TIMESTREAMS TO MAPS

Consider a single detector at the focus of an optical system that can move in az & el (a "telescope")

define $A(\theta, \phi, t) \equiv$ where on sky detector is "pointed" at time t

ASIDE # 1: don't get too confused when people talk about detectors as if they were transmitters. The math is all equivalent, and it's a handy way to express some important concepts

Pretend a couple of things:

1) instead of finite-resolution instrument observing full-bandwidth sky, pencil-beam instrument observing beam-convolved sky

2) instead of continuous distribution of brightness $T(\theta, \phi)$, sky is discrete pixels of brightness T^α.
[$\alpha \equiv$ pixel index]

- also discretize time: $A(\theta, \phi, t) \rightarrow A^i_\alpha$

Now we can write brightness "seen" by detector as a function of time (or, equivalently, for time sample i):

$$s^i = A^i_\alpha T^\alpha$$ (implied sum over α

ie, $s(t_i) = \sum_\alpha A(\theta_\alpha, \phi_\alpha, t_i) T(\theta_\alpha, \phi_\alpha)$)

- $A^i_\alpha \equiv$ "pointing matrix," projects sky into time-ordered data. Each row usually has only one non-zero element (for the one sky pixel α that was pointed at during time sample i).

Actual detector data will be a combo of this signal and some noise:

$$d^i = s^i + n^i = A^i_\alpha T^\alpha + n^i$$

(i.e., $d(t_i) = \sum_\alpha A(\theta_\alpha, \phi_\alpha, t_i) T(\theta_\alpha, \phi_\alpha) + n(t_i)$)

Note this eqn. is linear in T, so it is trivially solved. Lots of ways to derive this, but let's use χ^2. For a given estimate of sky brightness $\bar{T}^\alpha$ (bar indicates estimated quantity), the residual between model & data is:

$$r^i = d^i - m^i = A^i_\alpha T^\alpha + n_i - A^i_\alpha \bar{T}^\alpha$$
$$= A^i_\alpha (T^\alpha - \bar{T}^\alpha) + n_i$$

and the χ^2 of this model is

$$\chi^2 = r^i W_{ij} r^j \quad \left(\equiv \sum_{i,j} r(t_i)\, w(t_i, t_j)\, r(t_j) \right)$$

where W_{ij} is some weight matrix. The minimum-variance solution will be when $W_{ij} = (C^{ij})^{-1}$, where C^{ij} is the TOD noise covariance matrix $\langle n^i n^j \rangle$.

To get the best-fit solution, see where χ^2 is minimized or $\frac{\partial \chi^2}{\partial \bar{T}} = 0$:

$$\frac{\partial \chi^2}{\partial \bar{T}} = \frac{\partial r^i}{\partial \bar{T}} W_{ij} r^j + r^i W_{ij} \frac{\partial r^j}{\partial \bar{T}}$$

(Dropping subscripts for clarity & time)

$$= 2 \frac{\partial r}{\partial \bar{T}} W r = 2 \frac{\partial m}{\partial \bar{T}} W (d - m) = 2AW(d - A\bar{T}) \equiv 0$$

$$\rightarrow AWd = AWA\bar{T}$$

$$\hookrightarrow \bar{T}_{best} = (AWA)^{-1} AWd$$

$\overline{T}_{best} = (AWA)^{-1} AWd$ "Mapmaking equation"
- but it's just linear least-squares

If TOD noise is "white" (uncorrelated between TOD points $\longleftrightarrow$ flat in Fourier space),
$C^{ij} = \langle n^i n^j \rangle = \sigma^2_{noise} \delta^{ij}$, and $W_{ij} = \frac{1}{\sigma^2_{noise}} \delta_{ij}$,
and $\overline{T}^{\alpha}_{best} = \frac{1}{N_{obs}} \sum\limits_{A^i_\alpha = 1} d^i$ - "bin and average"

If not:
- C^{ij} is a dense, $N_{TOD} \times N_{TOD}$ matrix
- $AWA \equiv W_{\alpha\beta}$ is a dense, $N_{pixel} \times N_{pixel}$ matrix

these are huge* numbers, no way to invert

- two solutions:
  1) iterative approximations to ML
  2) filter 'til it's white, bin and average

#1 gives a less biased map (still biased by beam but not by filtering) at the cost of lots of computing time in the mapmaking step.

#2 is conceptually simple and takes little time & CPU for mapmaking but lots to simulate effect of filtering

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* except for very low-resolution maps and TOD

Starting-point refs for mapmaking:

general & filter-bin-average method: Hivon et al. (2002) ("MASTER")

ML approximation and generalization to polarization: Jarosik et al. (2007), Section 3

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MAPS TO POWER SPECTRA

What do we mean by "power spectrum" in the CMB world?

Full-sky map (Mollweide projection)

$\xrightarrow{\text{SHT}}$ $a_{\ell m}$

"quantum numbers"

ℓ: isotropic angular frequency

m: azimuthal frequency

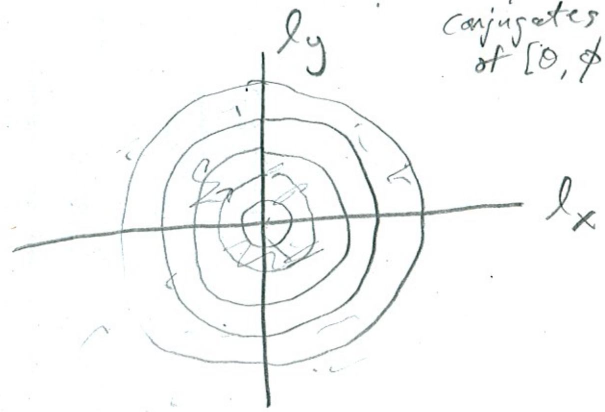
$$C_\ell = \langle |a_{\ell m}|^2 \rangle \Big|_m$$

in more familiar terms:

$T(\theta, \phi)$

Fourier Transform $\longrightarrow$

$T(\vec{l})$, $\vec{l} \equiv 2\pi[u,v]$ — conjugates of $[\theta, \phi]$



partial-sky map in some flat projection

$$C_l = \sum'_{\sqrt{l_x^2 + l_y^2} \in l} |T(\vec{l})|^2$$

yes, it's that easy. Hard part is correcting for beam, filtering, masking, systematics, foregrounds, etc.

