Voltage Bias

\[ V_{\text{total}} = I R_b \frac{R_{\text{TES}}}{R_{\text{TES}} + R_b} \]

\[ R_{\text{total}} = \frac{R_b R_{\text{TES}}}{R_{\text{TES}} + R_b} \]

If \( R_b \ll R_{\text{TES}} \)

\[ V_{\text{total}} = I R_b \frac{R_{\text{TES}}}{R_{\text{TES}} + R_b} \]

Vbias requirement is set by bolometer properties

\[ \Rightarrow P_{\text{optical}} \sim P_{\text{electrical}} \]

\[ R_{\text{TES}} \]

SPT-36 example 128 bolometer

\[ P_{\text{optical}} \sim 7 \text{pW} \]

\[ P_{\text{electrical}} \sim 7 \text{pW} \Rightarrow V_{\text{bias}} \sim 3 \mu\text{V} \]

\[ \Rightarrow I_{\text{TES}} \sim 3 \mu\text{A} \]

Assuming \( R_{\text{TES}} \) fluctuates by 1% RMS in reaction to sky signal (this is big)

\[ \Rightarrow \text{need to be able to read out 30 nA signal} \]

16 bit ADC \( \frac{4V}{2^{16}} \sim 15 \mu\text{V resolution} \)

\[ \Rightarrow \text{need amplification w/o introducing noise} \]
The SQUID

→ superconducting quantum interference device
→ extremely sensitive to external magnetic flux
→ can detect \( \Delta \Phi \sim 10^{-4} \Phi_0/\sqrt{\text{Hz}} \)

\[ \Phi_0 = \frac{\hbar}{2e} \rightarrow \text{flux quantum} \]
\[ \sim 2 \times 10^{-15} \text{ Webers} \]

But also drawbacks
→ dynamic range
→ cryogenics

1 G = 10^{-4} T

1 T = 1 W/m²

Earth magnetic field \( \sim 10^{-4} \) W/m²

→ low noise
→ high forward gain
→ large operational bandwidth

Building block of a SQUID 1) is the Josephson Junction

2) Flux quantization in a superconducting ring

\[ 2\pi n = \oint \Phi_0 \, dl \rightarrow \Phi \]

Cooper pairs can tunnel through the barrier

2 equations to describe behavior

1) \( I_J = I_c \sin S \)

Current phase relation

\( I_c = \text{critical current} \rightarrow \) maximum supercurrent

\( S = \Phi_1 - \Phi_2 = \text{phase difference} \)

between the wavefunctions of \( S1852 \)

\( \pm = \text{supercurrent across the junction} \)

The Josephson Junction

\[ \frac{\hbar}{2e} = \text{cooper pair density} \]

\( \Phi_c = \text{center of point motion of cooper pairs} \)

Superconductor Oxide Barrier Superconductor

Cooper pairs can tunnel through the barrier
\[ \frac{d\phi}{dt} = \frac{2eU}{h} = \frac{2\pi}{\Phi_0} U \]
where
\[ \Phi_0 = \frac{h}{2e} = \text{magnetic flux quantum} \]

If the phase across the junction varied, a potential develops resistively & capacitively shunted junction.

More realistic model:
- stray capacitance due to junction geometry
- resistive shunt due to non-zerog quasiparticles (single electrons) & purposeful to prevent hysteresis.

\[ I = I_f + I_r + I_c \]
\[ = I_c \sin \theta + \frac{U}{R} + C \frac{dU}{dt} \]
Inset: \( \cos \theta = \frac{U}{R} \frac{dU}{dt} \)

Note: ignore noise

Analogue to damped harmonic oscillator w/ non-linear force term.

\[ \Rightarrow \text{Solution} \ U = -I\theta - I_c \cos \theta \]

Voltage as a function of phase difference leads to washboard potential.
- To allow potential to develop bias current tricks it to allow voltage to develop
$\langle S_0 | \Rightarrow | S_0 \rangle = 0$

$\langle S_1 | \Rightarrow | S_1 \rangle \neq 0$

---

**The DC SQUID**

2 Josephson junctions in parallel

Max current in the loop $= I_{c1} + I_{c2}$

Bias w/ $I > I_c$

Read $V$ as a function applied $\Phi$

$\Phi$ modulates $I_c \Rightarrow$ which changes $M$
When \( I_c \) is max, \( V \) is min
\( I_c \) is min, \( V \) is max

Flux wants to be integer multiple \( \Phi_0 \)

Current through each junction is:

\[
I_1 = \frac{I}{2} + J \\
I_2 = \frac{I}{2} - J
\]

\( \Rightarrow \) Back to the RCST model

\[
I = I_c \sin \delta + \frac{\Phi}{2e} + \frac{Ch}{2e} \delta
\]

Phase differences across the junctions are related to the total flux:

\( \Rightarrow 2 \) components

1) External magnetic field
2) Circulating current

\[
\delta_2 - \delta_1 = 2 \frac{I}{I_c} \left( \frac{\Phi_0}{\Phi} - 2J \right)
\]

In the static case, can solve for \( I_c(\phi_A) \)

\[
I_{cp} = 2I_c \left| \cos \left( \frac{\pi \Phi_A}{\Phi_0} \right) \right|\]

(simplified assumptions)
Must solve current equations for non-static case to allow voltage to develop

Magnetic flux coupled via inductor input coil

Can maximize $\Delta V$ for $\Delta \Phi_A$

$\Rightarrow$ find $\frac{\partial V}{\partial \Phi_A}$ is max

Can use $V_{PP}$ as a proxy

Transimpedance $\Rightarrow$ quoted in $\Omega$

$Z_{se} = M \frac{\partial V}{\partial \phi}$

$\Rightarrow$ output voltage given input current
The RF SQUID

Single junction
Inductively coupled to
a resonant circuit
which is driven
at its resonant
frequency.

