Fourier Transform Spectroscopy and Interference

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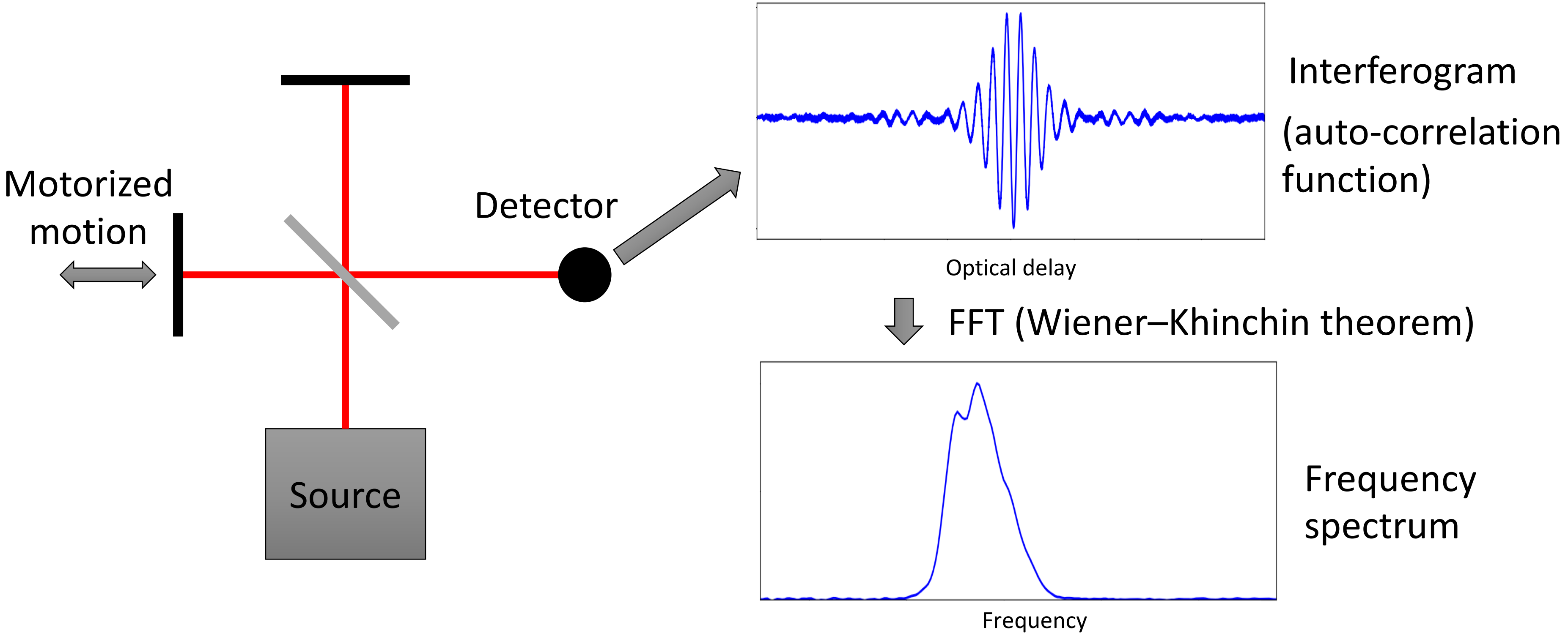
2022 CMB Instrumentation Summer School, University of Chicago

Fourier-Transform Spectrometer (FTS)



- Detector measures power vs. optical delay (auto-correlation function of the source's radiation).
- FFT of power vs. optical delay is the power spectrum of the source, including the detector's response function.
- We can add/remove materials in the optical path to probe the transmission

How an FTS works

- Electrical field from one arm is $E(t)$, where t is the time.
- Electrical field from the other arm is $E(t+\tau)$, τ is the added delay.
- The power measured by the detector with delay τ is

$$\int_{-\infty}^{\infty} |E(t) + E(t+\tau)|^2 dt \sim \text{Constant} + 2 \boxed{\int_{-\infty}^{\infty} \overline{E(t)} E(t+\tau) dt}$$

Auto-correlation function
$C(\tau)$

How an FTS works

Wiener-Khinchin Theorem

Recall the definition of the autocorrelation function $C(t)$ of a function $E(t)$,

$$C(t) \equiv \int_{-\infty}^{\infty} \overline{E}(\tau) E(t+\tau)\, d\tau.$$

Also recall that the Fourier transform of $E(t)$ is defined by

$$E(\tau) = \int_{-\infty}^{\infty} E_\nu\, e^{-2\pi i \nu \tau}\, d\nu,$$

giving a complex conjugate of

$$\overline{E}(\tau) = \int_{-\infty}^{\infty} \overline{E}_\nu\, e^{2\pi i \nu \tau}\, d\nu.$$

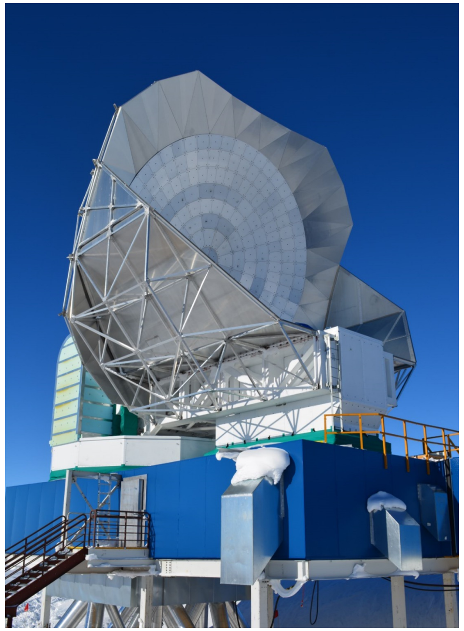
Plugging $\overline{E}(\tau)$ and $E(t+\tau)$ into the autocorrelation function therefore gives

$$\begin{aligned}
C(t) &= \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} \overline{E}_\nu\, e^{2\pi i \nu \tau}\, d\nu\right]\left[\int_{-\infty}^{\infty} E_{\nu'}\, e^{-2\pi i \nu' (t+\tau)}\, d\nu'\right] d\tau \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \overline{E}_\nu E_{\nu'}\, e^{-2\pi i \tau(\nu'-\nu)}\, e^{-2\pi i \nu' t}\, d\tau\, d\nu\, d\nu' \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \overline{E}_\nu E_{\nu'}\, \delta(\nu'-\nu)\, e^{-2\pi i \nu' t}\, d\nu\, d\nu' \\
&= \int_{-\infty}^{\infty} \overline{E}_\nu E_\nu\, e^{-2\pi i \nu t}\, d\nu \\
&= \int_{-\infty}^{\infty} |E_\nu|^2\, e^{-2\pi i \nu t}\, d\nu \\
&= \mathcal{F}_\nu\left[|E_\nu|^2\right](t),
\end{aligned}$$

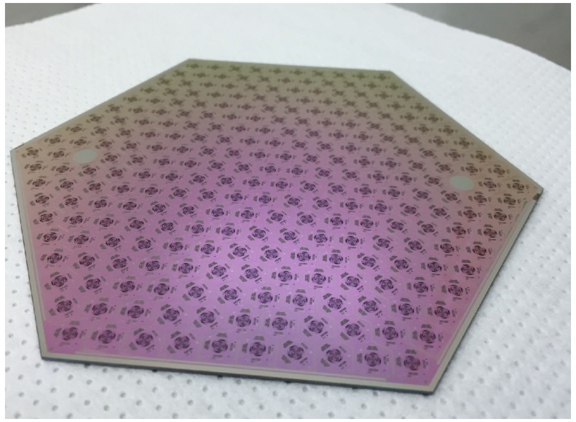
Taken from Walfram Mathworld

- The autocorrelation is simply given by the Fourier transform of $|E(\nu)|^2$ or power spectrum density, where ν is the frequency.
- An (inverse) Fourier transform converts the auto-correlation function to the power spectrum of the measured radiation field.
- Here they swapped variable t and τ from our notation.

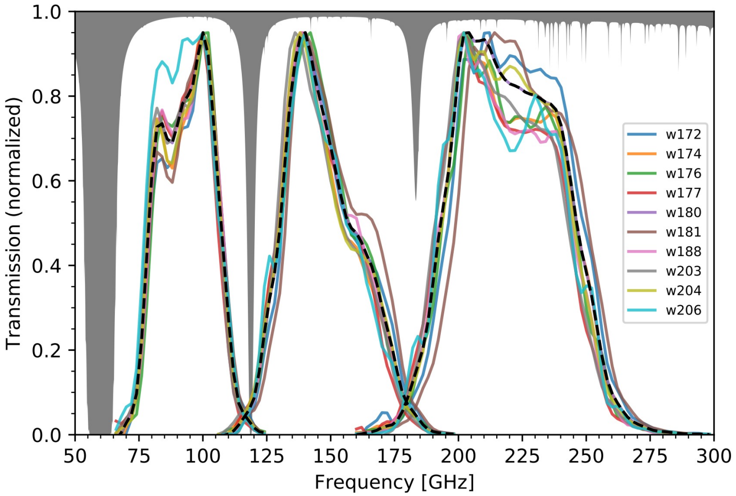
Application – detector frequency band characterization



South Pole Telescope (SPT), a 10m telescope for measuring the CMB.



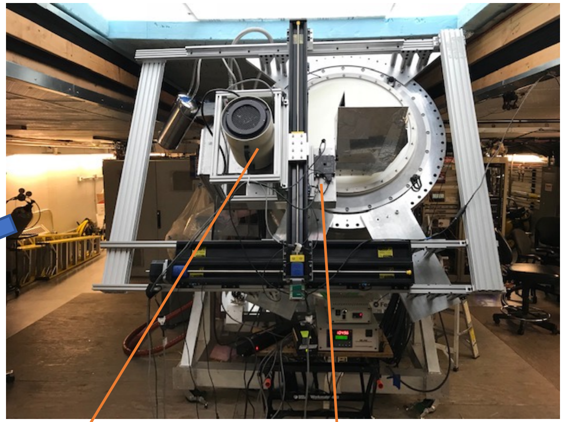
SPT detectors, with frequency bands centered at 95GHz, 150GHz, and 220GHz



Measurement of detector frequency bands, separated by wafer and frequency.