ASIDE #2: POWER SPECTRUM NORMALIZATION & PARSEVAL'S THEOREM & INVARIANTS

$$\langle T^2_{map} \rangle = \sum'_l \frac{2l+1}{4\pi} C_l \approx \sum_l \frac{l C_l}{2\pi}$$

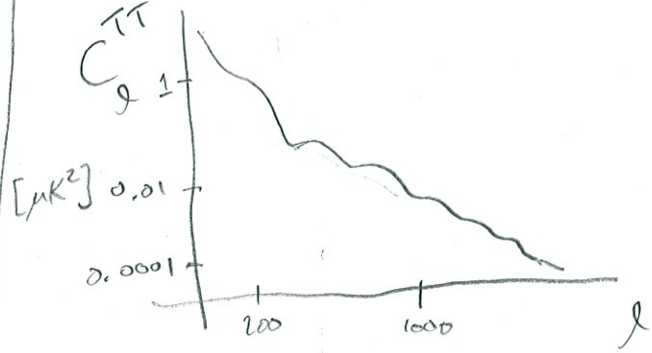
$$\langle T^2_{TOD} \rangle = \int_0^{f_{Nyquist}} df \text{ "PSD"} \approx NEP^2 \cdot \Delta f$$

$\longleftarrow$ power spectral density in $\frac{\text{units}^2}{\text{Hz}}$

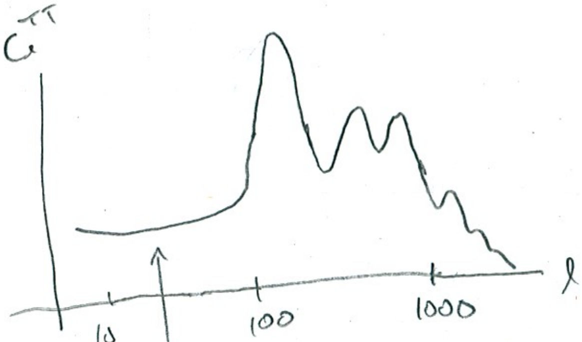
NEP in $\frac{W}{\sqrt{Hz}}$, NET in $K\text{-}\sqrt{s}$, map noise in μK-arcmin

ASIDE # 3 : C_ℓ vs. "D_ℓ"

an abomination that serves only to thwart intuition



$D_\ell^{TT} = \frac{\ell(\ell+1)}{2\pi} C_\ell^{TT}$



this is flat in D_ℓ for "scale-free" primordial power spectrum

Methods: Everyone does "pseudo-C_ℓ"

(FT or SHT, square, bin, correct for noise, beam, filtering, mask, etc.)

except at very low ℓ.

- reference: Hivon et al. again ("MASTER")

One advance since 2002: now usually do cross-spectra of maps from independent data (e.g. different observing times) to avoid noise bias. Ref: Polenta et al. (2005)

ASIDE #4: NOISE VARIANCE & SAMPLE VARIANCE

When you measure the power spectrum of the CMB, you are measuring a <u>variance</u>, so there's "noise" in the signal, too.

- Ex: take a set of Gaussian-distributed random numbers and estimate their mean. If the mean is zero or the mean is 5, does the uncertainty on the mean change? Now estimate the variance - does the uncertainty on the variance change if the variance is 5 vs the variance being 1?

"Knox formula" (Knox, 1995)

signal variance, noise variance

$$\delta C_\ell = \underbrace{\frac{2}{(2\ell+1) f_{sky}}}_{\#\text{ of modes}} \left[C_\ell + C_N \right]$$

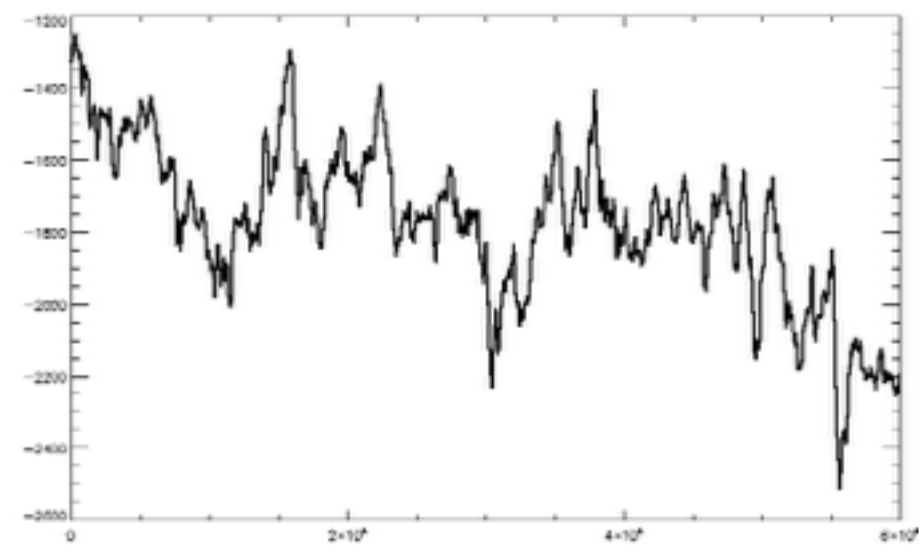
- equiv to $Var(Var(x)) = \frac{2}{N} Var(x)$

So if $C_\ell \approx C_N$, your noise & sample variance are contributing equally. And $C_\ell \approx C_N$ is equivalent to saying you have $S/N \approx \underline{1}$ on individual sky modes.

