Making massive spin-2 particles from gravity during inflation

Andrew Long
Rice University
@ Adventures of Rocky & Friends
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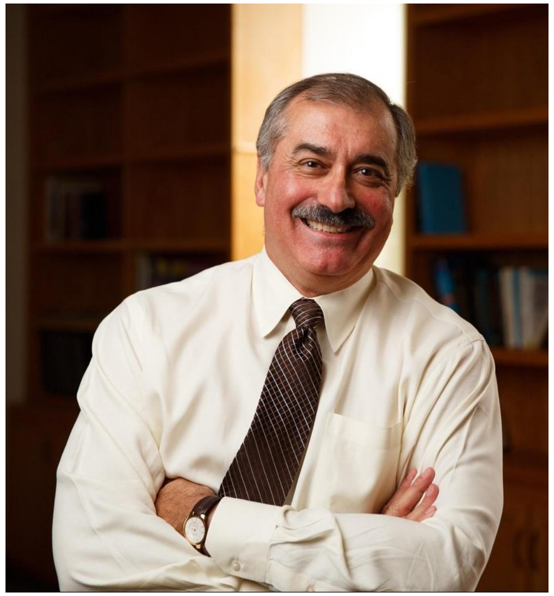
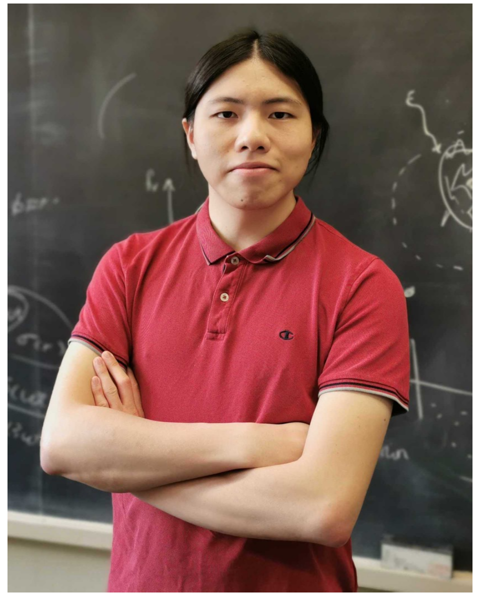
Collaborators

Rocky Kolb



dark matter & inflation

Siyang Ling



grad student @ Rice
QFT in curved spacetime

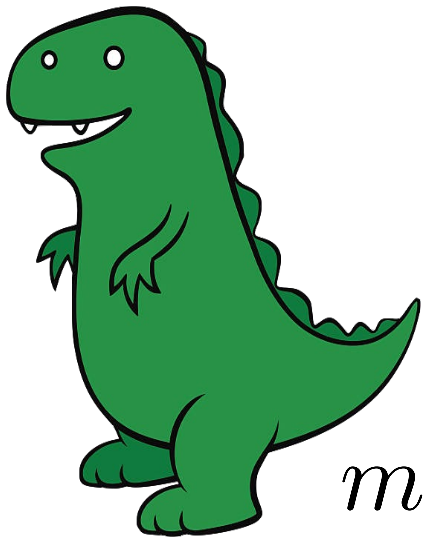
Rachel Rosen

gravity & QFT

read along: 2302.04390

Years of fruitful collaboration

1708.04293 - Kolb & AL - *Higgs portals*
1812.00211 - Chung, Kolb, & AL - *super-Hubble mass*
2009.03828 - Kolb & AL - *spin-1*
2101.11621 - Ling & AL - *alpha attractor*
2102.10113 - Kolb, AL, & McDonough - *spin-3/2*
2103.10437 - Kolb, AL, & McDonough - *spin-3/2*
2206.14204 - Hashiba, Ling, & AL - *Stokes phenom*
2209.01713 - Basso, Chung, Kolb, & AL - *interference*
2211.14323 - Kolb, AL, McDonough, & Payeur - *kination*
2302.04390 - Kolb, Ling, AL, & Rosen - spin-2



$m > H_{\rm inf}$

(1) Introduction & motivation

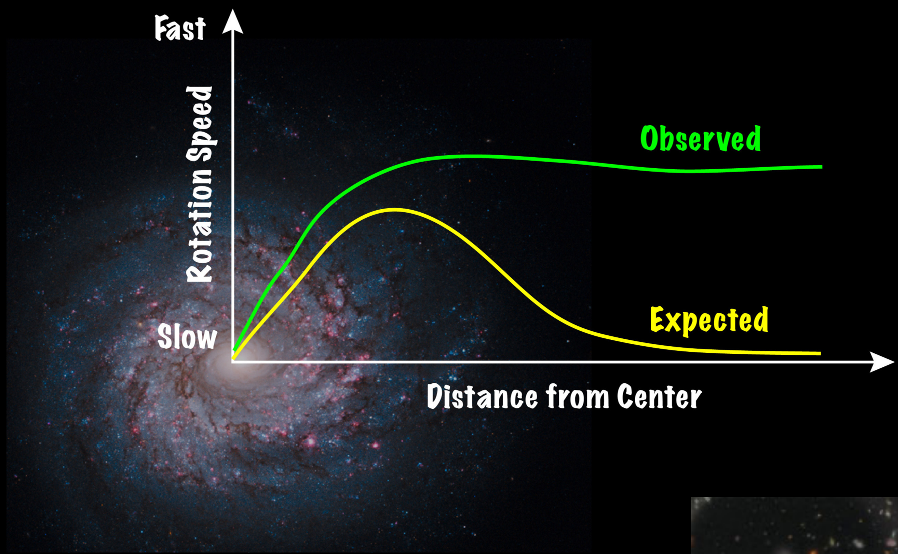
(2) Cosmological gravitational particle production

(3) CGPP of massive spin-2 particles

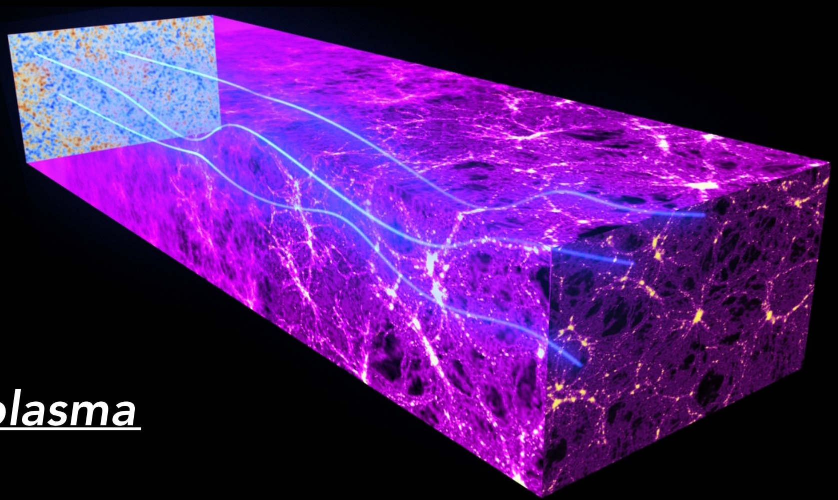
(4) Summary & conclusion

dark matter pulls on things

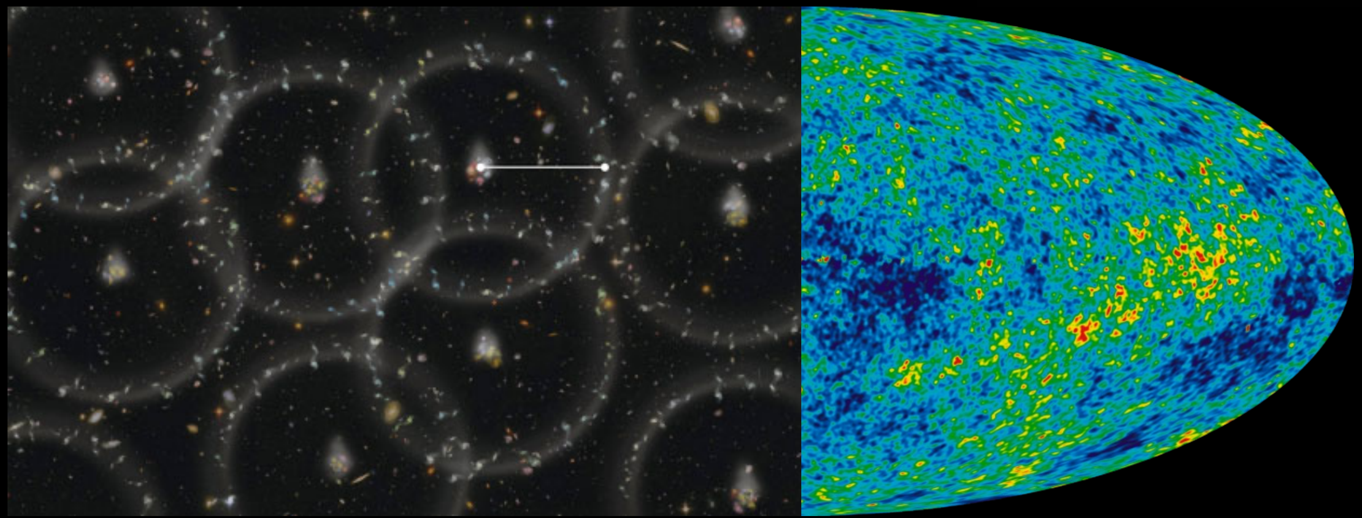
Dark matter pulls on stars in galaxies
(galactic rotation curves)



