
# INTERESTING ASPECTS OF MULTI-VALUED HAMILTONIANS

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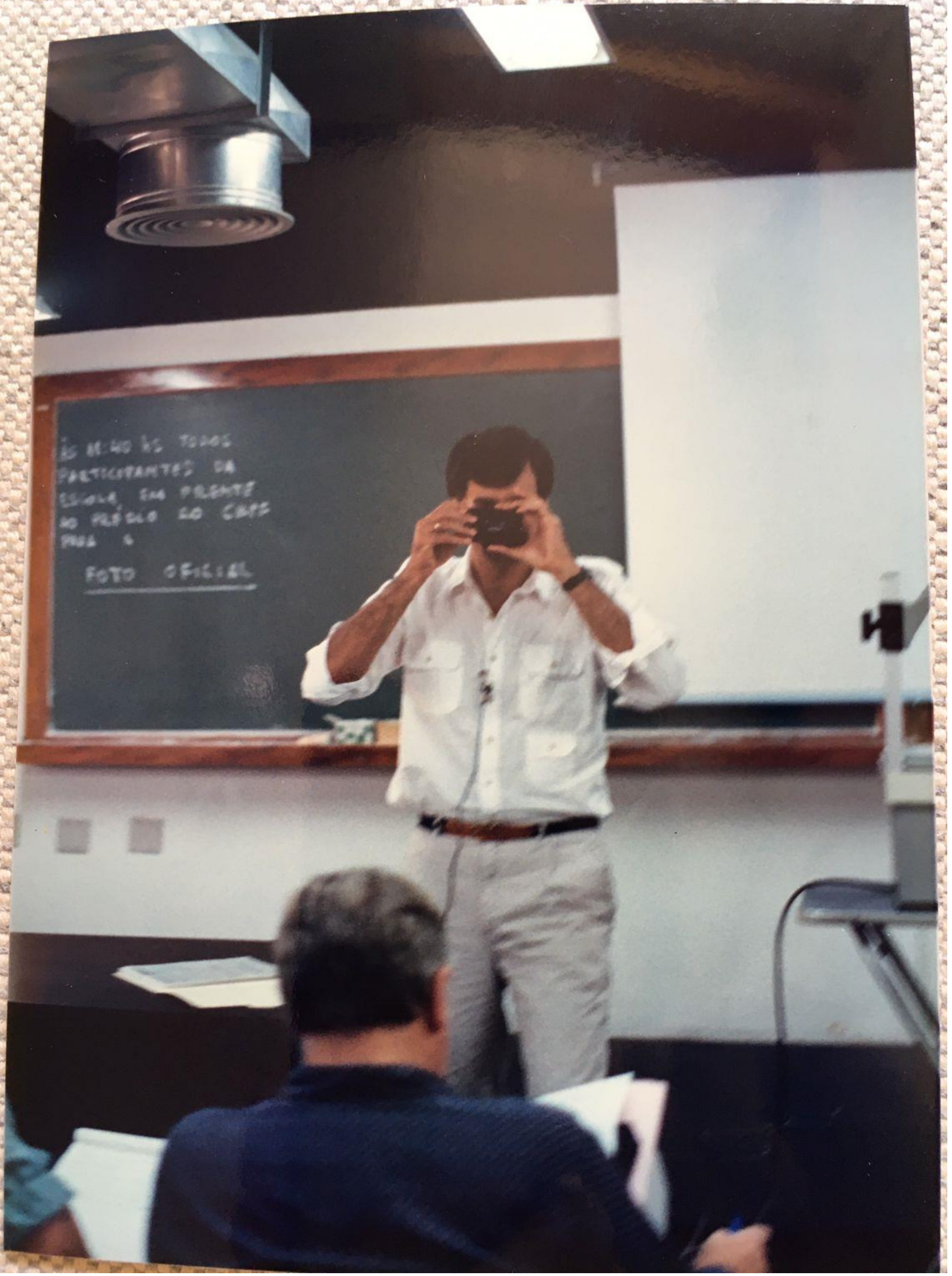
**together with Alexsandre Ferreira and Jorge Zanelli**

First visit of Rocky to Rio: 1987.