Detector spectral band calibration at the South Pole

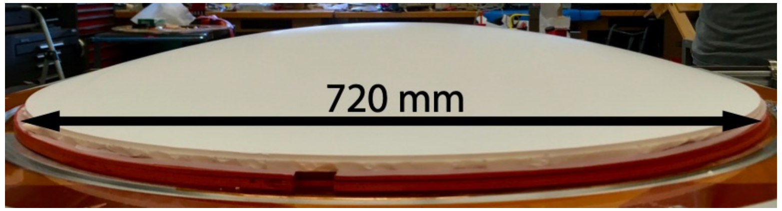
This FTS mounted on a X-Y stage



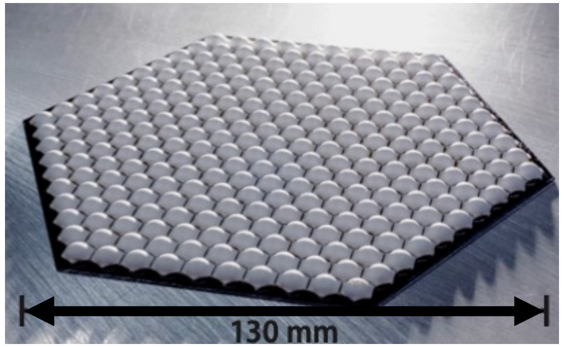
1300K Thermal source

FTS

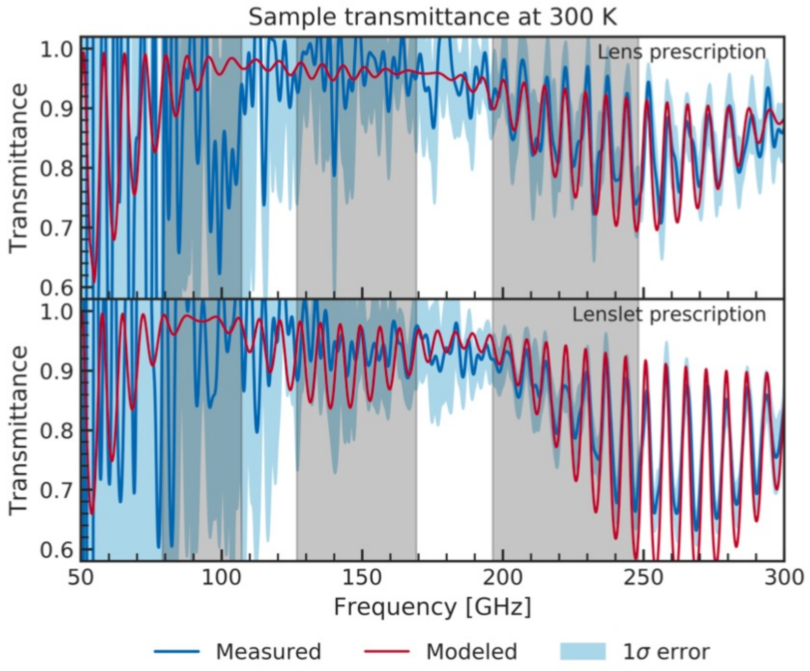
Application – optical system characterization



A lens for SPT-3G



A lenslet array for SPT-3G

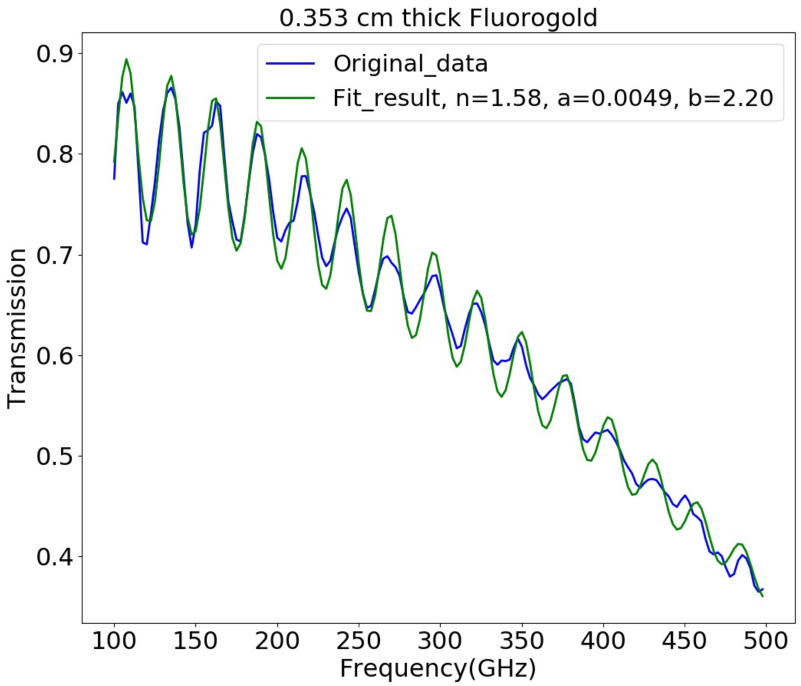


Transmissions of the 3-layer AR-coated lens and lenslet

Testing setup

Nadolski, A, et ,al, Applied Optics 59.10 (2020): 3285-3295

Application: optical material characterization



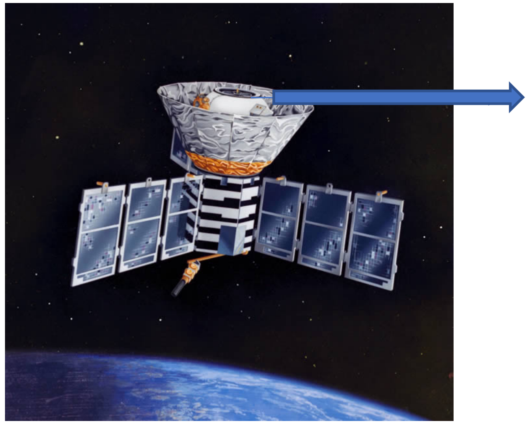
We used the FTS to measure the refraction index and loss properties of Fluorogold.

Green line is fit to equations from Halpern et, al. 1986

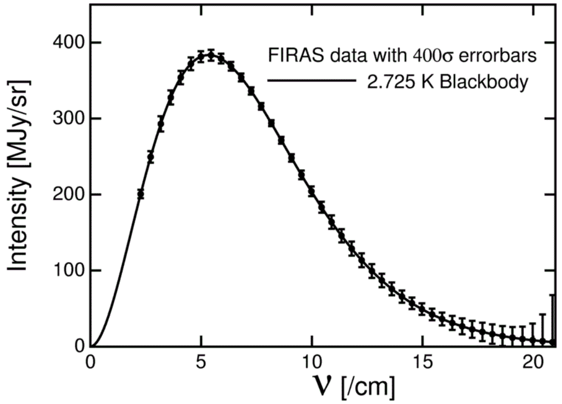
Can be used for lens and AR coating transmission measurements.

Application – measure power spectrum of astrophysical and other sources

Measure the Cosmic Microwave Background (CMB) spectrum



COBE satellite

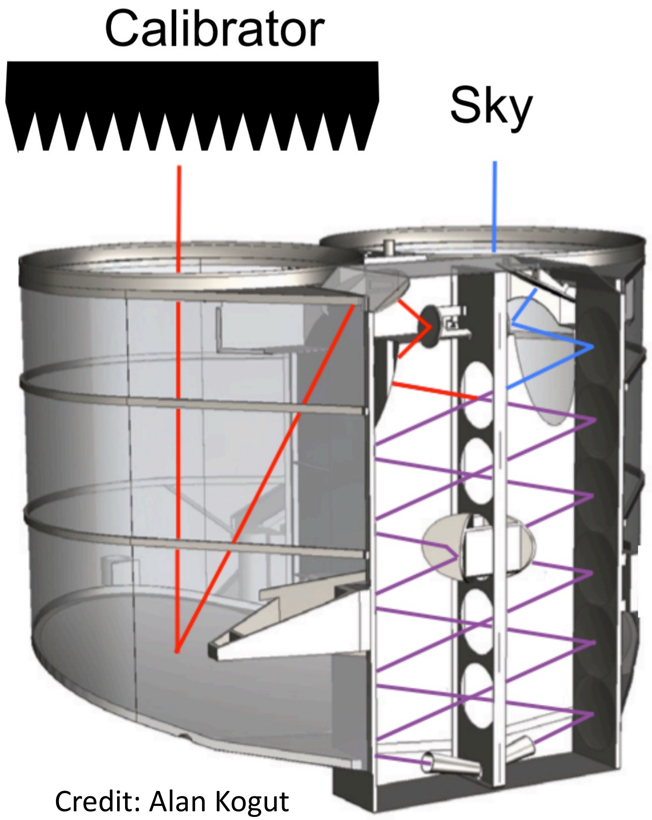


Credit: NASA COBE team

No spectral distortion beyond blackbody was detected.

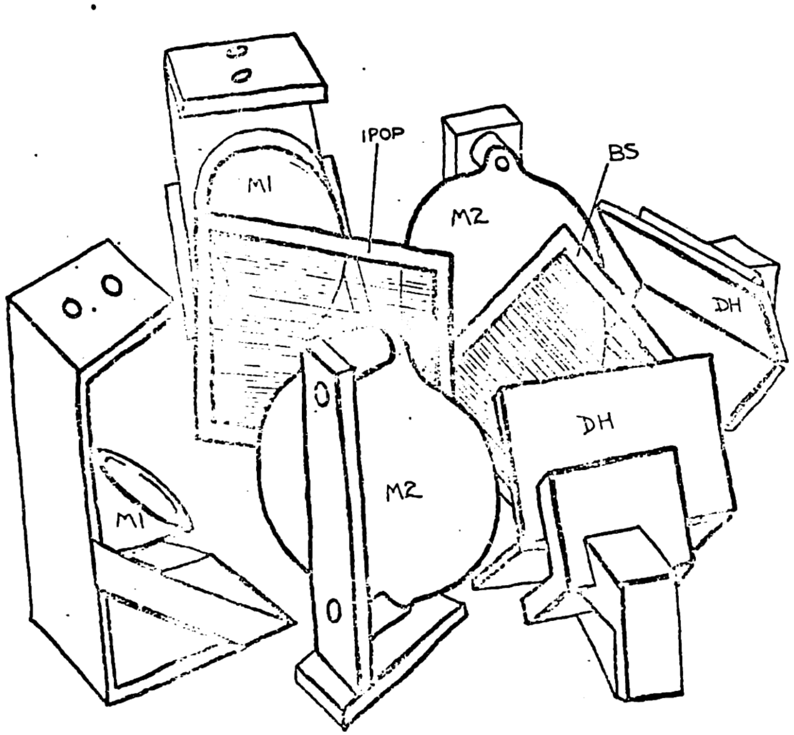
PIXIE was proposed

The Primordial Inflation Explorer (PIXIE)

