NEP and NET

Bradford Benson

August 8, 2017

Photon Noise

- CMB experiments today are effectively photon noise limited, i.e., limited by the random arrival times of the photons we are measuring

$$n = \frac{\eta}{exp(h\nu/kT) - 1} = \left[\frac{photons}{s\ Hz}\right]$$

- At 150 GHz for a 20 K RJ source, with 0.30 efficiency:

$$\frac{h\nu}{k_B T} = \frac{6.626\, 10^{-34}\,(150\, 10^9\ Hz)}{1.38\, 10^{-23}(20K)} \sim 0.3$$

$$n \approx \frac{0.3}{1 + 0.3 - 1} \sim 1\, \frac{photons}{s\, Hz}\ \text{per mode}$$

- So for 30 GHz bandwidth -> 3 x 10^{10} photons / sec, i.e., we are measuring lots of photons per sec.

Noise Equivalent Power (NEP)

- Variance in photons will be:

$$< \Delta n^2 >= n + n^2$$

- Variance in the photon power will be: $(h\nu)^2 < \Delta n^2 >$
- The noise equivalent power will be the variance in the photon power, for two polarization modes, integrated over frequency, per 1 sec of integration:

$$NEP_\gamma^2 = \int 4(h\nu)^2 n d\nu + \int 4(h\nu)^2 n^2 d\nu = [W^2/Hz]$$

- Power is $P_\nu = 2nh\nu$, so can re-write above as:

$$NEP_\gamma^2 = \int 2h\nu P_\nu d\nu + \int P_\nu^2 d\nu$$

- First term is "Poisson" term, and the second term is the "Bunching" term, due to correlated nature of photons emission

Photon Noise (NEP)

- Assuming spectrum does not very significantly over band, NEP can be approximated as:

$$NEP_\gamma^2 \approx 2P_{opt}h\nu_0 + P_{opt}^2/\Delta\nu$$

- For our 150 GHz detector with 40 GHz bandwidth and 6.5 pW of loading, this gives:

$$NEP_\gamma^2 \approx 1.26 \times 10^{-33} + 1.06 \times 10^{-33} [W^2/Hz]$$

- Poisson and bunching terms are approximately equal, which is not that surprising consider that is n ~ 1 (i.e., n << 1 infers Poisson term dominates, n >> 1 infers bunching term dominates)

$$NEP_\gamma \approx 4.8 \times 10^{-17} \, [W/\sqrt{Hz}] = 48 \, [aW/\sqrt{Hz}]$$

Thermal Carrier ("G") Noise

- The detector has an intrinsic noise from thermal fluctuations across the thermal link, often called "G" or "Phonon" noise:

$$NEP_G^2 = 4k_B T_b^2 G$$

- To design a bolometer want the following properties:
  1) Design "G" to match expecting loading (Popt)
  2) Make band-width x efficiency product as large as possible
  3) Keep G-noise < Photon-noise

- Yesterday, said that typical parameters are G ~ 100 pW/K and Tb ~ 0.5 K, which implies:

$$NEP_G \approx \sqrt{4k_B(0.5K)^2(100pW/K)} \sim 37\,[aW/\sqrt{Hz}]$$

- On order of photon noise. In practice, try to tune G/Psat or Tc as low as possible, to minimize G-noise.

Readout Noise

- Readout noise also can contribute to system noise.
- For example, SQUID noise is usually quoted in Noise Equivalent Current (NEI) as ~4 pA/sqrt(Hz).
- To covert to NEP need to multiply by voltage bias of detector, typically ~4 uV for a 1 Ohm detector (Pelec ~ 15 pW), i.e., the SQUID will provide NEP ~ 16 aW/sqrt(Hz)
- Rule of thumb: smaller voltage bias will typically reduce size of readout noise when referenced to bolometer:
  - e.g., main reason why many groups design TES with smaller resistances ~30 mOhm, to keep voltage bias as small as possible.