The V Brazilian School of Cosmology and Gravitation


PARTICIPANTES DA
ESCOLA, EM FRENTE
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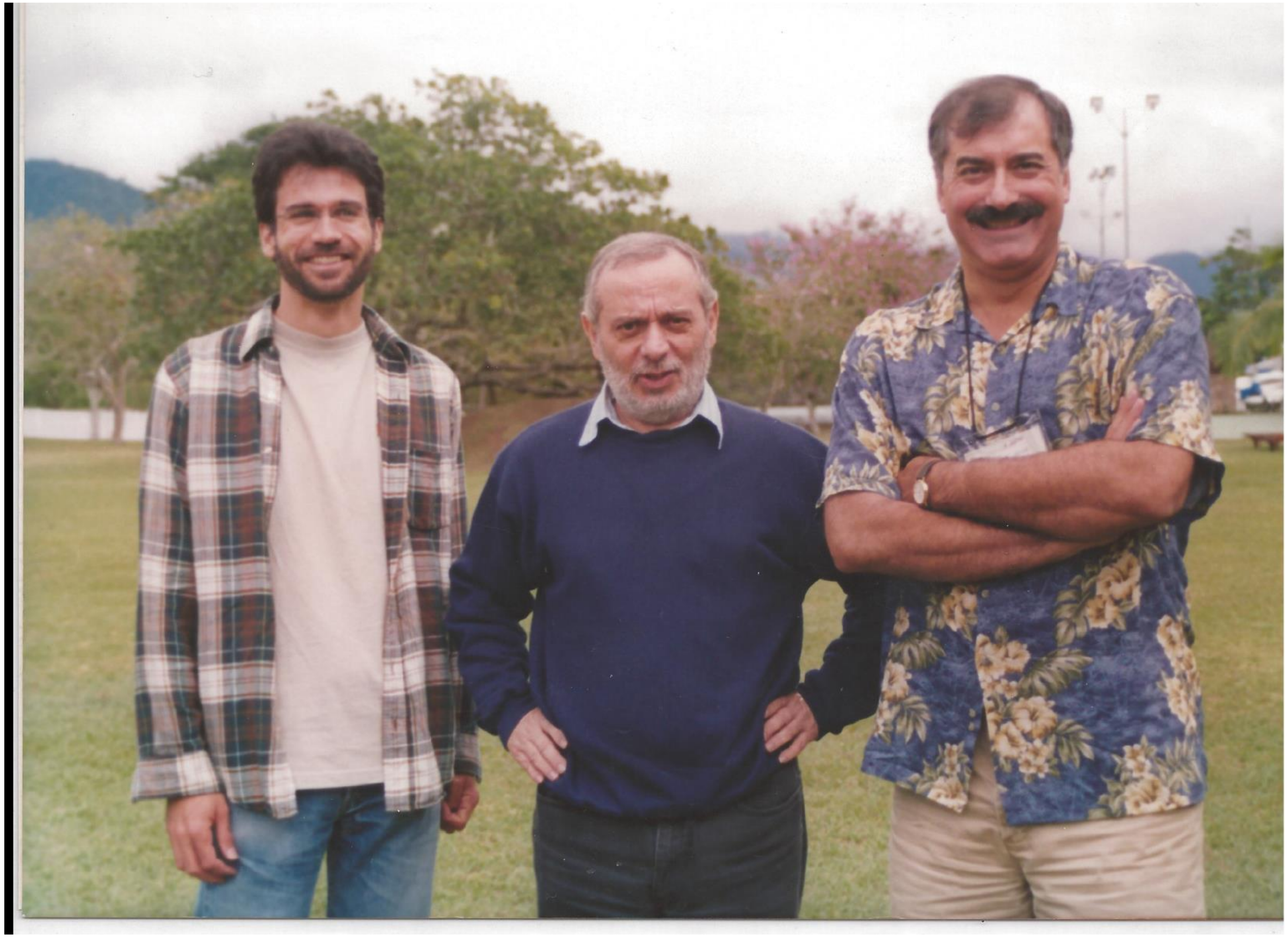
He came back in 2002,

