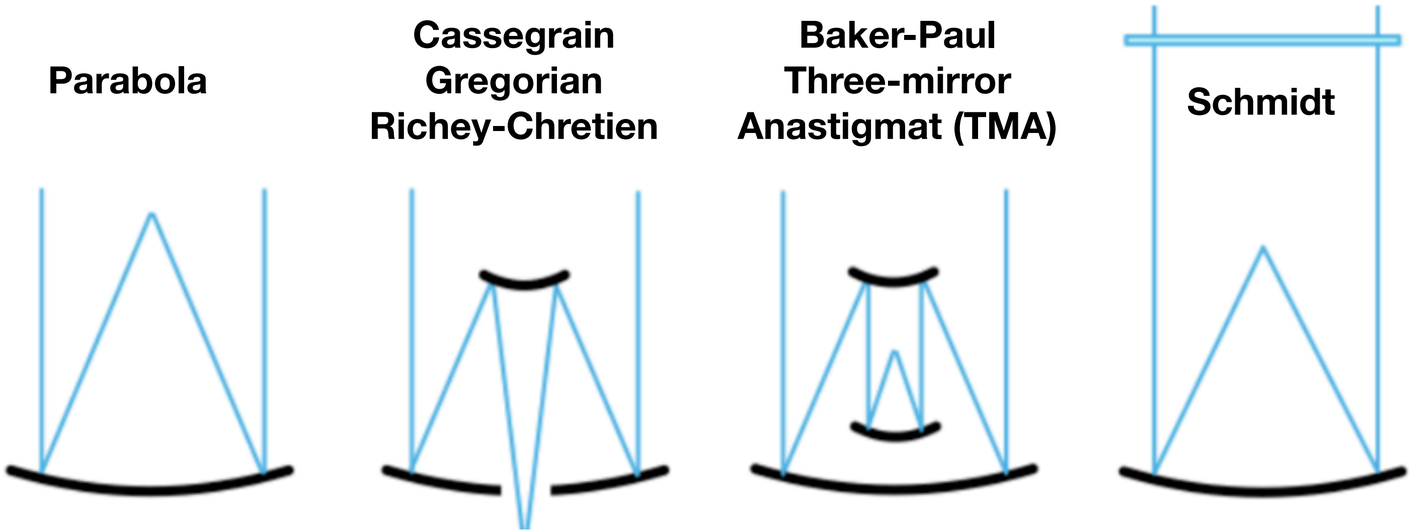
Correcting Aberrations



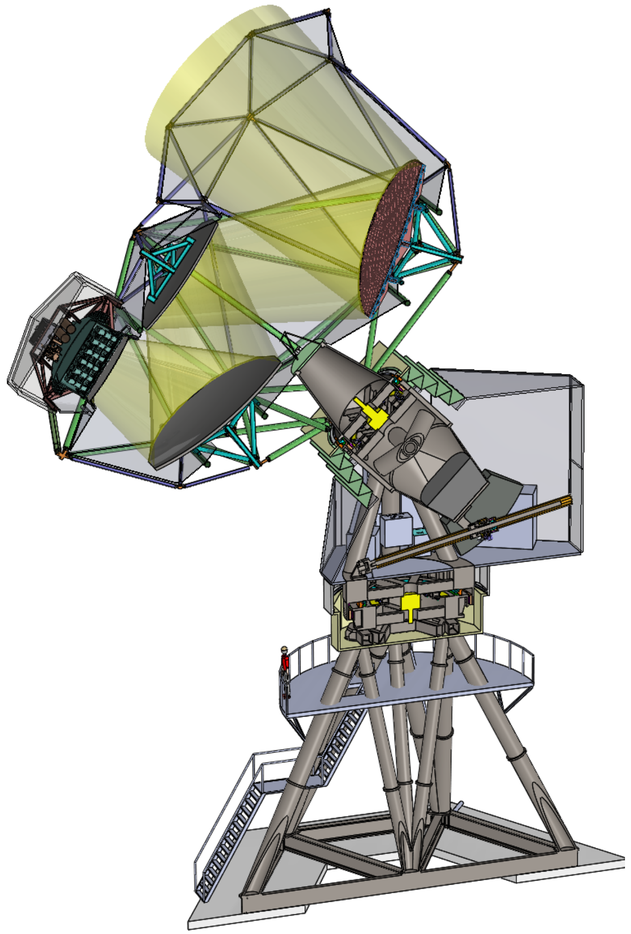
- Generally possible to correct for one aberration per optical surface
- **Parabolic** mirror: corrects spherical aberration
- **Cassegrain** or **Gregorian** (2-mirror) corrects for only spherical aberation
- **Richey-Chretien** (2-mirror) corrects spherical aberration and coma by using a hyperbolic primary (in place of parbolic primary in Cassegrain ssystem)
  - Most modern telescopes are RC, e.g., Hubble, Keck, VLT
- **Baker-Paul / TMA** (3-mirror) corrects for spherical aberration, coma, and astigmatism
  - Gives wider field-of-fiew, e.g., LSST, JWST, WFIRST, SP-TMA
- **Schmidt** telescope (spherical mirror+thin lens/corrector plate)
  - Efficient way to achieve large field-of-view with only one reflector (e.g., Kepler)

Survey / Mapping Speed

- Parameter most important for survey speed is the telescope throughput (AΩ), sometimes called etendue.
- AΩ = (collecting area [m2]) x (field of view [deg2])
- Survey speed = AΩ

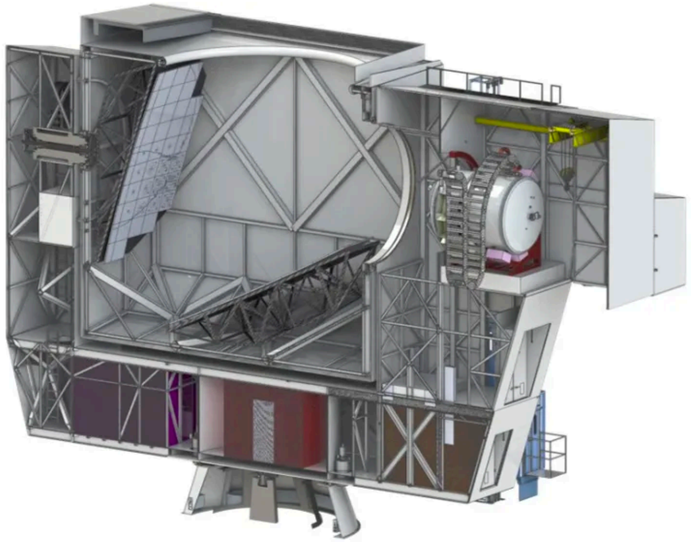
Three-Mirror Antastigmat (TMA) and Cross-Dragonne (CD) Designs

Three-Mirror Anastigmat (TMA) Design
Padin, Applied Optics (2018) , 57(9)
Gallardo et al. (2022), arxiv: 2207.10012



TMA and CD designs have ~10-20x larger throughput than either SPT or ACT!

Cross-Dragonne (CD) Design
Parshley et al. arXiv:1807.06678
Parshley et al. arXiv:1807.06675
Galitzki et al. arXiv:1808.04493
Gudmundsson et al. arXiv:2009.10138



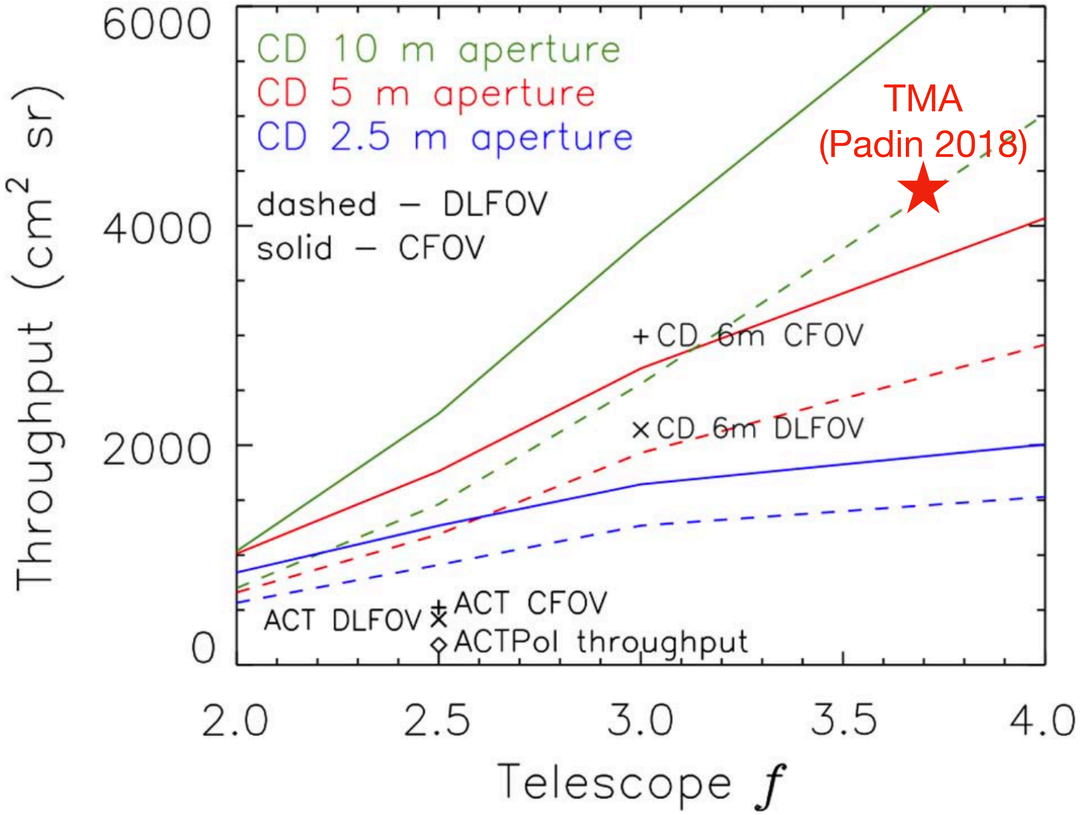


Fig. 2. Telescope f versus throughput for different telescope apertures (2.5 m—blue/lowest, 5 m—red, 10 m—green/highest). Solid (dashed) lines indicate the telescope 150 GHz CFOV (DLFOV). Also shown are points indicating the 150 GHz throughput of the ACT FOVs, ACTPol, and the CD 6 m design in Figs. 3 and 4.