NEP to NET (Noise-Equivalent Temperature)

- Need to convert NEP to noise-equivalent temperature, i.e., how big a power fluctuation on the bolometer we would expect for given temperature fluctuation:

$$NET = NEP\frac{dT}{dP}$$

- Easy to do in RJ limit:

$$P_{RJ} = 2k_B T_{RJ}(\eta\Delta\nu)$$

$$\frac{dP_{RJ}}{dT_{RJ}} = 2k_B(\eta\Delta\nu) \sim 2k_B(0.3 \times 40e9)$$

- Implies that:

$$NEP \approx 50aW/\sqrt{Hz} \longrightarrow NET_{RJ} \approx 150\,\mu K/\sqrt{Hz}$$

NEP to NET (Noise-Equivalent Temperature)

- More helpful to work in CMB units, using blackbody equation:

$$B(\nu, T) = \frac{2h\nu^3}{c^2} \frac{1}{exp(h\nu/kT) - 1} \qquad x = \frac{h\nu}{k_B T_{CMB}}$$

$$\frac{dP}{dT_{CMB}} = 2k_B(\eta\Delta\nu)\frac{x^2 e^x}{(e^x - 1)^2}$$

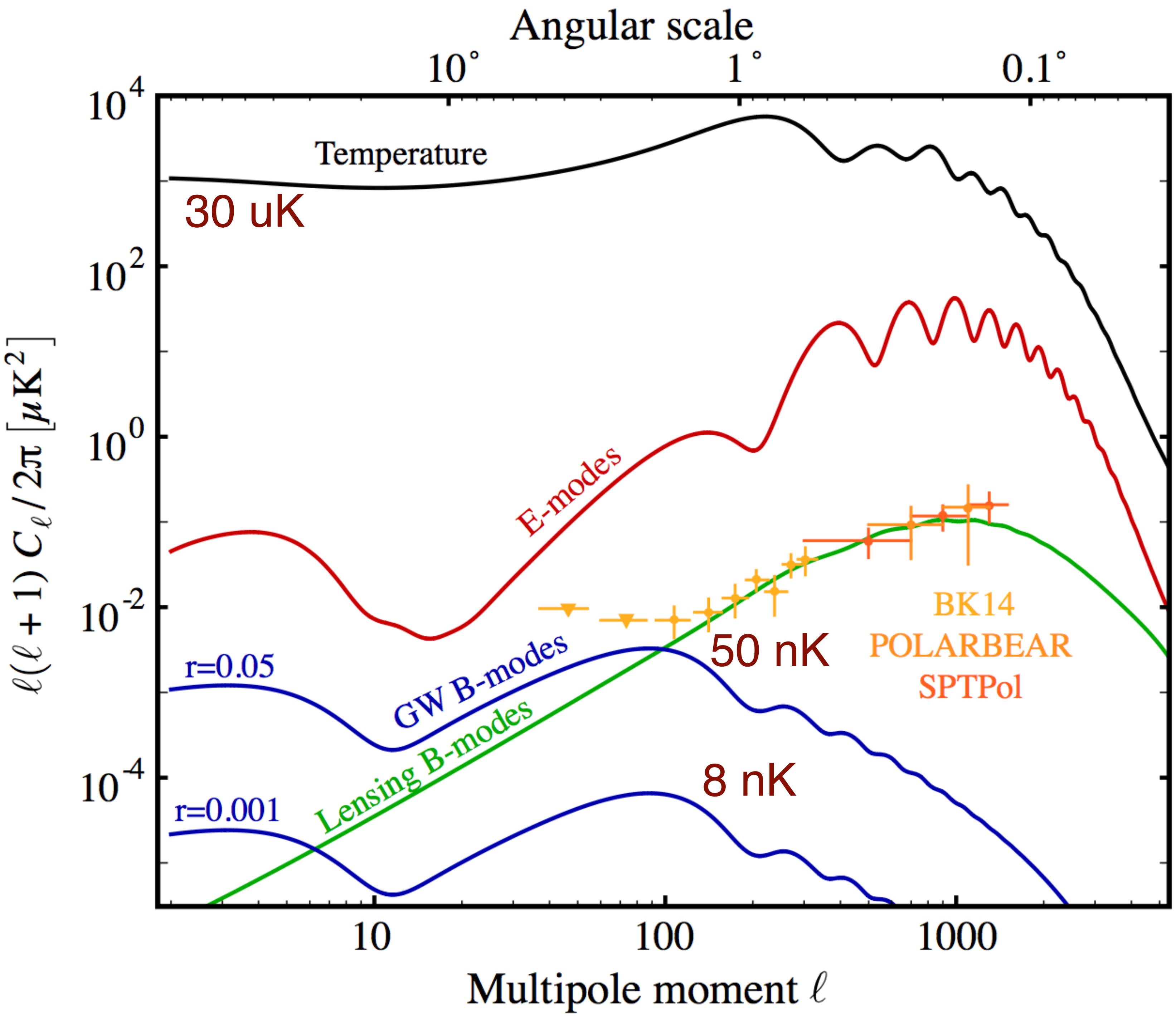
- At 150 GHz with Tcmb=2.725, implies x = 2.64, and:

$$NEP \approx 50aW/\sqrt{Hz} \longrightarrow NET_{CMB} = 262\,\mu K/\sqrt{Hz}$$

- Conversion between RJ and CMB temp at 90, 150, 220 GHz are factors of ~ 1.23, 1.74, and 3.08, respectively, i.e., correction gets larger the further away from RJ part of spectrum.

Designing CMB-S4

- One of primary goals of CMB-S4 is to detect inflationary gravity waves through B-mode spectrum.



- Power spectrum gives you sense of how big the signals you are looking for.
- In temperature, large-scale anisotropies correspond to 30 uK RMS (i.e., 1/100,000 of Tcmb)
- Peak in B-mode spectrum at ell ~ 80 corresponds to 50 and 8 nK signal for r = 0.05 and r=0.001, respectively.

CMB-S4 Science Book: arxiv:1610.02743

How many detectors do we need?

- NET per detector tells us size of fluctuation we can see in a unit observing time. Assume polarization sensitive detectors (i.e., NET increases by ~sqrt(2), and convert 1/sqrt(Hz) to sqrt(sec) by dividing NET by sqrt(2))

$$NET_{bolo} \approx 260\,\mu K\sqrt{s}$$

- Useful measure of sensitivity of the map for B-modes is nK-deg, i.e., what is the RMS of the map on 1-deg scales

$$Noise_{map}[nK\,deg] \approx \frac{NET_{bolo}\sqrt{SkyArea(deg^2)}}{\sqrt{N_{bolo}\,t}}$$

- Inflationary B-mode signal peaks at ~1-deg, so we want to get down to at a level such that image fluctuations with S/N ~ few, i.e., to get σ(r) ~ 0.001 need noise ~ 8 nK-deg / 3 ~ 3 nK-deg

How many detectors do we need?

$$Noise_{map}[nK\,deg] \approx \frac{NET_{bolo}\sqrt{SkyArea(deg^2)}}{\sqrt{N_{bolo}\,t}}$$

- Sky Area: All-sky is ~40,000 deg^2, typically thought we should focus on smaller patch with clean foregrounds ~2000 deg^2
- Observing time: 1 year = 3 x 10^7 sec, lets just assume a 4-yr survey with 25% obs. efficiency (close to typical)

$$N_{bolo} = \left(\frac{300\mu K\sqrt{s}}{0.003\mu K\,deg}\right)^2 \frac{2000\,deg^2}{3\times 10^7\,s} \approx 600,000$$

- So we expect CMB-S4 to require an instrument with ~order half million bolometers