The X Brazilian School of Cosmology and Gravitation



and in 2022,

The XVIII Brazilian School of Cosmology and Gravitation

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# FIRST ORDER FORMALISM FOR QUANTUM GRAVITY

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**String theory should lead to no ghost higher order corrections supposed to contain a Gauss-Bonnet term in d-dimensions →**

If we write the lagrangian density as

$$\mathscr{L} = k^{-1}R + \alpha\left(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2\right), \tag{1.2}$$

we obtain, after a tedious calculation, the action in ADM form

$$\begin{aligned} I = \int \mathrm{d}t\,\mathrm{d}^{d-1}x\, Nh^{1/2}\Big\{ & k^{-1}(\operatorname{tr}K^2 - K^2 + \tilde{R}) \\ & + \alpha\Big[\left(2\operatorname{tr}K^4 - (\operatorname{tr}K^2)^2 + 2K^2\operatorname{tr}K^2 - \tfrac{8}{3}K\operatorname{tr}K^3 - \tfrac{1}{3}K^4\right) \\ & + \left(-4\tilde{R}_{ijkl}K^{ij}K^{kl} - 8\operatorname{tr}(K^2\tilde{R}) - 2\tilde{R}K^2 + 2\tilde{R}\operatorname{tr}K^2\right. \\ & \left. + 8K(\operatorname{tr}K\tilde{R}) + \left(\tilde{R}_{ijkl}\tilde{R}^{ijkl} - 4\tilde{R}_{ij}\tilde{R}^{ij} + \tilde{R}^2\right)\right]\Big\} + \text{s.t.}, \end{aligned}$$

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# Quantum mechanics for multivalued Hamiltonians

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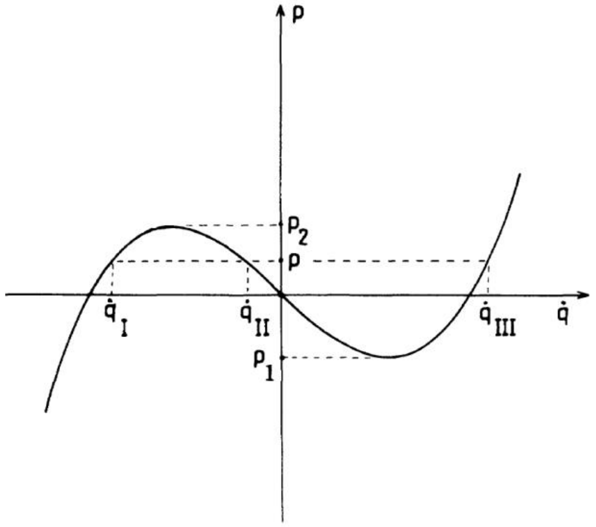
When the Lagrangian is not quadratic in the velocities, the situation may arise that the expression for the velocities in terms of the momenta is multivalued. As a consequence, the classical motion is unpredictable since at any time one can jump from one branch of the Hamiltonian to another. Yet, the quantum theory turns out to be perfectly smooth, with wave functions which are regular functions of time. We show that the path integral automatically picks up a unique combination of the branch Hamiltonians, which is a natural generalization of the Brouwer degree of the Legendre map.

$$S \equiv \int L\,dt = \int \left(\tfrac{1}{4}\dot{q}^4 - \tfrac{1}{2}\alpha\dot{q}^2\right)dt \ .$$

$$p(\dot{q}) \equiv \frac{\partial L}{\partial \dot{q}} = \dot{q}^3 - \alpha\dot{q}$$




In fact, this feature appears in several interesting physical systems, such as the Lovelock extension of General Relativity in higher dimensions, classical time crystals, k-essence fields, Horndeski theories, compressible fluids, nonlinear electrodynamics, and so on.