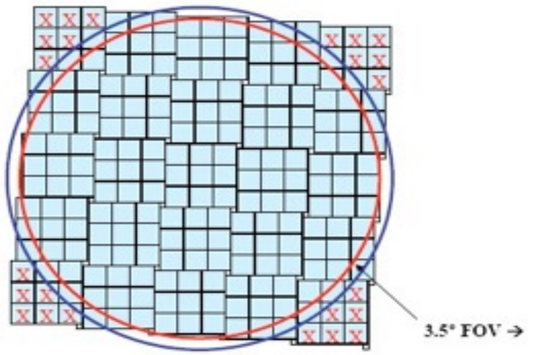
(Niemack 2016 arXiv:1511.04506)

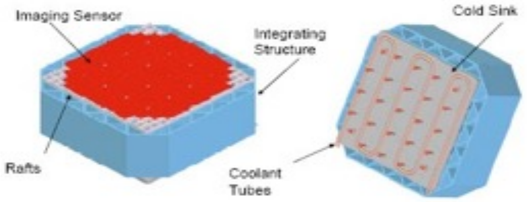
Large Synoptic Survey Telescope

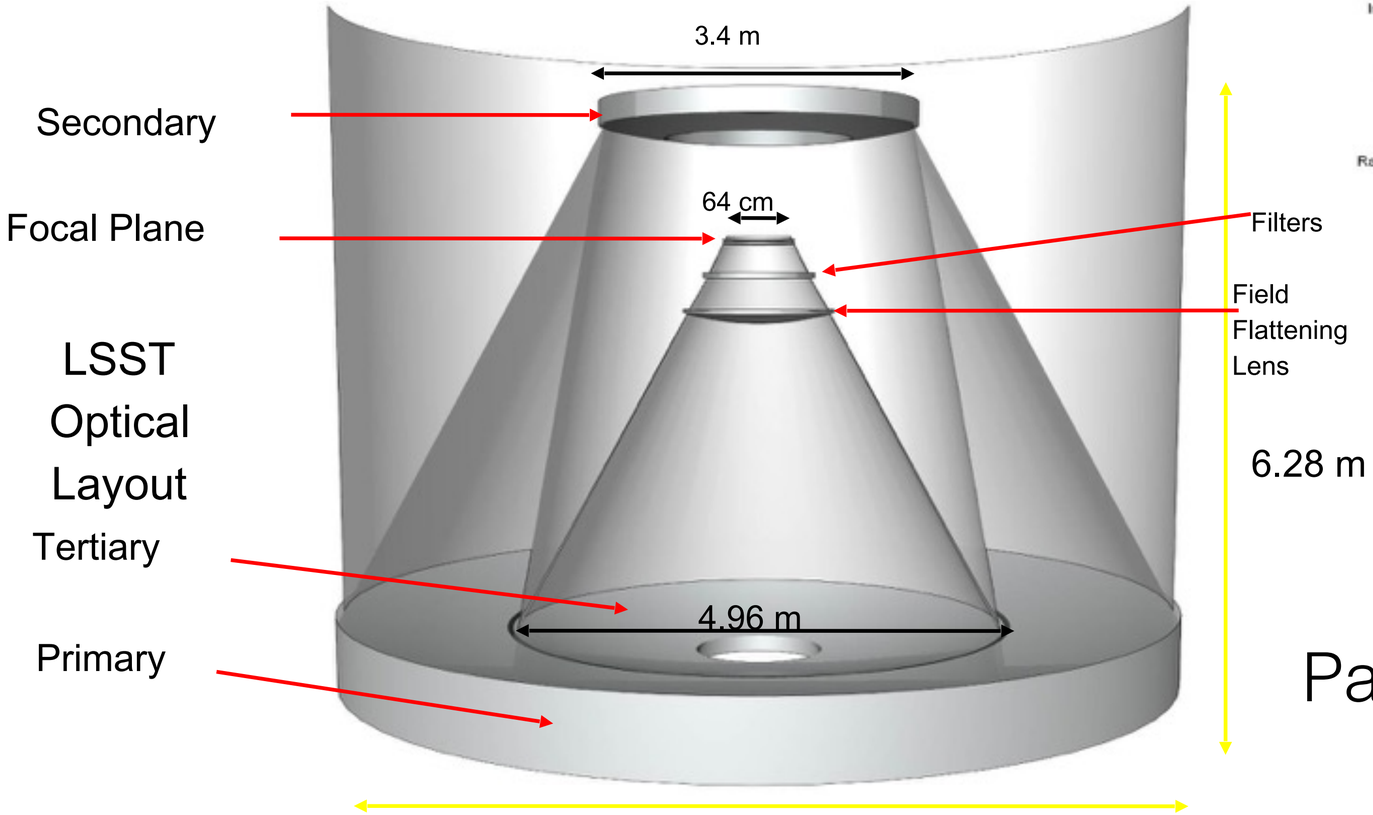
3.5° field of view for all-sky survey

Primary and Tertiary mirrors to be made at Arizona *on the same substrate*

200 4k x 4k detectors

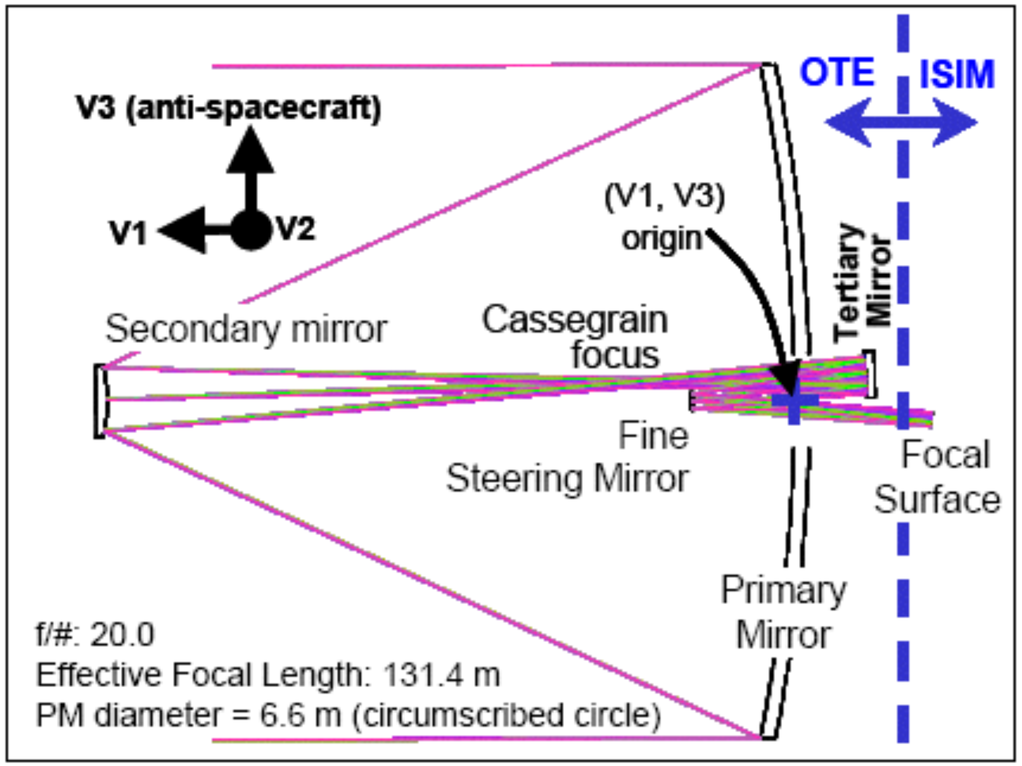


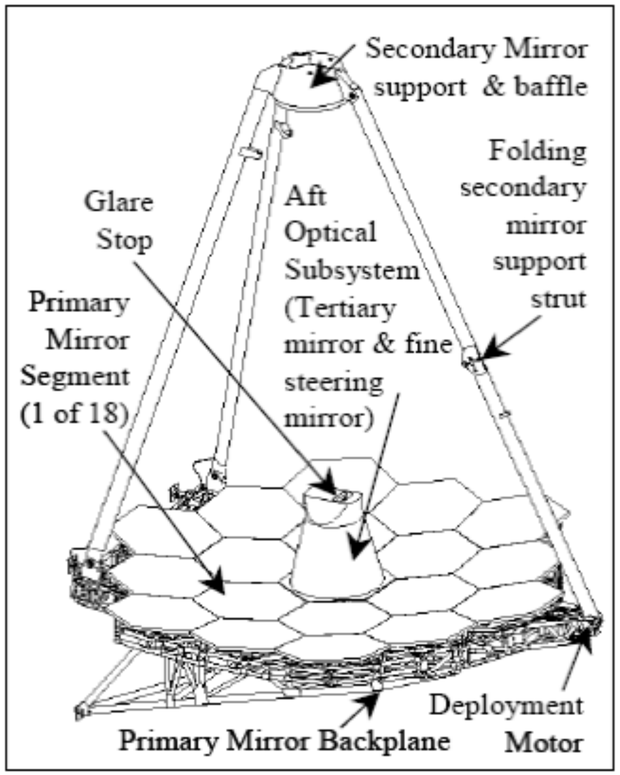


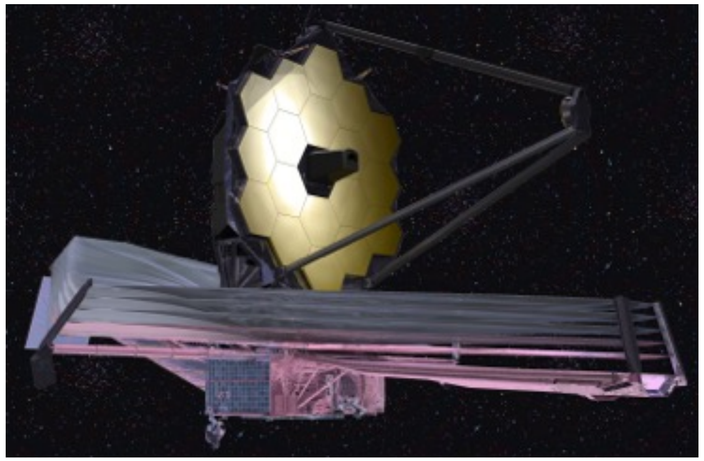


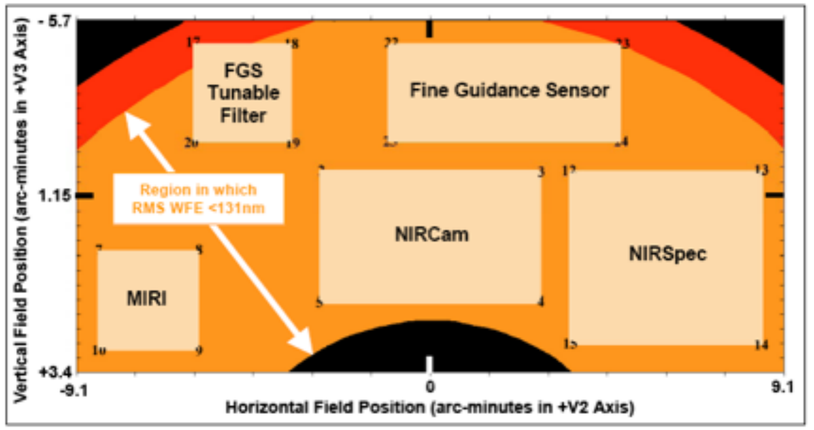
Paul-Baker Design

JWST TMA


OTE
ISIM
V3 (anti-spacecraft)
V1
V2
(V1, V3)
origin
Tertiary
Mirror
Secondary mirror
Cassegrain
focus
Fine
Steering Mirror
Focal
Surface
Primary
Mirror
f/#: 20.0
Effective Focal Length: 131.4 m
PM diameter = 6.6 m (circumscribed circle)