Standard

Dissipationless

Loop has self
inductance, current
that tunnel across junction
also drives flux in the
loop.

Higher noise than DC SQUID
not require AC readout
simpler fab

Tiny coupling into the SQUID
Self inductance depends on
mean flux
Charge resonance at the tank
Simple Readout w DC SQUID

But really, we use arrays of SQUIDs,
  ⇒ can yield few mV of signal

Series SQUID array
  ⇒ voltages increase w N
  ⇒ noise adds quadratically
    \[ \sigma_{\text{SSA}}^2 = N \sigma_s^2 \Rightarrow \sigma_{\text{SSA}} = TN^{1/2} \sigma_s \]
  ⇒ win in S/N by \( T \)

Caveat for SSA
  ⇒ increased sensitivity to trapped flux
  & external magnetic fields
  ⇒ don't want gradients across the SSA
  ⇒ so that they add coherently
For CMB experiments

**Multiplexing**

Multiplexing required

1) Time division multiplexing
2) Frequency division multiplexing  \( \rightarrow \) DC squids
3) \( \mu \)-multiplexing  \( \rightarrow \) RF squids
4) MKIDs

First 3 all use SQUIDs, but in different ways.
Will talk about #4 later (also Erik Shirokorot)

TESs require subkelvin operation  \( \rightarrow \) limited power dissipation on cryocooler

\( \Rightarrow \) 2 wires per detector not possible as focal plane sizes increase
(\& we are there now)

**TDM**

\( \Rightarrow \) TES is DC biased

First stage SQUID inductively coupled to each TES

SQUIDs are biased sequentially
\( \rightarrow \) A, then B
the summing coils couples into 2nd stage squid
\( \rightarrow \) couples into on SSA for amplification

Uses a common bias line
Uses flux lock loop in multiple stage

- feeds back on output of SSA
to keep it constant

Updated TDM architecture for Adv. Act (eliminates SSA)

- uses flux activated switches for row select
  (Array of Josephson junctions)

Example of switching
rate ⇒ row rate = 560 kHz for ACT
full array = 15.2 kHz
every 65 μs

limitation ⇒ limited sampling rate
⇒ high noise is aliased into signal band
⇒ results in increased noise

⇒ can be mitigated by bandwidth
limiting signals from TES above Nyquist

Adv. Act 64 rows x 32 columns

Nyquist
inducers
Frequency Division Multiplexing

This is the one you'll use in the lab this afternoon.

TES is AC biased,

\[ I(t) = A \cos \omega t \]

current through TES from voltage bias

optical signal \[ I_y = A_y \cos \omega_y t \]

Define the \( \text{IM} = A \cos \omega t (1 + A_y \cos \omega_y t) \) modulated waveform

\[ \text{original carrier} \]

\[ \text{sky signal} @ \omega_c \pm \omega_y \rightarrow \text{the sidebands} \]

Can multiply by unmodulated reference at original bias frequency to demodulate down to base band & measure \[ \int A_y \cos \omega_y t \, dt \]

In FDM \( \rightarrow \) each TES in a module is assigned a unique resonant frequency

\[ R_{\text{TES}} \]

\[ f_0 = \frac{1}{2\pi L C} \]

\[ \Delta f = \frac{R}{2\pi L} \]
Make a parallel network that is summed together and then inductively coupled into a DC SQUID

AC Biased TES
DC SQUID \rightarrow \text{for amplification}

Feedback

Old generation \rightarrow \text{combo of ELL & injected mixer}

New generation
\rightarrow \text{digital waveform w/ active feedback (subtract carriers & sidebands)}
\rightarrow \text{mixer becomes the signal you readout}

Limitations

How many channels can pack into a given bandwidth w/ acceptable crosstalk

Macroscopic components

\Delta f_{\text{elec}} > 5.8 \Delta f_{\text{thermal}}

Electrical circuit bandwidth must be 5.8x > TES bandwidth
\rightarrow \text{or TES could oscillate at frequencies a circuit couldn't respond}
dissipationless rf SQUID used to provide gain between TES & HEMT

→ self inductance depends on the flux

→ couple to microwave resonator (GHz frequencies)

=> each w/ unique resonant frequency

TES is DC biased w/ parallel shunt resistor

→ multiple TESs share single pair of bias wires

Modulations in TES resistance modulate flux through SQUID

→ couples to the resonator & modulates its resonant frequency

Measure the shift in a fixed tone → single coax

SQUID noise
HEMT noise
TLS noise

Mustang 27 215 elements

SO

NB: HEMT = high electron mobility transistor

→ cryogenic amplifier