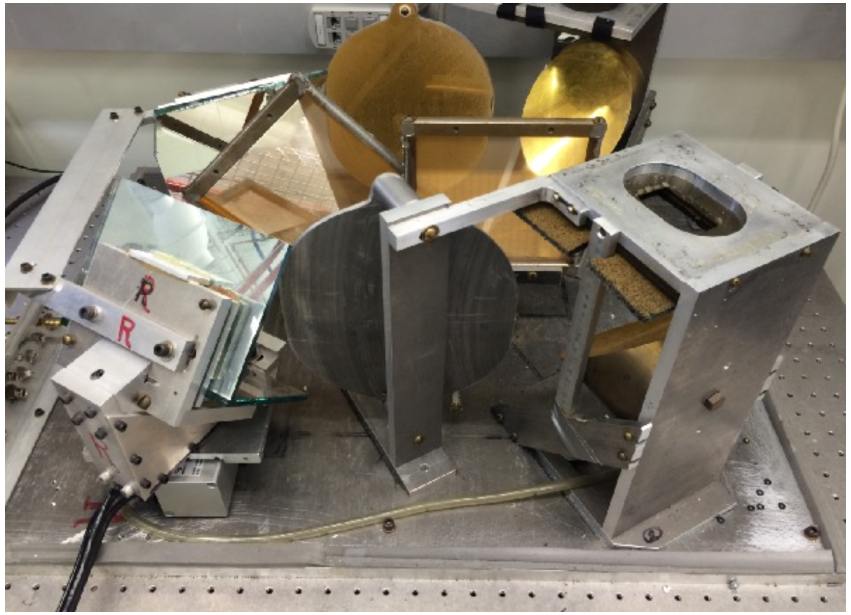


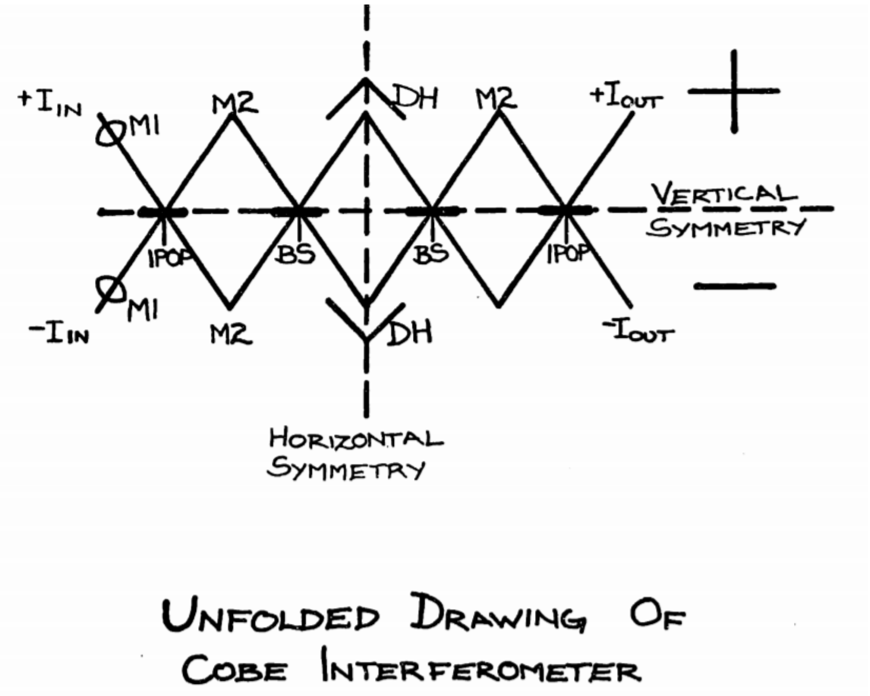
Credit: Alan Kogut

COBE FIRAS FTS

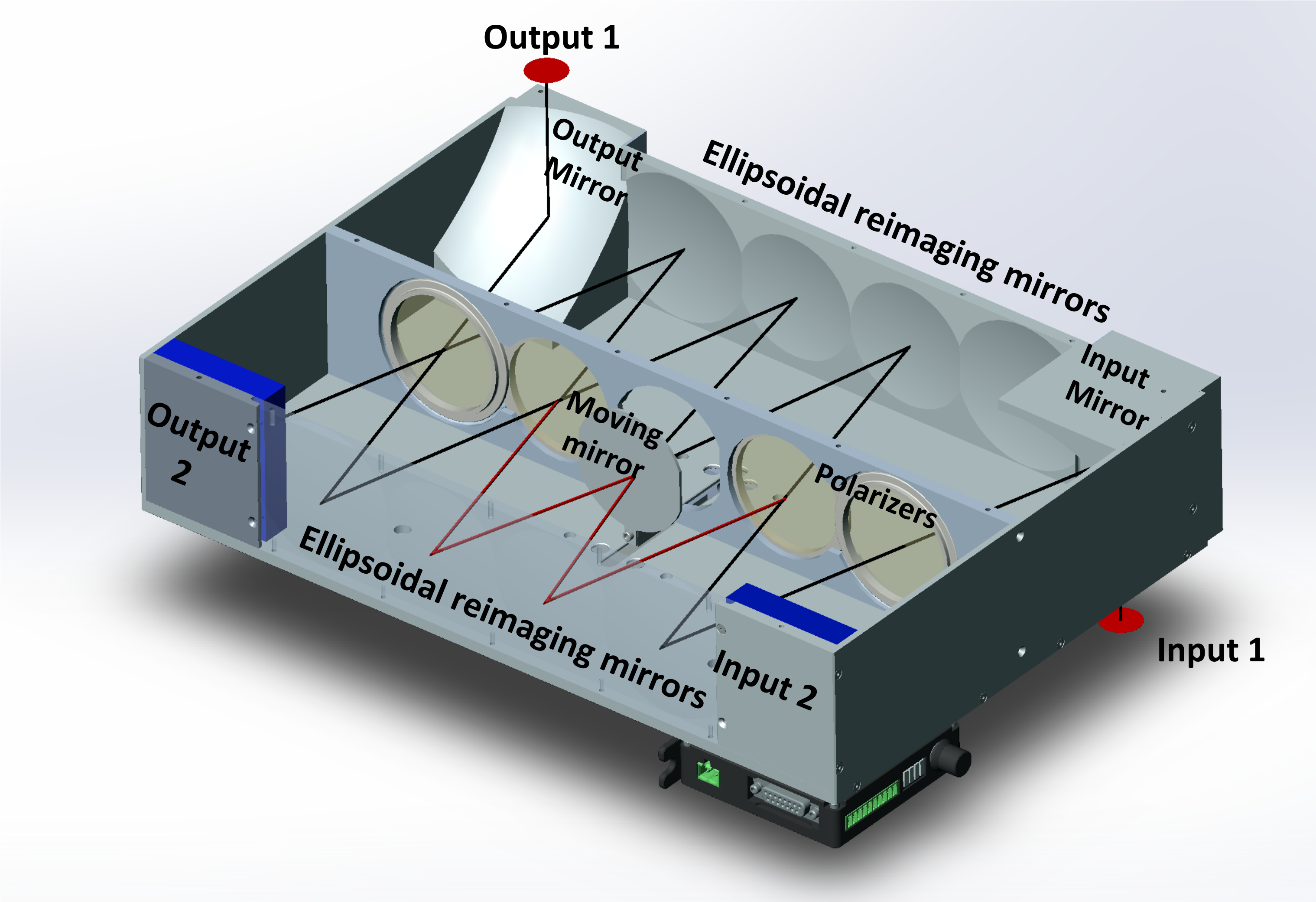


Shoemaker Thesis, 1980



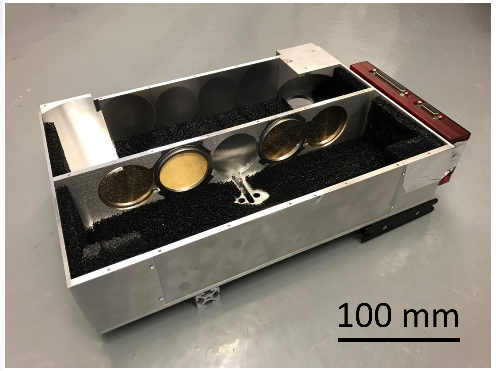


PIXIE-style FTS



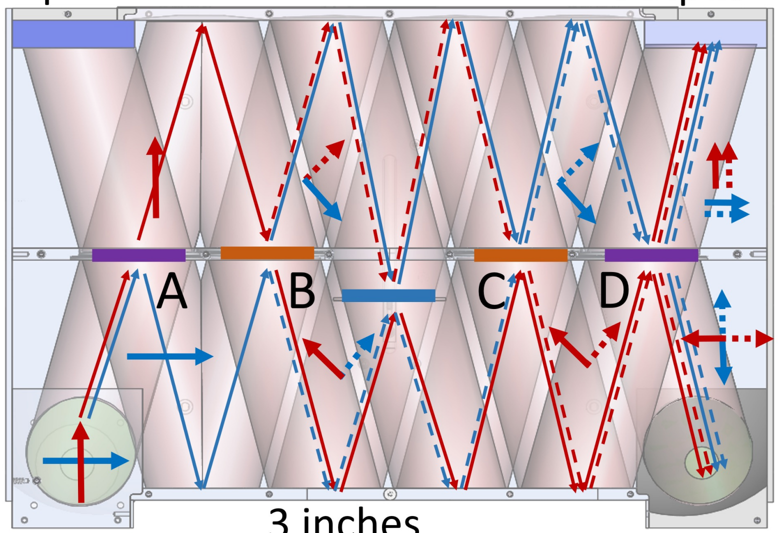
Frequency range: 50-330 GHz, étendu: 100 $\mathrm{mm}^2\mathrm{sr}$, resolution: 4GHz.
Driven by detector characterization, can be tuned for other purposes

- Symmetric design: systematics control
- Two inputs and two outputs: differential output
- Two polarizations: no polarization loss
- Ellipsoidal mirrors: high density of beams
- Moving mirror: add optical delay

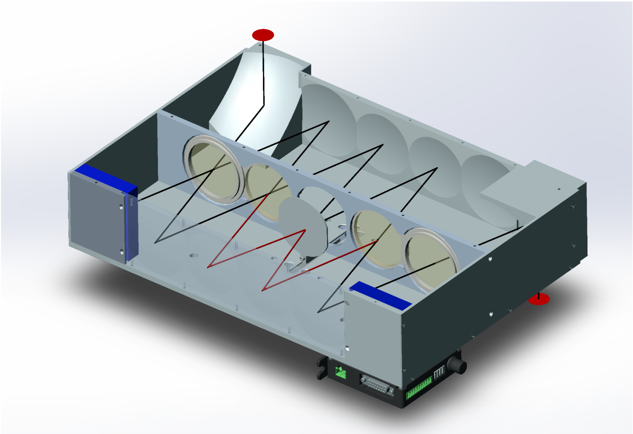


Fabricated FTS
380x250x76 mm

Polarization transfer

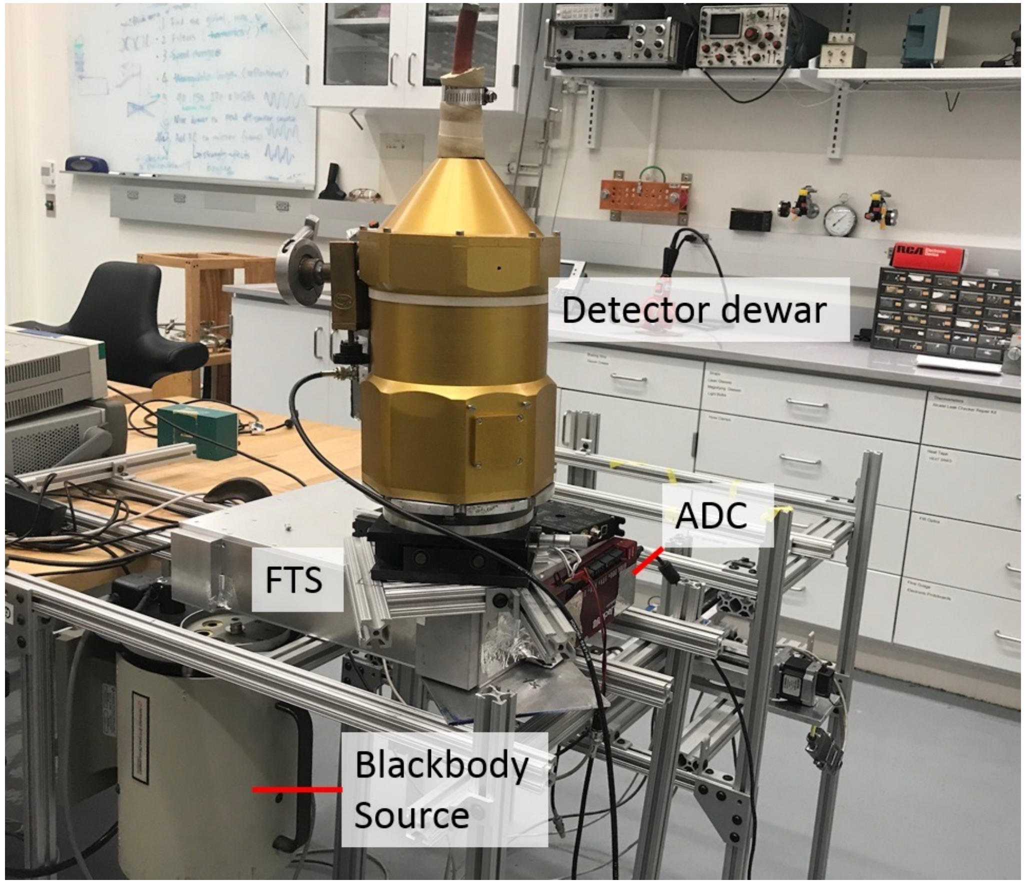


Thin arrows: optical paths.
Thick arrows: polarizations



- Trace radiation from Input 1
- Polarizer A, B, C and D are made from gold plated tungsten wires
- Polarizer A polarizes the input radiation
- Polarizer B is 45° to polarizer A and mixes the radiation.
- Center mirror (Blue) Add phase delay.
- Polarizer C is vertical to B and recombines the polarizations.
- Polarizer D is parallel to A and splits radiation to two outputs.