Secondary Mirror
support & baffle
Folding
secondary
mirror
support
strut
Glare
Stop
Aft
Optical
Subsystem
(Tertiary
mirror & fine
steering
mirror)
Primary
Mirror
Segment
(1 of 18)
Deployment
Motor
Primary Mirror Backplane



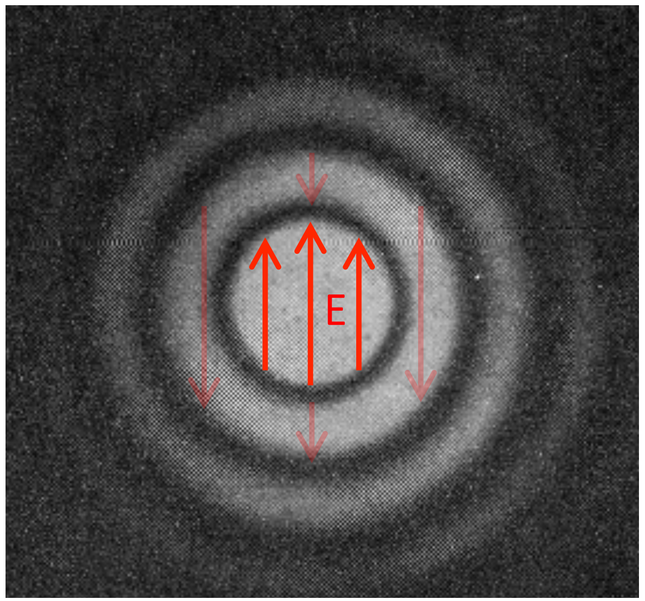

- 5.7
1.15
+3.4
-9.1
0
9.1
Vertical Field Position (arc-minutes in +V3 Axis)
Horizontal Field Position (arc-minutes in +V2 Axis)
FGS
Tunable
Filter
Fine Guidance Sensor
Region in which
RMS WFE <131nm
NIRCam
NIRSpec
MIRI

Telescopes: Putting it all together

- Physical Size:
  - Keep f/# small (<~2) to keep telescope size small; for 6-10 m class telescope want f# < ~2 to keep overall telescope of reasonable size
  - Small f/# (<~2) also keeps focal plane reasonable size to fit in a cryostat. For example, CD/TMA for SO/CMB-S4 has a ~2-m diameter focal plane with ~5-mm pixels (i.e., with pixel-size >1.2 (f/#) λ)
- Surface quality:
  - Keep RMS surface accuracy of mirrors < λ/40 to minimize loss in the main beam
    - Ruze formula: Loss = 1 - $\exp[-(4\pi\sigma_{rms} / \lambda)^2]$, with σ_{rms} = RMS surface variation
    - e.g., for λ=1-mm and σ_{rms} = λ/40 = 25-um -> 10% loss
- Mechanical Support:
  - Deflection of circular plate ~ $(\text{diameter})^4 / (\text{thickness})^2 = D^4 / t^2$
  - So need $t \sim D^2$ to keep deformation constant i.e., going from D=1-m to 10-m, requires a 100x stiffer support!
- Thermal Gradients
  - Keep thermal graidents across the mirror low, e.g., a 5-K gradient would lead to a 20-um deflection, either during observations or during machining

Telescope Airy Pattern

- Beam on the sky (or at focal plane) is the Fourier transform of the primary mirror illumination
  - For top-hat illumination, it will be an Airy Disk of width $\theta \sim 1.2\,\lambda/D$



Field amplitude

$$E(\theta) = \frac{2J_1\left(\frac{2\pi}{\lambda}\frac{D}{2}\theta\right)}{\frac{2\pi}{\lambda}\frac{D}{2}\theta}$$

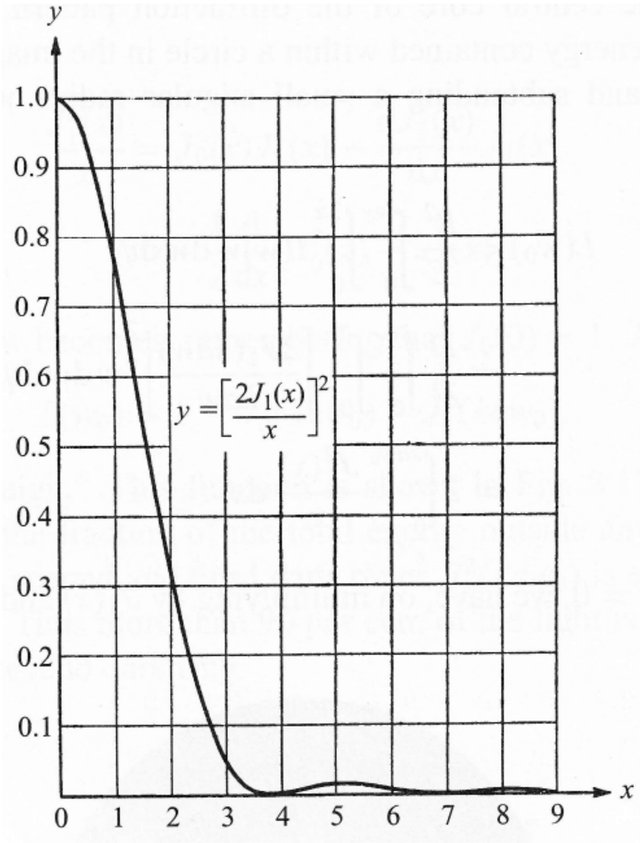
1st null $\frac{2\pi}{\lambda}\frac{D}{2}\theta = 1.22\pi$

$\theta = 1.22\frac{\lambda}{D}$

half power $\frac{2\pi}{\lambda}\frac{D}{2}\theta = 1.6165$

$\theta = 0.515\frac{\lambda}{D}$

Need to couple this intensity/field pattern to a detector with well-defined polarization and frequency range.



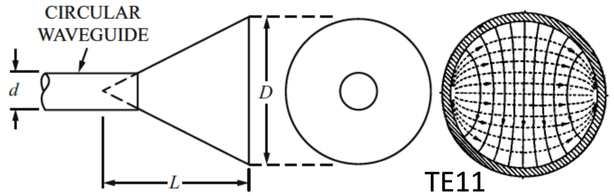
Born & Wolf, Principles of Optics

Airy, Trans. Camb. Philosophical Soc., 5 1835

Horn-Coupling

- Most radio telescopes and many CMB experiments (e.g., SPT-SZ, SPTpol, ACTpol, AdvACT, SO, CMB-S4) use horn-coupling
- Use transmission line components to define frequency bands and polarization (e.g., waveguide, microstrip filters)
- Conical horn
  - Simple to manufacture.
  - Need D > ~1.5 λ.
  - Flat-phase at aperture, need $L > 2\ D^2 / \lambda \sim 8\ F^2$

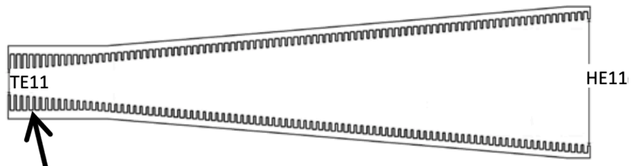
Conical



TE11
E solid, H dashed

- Corrugated horn
  - Difficult to manufacture (λ/4 grooves)
  - Excllent beam symmetry with low cross-pol.

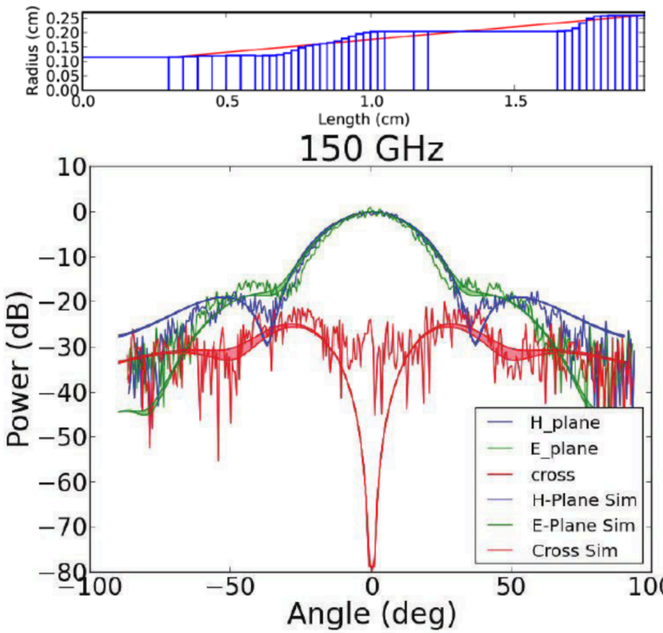
Corrugated



Mode converter generates ~15% TM11

- Smooth-walled / Spline-profile
  - Relatively easy to make, but tend to be longer than corrugated.
  - Good beam symmetry and cross-pol