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# Classical Time Crystals

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We consider the possibility that classical dynamical systems display motion in their lowest-energy state, forming a time analogue of crystalline spatial order. Challenges facing that idea are identified and overcome. We display arbitrary orbits of an angular variable as lowest-energy trajectories for nonsingular Lagrangian systems. Dynamics within orbits of broken symmetry provide a natural arena for formation of time crystals. We exhibit models of that kind, including a model with traveling density waves.

$$L_2(\phi, \dot{\phi}) = -\frac{\kappa_2}{2}\dot{\phi}^2 + \frac{\lambda_2}{4}\dot{\phi}^4$$

Phys. Rev. D 105, 084064 (2022)

$$V(\phi) = \frac{\lambda}{4}\phi^4 - \frac{\omega}{2}\phi^2,$$

# Dynamical dimensional reduction in multi-valued Hamiltonians

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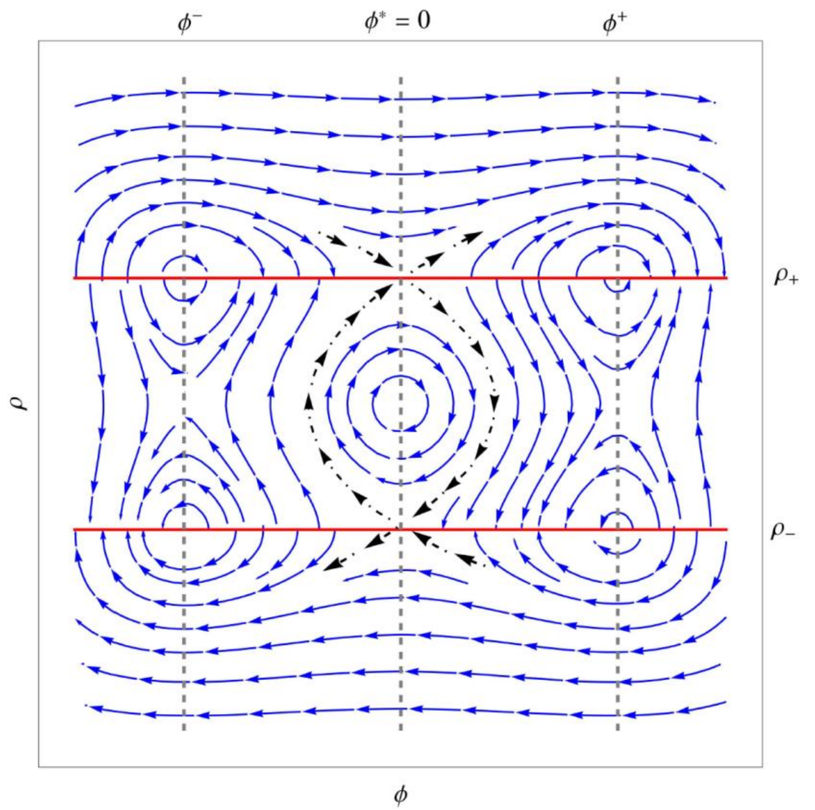
(Dated: May 3, 2022)

$$\rho = \dot{\phi}.$$

Several interesting physical systems, such as the Lovelock extension of General Relativity in higher dimensions, classical time crystals, k-essence fields, Horndeski theories, compressible fluids, and nonlinear electrodynamics, have apparent ill defined sympletic structures, due to the fact that their Hamiltonians are multi-valued functions of the momenta. In this paper, the dynamical evolution generated by such Hamiltonians is described as a degenerate dynamical system, whose sympletic form does not have a constant rank, allowing novel features and interpretations not present in previous investigations. In particular, it is shown how the multi-valuedness is associated with a dynamical mechanism of dimensional reduction, as some degrees of freedom turn into gauge symmetries when the system degenerates.