Characterization



Testing setup

- Input1 is coupled to a radiation source (broadband blackbody, or narrow-band Gunn oscillator)
- Output 1 is coupled to a bolometer (within the detector dewar)
- The other ports are covered with HR20 absorber

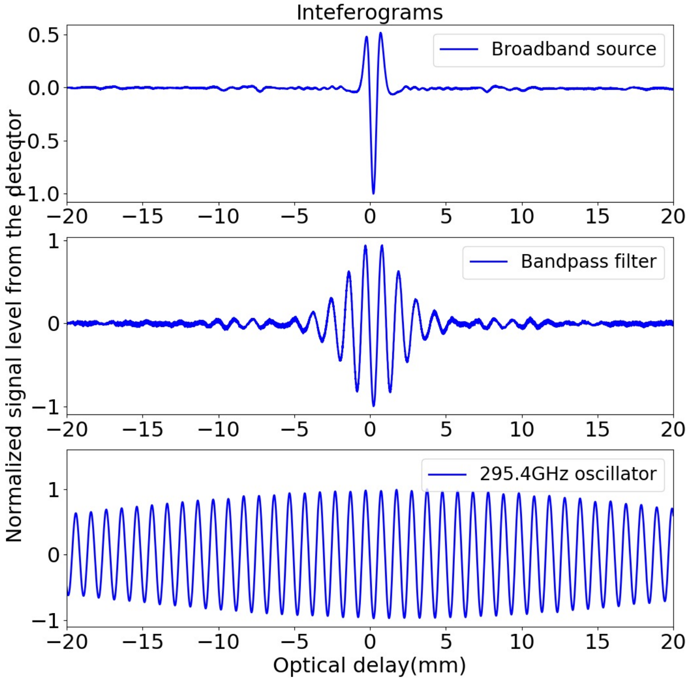
Tests

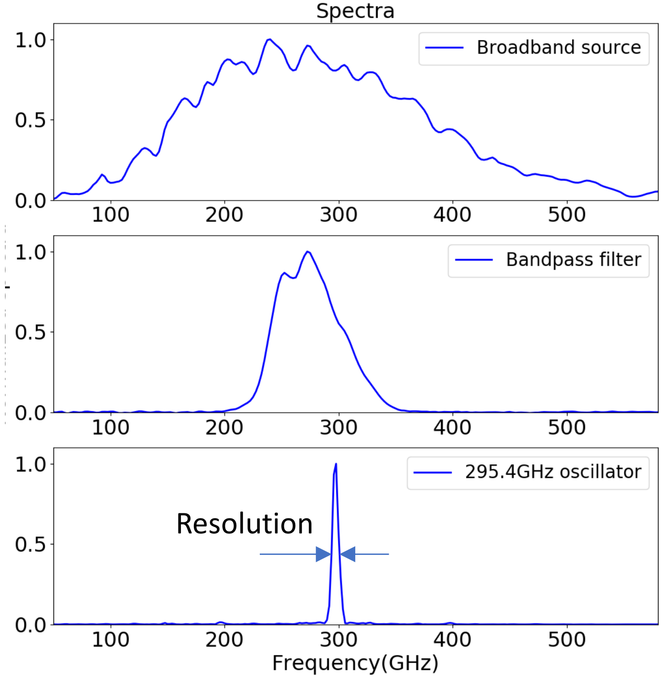
- Sample spectra measurements
- Transfer efficiency
- Frequency resolution and shift (accuracy)
- Instrument delay window function

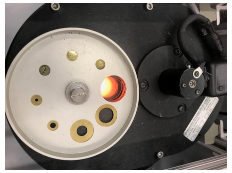
Sample interferograms and spectra

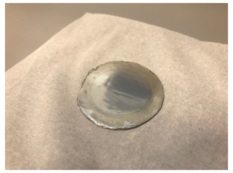
FFT

Interferograms of different sources → Spectra






1300K Blackbody

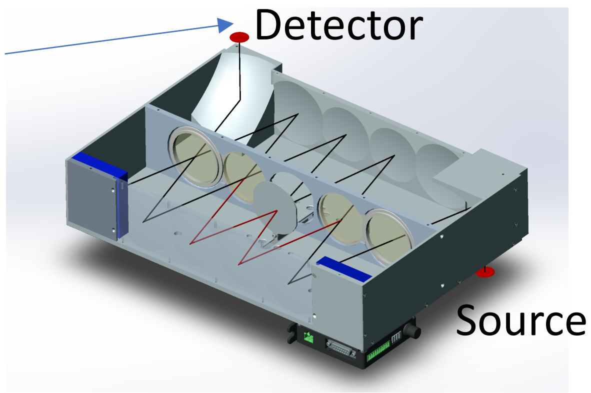

Metal-mesh filter


Gunn oscillator

- The bands for these sources match our expectations.
- The narrower the band, the longer the coherence length.