NOTE: in the absence of signal, no sample variance (duh), so if you're searching for, e.g., B modes, drill deep until you see something.

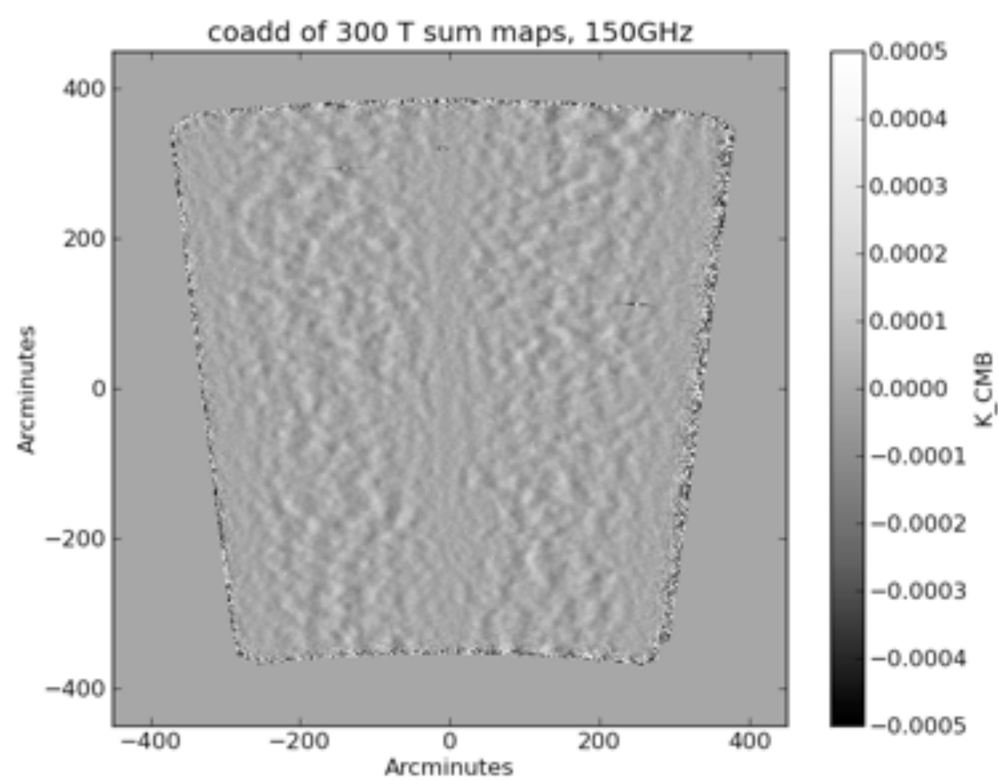
SPT Data: What is it?

- Fundamentally: currents from O(1000) detector elements.



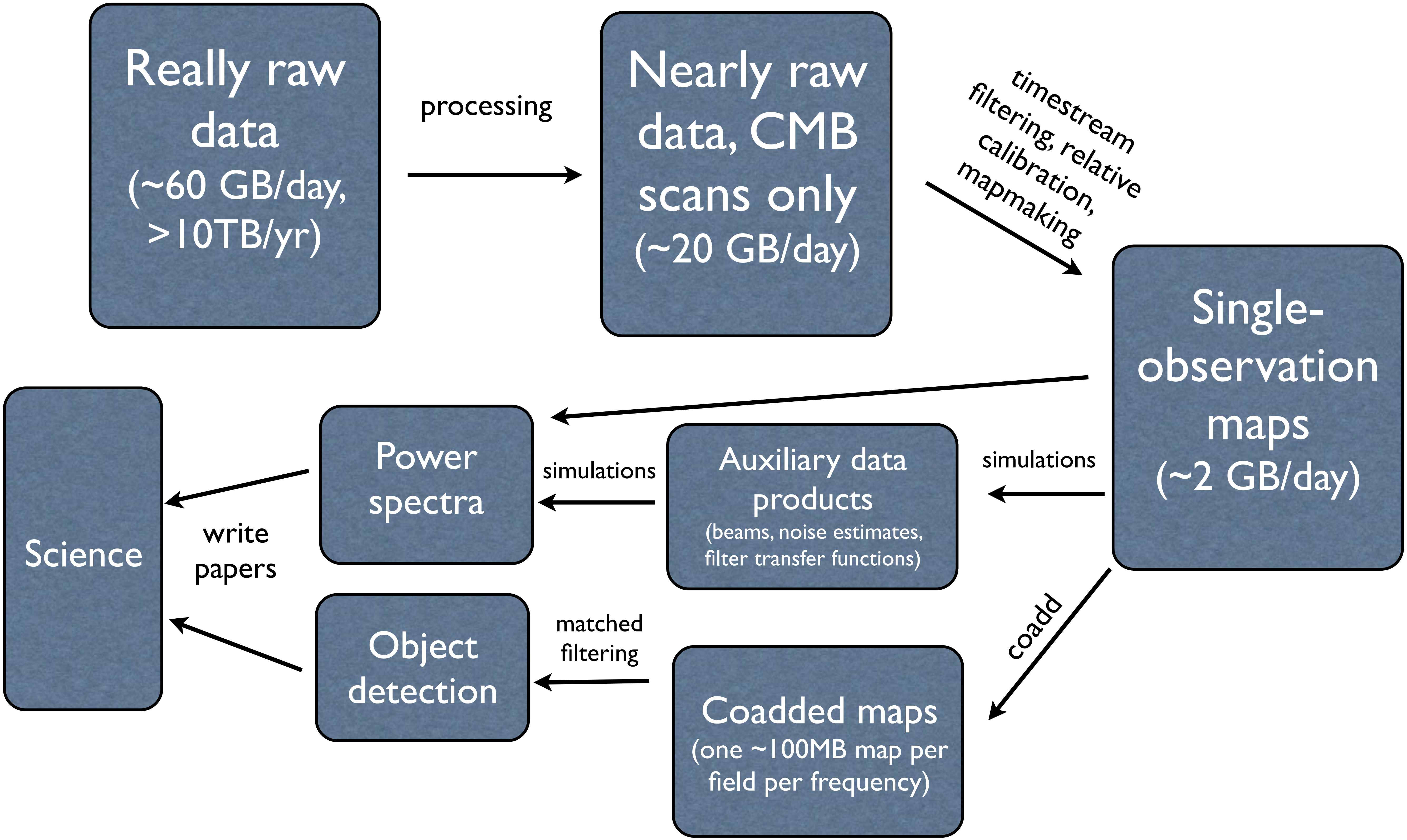
10 minutes of SPT-SZ data from a single bolometer