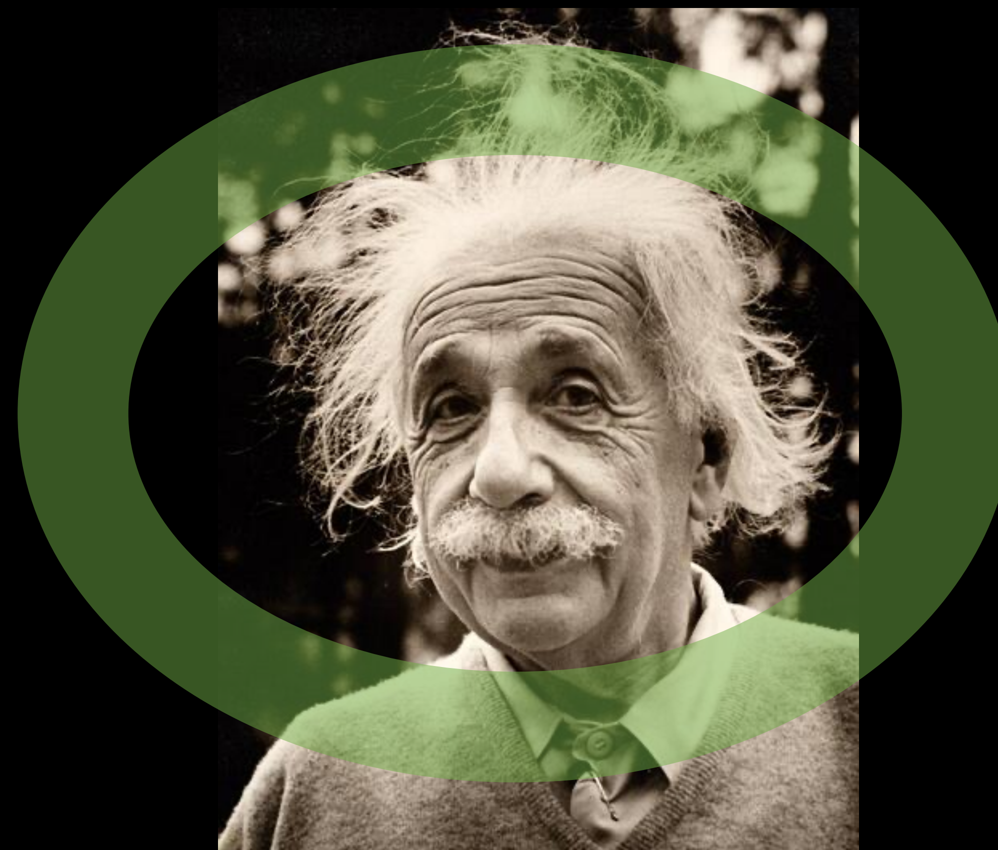
Dark matter pulls on light
(gravitational lensing)



Dark matter pulled on e^-p^+ plasma
(CMB & large scale structure)



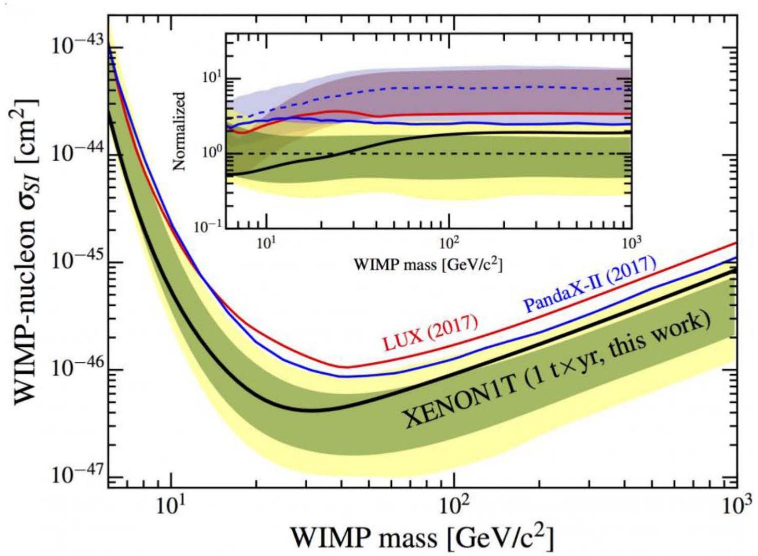
don’t need a dark force



perhaps that’s all there is to it?

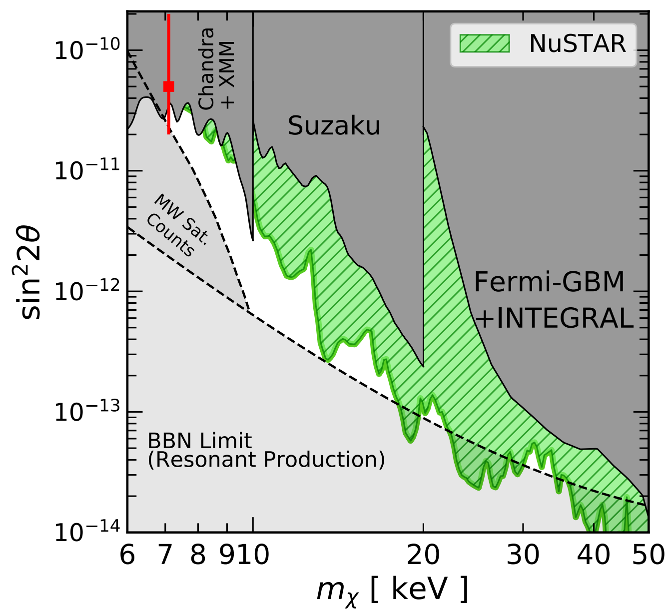
No dark matter bumping into things

(direct detection; 1805.12562)



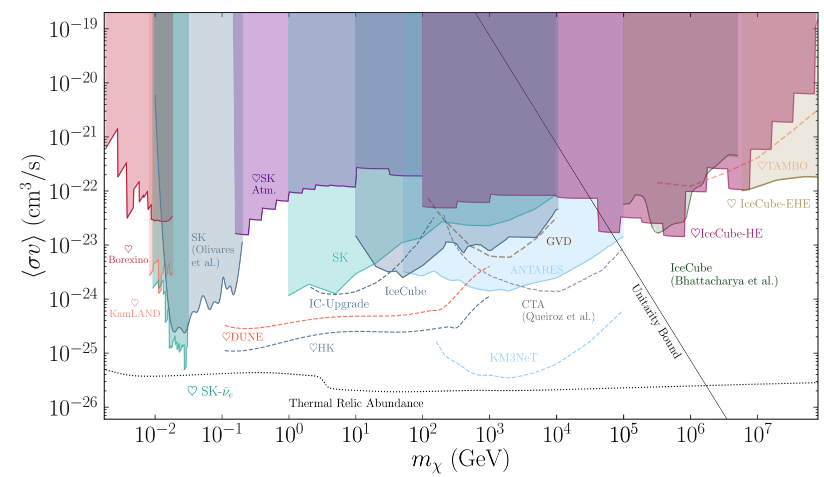
No dark matter decaying into things

(X-ray emission; 1908.09037)



No dark matter bumping into itself

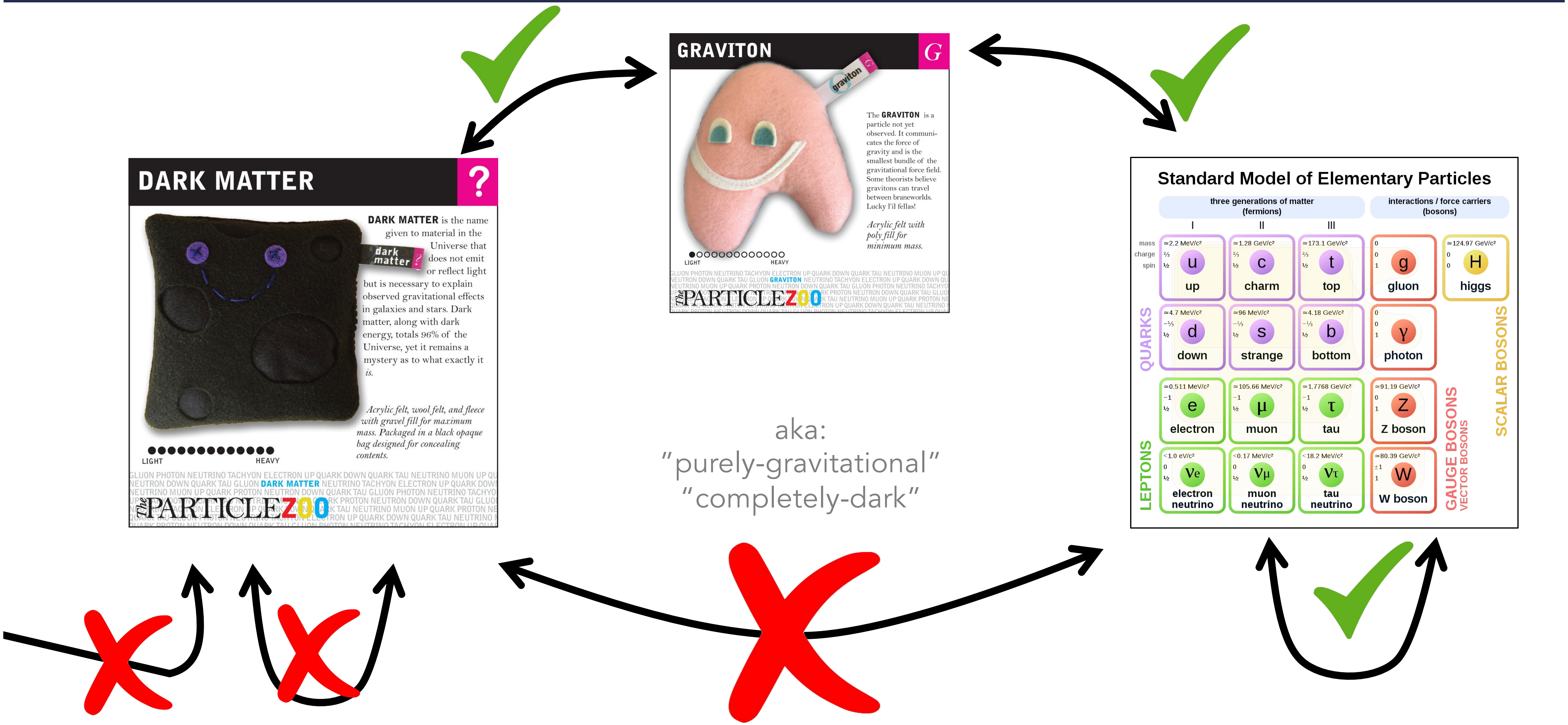
(annihilation to ν’s; 1912.09486)



(notwithstanding hints of new physics, there’s no overwhelming evidence) hypothesis:

dark matter only interacts gravitationally

The hypothesis:



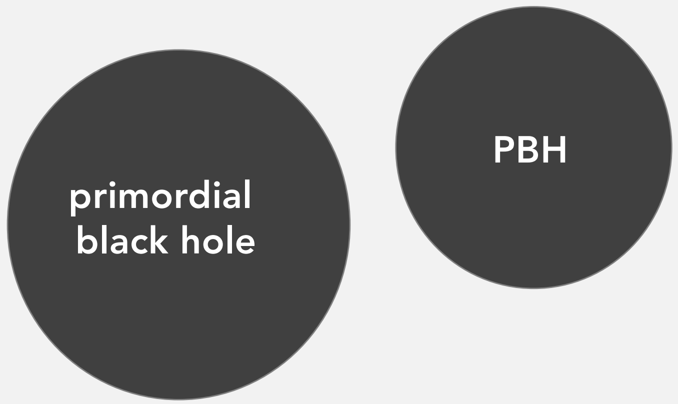
the problem:

where did all the dark matter come from?

(how do we use gravity to make dark matter?)