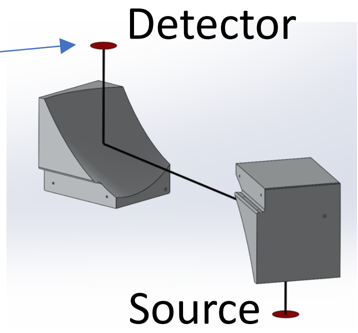
Transfer efficiency

2x Output through the FTS



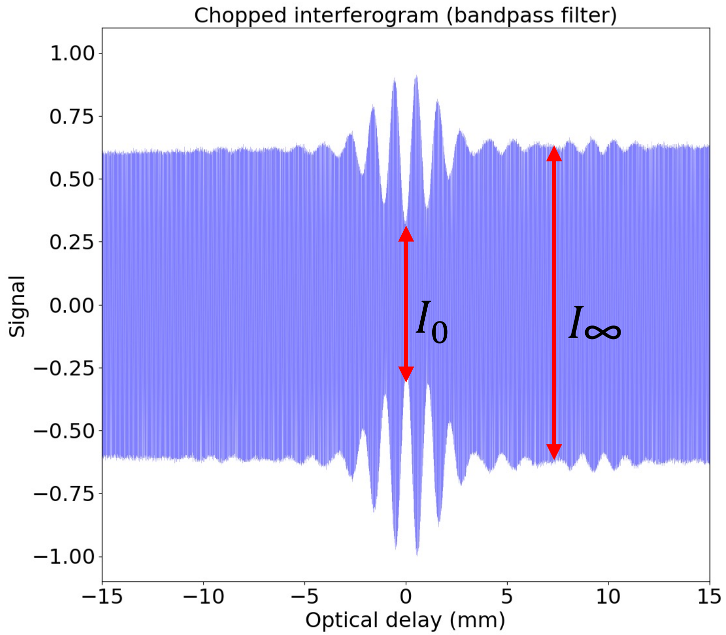
Divided by

Output through two coupling mirrors



Measured transfer efficiency with mirror centered: 92±5%

Modulation contrast



Destructive interference at the center

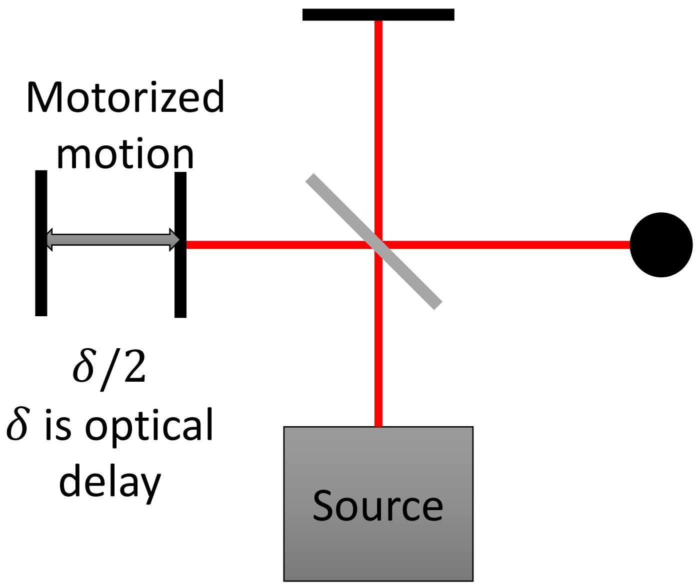
Modulation contrast definition

$C = I_0 / I_\infty$ - 1

Ideally, I_0=0, C=-100%

Measured modulation contrast: -55±3%

Frequency resolution – ideal case

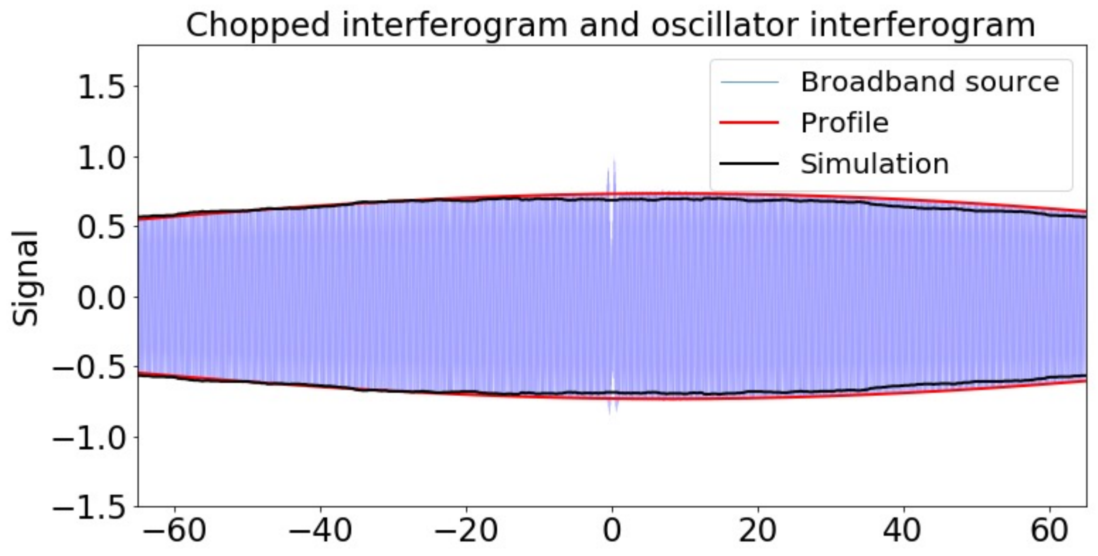


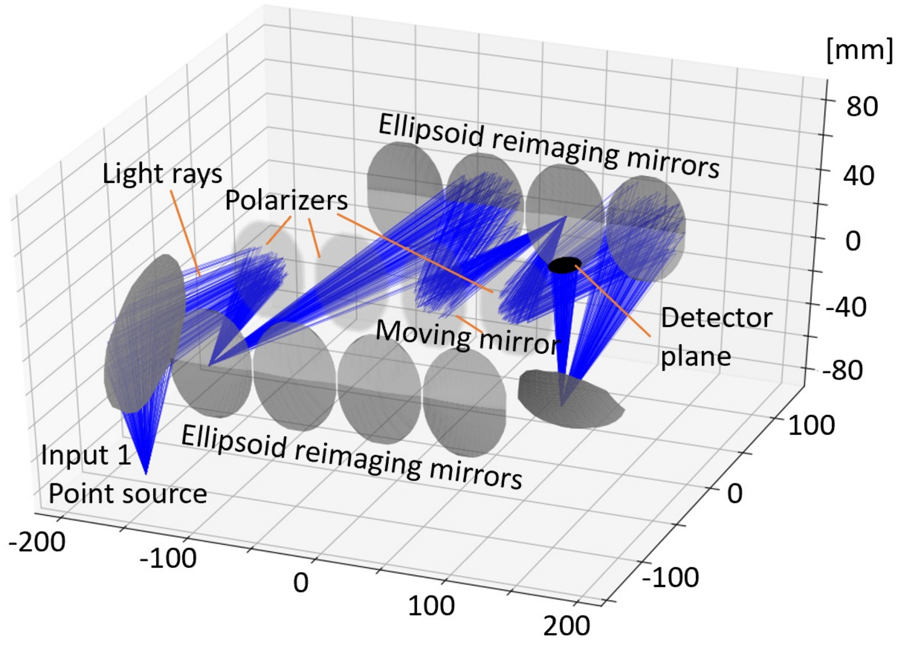
- The mirror movement has finite limits such that the interferogram is multiplied by $A(\delta) = 1 \text{ for } \delta \text{ from } -\delta_{\max}$ to $\delta_{\max}$ and 0 elsewhere
- The result is convolving the power spectrum by the instrument line shape function $2\,\delta_{\max}\dfrac{\sin(2\pi\nu\delta_{\max})}{2\pi\nu\delta_{\max}}$
- Rayleigh criterion: adjacent spectral lines are resolved when the peak of one line is the first zero of a nearby line. Frequency resolution is $1/2\delta_{\max}$

Frequency resolution – limiting factors (non-idealities)

- Frequency spectrum is the FFT of the optical power vs. delay (interferogram)
- Frequency resolution is the FFT FWHM of the window function on the interferogram
- From the instrument, scanning weight, and later apodization
- Limiting factors from instrument
  - Maximum optical delay (scan length), set by hardware
  - Transfer efficiency loss – more beam is lost at higher delay
  - Coherence reduces at higher optical delays
    - Recombined beam from the two optical paths separate as delay increases
    - Non-uniform optical delays for light rays within the beam
  - Frequency shift for extended sources
    - Light rays within a finite area travel along different paths and have different delays

Limiting factor 1 - transfer efficiency loss

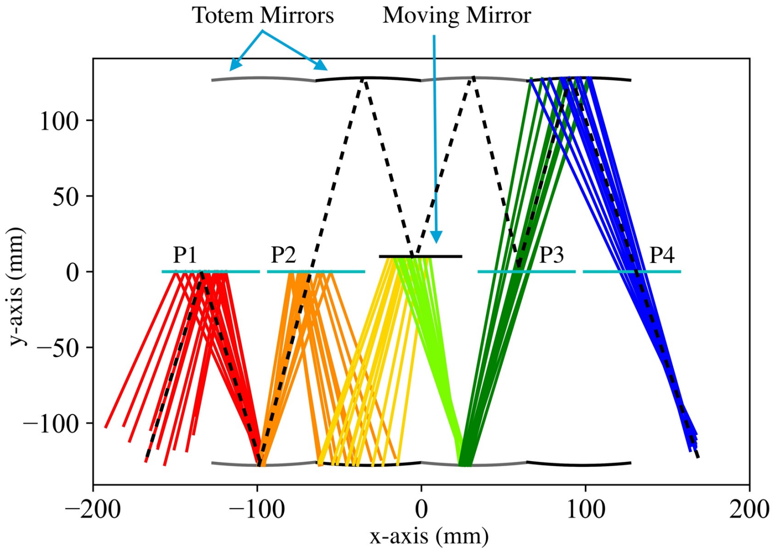


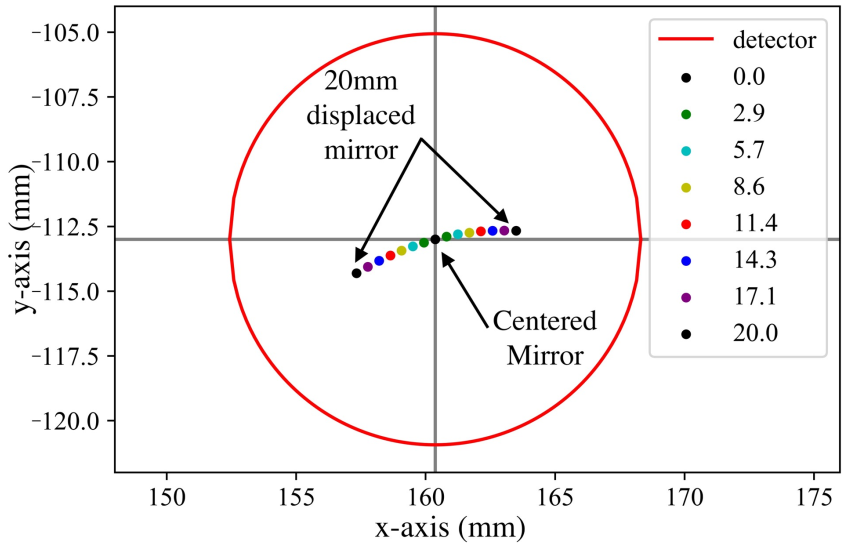


- Blue is 100% modulated interferogram
- Red is the profile of the blue, proportional to the transfer efficiency
- Black is simulation

- Geometric effect
- Can be simulated by tracing the light rays and discard those not captured by the optical elements

Limiting factor 2: coherence loss – separation of the recombined beams

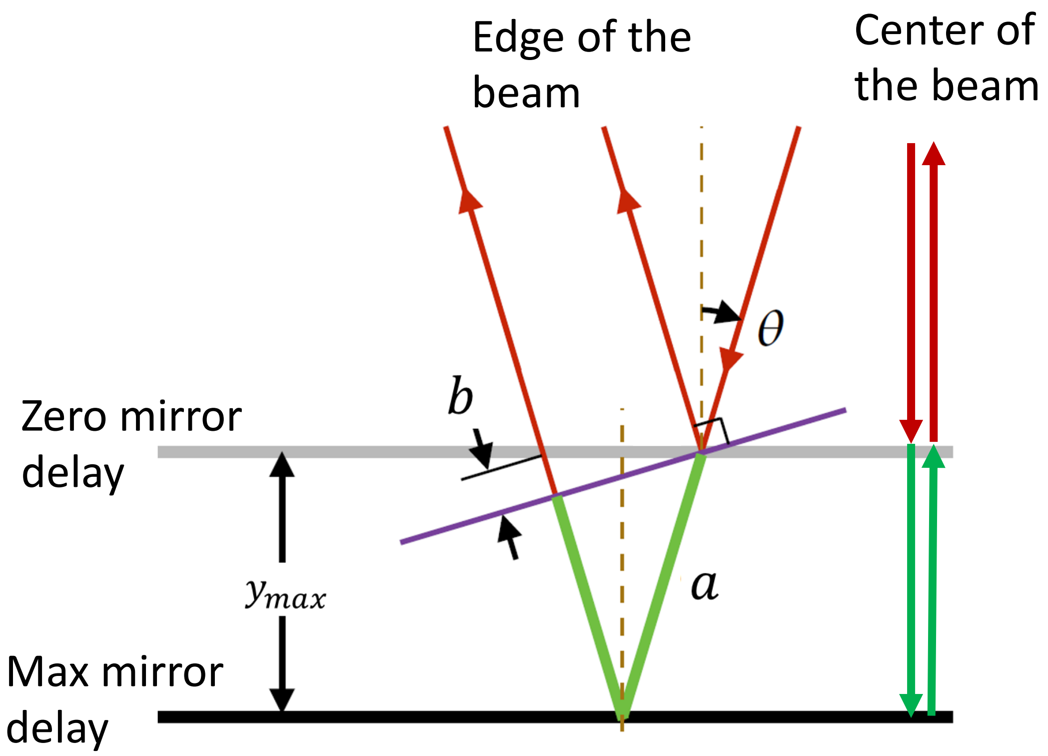




- Dashed and Solid lines are the two interfering light paths
- When the moving mirror moves off the center, the top-bottom symmetry is broken
- The distribution traces the beam profile
- Focusing optics beyond the box not shown

- Output focal plane
- Same-colored dots correspond to the recombined beam center locations of the interfering light paths at a given mirror delay
- Recombined locations separate at higher mirror delays -> less interference

Limiting factor 3: coherence loss – non-uniform delays within the beam (non-zero solid angle)



- Light rays at the center and edge of the beam have different optical delays at the same mirror displacement

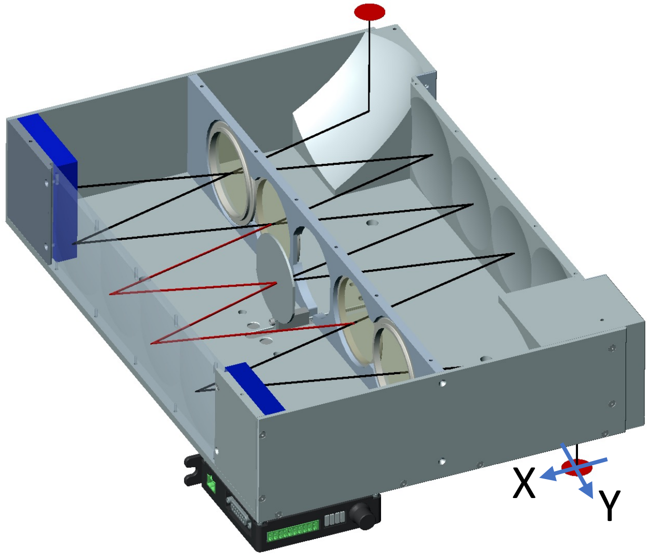
- We designed the instrument such that the the path length difference between beam center and beam edge is one wavelength at the maximum delay

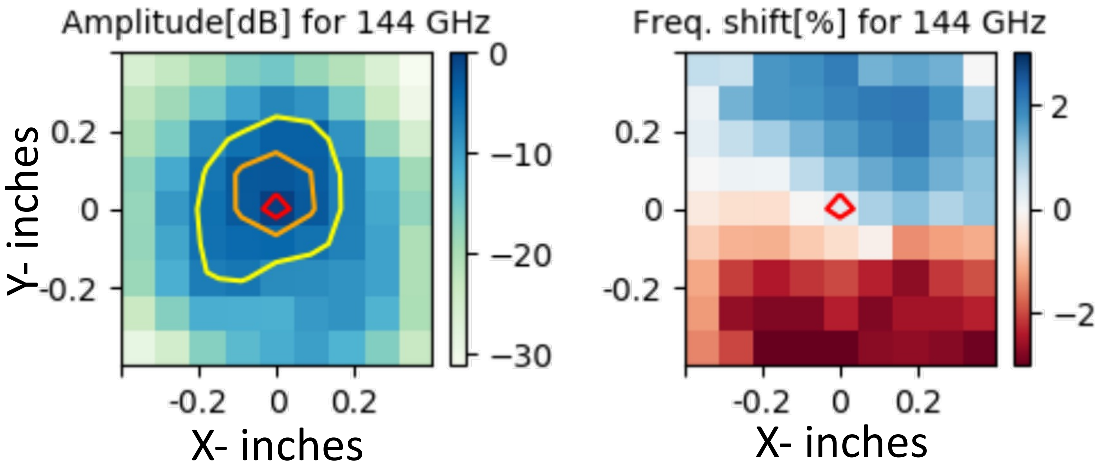
- Any FTS with a finite beam has this effect

- Our FTS has a large throughput, so we need to balance the nonidealities.