- Main pipeline products: best-fit temperature at pixelized locations on the sky: Maps.



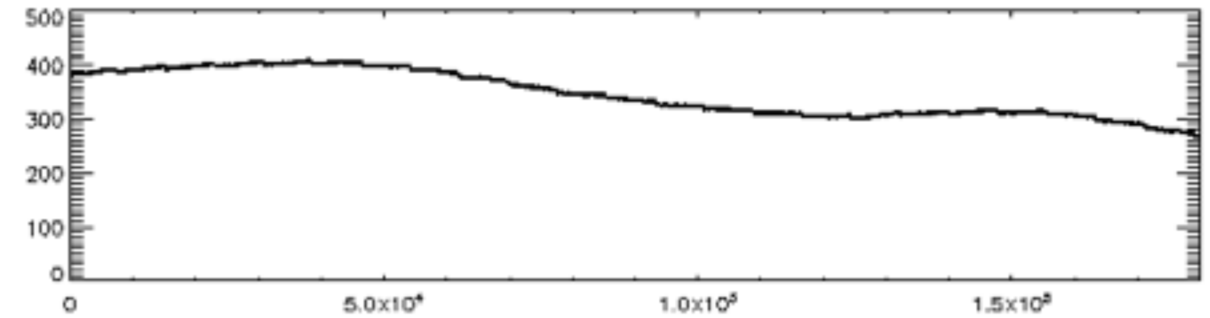
100 deg^2 field observed by SPTpol for <1 month