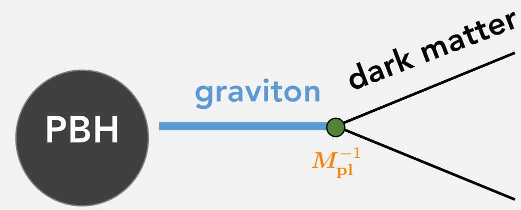
ideas …

the DM is a collection
of primordial black holes



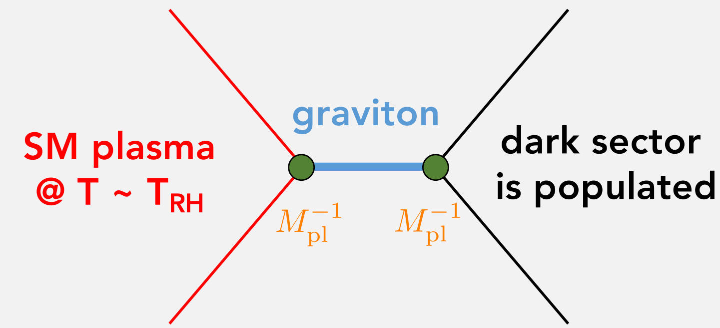
[many authors, esp. after LIGO BBH merger (2016)]

the DM is produced
from PBH evaporation



[Hooper, Krnjaic, & McDermott (2019)]

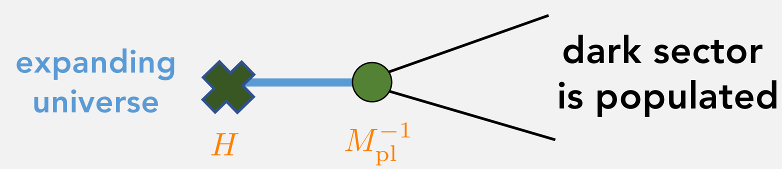
the DM is a produced
from thermal freeze-in



[Garny, Sandora, & Sloth (2015); Mambrini & Olive (2021)]

this talk:

the DM is produced from
cosmological expansion
during (or at the end of) inflation



[Kuzmin & Tkachev (1999); Chung, Kolb, & Riotto (1999)]

(1) Introduction & motivation

(2) Cosmological gravitational particle production

(3) CGPP of massive spin-2 particles

(4) Summary & conclusion

Cosmological gravitational particle production

[Schrodinger (1939); Parker (1965, 68); Fulling, Ford, & Hu; Zel'dovish; Starobinski; Grib, Frolov, Mamaev, & Mostepanenko; Mukhanov & Sasaki, Birrell & Davies…]

quantum filed theory in curved spacetime applied to spectator fields in an expanding universe

cosmological expansion

$a(t)$

STEP 1

time-dependent Hamiltonian

$\hat{H}(t)$

STEP 2

mixing of + and – frequency modes

$$\ddot{\chi}_{\boldsymbol{k}}(t) + \omega_k^2(t)\,\chi_{\boldsymbol{k}}(t) = 0$$

STEP 3

particle production

$$n_k = |\beta_k|^2$$

Example: scalar field in FRW background

$$ds^2 = a(\eta)^2 \left[d\eta^2 - d\boldsymbol{x}^2 \right]$$

covariant action

$$S[\varphi(x), g_{\mu\nu}(x)] = \int d^4x \sqrt{-g} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - \frac{1}{2} m^2 \varphi^2 + \frac{1}{2} \xi \varphi^2 R \right]$$

action in an FRW background

$$S[\varphi(\eta, \boldsymbol{x})] = \int_{-\infty}^{\infty} d\eta \int d^3\boldsymbol{x} \left[\frac{1}{2} a^2 (\partial_\eta \varphi)^2 - \frac{1}{2} a^2 (\boldsymbol{\nabla} \varphi)^2 - \frac{1}{2} a^4 m^2 \varphi^2 + \frac{1}{2} a^4 \xi \varphi^2 R \right]$$

field rescaling & Fourier transform

$$\varphi(\eta, \boldsymbol{x}) = a(\eta)^{-1} \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} \chi_{\boldsymbol{k}}(\eta) \, e^{i\boldsymbol{k}\cdot\boldsymbol{x}}$$

action for canonically-normalized field

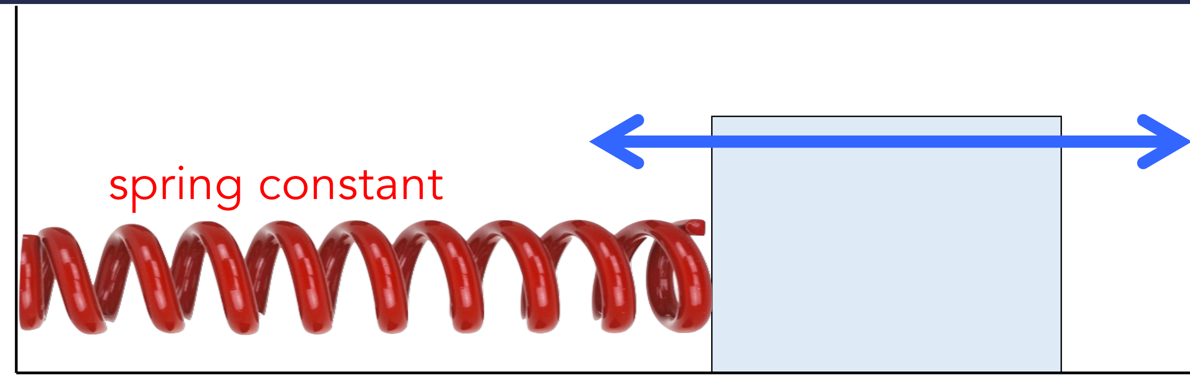
$$S[\chi_{\boldsymbol{k}}(\eta)] = \int_{-\infty}^{\infty} d\eta \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} \left[\frac{1}{2} \left| \partial_\eta \chi_{\boldsymbol{k}} \right|^2 - \frac{1}{2} \omega_k^2 \left| \chi_{\boldsymbol{k}} \right|^2 \right]$$

time-dependent frequency

$$\omega_k^2(\eta) = k^2 + a(\eta)^2 m^2 + \left(\tfrac{1}{6} - \xi \right) a(\eta)^2 R(\eta)$$

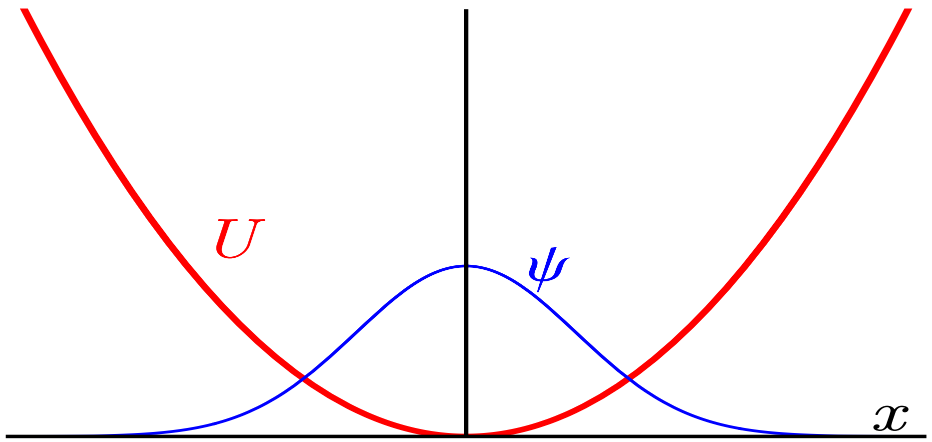
This problem looks like
a harmonic oscillator
with a time-dependent frequency

An analogy with 1D quantum mechanics

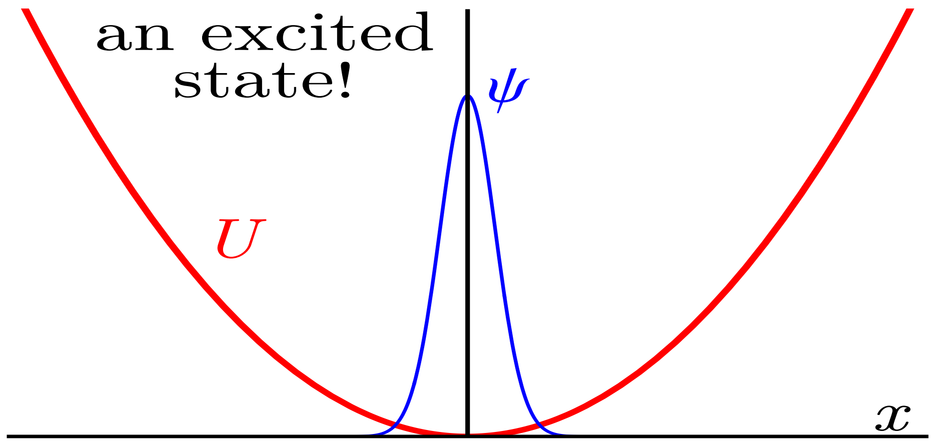


Each fourier mode of the field is an oscillator

Spring constant is varied slowly (adiabatically)



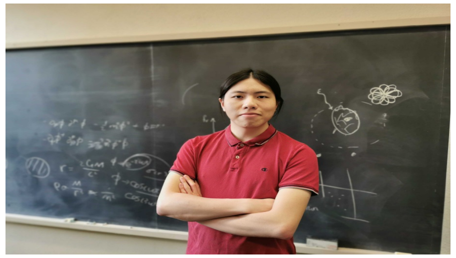
Spring constant is varied abruptly (*non*-adiabatically)