Limiting factor 4: frequency shift for extended sources (nonzero source area)

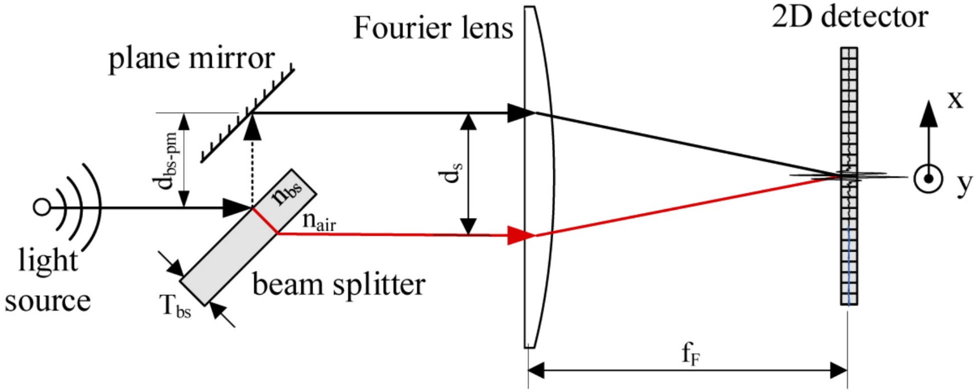
- Light rays from different source locations have slightly different delays
- Frequency shift is mapped using a Gunn oscillator.
- The frequency shift is ± 4 GHz.
- The FWHM widths for the interference intensity map are 8, 5, and 3 mm (0.3, 0.2, 0.1 in) for 90, 144, and 294 GHz sources.
- Can be pre-calibrated with a Gunn oscillator or simulation (geometric effect)



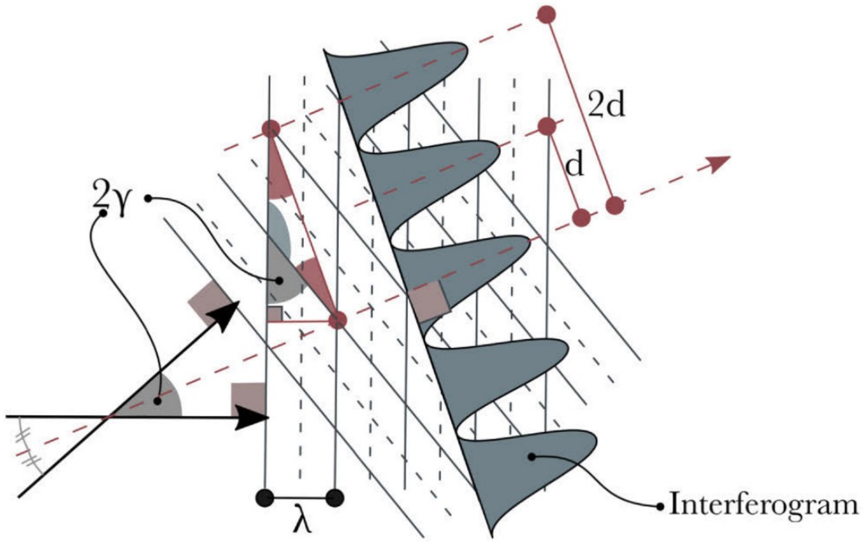


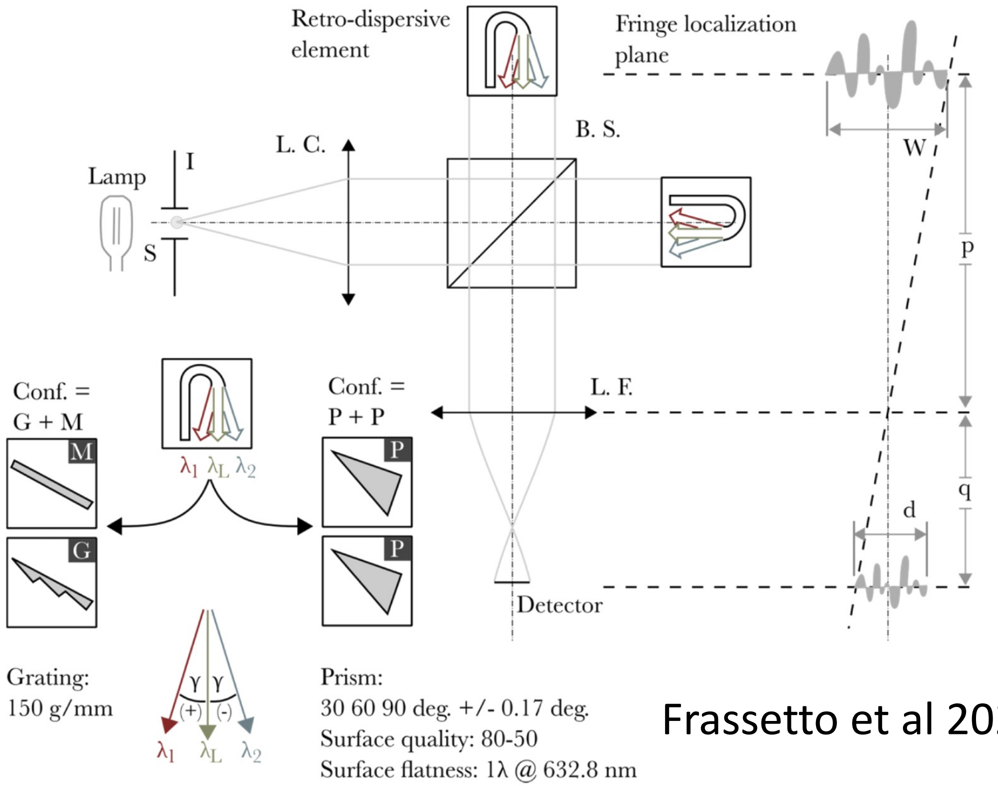
Measured amplitude and frequency vs. source location

Other types of FTS - Static FTS



Kohler et al 2020





Frassetto et al 2021

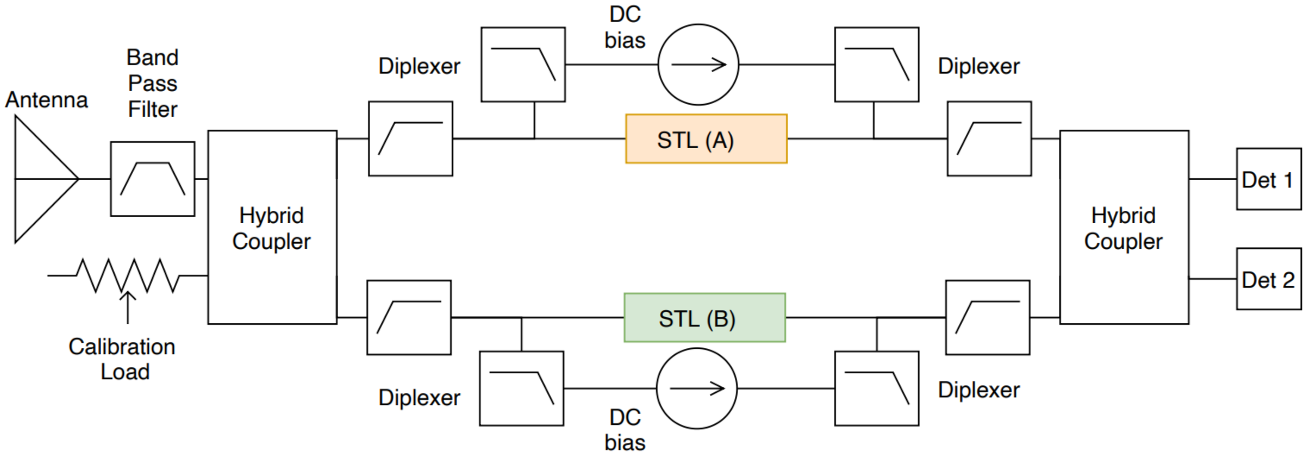
- Create the path length difference in spatial space
- Use a 2D detector array to measure the signal
- The path length difference can be created by different optical elements, such as grating, prism, or a dispersing light source with a lens.

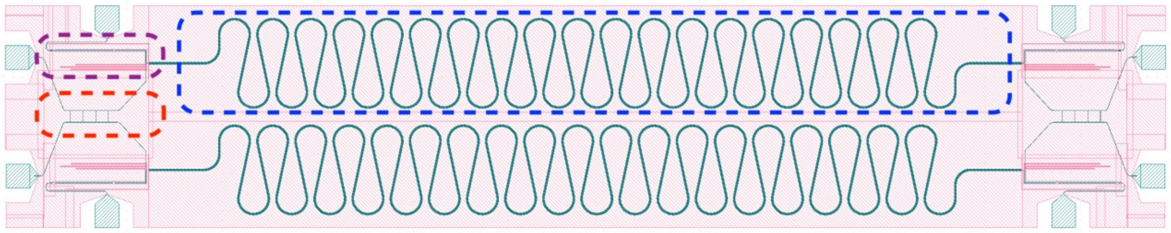
Other types of FTS – on-chip FTS

- Thermal-optical or electro-optical modulation.

$$\Delta\tau(I) = \ell\left(\sqrt{\mathscr{L}(I)\mathscr{C}} - \sqrt{\mathscr{L}(I=0)\mathscr{C}}\right) \approx \frac{L_{\square}\ell}{Z_0 w}\left(\sqrt{[1+(I/I_*)^2+(I/I'_*)^4]} - 1\right)$$

- Example: Superconducting On-chip Fourier Transform Spectrometers (SOFTS)