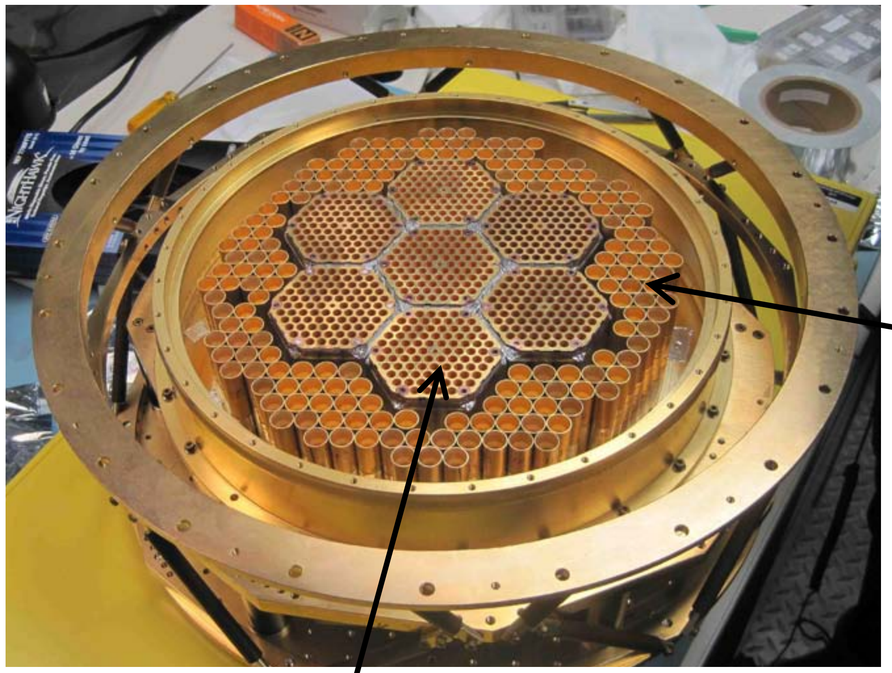
Smooth-Walled

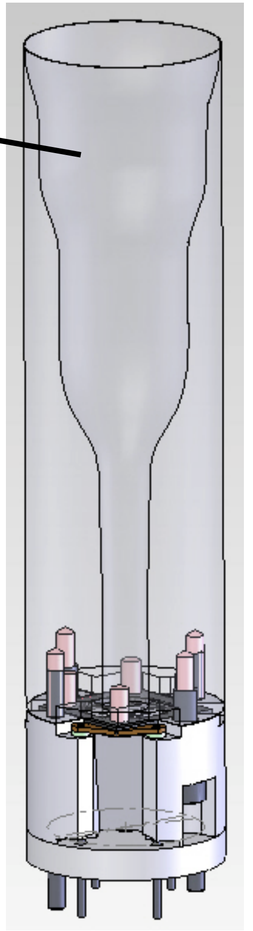


AdvACTpol 90/150GHz horn
Simon et al., SPIE 991416 2016

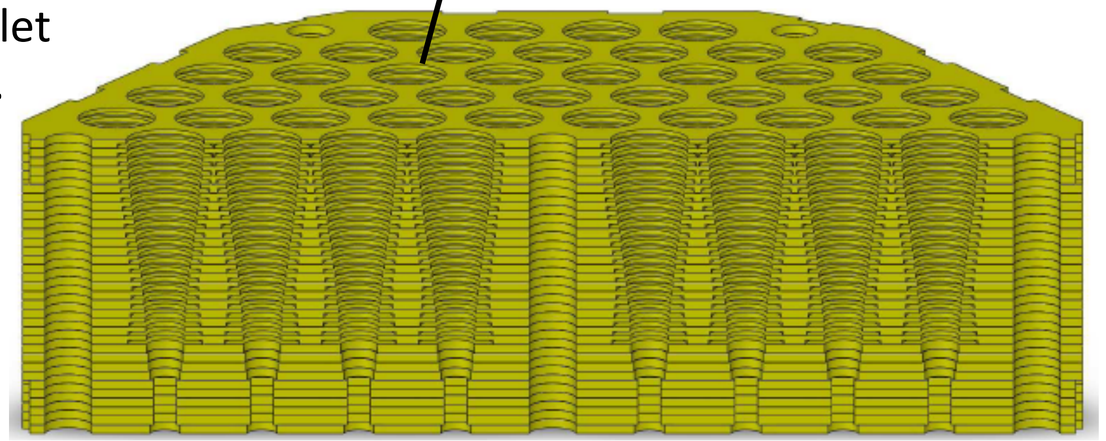
SPTpol horns

Profiled smooth-wall horns at 90GHz.
2.3Fλ
Machined in Cu with profiled reamer, then Au plated.



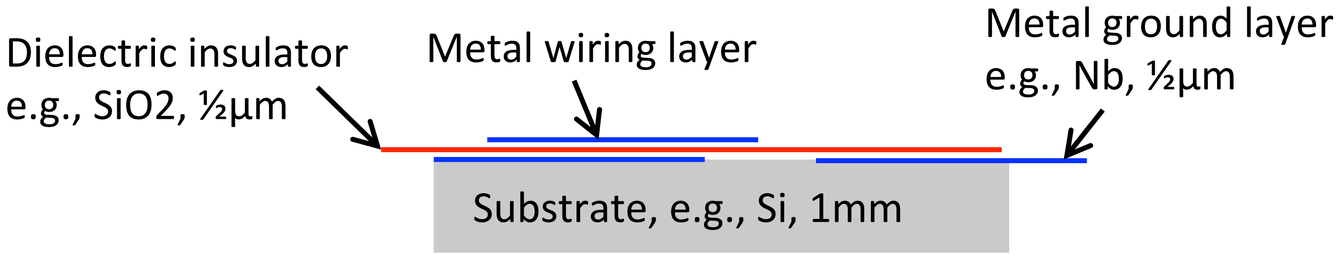


Corrugated platelet horns at 150GHz.
1.6Fλ
Stack of micro-machined Si wafers, Cu/Au plated.



Planar Antennas

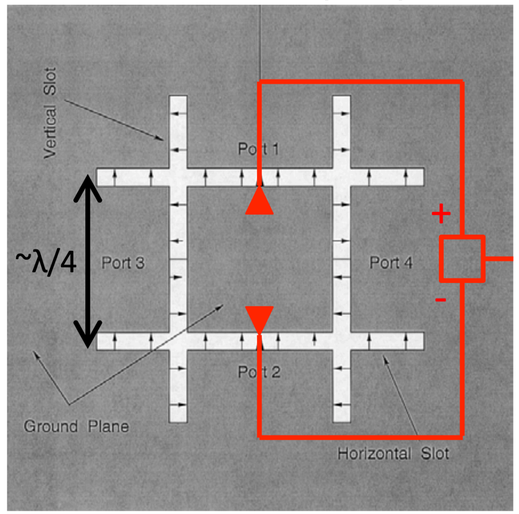
Printed metal structures on a dielectric substrate



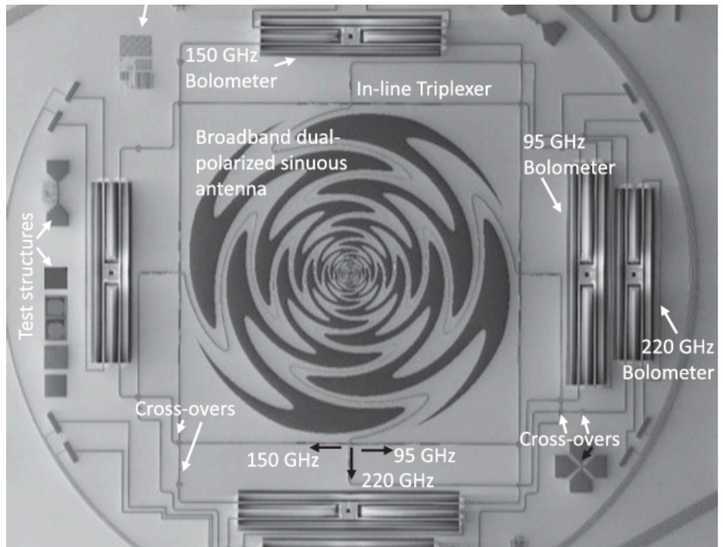
Easy to integrate planar antennas, filters, detectors etc. on wafer

Cover most of the substrate with ground, leaving holes only where there are antennas; feed signals from the ground side

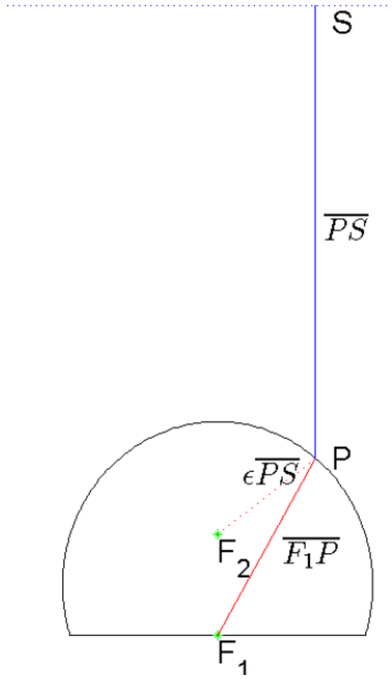
Slot-Antenna
Polarbear-1, Superspec

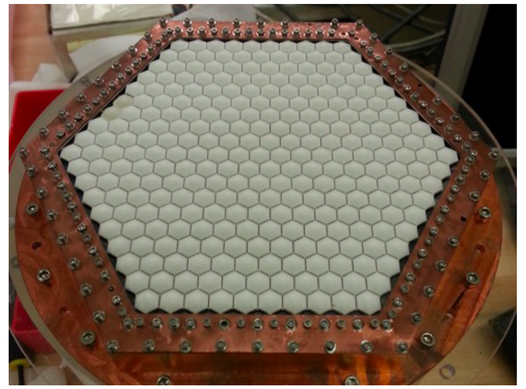


Sinuous-Antenna
SPT-3G, Simons Array, Litebird

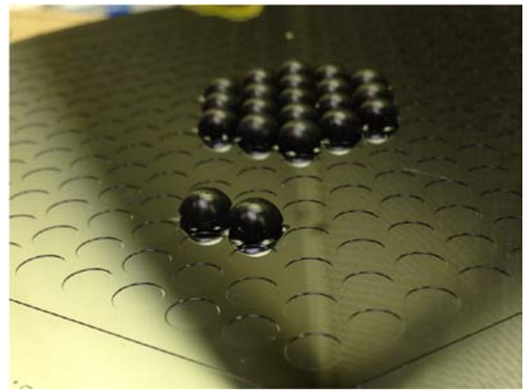


Use lenslet to improve coupling (focus) to planar-antenna





SPT-3G lenslet array

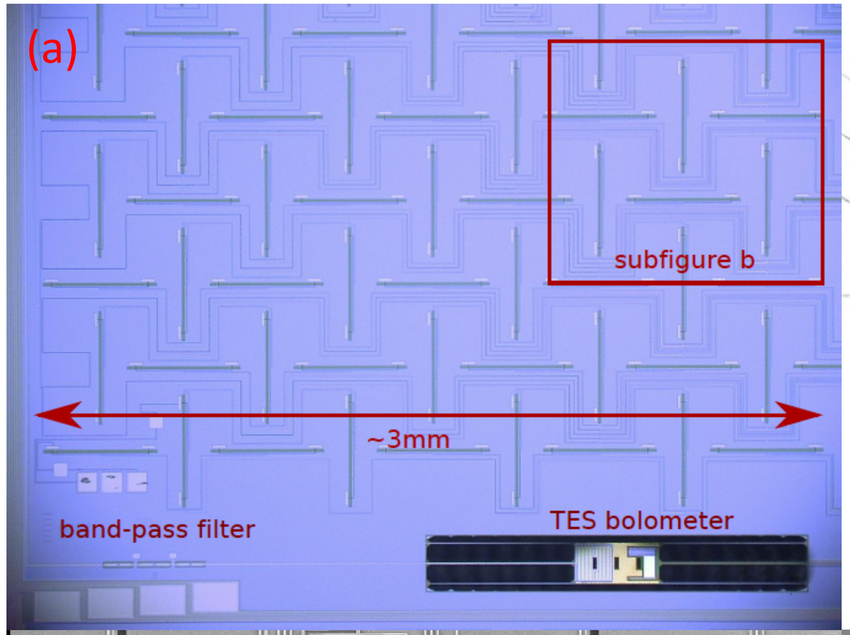


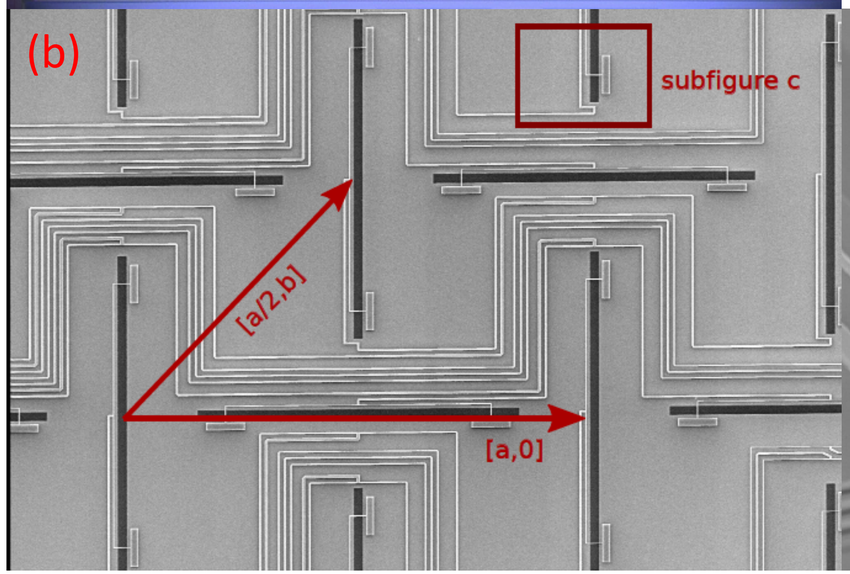
PB lenslets on seating wafer, Suzuki thesis 2013