an s=0 example:
scalar dark matter from
T-model alpha attractor

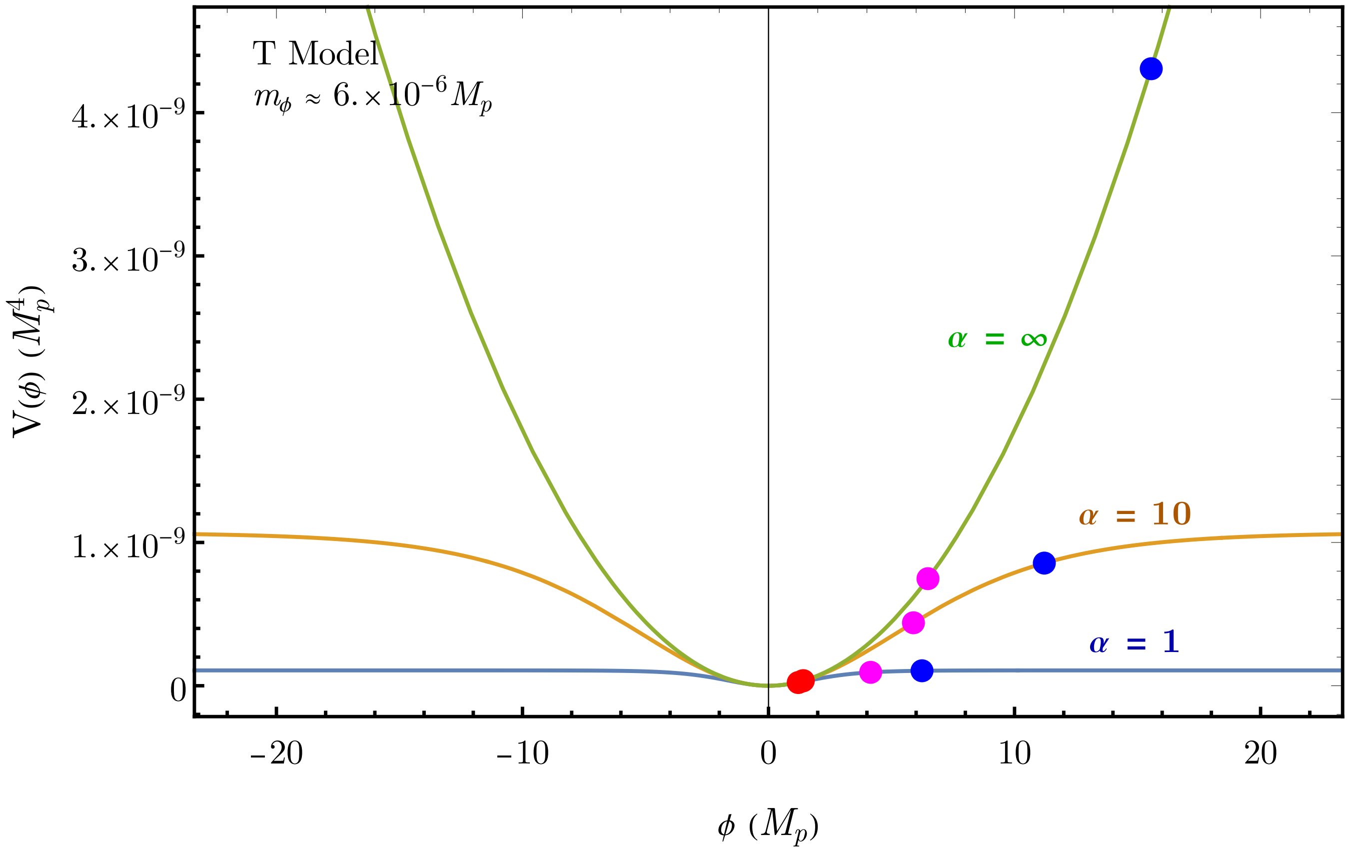
T-model alpha attractor (s=0 GPP)

[Ling & AL (2101.11621)]



T-model alpha attractor

$$V_T(\phi) = \alpha \mu^2 M_p^2 \tanh^2 \frac{\phi}{\sqrt{6\alpha} M_p}$$



$$\Rightarrow \begin{cases} \phi(t) \\ a(t) \end{cases}$$

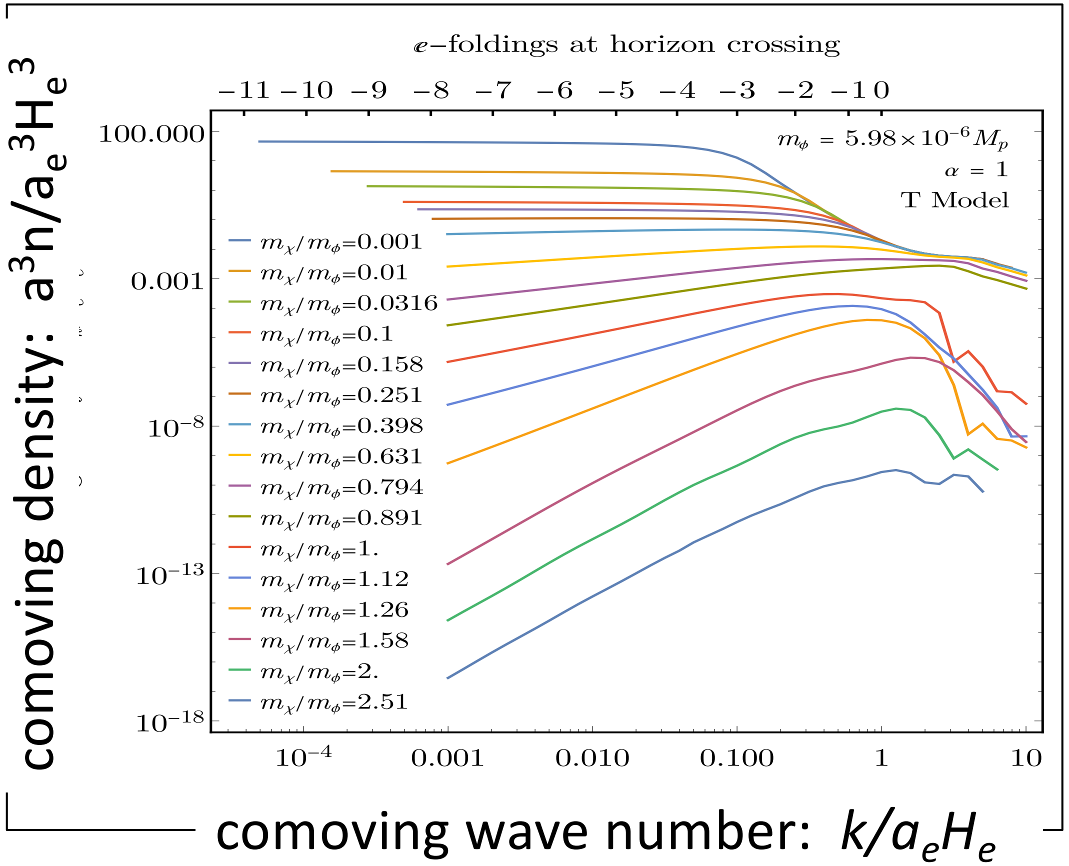
T-model alpha attractor (s=0 GPP)

[Ling & AL (2101.11621)]

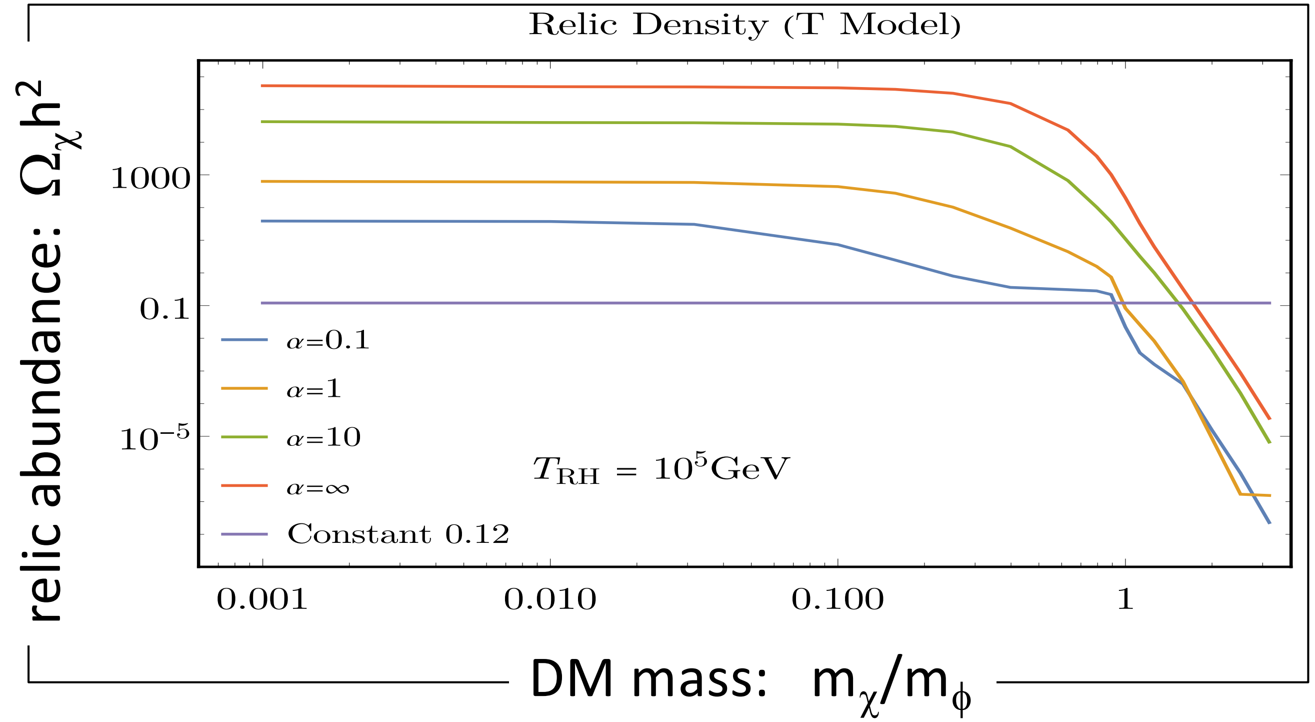
$$dn = \frac{d^3\boldsymbol{k}}{(2\pi)^3}\,|\beta_k|^2 = \frac{k^2 dk}{2\pi^2}\,|\beta_k|^2$$

$$\Omega_\chi h^2 \simeq (0.114)\left(\frac{m_\chi}{10^{10}\,\mathrm{GeV}}\right)\left(\frac{H_e}{10^{10}\,\mathrm{GeV}}\right)\left(\frac{T_{\mathrm{RH}}}{10^8\,\mathrm{GeV}}\right)\left(\frac{a^3 n}{a_e^3 H_e^3}\right)$$