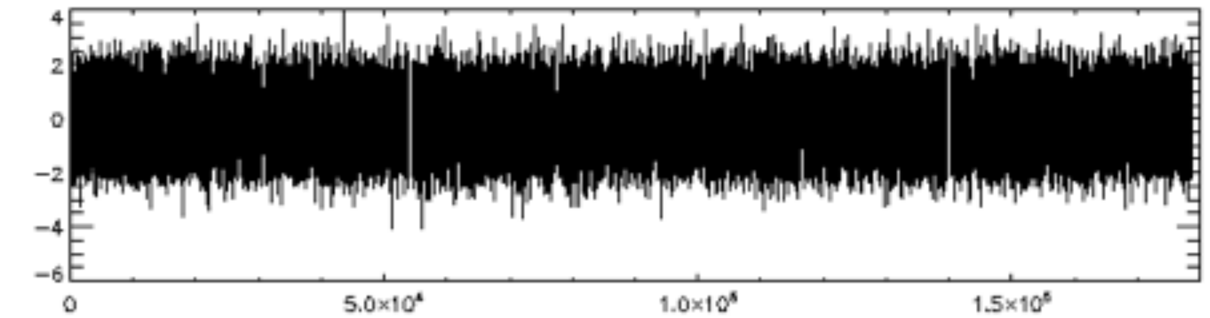
The SPT-SZ / SPTpol Pipeline



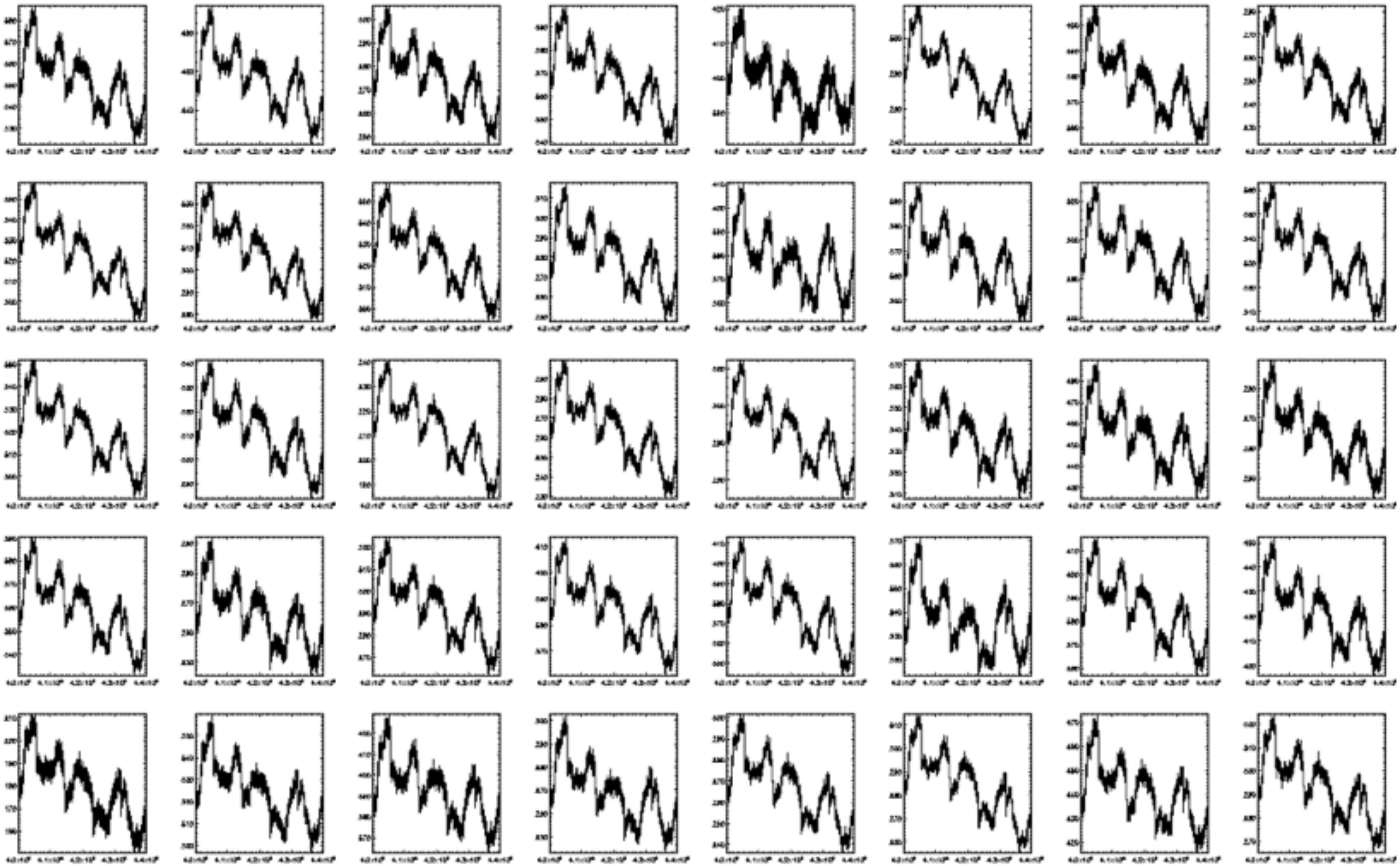
Data Processing

- ❖ Undo detector time response.
- ❖ De-spike.
- ❖ Fourier-domain filtering (includes notching of spectral lines).
- ❖ Remove large-scale common mode and long-timescale drifts.