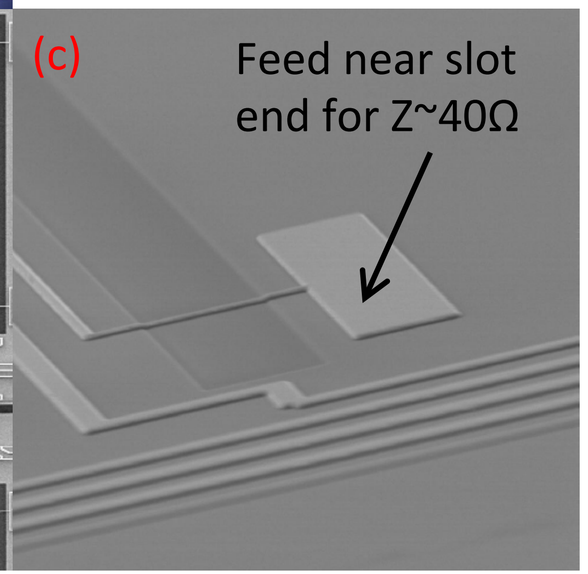
Phased arrays

BICEP2/Keck 8×8 slot array
Ade et al., Ap.J., 812 2015

Good control of angular response







Array of N slots

$\frac{N-1}{2}x$ … $2x$, x, 0, $-x$ … $-\frac{N-1}{2}x$

θ, $x\sin\theta$

Amplitude pattern

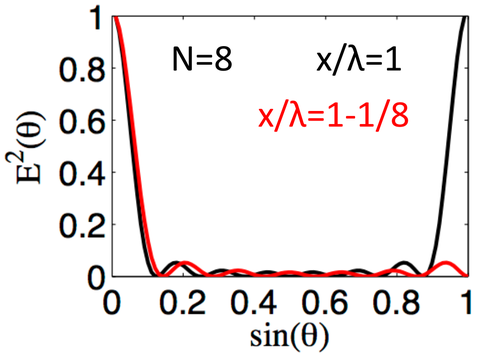
$$E(\theta) = \sum_{M=-\frac{N-1}{2}}^{\frac{N-1}{2}} e^{-i\frac{2\pi}{\lambda}Mx\sin\theta}$$

$$= \frac{\sin\left(N\pi\frac{x}{\lambda}\sin\theta\right)}{\sin\left(\pi\frac{x}{\lambda}\sin\theta\right)}$$

1st grating lobe at $\sin\theta = \frac{\lambda}{x}$

lobe width $\sim \frac{\lambda}{Nx}$

avoid lobe by making $x < \lambda\left(1-\frac{1}{N}\right)$



Effect of Pixel Size on Focal Plane Speed

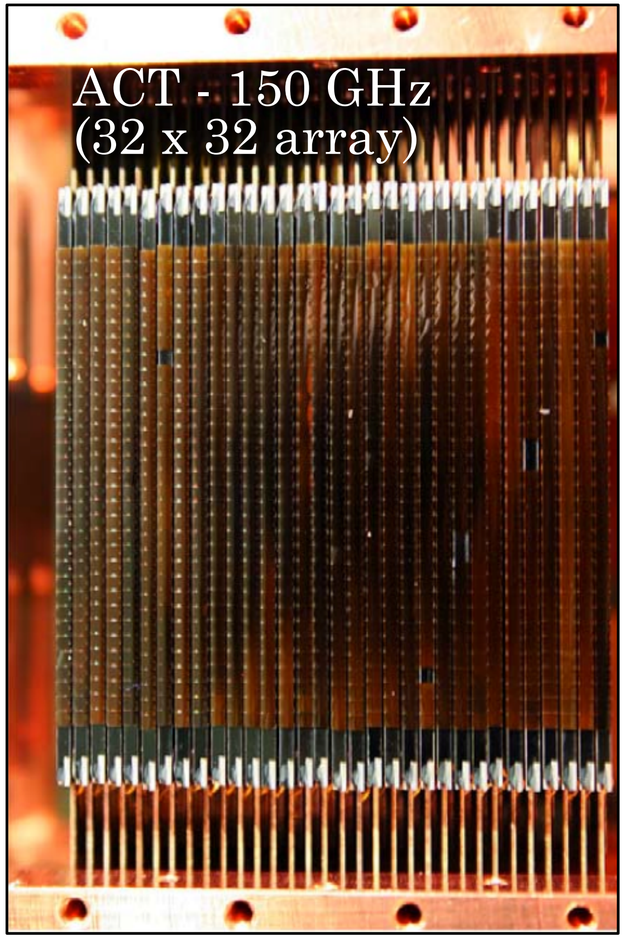
ACT 150 GHz focal plane
- 0.5 F λ pixel size
- ~700 “working” bolometers
- NET_{bolo} ~ 900 uK $s^{1/2}$
- NET_{150} ~ 35 uK $s^{1/2}$

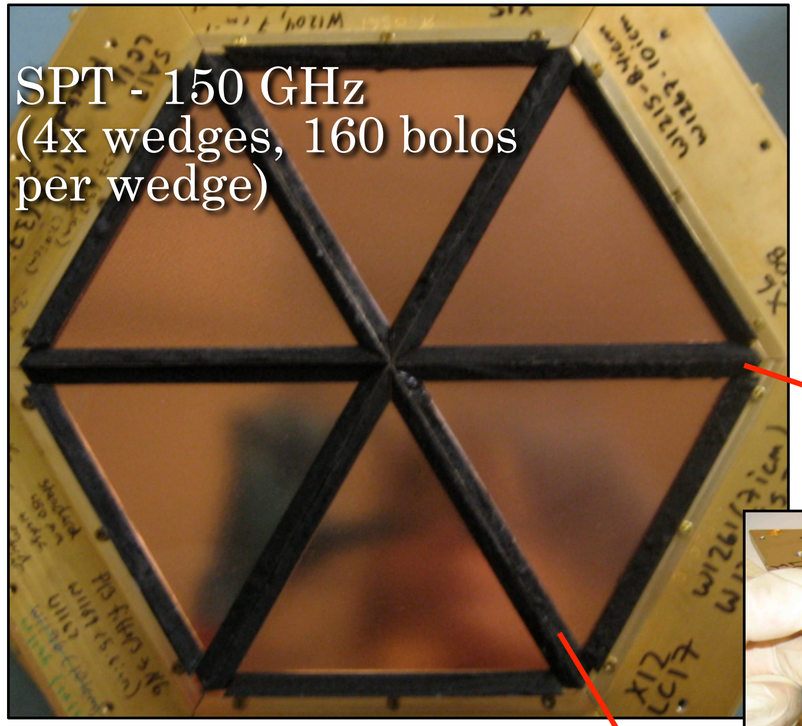
SPT 150 GHz focal plane
- 1.7 F λ pixel size

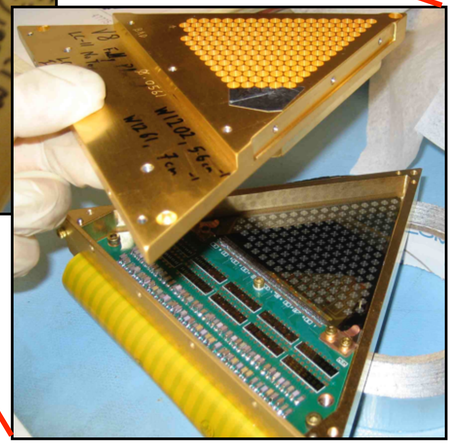
~450 “working” bolometers
- NET_{bolo} ~ 450 uK $s^{1/2}$
- NET_{150} ~ 20 uK $s^{1/2}$

→

SPT was ~3x faster than ACT, largely from more efficient bolometer spacing!