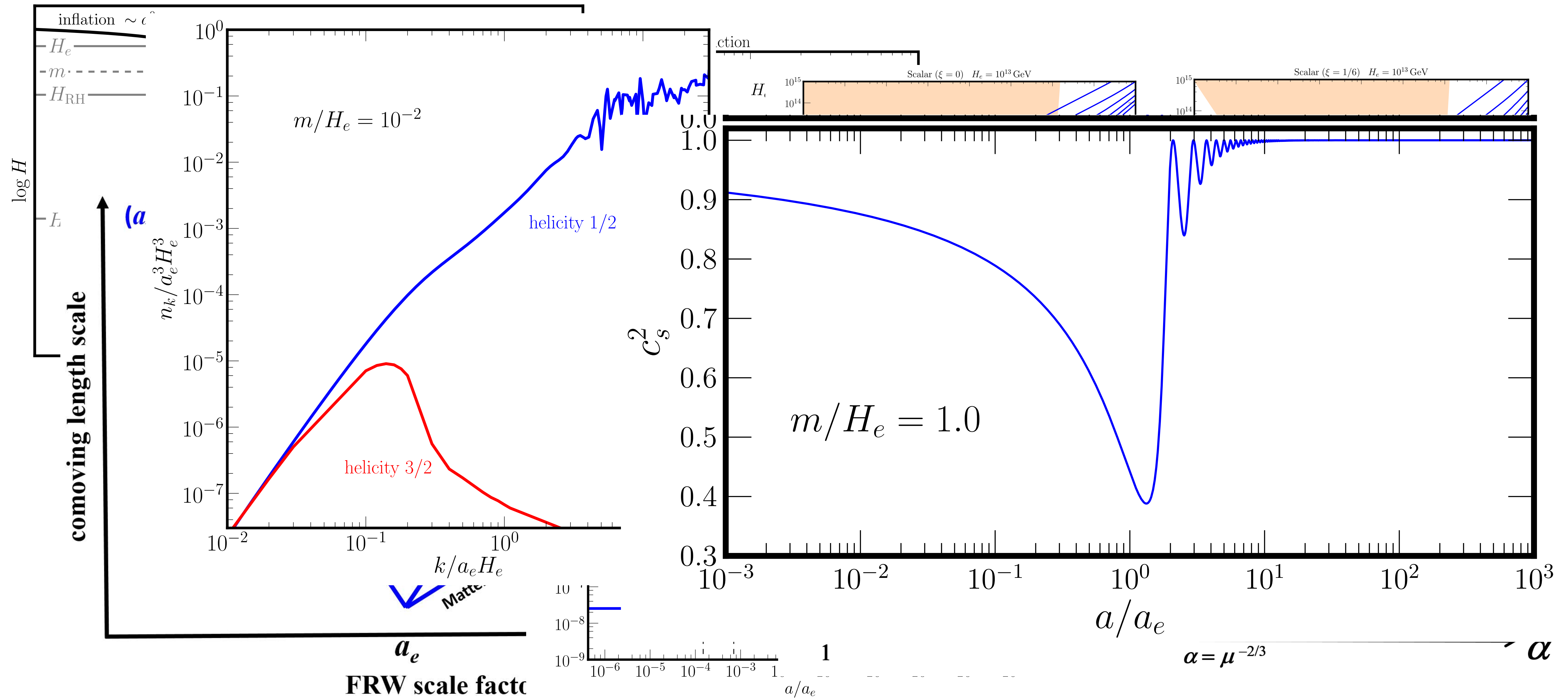
number density spectrum



relic abundance



Collaboration with Rocky: a collage of plots & graphs



what about higher-spin particles?

Beyond scalar fields

spin-0 (real scalar boson)

$$\mathscr{L} = \frac{1}{2} g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}m^2\varphi^2$$

spin-1/2 (Dirac fermion)

$$\mathscr{L} = \frac{i}{2}\bar{\Psi}\underline{\gamma}^\mu(\nabla_\mu\Psi) - \frac{1}{2}m\bar{\Psi}\Psi$$

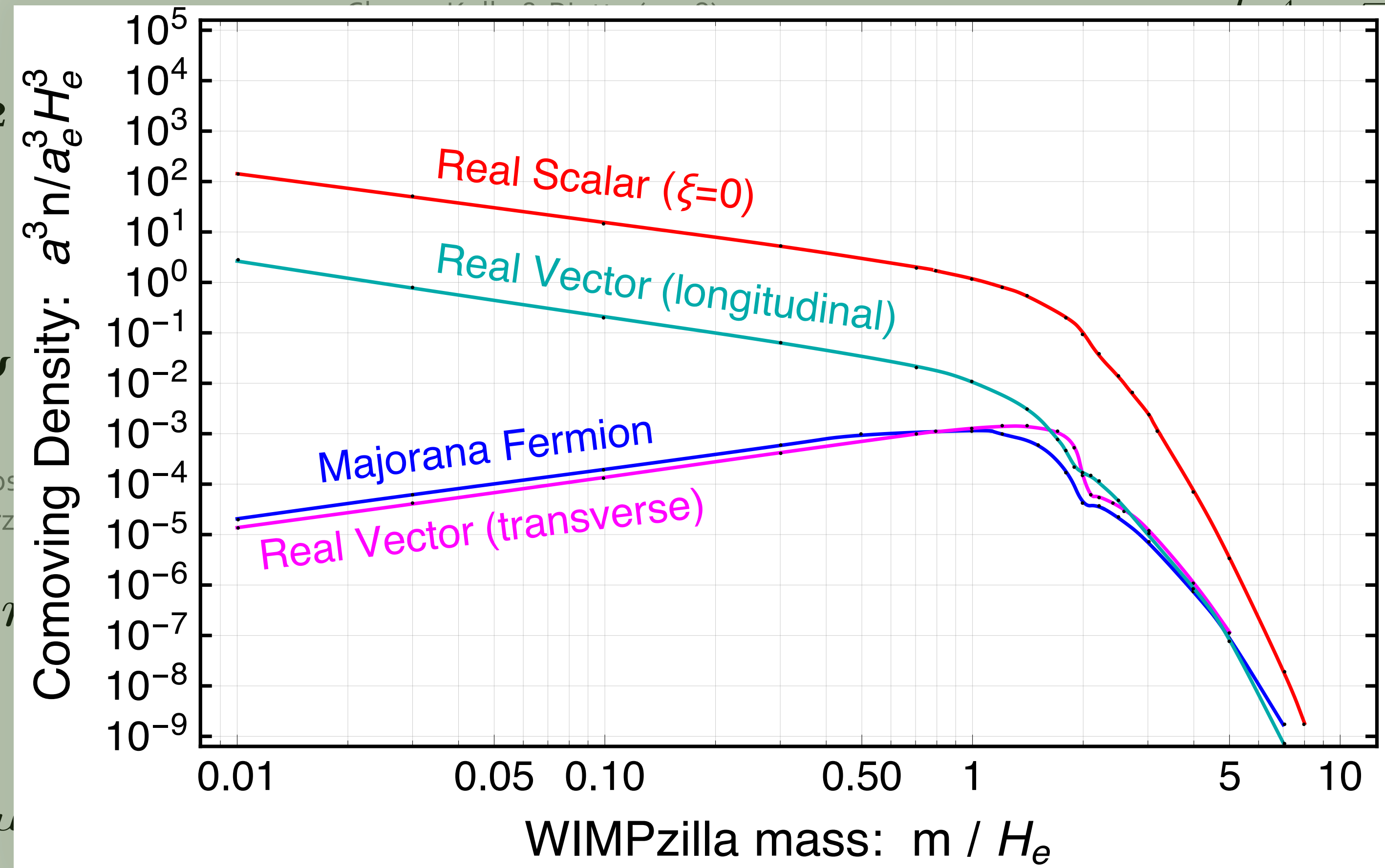
spin-1 (real vector boson)

Dimopoulos
Ahmed, Grz

$$\mathscr{L} = -\frac{1}{4}g^{\mu\alpha}g^{\nu\beta}F_{\mu\nu}F_{\alpha\beta} + \frac{1}{2}$$

spin-3/2 (Rarita-Schwinger fermion)

$$\mathscr{L} = \frac{i}{4}\bar{\Psi}_\mu(\underline{\gamma}^\mu\underline{\gamma}^\rho\underline{\gamma}^\sigma - \underline{\gamma}^\sigma\underline{\gamma}^\rho\underline{\gamma}^\mu$$



Kolb, Ling, AL, & Rosen (2302.04390)

spin-2 (Fierz-Pauli boson)

$$\mathscr{L} = \tfrac{1}{2}\nabla h_{\mu\nu}\nabla h^{\mu\nu} - \tfrac{1}{2}m^2 h_{\mu\nu}h^{\mu\nu} + \cdots ???$$

(how to formulate massive spin-2 on FRW bkg?)

(1) Introduction & motivation

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What's the minimal (ghost-free) theory
that describes both
a massless graviton
and a massive spin-2 field
on an FRW background?

Constructing the theory

Start with GR coupled to matter

$$S[g_{\mu\nu}] = \int \mathrm{d}^4x \sqrt{-g} \left[\tfrac{1}{2} M_P^2 \, R[g] - \tfrac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - V(\phi) \right]$$

Linearize around an FRW background

$$g_{\mu\nu} = \bar{g}^{(\mathrm{FRW})}_{\mu\nu} + \frac{2}{M_P} u_{\mu\nu} \quad \text{and} \quad \phi = \bar{\phi}^{(\mathrm{FRW})} + \varphi_u$$

Quadratic action is ghost-free

$$\begin{aligned}
\mathcal{L}^{(2)}_{\mathrm{massless}} &= \mathcal{L}^{(2)}_{uu} + \mathcal{L}^{(2)}_{u\varphi_u} + \mathcal{L}^{(2)}_{\varphi_u\varphi_u} \\
\mathcal{L}^{(2)}_{uu} &= -\tfrac{1}{2}\nabla_\lambda u_{\mu\nu}\nabla^\lambda u^{\mu\nu} + \nabla_\mu u^{\nu\lambda}\nabla_\nu u^\mu{}_\lambda - \nabla_\mu u^{\mu\nu}\nabla_\nu u + \tfrac{1}{2}\nabla_\mu u \nabla^\mu u \\
&\quad + \left(\bar{R}_{\mu\nu} - \tfrac{1}{M_P^2}\nabla_\mu\bar{\phi}\nabla_\nu\bar{\phi}\right)\left(u^{\mu\lambda}u_\lambda^{\ \nu} - \tfrac{1}{2}u^{\mu\nu}u\right) , \\
\mathcal{L}^{(2)}_{u\varphi_u} &= \tfrac{1}{M_P}\left[\left(\nabla_\mu\bar{\phi}\nabla_\nu\varphi_u + \nabla_\nu\bar{\phi}\nabla_\mu\varphi_u\right)\left(u^{\mu\nu} - \tfrac{1}{2}\bar{g}^{\mu\nu}u\right) - V'(\bar{\phi})\varphi_u u\right] , \\
\mathcal{L}^{(2)}_{\varphi_u\varphi_u} &= -\tfrac{1}{2}\nabla_\mu\varphi_u\nabla^\mu\varphi_u - \tfrac{1}{2}V''(\bar{\phi})\varphi_u^2 \ .
\end{aligned}$$

(massless spin-2 graviton + inflaton perturbation)

Add a Fierz-Pauli mass term

$$\begin{aligned}
\mathcal{L}^{(2)}_{\mathrm{massive}} &= \mathcal{L}^{(2)}_{vv} + \mathcal{L}^{(2)}_{v\varphi_v} + \mathcal{L}^{(2)}_{\varphi_v\varphi_v} \\
\mathcal{L}^{(2)}_{vv} &= -\tfrac{1}{2}\nabla_\lambda v_{\mu\nu}\nabla^\lambda v^{\mu\nu} + \nabla_\mu v^{\nu\lambda}\nabla_\nu v^\mu{}_\lambda - \nabla_\mu v^{\mu\nu}\nabla_\nu v + \tfrac{1}{2}\nabla_\mu v \nabla^\mu v \\
&\quad + \left(\bar{R}_{\mu\nu} - \tfrac{1}{M_P^2}\nabla_\mu\bar{\phi}\nabla_\nu\bar{\phi}\right)\left(v^{\mu\lambda}v_\lambda^{\ \nu} - \tfrac{1}{2}v^{\mu\nu}v\right) \\
&\quad - \tfrac{1}{2}m^2\left(v^{\mu\nu}v_{\mu\nu} - v^2\right) , \\
\mathcal{L}^{(2)}_{v\varphi_v} &= \tfrac{1}{M_P}\left[\left(\nabla_\mu\bar{\phi}\nabla_\nu\varphi_v + \nabla_\nu\bar{\phi}\nabla_\mu\varphi_v\right)\left(v^{\mu\nu} - \tfrac{1}{2}\bar{g}^{\mu\nu}v\right) - V'(\bar{\phi})\varphi_v v\right] , \\
\mathcal{L}^{(2)}_{\varphi_v\varphi_v} &= -\tfrac{1}{2}\nabla_\mu\varphi_v\nabla^\mu\varphi_v - \tfrac{1}{2}V''(\bar{\phi})\varphi_v^2 \ .
\end{aligned}$$