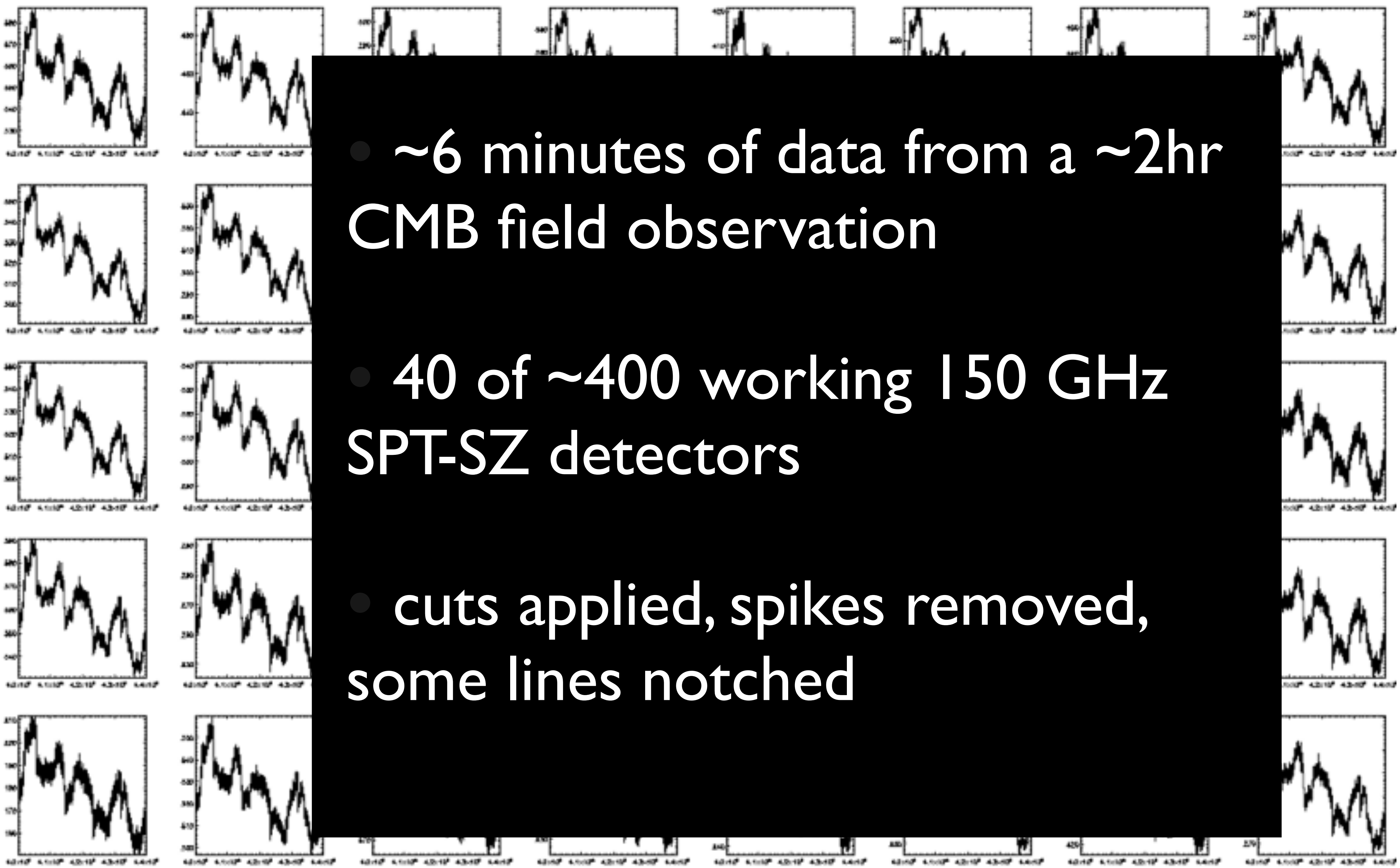




Minimally processed data

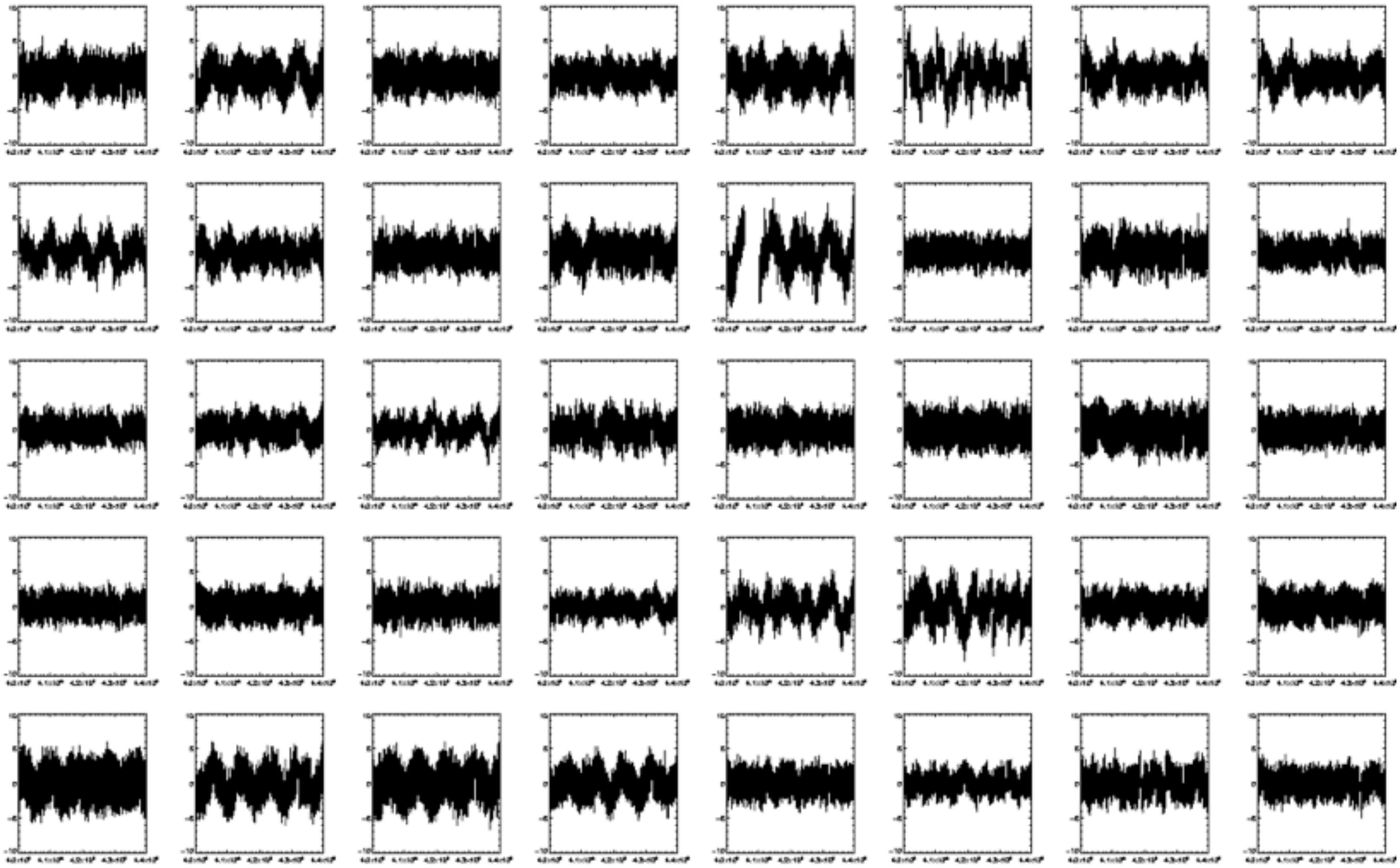


Minimally processed data

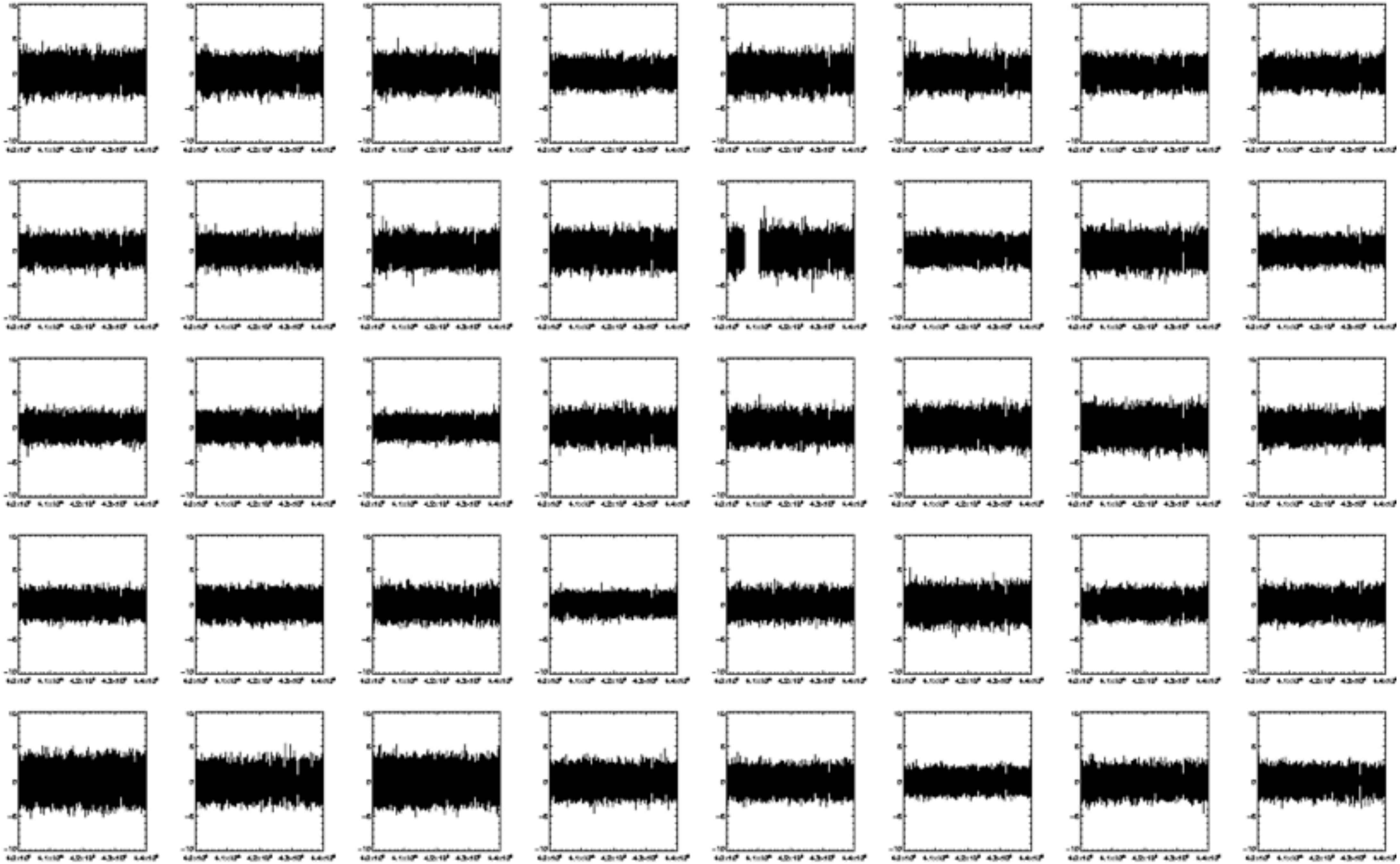


- ~6 minutes of data from a ~2hr CMB field observation
- 40 of ~400 working 150 GHz SPT-SZ detectors
- cuts applied, spikes removed, some lines notched

Common mode removed



High-order polynomial removed



Make Maps

- Calculate sky pointing for every detector at every timestream point.
- Apply calibration to each detector.
- Weight each detector's data by inverse variance (single number for SPT-SZ; 3x3 matrix for SPTpol).
- Take weighted mean of all measurements of sky temperature/polarization at every (pixelized) observed sky location.