$$L = \frac{\beta}{4}\dot{\phi}^4 - \frac{\kappa}{2}\dot{\phi}^2 - V(\phi),$$

$$p_\phi = \beta\dot{\phi}^3 - \kappa\dot{\phi},$$

$$(3\beta\dot{\phi}^2 - \kappa)\ddot{\phi} = -V'(\phi).$$

# K-essence: application to Cosmology

Our model Universe is a spatially flat FLRW space-time filled with a $k$-essence scalar field:

$$\mathcal{L} = f(\phi)k(X) - V(\phi). \tag{1}$$

The kinetic term is an *a priori* arbitrary function $k(X)$ of the canonical kinetic term $X = (\nabla\phi)^2/2 = \dot{\phi}^2/2$, to be conveniently chosen for our purposes. Moreover $f(\phi) > 0$, for not to be hunted by ghosts. Analogously to perfect fluids, the field has pressure $p = \mathcal{L}$, and energy density

$$\varepsilon = 2X\mathcal{L}_{,X} - \mathcal{L} = f(2Xk_{,X} - k) + V. \tag{2}$$

The dynamics of the model is found through Einstein's equations

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\varepsilon, \quad \varepsilon_{,X}\ddot{\phi} = -3Hp_{,X}\dot{\phi} - \varepsilon_{,\phi}. \tag{3}$$

$$w = \frac{fk - V}{f(2Xk_{,X} - k) + V},$$

$$c_s^2 = \frac{p_{,X}}{\varepsilon_{,X}} = \frac{k_{,X}}{2Xk_{,XX} + k_{,X}} \geq 0,$$