(massive spin-2 + inflaton perturbation)

This action also arises from ghost-free bigravity

Hassan & Rosen (2012)

Field content: two metrics & two scalars

$$g_{\mu\nu}\ , \quad f_{\mu\nu}\ , \quad \phi_g\ , \quad \phi_f$$

A theory of bigravity with a minimal coupling to matter

$$S = \int \mathrm{d}^4x \, \Big[\tfrac{1}{2} M_g^2 \sqrt{-g}\, R[g] \ + \ \tfrac{1}{2} M_f^2 \sqrt{-f}\, R[f] \qquad \text{(metric kinetic terms)}$$

$$- m^2 M_*^2 \sqrt{-g}\, V(\mathbb{X};\, \beta_n) \qquad \text{(metric interactions)} \qquad \left(\begin{array}{c} M_*^{-2} = M_g^{-2} + M_f^{-2} \\ M_P^2 = M_g^2 + M_f^2 \end{array} \right)$$

$$+ \sqrt{-g}\, \mathcal{L}_g(g, \phi_g) \ + \ \sqrt{-f}\, \mathcal{L}_f(f, \phi_f) \Big] \qquad \text{(coupling to matter)}$$

Matter-sector Lagrangians

$$\mathcal{L}_g(g, \phi_g) = -\tfrac{1}{2} g^{\mu\nu} \nabla_\mu \phi_g \nabla_\nu \phi_g - V_g(\phi_g)$$

$$\mathcal{L}_f(f, \phi_f) = -\tfrac{1}{2} f^{\mu\nu} \nabla_\mu \phi_f \nabla_\nu \phi_f - V_f(\phi_f)$$

This approach includes interactions
… allows for questions about stability

Let's study CGPP for each polarization mode

tensor modes ~ helicity = +-2
vector modes ~ helicity = +-1
scalar modes ~ helicity = 0

SVT decomposition of massive metric perturbations

Massive metric perturbations

$$v_{00} = a^2 E, \quad v_{0i} = a^2(\partial_i F + G_i), \quad v_{ij} = a^2(\delta_{ij} A + \partial_i \partial_j B + \partial_i C_j + \partial_j C_i + D_{ij})$$

Constraints

$$\partial_i C_i = 0, \quad \partial_i G_i = 0, \quad \partial_i D_{ij} = 0, \quad \text{and} \quad D_{ii} = 0$$

The action decomposes into scalar, vector, and tensor sectors:

$$\sqrt{-\bar{g}}\, \mathcal{L}^{(2)}_{\text{massive}} = L_S + L_V + L_T$$

$$L_T = \text{only a function of } D_{ij}$$

$$L_V = \text{only a function of } C_i \text{ and } G_i$$

$$L_S = \text{only a function of } A,\, B,\, E,\, F, \text{ and } \varphi_v$$

Tensor & vector polarization modes

Quadratic action

$$L_T = \frac{1}{2} a^2 \Big[D'_{ij} D'_{ij} - \partial_k D_{ij} \partial_k D_{ij} - a^2 m^2 D_{ij} D_{ij} \Big]$$

$$L_V = a^2 \Big[\partial_j (G_i - C'_i) \partial_j (G_i - C'_i) + a^2 m^2 (G_i G_i - \partial_j C_i \partial_j C_i) \Big]$$

Equations of motion

tensor sector:

$$\tilde{\chi}''_s(\eta, \boldsymbol{k}) + \omega_k^2(\eta)\, \tilde{\chi}_s(\eta, \boldsymbol{k}) = 0 \qquad \text{for } s = +, \times$$

$$\text{where} \quad \omega_k^2(\eta) = k^2 + a^2 m^2 - 2a^2 H^2 - aH'$$

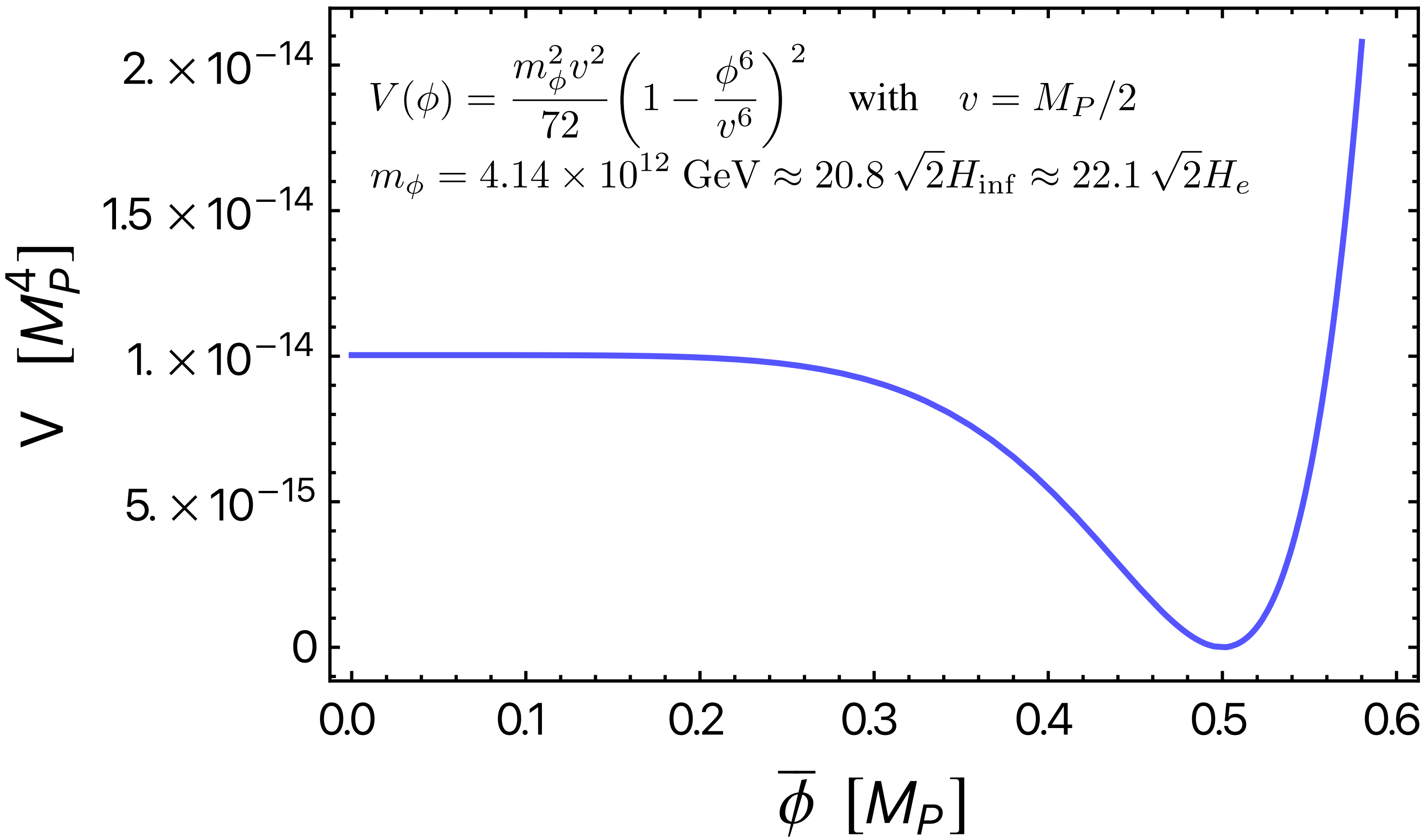
vector sector:

$$\tilde{\chi}''_s(\eta, \boldsymbol{k}) + \omega_k^2(\eta)\, \tilde{\chi}_s(\eta, \boldsymbol{k}) = 0 \qquad \text{for } s = +, -$$

$$\text{where} \quad \omega_k^2(\eta) = k^2 + a^2 m^2 - f''/f, \quad f = a^2/\sqrt{k^2 + a^2 m^2}$$