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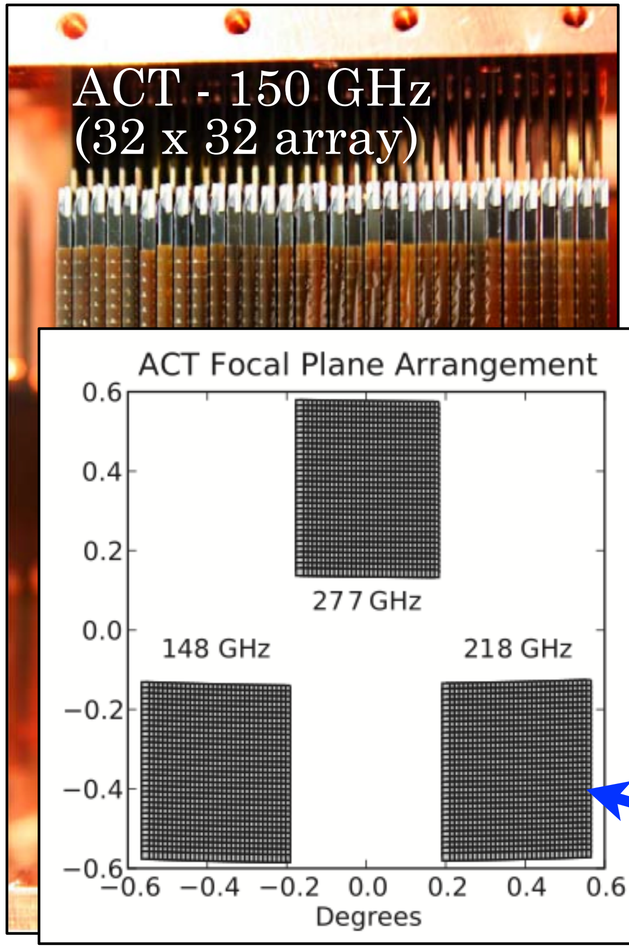
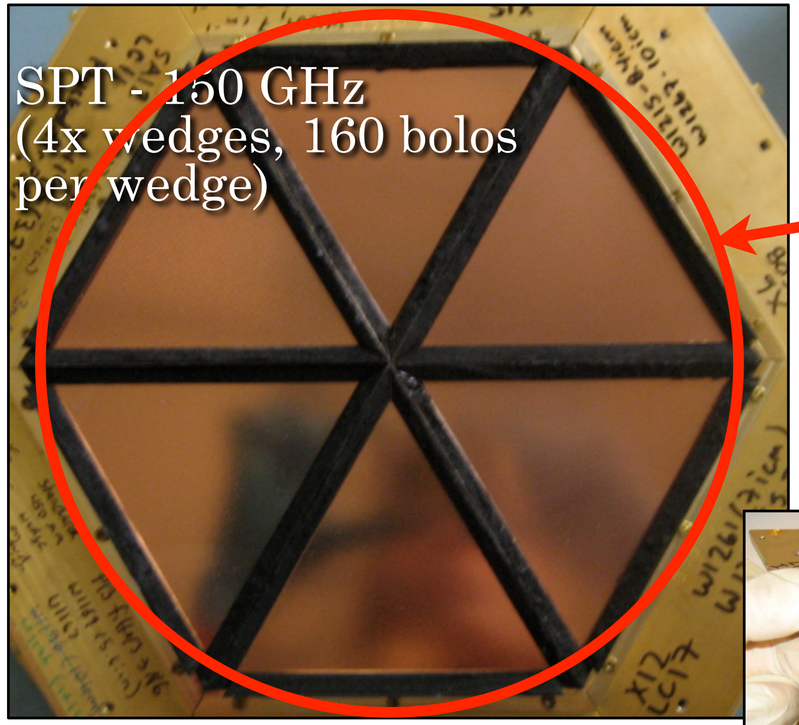
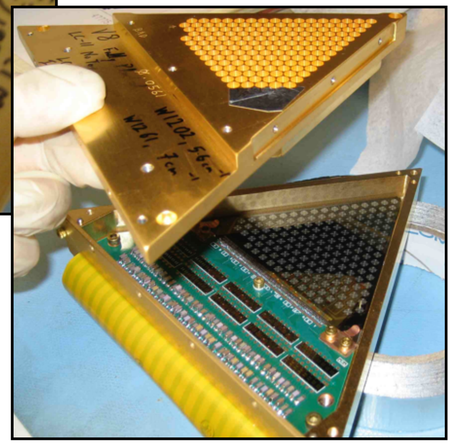
→ **SPT was ~3x faster than ACT, largely from more efficient bolometer spacing!**



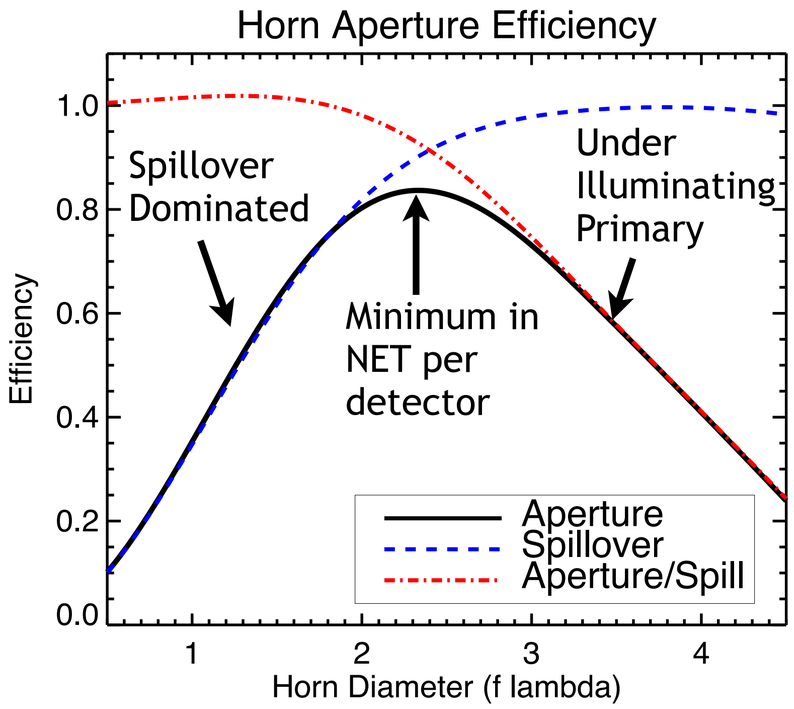


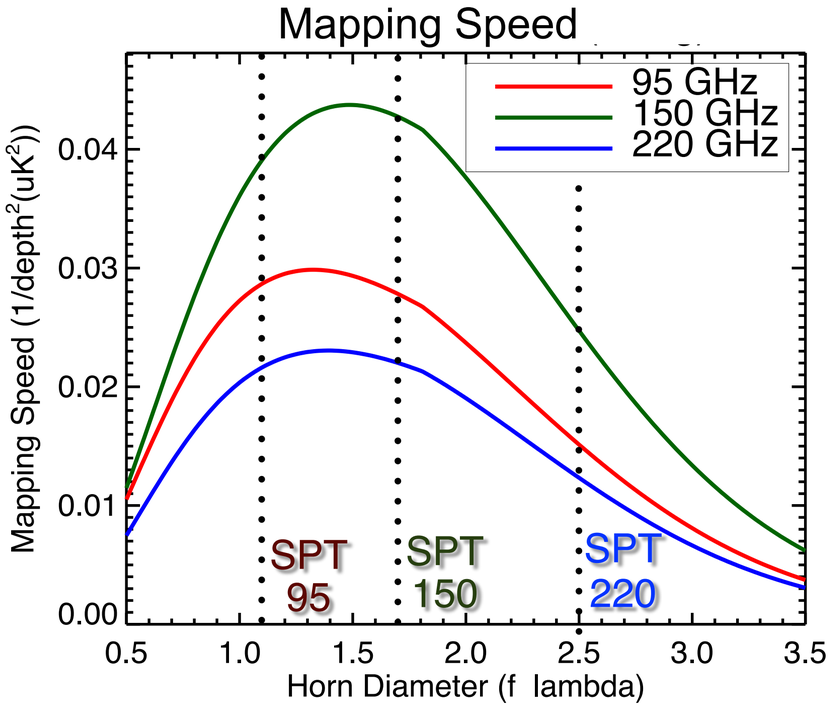
GOOD use of detectors and focal plane space

BAD use of detectors and focal plane space



Optimizing Mapping Speed



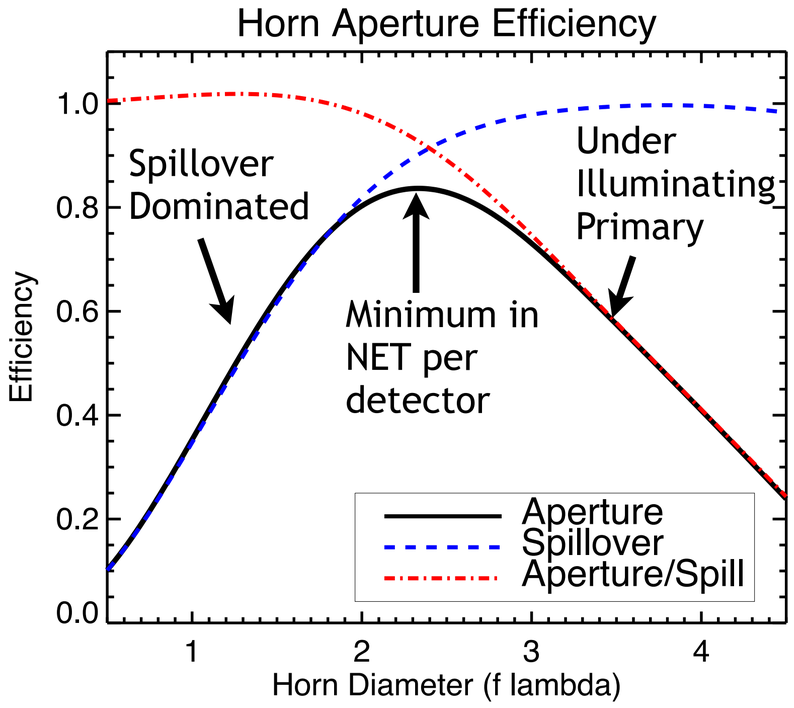


- Pixel size (or horn diameter) affects mapping speed
- Horn diameter of ***~2.2 F λ maximizes coupling to telescope and minimizes NET per detector*** (i.e., because it best-matches pixel to Airy disk / diffraction-size of telescope), but ***number of detectors increases as 1 / (horn diameter)²,*** so greater focal plane mapping speeds are typically possible with smaller pixels

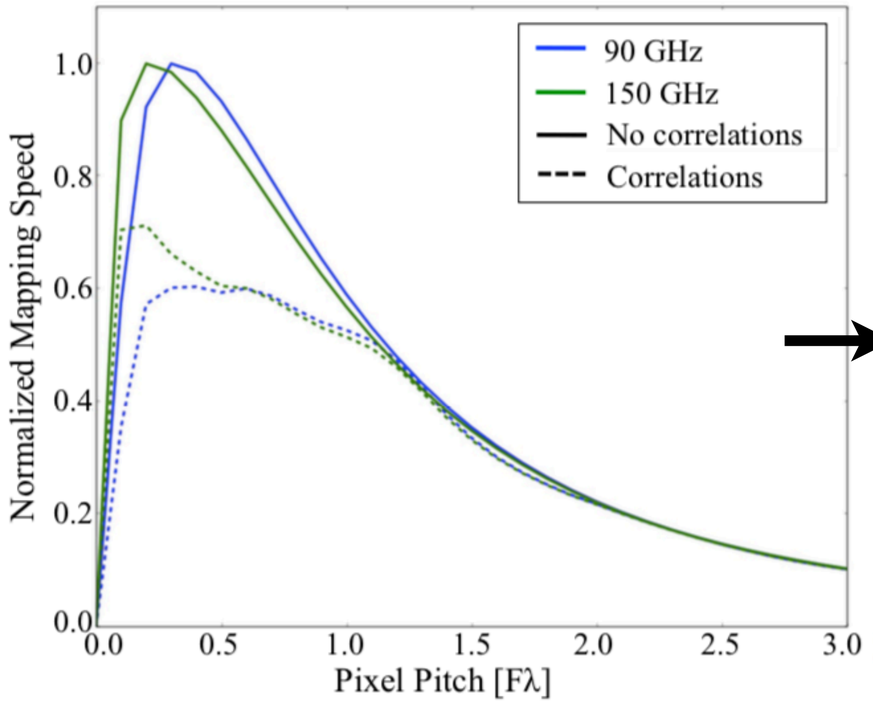
→ **For SPT-SZ, the mapping speed peaked at ~1.3 F λ horn diameter (4 mm at 150 GHz)**