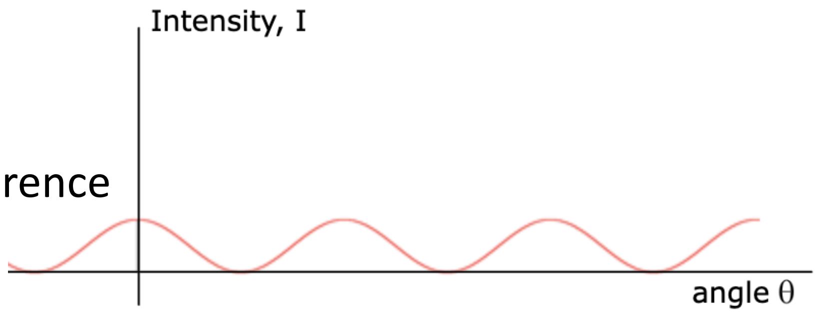




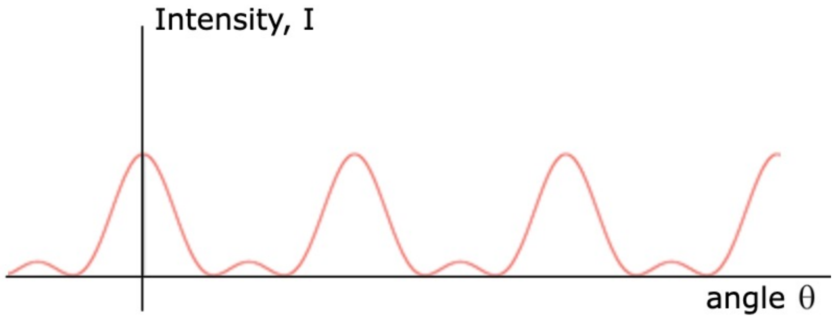
Basu Thakur et al 2021

Grating spectrometer

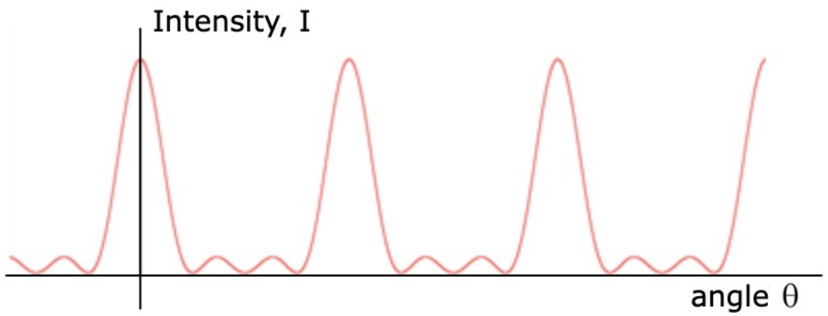
Two-slit interference



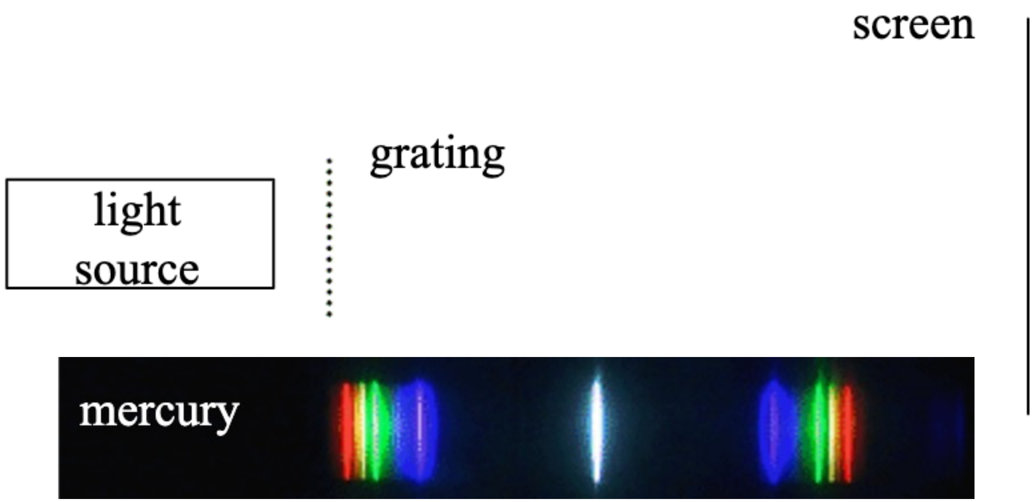
Three slits

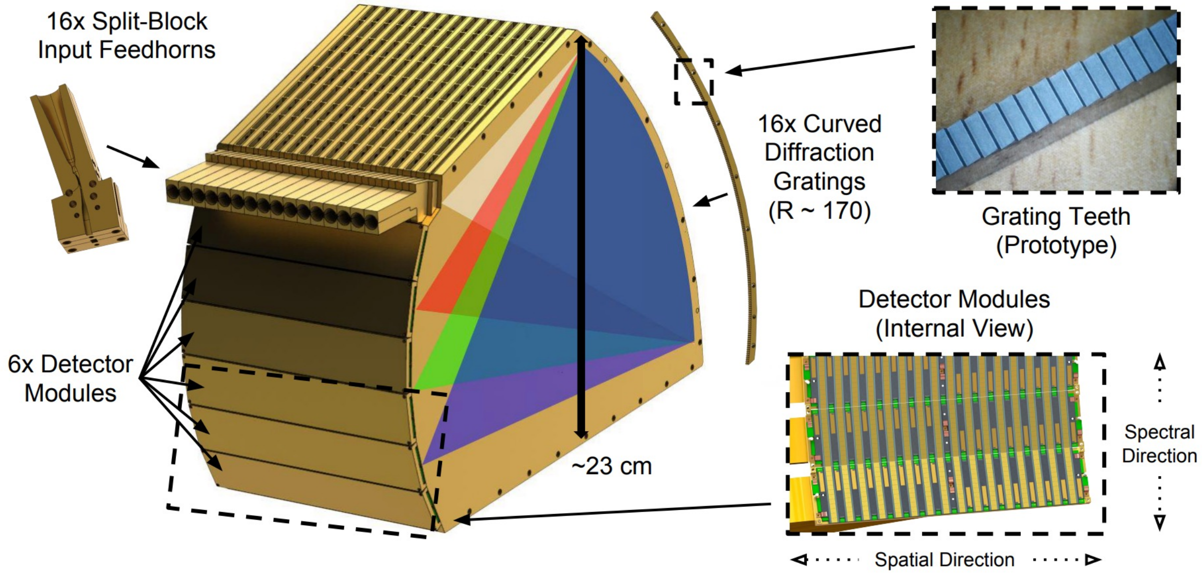


Four slits



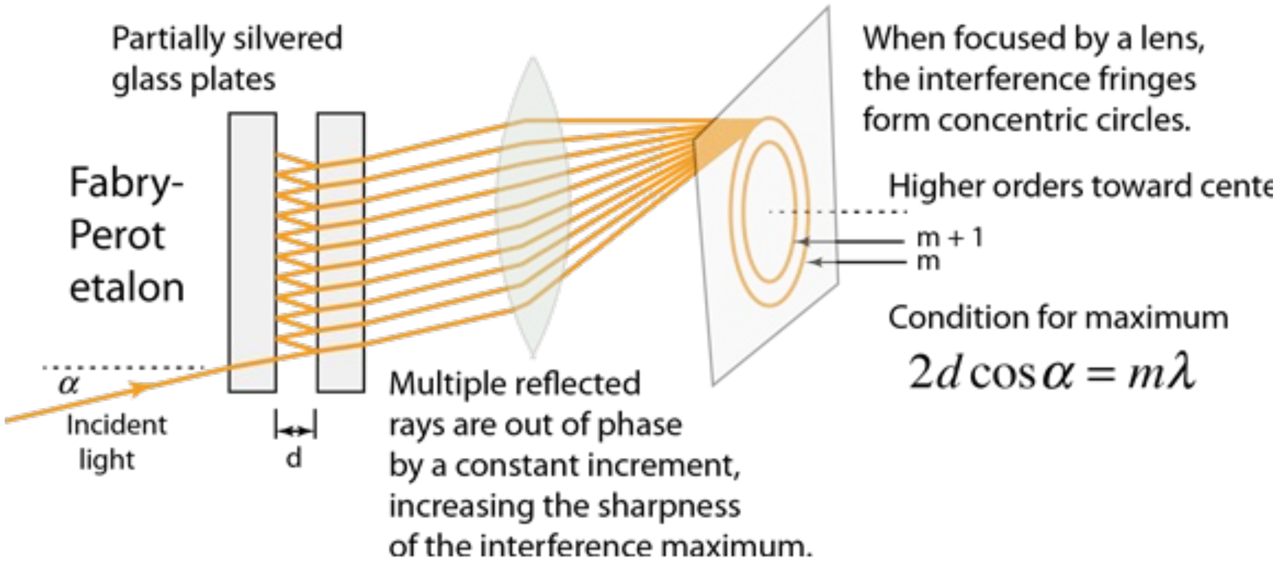
From UNSW Physics





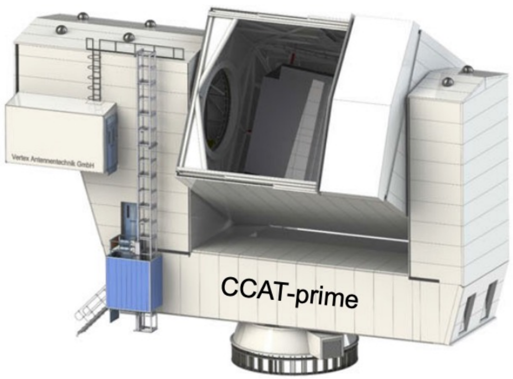
Example: TIME-Pilot spectrometer, Hunacek et al 2016