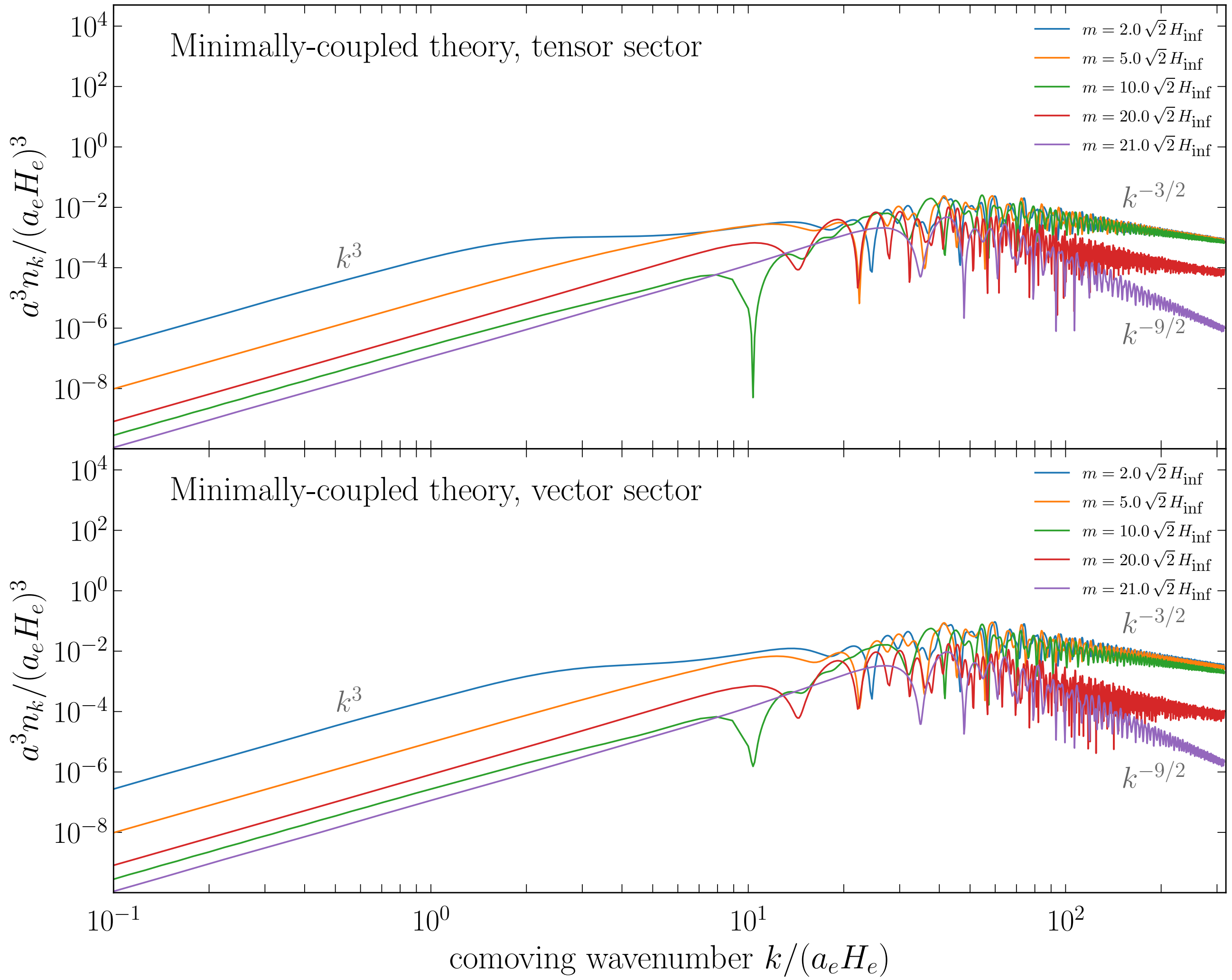
Numerical analysis

Assume a hilltop model of inflation



Spectra

$$a^3 n_k = \frac{k^3}{2\pi^2}|\beta_k|^2$$



Comments:

- The T- and V-sector spectra are very similar. This is b/c the equations of motion for non-rel modes (k<<am) are identical between T- and V-sector. Both give:

$$\omega_k^2 \approx a^2 m^2 - 2a^2 H^2 - aH'$$

- The low-k modes follow a power-law

$$n_k \propto k^\nu \quad \text{with} \quad \nu = 3 - 2\left[\tfrac{9}{4} - \tfrac{m^2}{H_{\rm inf}^2}\right]^{1/2}$$

which is familiar from studies of scalar spectators. This is b/c the T-sector EOM is identical to that of a scalar spectator. Low-k modes leave the horizon during inflation & expressions are easily derived.

- The high-k modes also follow a power law:

$$\text{for } m < m_\phi \quad \Rightarrow \quad \phi\phi \to \chi\chi \quad \Rightarrow \quad n_k \propto k^{-3/2}$$
$$\text{for } m < \tfrac{3}{2} m_\phi \quad \Rightarrow \quad \phi\phi\phi \to \chi\chi \quad \Rightarrow \quad n_k \propto k^{-9/2}$$

These modes never leave the horizon & CGPP can be understood as a scattering. Different channels are open depending on how the masses compare. [Ema, Nakayama, & Tang (2018)] , [Chung, Kolb, AL (2018)]

- Oscillatory features are the result of quantum interference between competing scattering channels. [Basso, Chung, Kolb, & AL (2022)]

What about the scalar pol. mode?

(it’s much more difficult)

Scalar polarization mode

Quadratic action

$$L_S = \text{only a function of } A,\, B,\, E,\, F, \text{ and } \varphi_v$$

After imposing constraints (and a LOT of algebra) there are only two propagating degrees of freedom

$$L_{S,\boldsymbol{k}} = K_\Pi\, |\tilde{\Pi}'|^2 + M_\Pi\, |\tilde{\Pi}|^2 + K_{\mathcal{B}}\, |\tilde{\mathcal{B}}'|^2 + M_{\mathcal{B}}\, |\tilde{\mathcal{B}}|^2 + \lambda_1\, \tilde{\Pi}^* \tilde{\mathcal{B}}' + \lambda_0\, \tilde{\Pi}^* \tilde{\mathcal{B}}$$

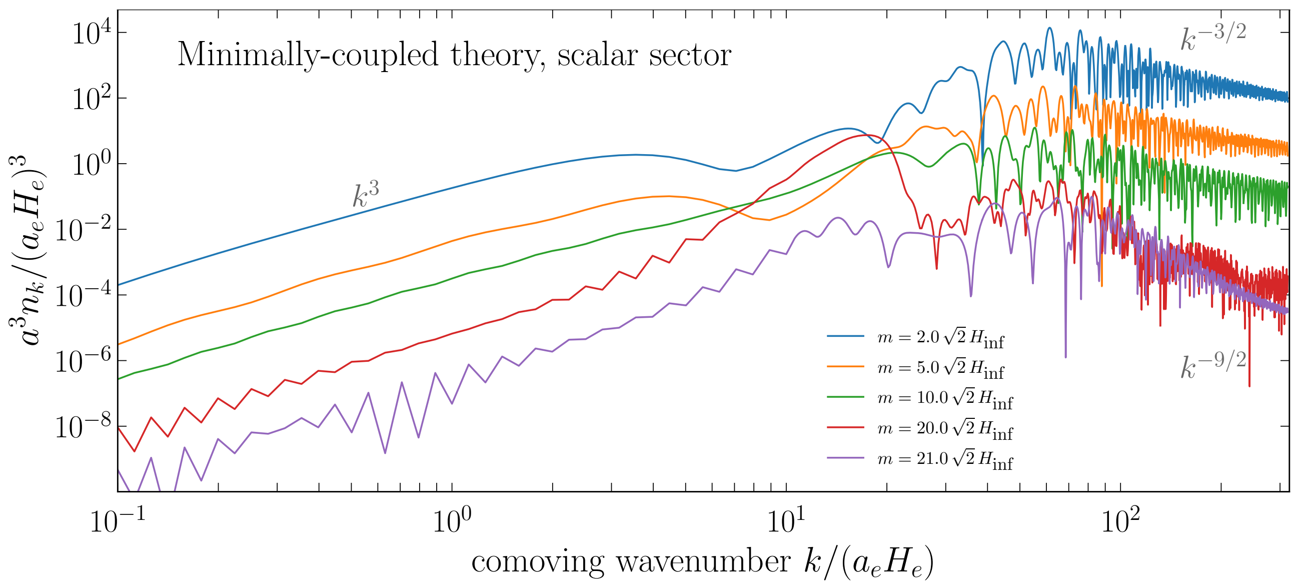
Where the kinetic coefficients are

$$K_\Pi = \frac{a^2}{2} \frac{k^4 + 3a^2\big(m^2 - m_H^2\big)k^2 + \frac{9}{4}a^4 m^2\big(m^2 - m_H^2\big)}{k^4 + 3a^2\big(m^2 - m_H^2\big)k^2 + \frac{3}{8}a^4 m^2\big(6m^2 - 4m_H^2 - \frac{m_H^4}{H^2}\big)}$$

$$K_{\mathcal{B}} = \frac{3a^6 m^2 (m^2 - m_H^2)}{4k^4 + 12a^2(m^2 - m_H^2)k^2 + 9a^4 m^2 (m^2 - m_H^2)}$$

and where we've defined: $m_H^2(\eta) = 2H^2 - (\bar{\phi}')^2/(aM_P)^2$

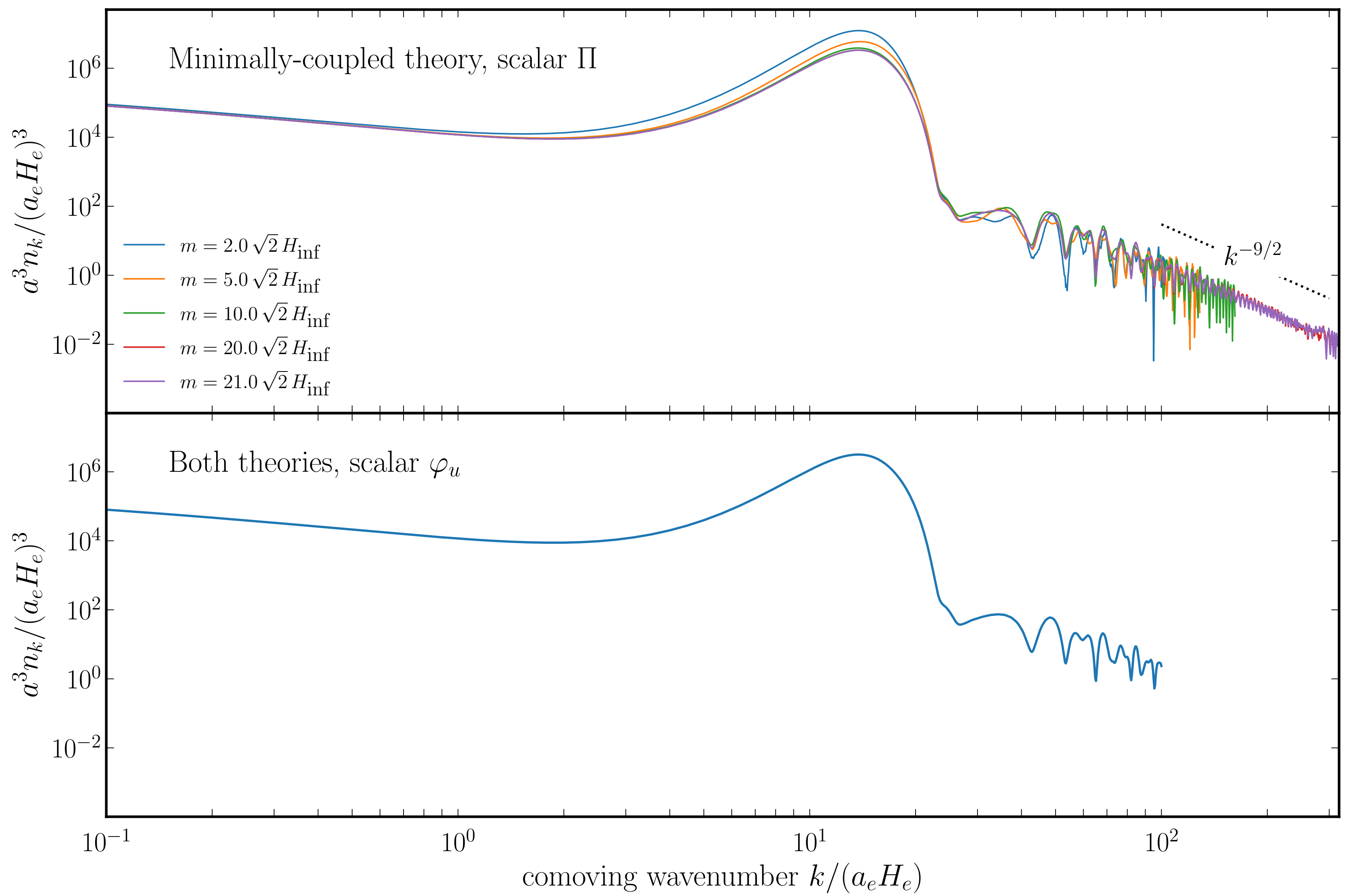
Spectrum



Comments:

- The S-sector particle production is generally larger than either the T- or V-sectors.
- Lowering the mass m enhances the S-sector particle production. This can be understood as an approach to the ghost instability at m = sqrt(2) H_{inf}.
- The same low-k and high-k power law behaviors are observed in the S-sector that we noted previously in the T- and V-sectors.