- For SPT-SZ, we kept horn size the same at all frequencies to minimize fab complexity, and to bracket maximum speed at 95 and 150 GHz

Optimizing Mapping Speed



Simons Observatory Mapping Speed
Hill et al. (2018), arxiv: 1806.04316



Optimization can depend on details of noise sources in instrument, e.g., if there is a noise penalty for smaller pixels

- Pixel size (or horn diameter) affects mapping speed

- Horn diameter of ***~2.2 F λ maximizes coupling to telescope and minimizes NET per detector*** (i.e., because it best-matches pixel to Airy disk / diffraction-size of telescope), but ***number of detectors increases as 1 / (horn diameter)²,*** so greater focal plane mapping speeds are typically possible with smaller pixels

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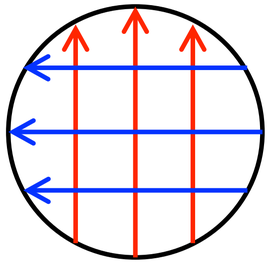
Refernces

- Clarricoats & Olver, Corrugated Horn for Microwave Antennas
- Simon et al. (2016), SPIE 991416 (AdvACT)
- McMahon et al. (2013), LTD13
- Grimes et al. (2007), EI Letters, 43
- Roger O’Brient thesis (2010)
- Toki Suzuki thesis (2013)
- Ade et al. (2015), ApJ, 812 (BICEP/KECK)
- Hill et al., (2018), arxiv: 1806.04316

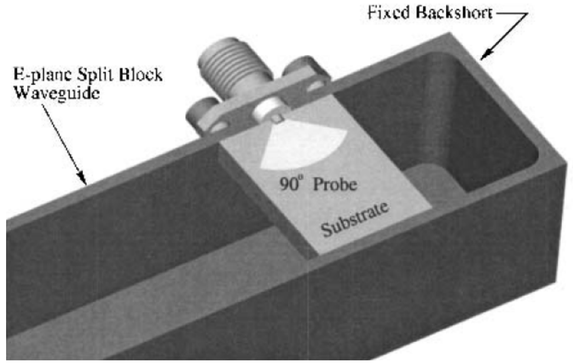
Extras

Orthomode transducer

Couples 2 orthogonal TE11 modes in circular guide to, e.g., mstrip

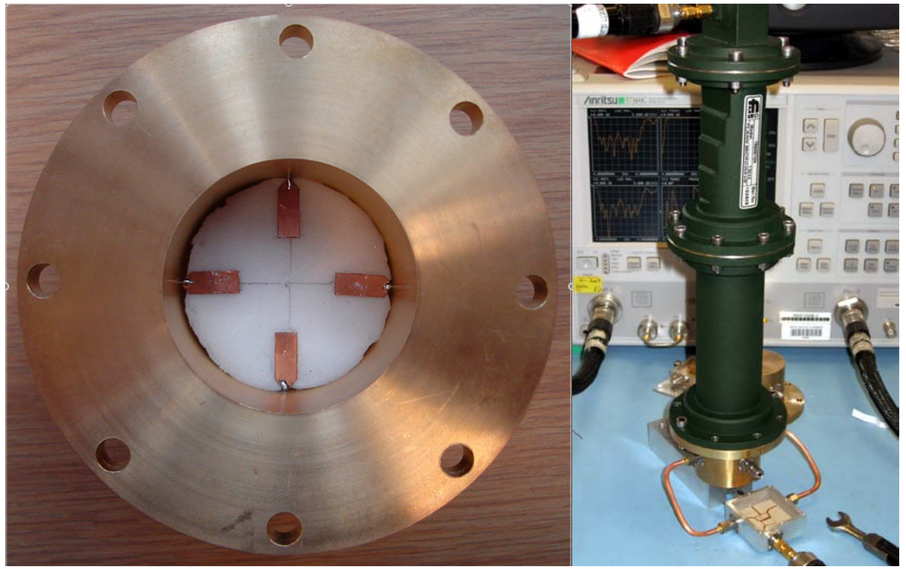


Simple transition for 1 polarization has a probe and a λ/4 backshort. Requires good ground connection at guide wall.



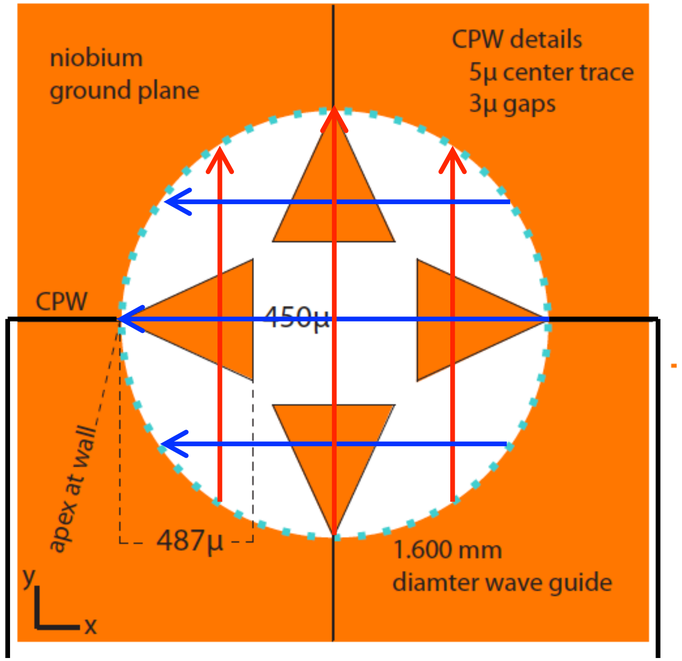
Kooi et al., Int J IR mm waves, 24 2003

4 probes give good symmetry, low xpol

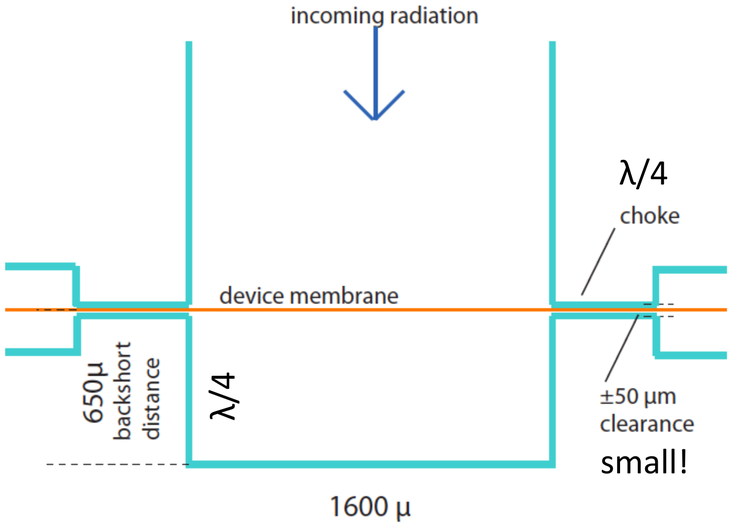


OMT for C-BASS
Grimes et al., El Lett, 43 2007

TOP VIEW



SIDE VIEW

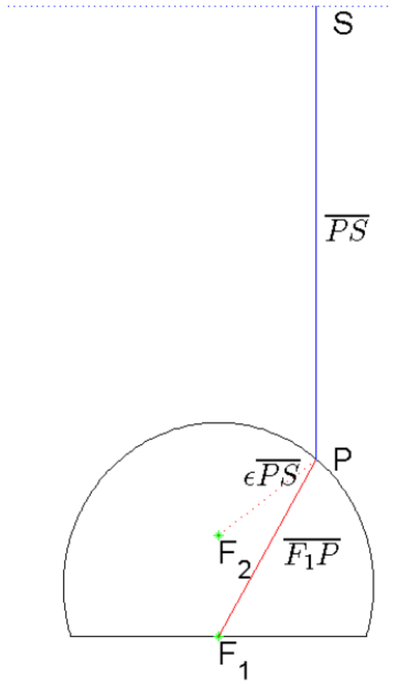


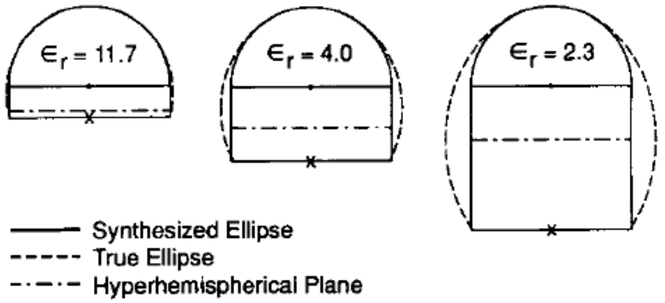
McMahon et al., LTD13 2009
Yoon et al., LTD13 2009

Lenslets

Lenslet improves coupling to a planar antenna on thick substrate
- No trapped rays
- Power in the dielectric is ~n^3 higher than in air
- Concentrate rays if absorber/antenna is smaller than Airy pattern

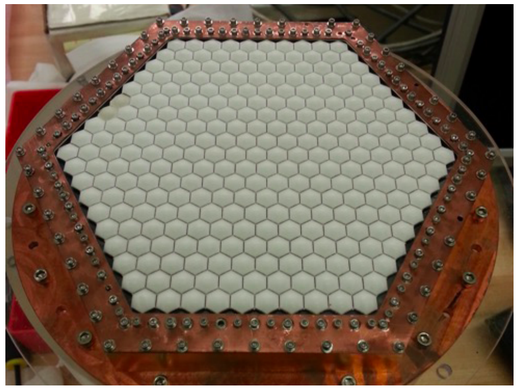
Use elliptical lens to focus a collimated beam



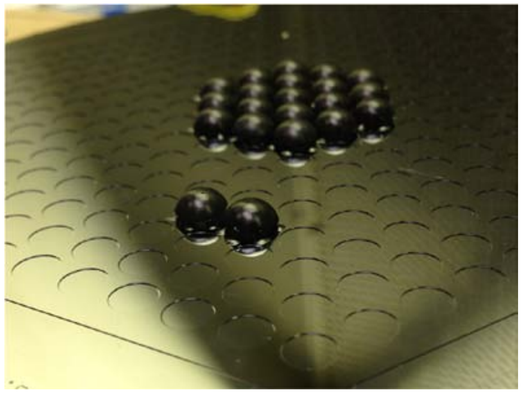


Make an elliptical lens by stacking a hemisphere on a plate
Rebeiz, Proc. IEEE, 80 1982

Elliptical lens focuses plane wave to a point at the far focus
eccentricity=1/n
O'Brient thesis 2010