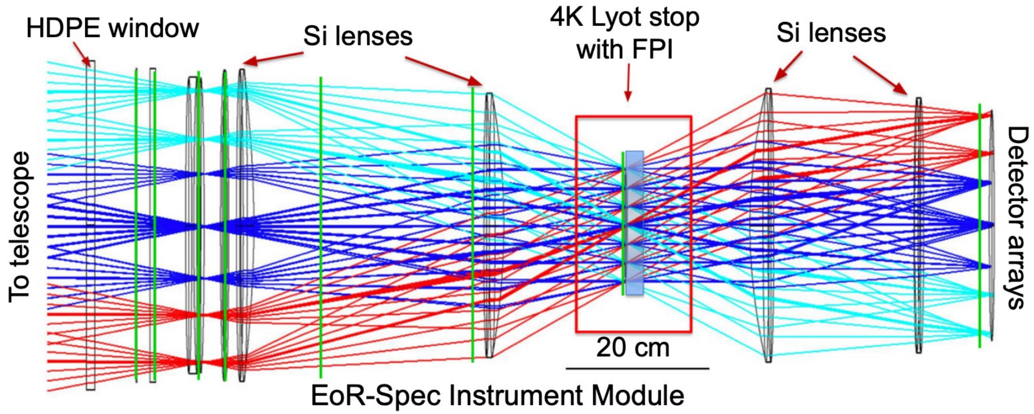
Fabry-Perot spectrometer



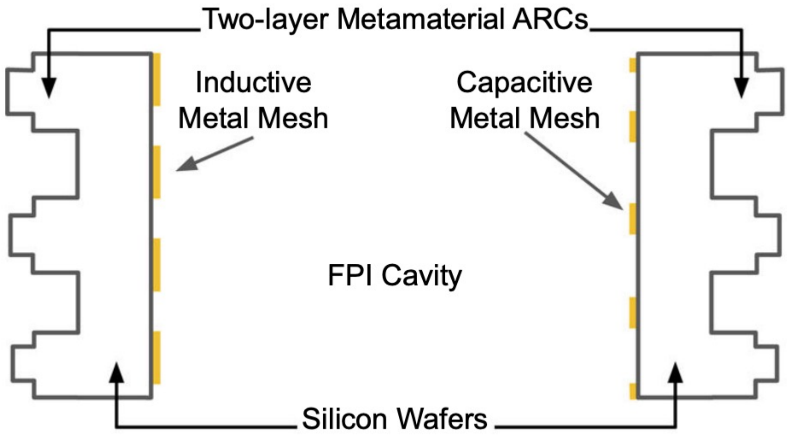
Taken from Hyperphysics

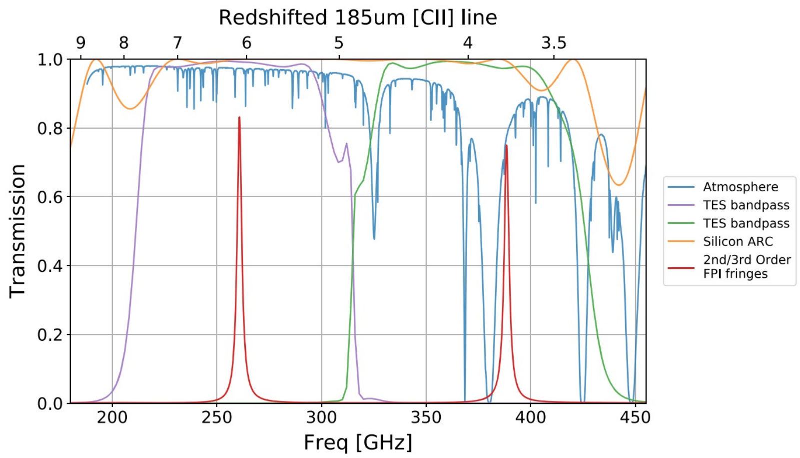
Example:
CCAT-prime





Operation mode:
Spatially mapping the sky at each step of the FPI's position

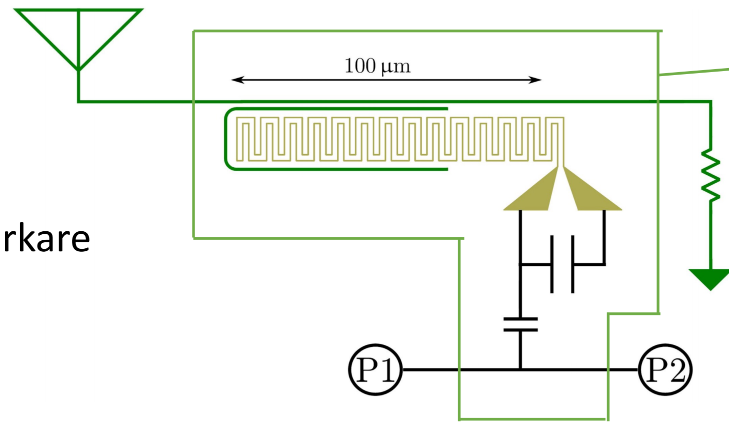




Cothard et al 2019

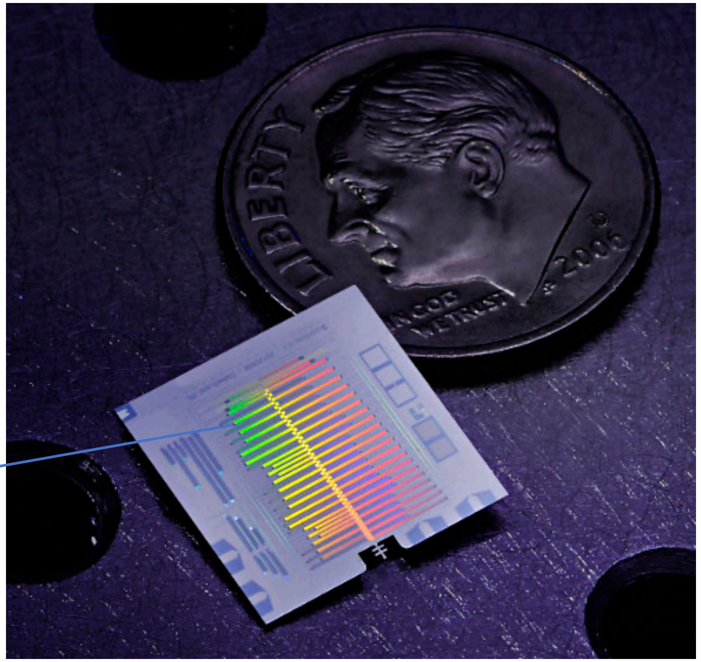
MKIDs spectrometers

K. Karkare

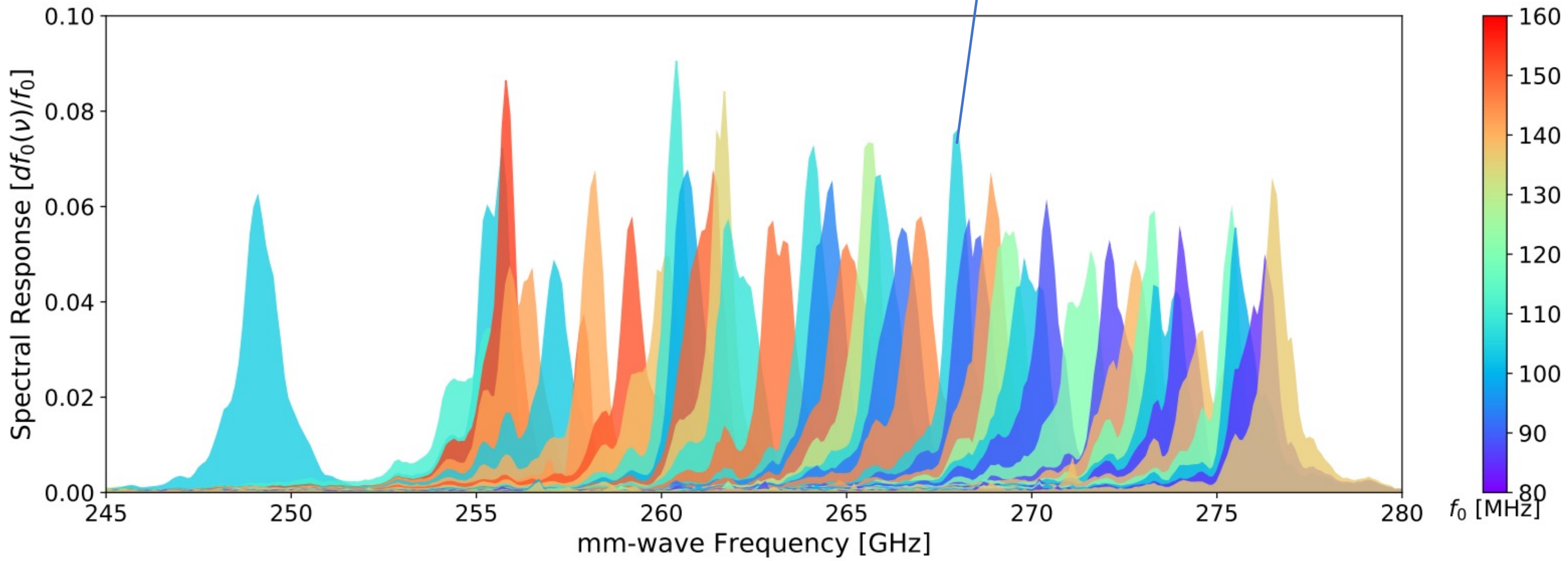


Replicate to get more channels

Each **filter channel** defines a **frequency channel**



E. Shirokoff



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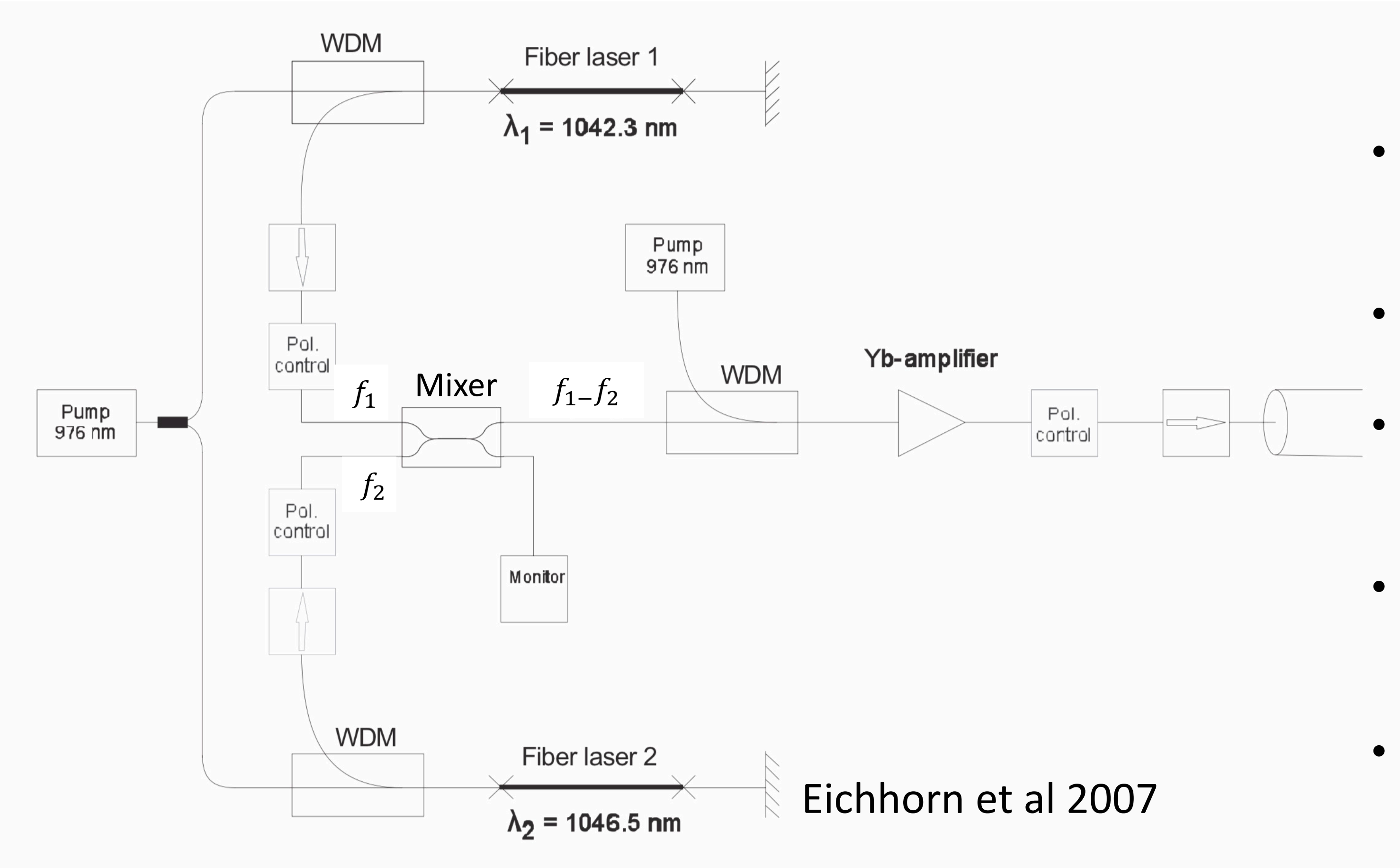
Fiber laser system for kHz-linewidth (sub)terahertz generation



$$\sin(2\pi f_1 t)\sin(2\pi f_2 t) = \frac{1}{2}\cos[2\pi(f_1 - f_2)t] - \frac{1}{2}\cos[2\pi(f_1 + f_2)t]$$

- Frequency can be tuned by thermally or electrically varying the grating pitch or stretching the cavity of a diode laser
- High-frequency resolution, used to characterize systems with high spectral resolution requirement
- Near-infrared laser light irradiates the photomixer at two adjacent frequencies.
- Applying a bias voltage to the metal electrodes then generates a photocurrent that oscillates at the beat frequency.
- An antenna structure surrounding the photomixer translates the oscillating photocurrent into the terahertz wave.
- Photomixer based on either GaAs or InGaAs/InP and require laser wavelengths below the semiconductor bandgap (i.e., around 0.8 µm or 1.5 µm, respectively).