Scalar sector - spectra



- We also calculate spectra for the inflaton-like scalar perturbations. This is just the usual quasi-scale invariant spectrum of curvature perturbations.

Phenomenological implications?

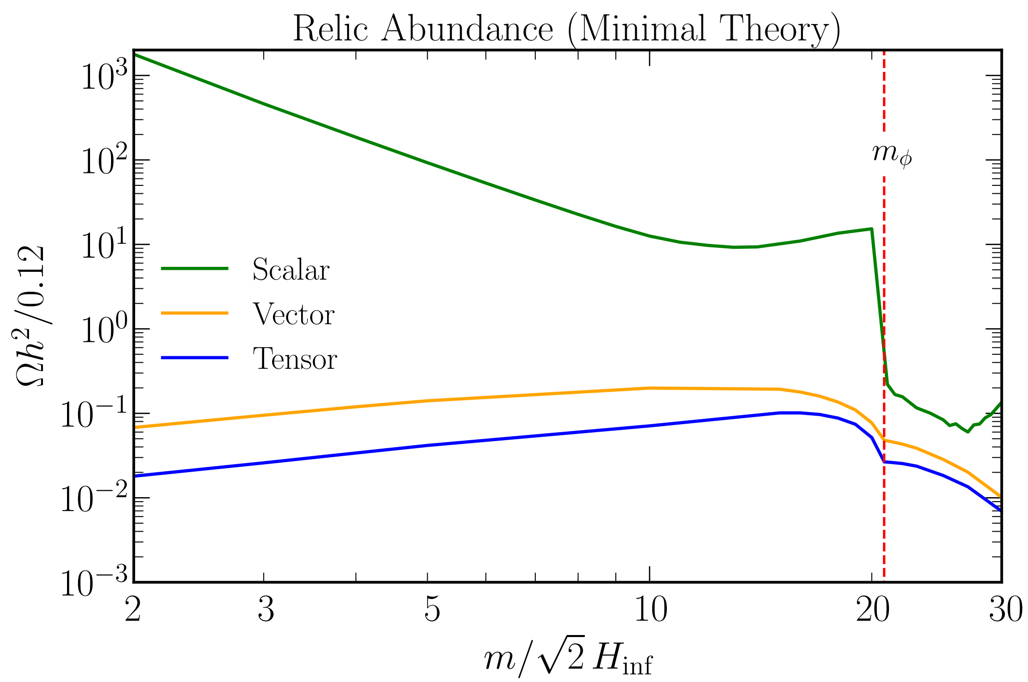
Implications for spin-2 dark matter

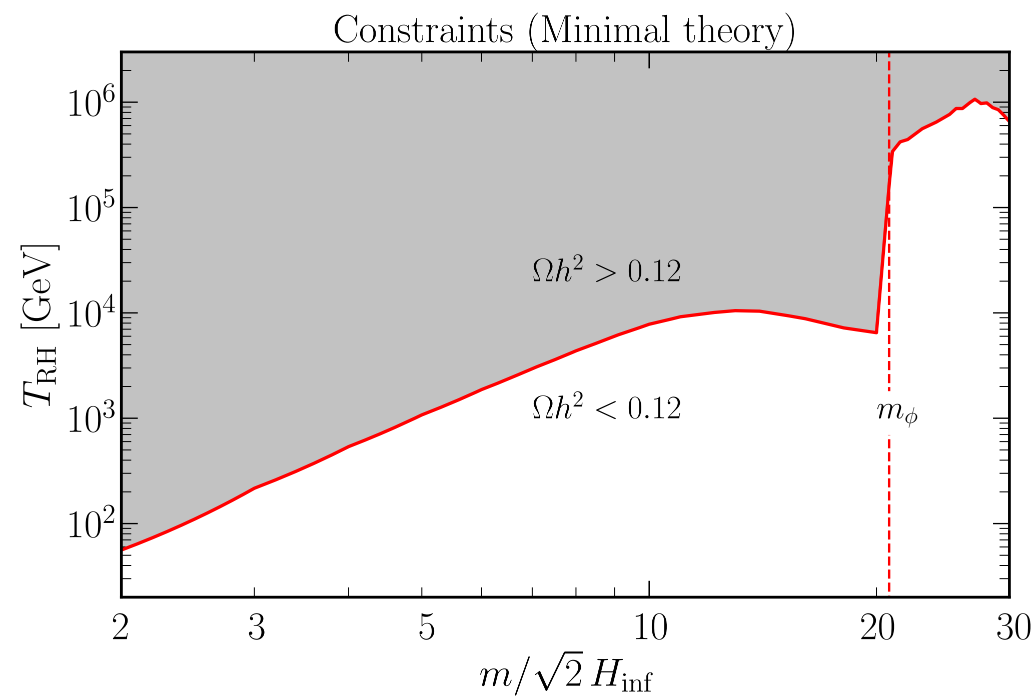
see also: Babichev et. al. (2016)

Assume: massive spin-2 particles are cosmologically long-lived (very non-trivial – ask me after!)

Relic abundance

$$\Omega h^2 \approx (0.114) \left(\frac{m}{10^{10}\ \mathrm{GeV}}\right) \left(\frac{H_e}{10^{10}\ \mathrm{GeV}}\right) \left(\frac{T_{\mathrm{RH}}}{10^{8}\ \mathrm{GeV}}\right) \left(\frac{a^3 n}{a_e^3 H_e^3}\right)$$





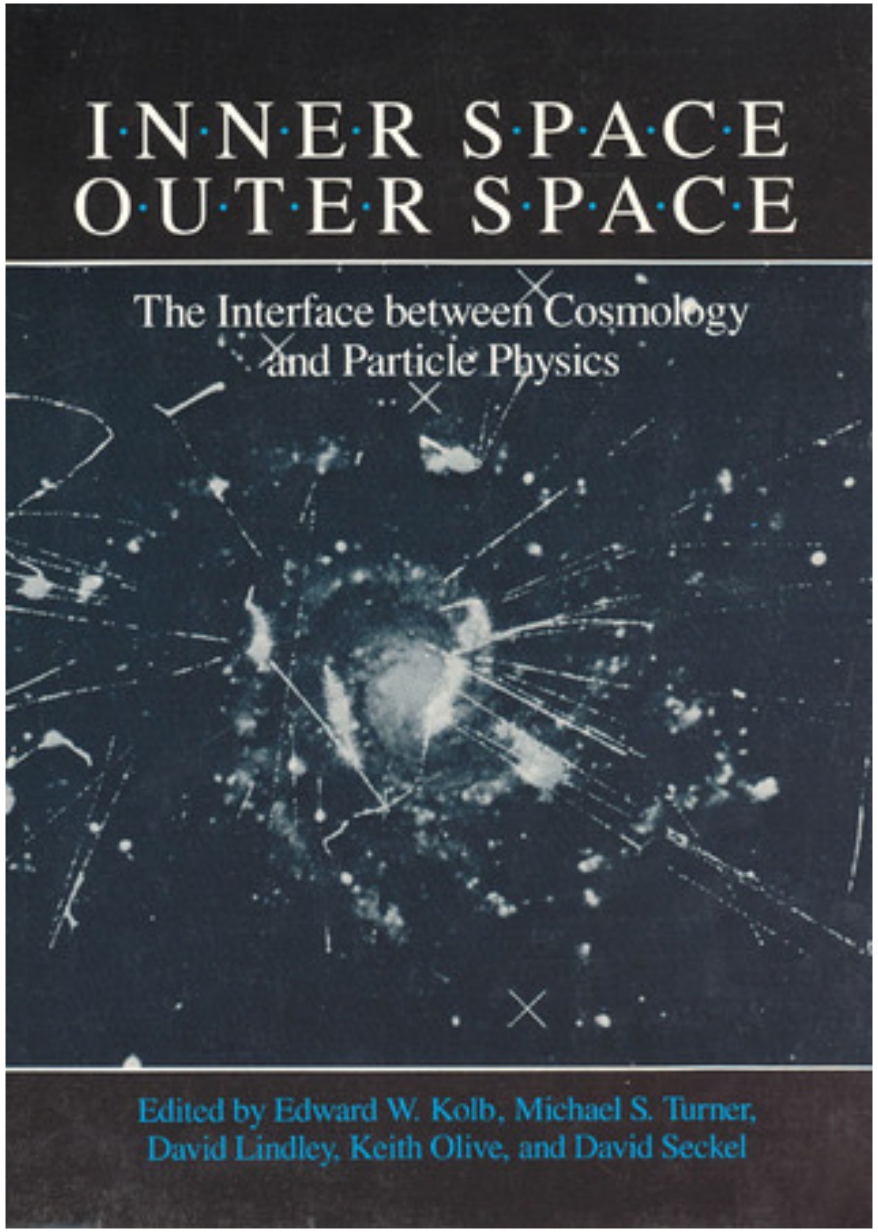
(1) Introduction & motivation

(2) Cosmological gravitational particle production

(3) CGPP of massive spin-2 particles

(4) Summary & conclusion

The Inner Space / Outer Space Interface



The Inner Space / Outer Space Interface

AN

write a haiku about the inner space / outer space interface for my friend and colleague physicist Rocky Kolb

Exploring within,
A universe of the mind,
Echoes in the stars.

Summary

We're interested in the phenomenon of CGPP with massive spin-2 particles

Lower spin fields have been studied extensively, but there's no comprehensive study of spin-2

To couple massive spin-2 to gravity, we study a theory of bigravity

Minimal ingredients: two metrics + two inflaton fields

(1) We derive the quadratic action for bigravity on an inflationary background

We use an SVT decomposition to isolate scalar, vector, and tensor degrees of freedom

(2) In the scalar sector, this decomposition reveals a ghost instability

We derive an FRW-generalized Higuchi bound

$$m^2 \geq 2H(\eta)^2 \left[1 - \epsilon(\eta)\right] \qquad \text{where} \qquad \epsilon(\eta) = -\dot{H}/H^2$$

(3) We numerically evaluate the spectra of gravitationally produced particles

If these massive spin-2 particles are cosmologically long-lived dark matter candidates, then CGPP provides a possible explanation for their origin in the early universe.