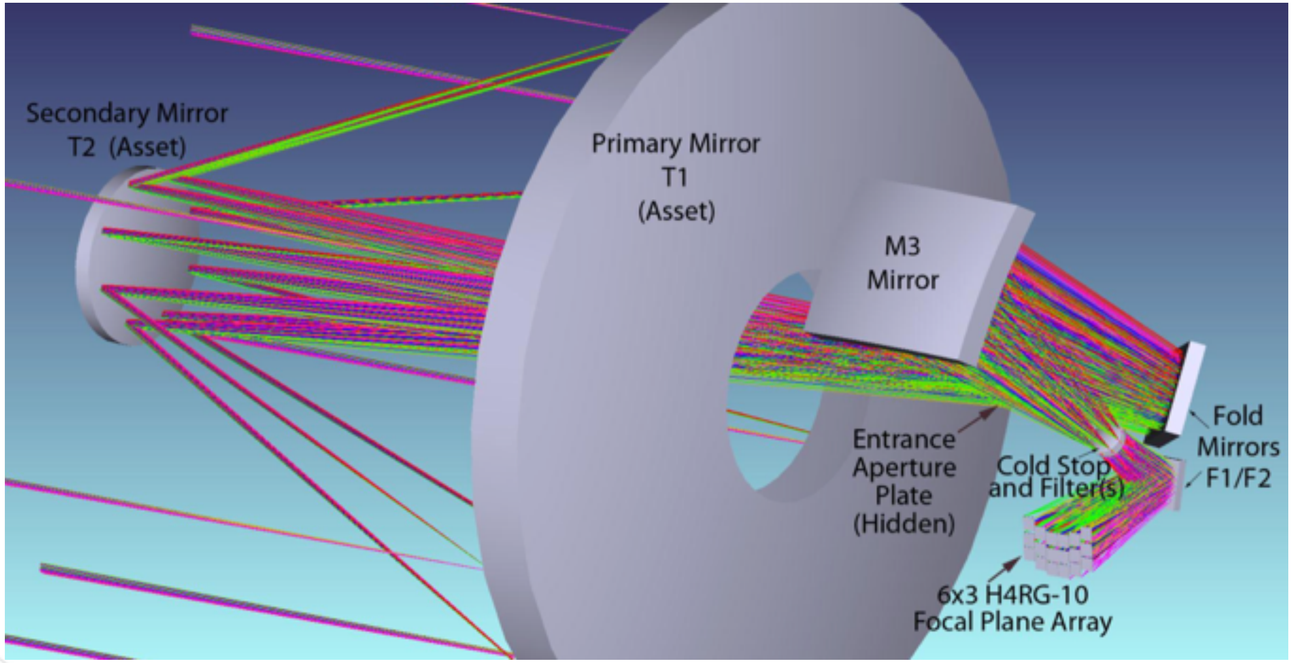
SPT-3G lenslet array

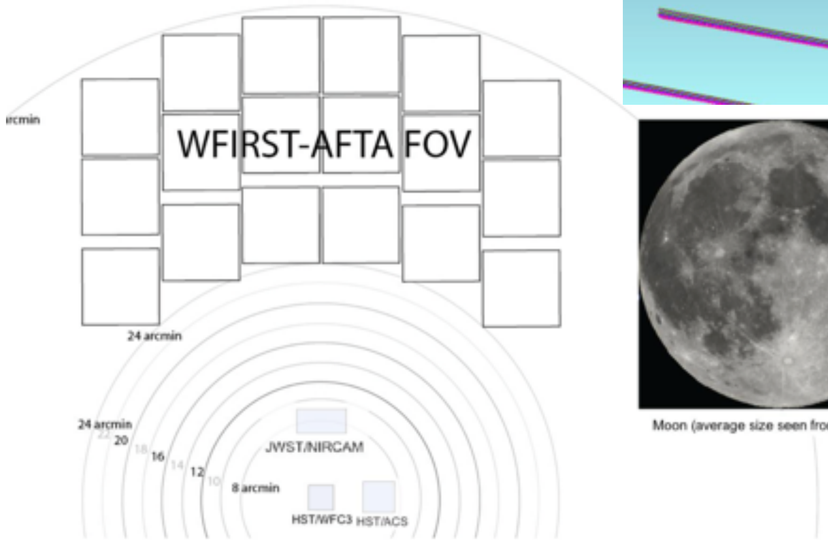


PB lenslets on seating wafer, Suzuki thesis 2013

TMA (Three Mirror Anastigmat)

WFIRST, annular FOV, 0.28 sq degrees,
2.4 m aperture, 0.8-2.0 μm bandpass





Moon (average size seen from Earth)

Multiple Instruments:

Wide-field Camera

High Contrast Exoplanet Imager

Integral-Field Spectrograph

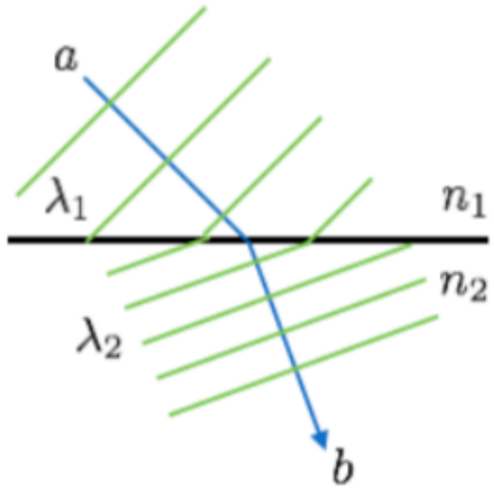
Reflection and Refraction

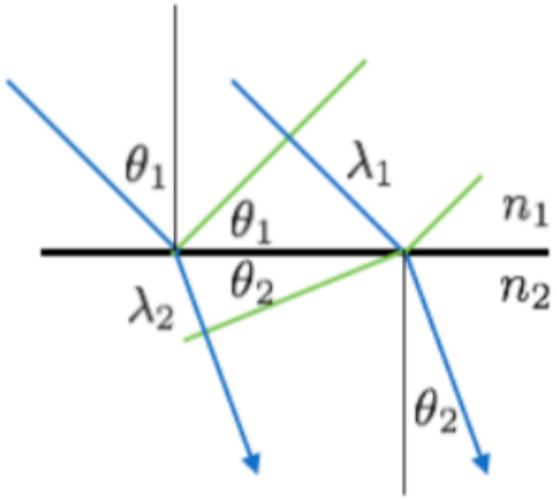
- The optical path difference between wavefronts must be the same in each medium, so:

$$n_1\lambda_1 = n_2\lambda_2$$

- Angles of incidence and refraction, measured from the normal ot the interface, are related by **Snell's law**:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$





Reflection and Refraction

- The refelcted and transmitted amplitudes are given by Fresnel equations.
- E-vector parallel to plane of incidence

$$r_p = \frac{n_2 \cos\theta_i - n_1 \cos\theta_t}{n_2 \cos\theta_i + n_1 \cos\theta_t}$$

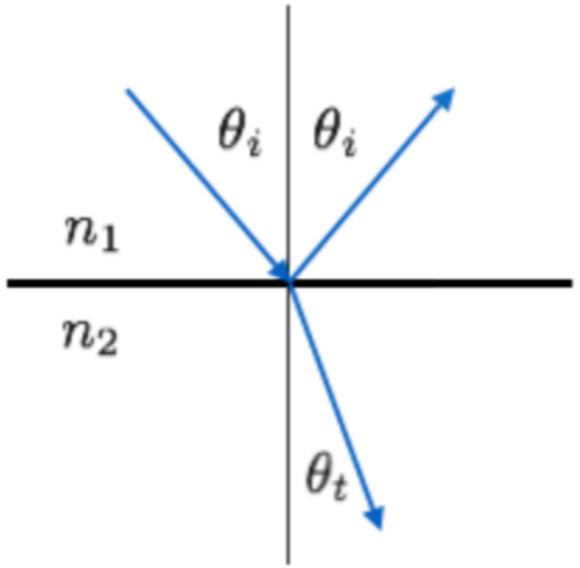
$$t_p = \frac{2n_1 \cos\theta_t}{n_2 \cos\theta_i + n_1 \cos\theta_t}$$

- E-vector perpindicular to plane of incipdence

$$r_s = \frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t}$$

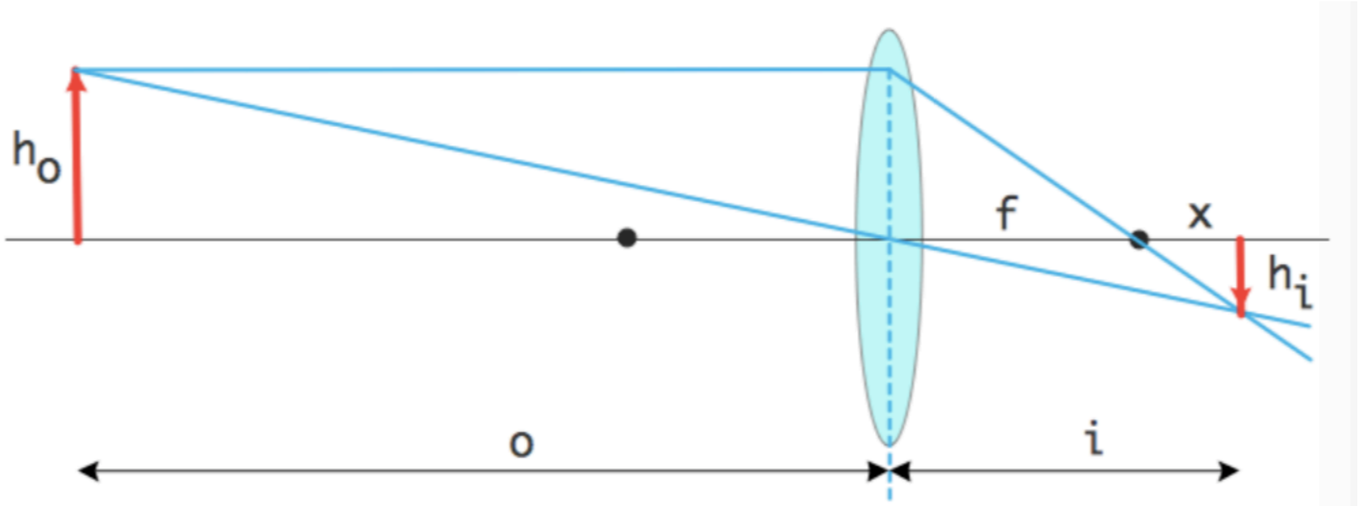
$$t_p = \frac{2n_1 \cos\theta_i}{n_1 \cos\theta_i + n_2 \cos\theta_t}$$

- The fraction of power reflected is
  $|r^2| \sim [(n_2 - n_1) / (n_2 + n_1)]^2$



$$r^2 + \frac{n_2 \cos\theta_t}{n_1 \cos\theta_i} t^2 = 1.$$

Lens Maker's Equation



- Replace sin θ with θ in Snell's law -> **Paraxial approximation**
- Assume refraction or reflection takes place in a plane -> **thin-lens approximation**
- From figure, we can see:

  $$(h_i/h_o) = (i/o) = (x/f) = (i-f)/f$$

- Gauss's image formula follows

  $$1/i + 1/o = 1/f$$