$$\boxed{\mathcal{L} = f(\phi)(X - X_d)^3 - V(\phi).}$$

**or any other power bigger than 3.**

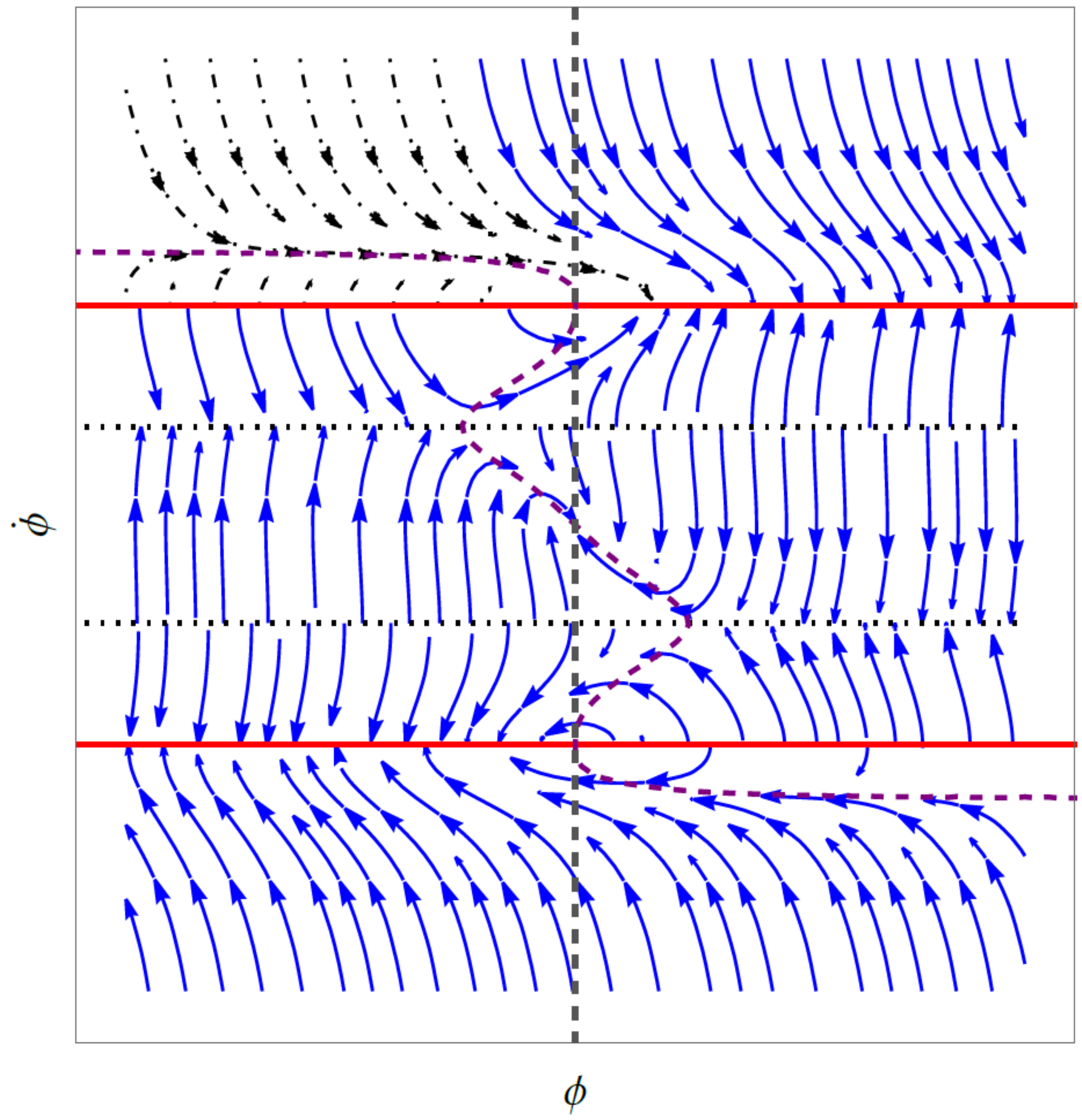



FIG. 5: Stream plot of the phase space for $f(\phi) = cte$ and $V = m^2\phi^2/2$. The red curves are $\dot{\phi} = \pm\sqrt{2}$, the black dotted ones are $\dot{\phi} = \sqrt{2/5}$. The dashed purple curve represents $\ddot{\phi} = 0$ and the dot-dashed stream curves are the solution which fall into the attractor.

$$X_d = 1 \qquad \Delta = X - 1$$

$$\boxed{p = f\Delta^3 - V, \quad \varepsilon = f\Delta^2(5X+1) + V,}$$

$$c_s^2 = \frac{\Delta}{5X-1},$$

$$\boxed{\frac{\mathrm{d}X}{\mathrm{d}\phi} = -\frac{4\Delta}{5X-1}\sqrt{3\pi f G X\left[\Delta^2(5X+1) + \frac{V}{f}\right]} - \frac{V_{,\phi}}{3f\Delta(5X-1)}.}$$

One would like to evaluate the typical value of the field when it reaches the degenerate surface in the future, the effective emerging cosmological constant, and the values of the new energy scales that lead to the present observed acceleration of the Universe. Two cases:

1) $f X_d^2 \ll 1: m = 10^3$ Tev, $\mu_d = 10$ kev

2) $f X_d^2 \gg 1: m = 10^3$ Tev, $\mu_d = 1$ Mev

In this second case, the attractor is very close to the degeneracy surface.

Summarizing the properties:

i) A single scalar field leading to primordial inflation and the $\Lambda$ of the $\Lambda$CDM model.
ii) Both primordial inflation and $\Lambda$ happens within a dense set of initial conditions.
iii) The model may not have singularities: it arises from de Sitter space, and ends as a de Sitter space with a much smaller cosmological constant.
iv) Many other Lagrangians may share the same properties, specially for case 2) above.
v) Speculations: quantum analysis, a scenario for Penrose cyclic model?

# THANKS ROCKY!!