Make Maps

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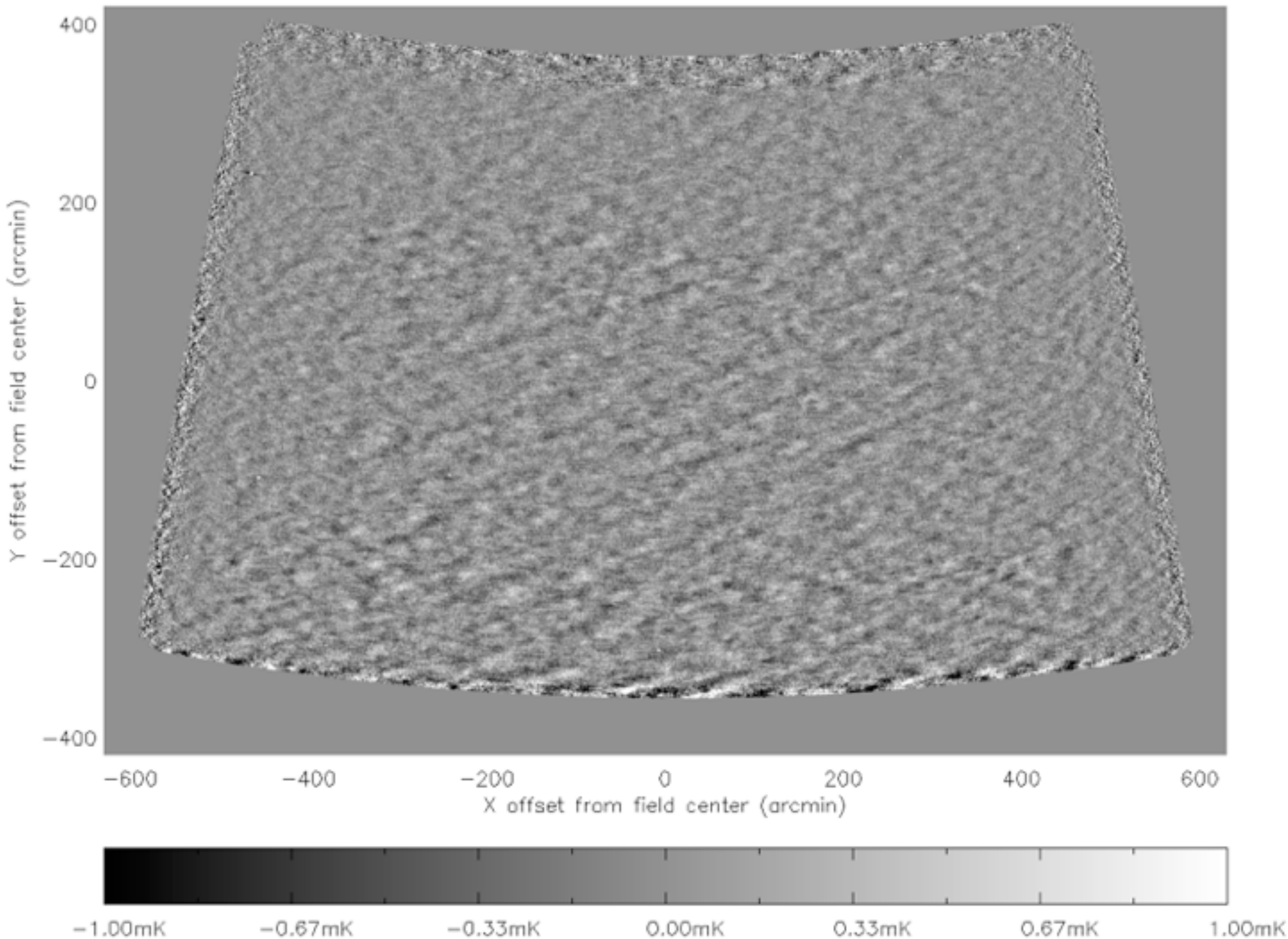
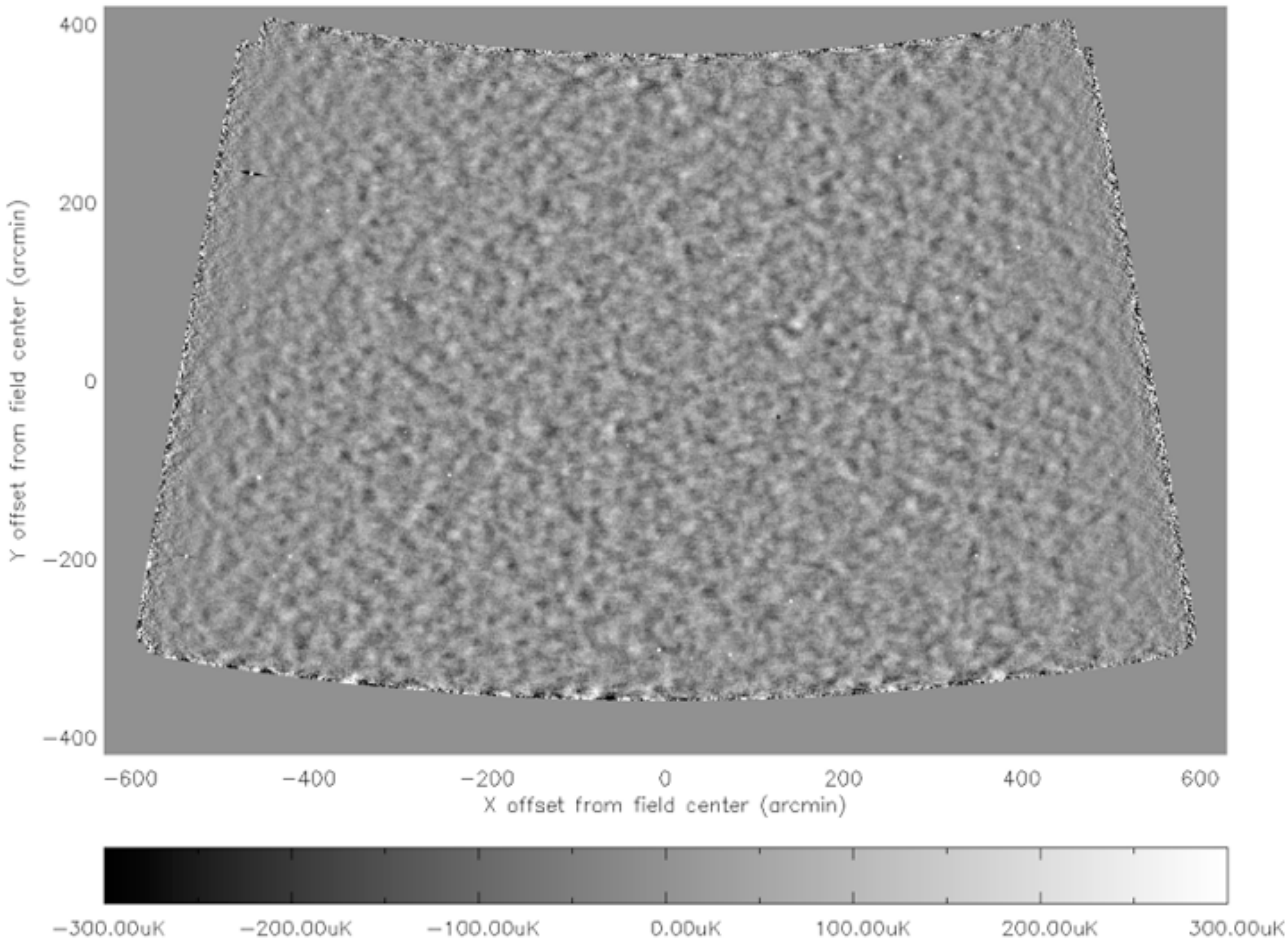
Mapmaking scheme is simple, but LOTS of effort into reconstructing pointing, optimizing calibration scheme, matching detectors within a polarization pixel, etc.

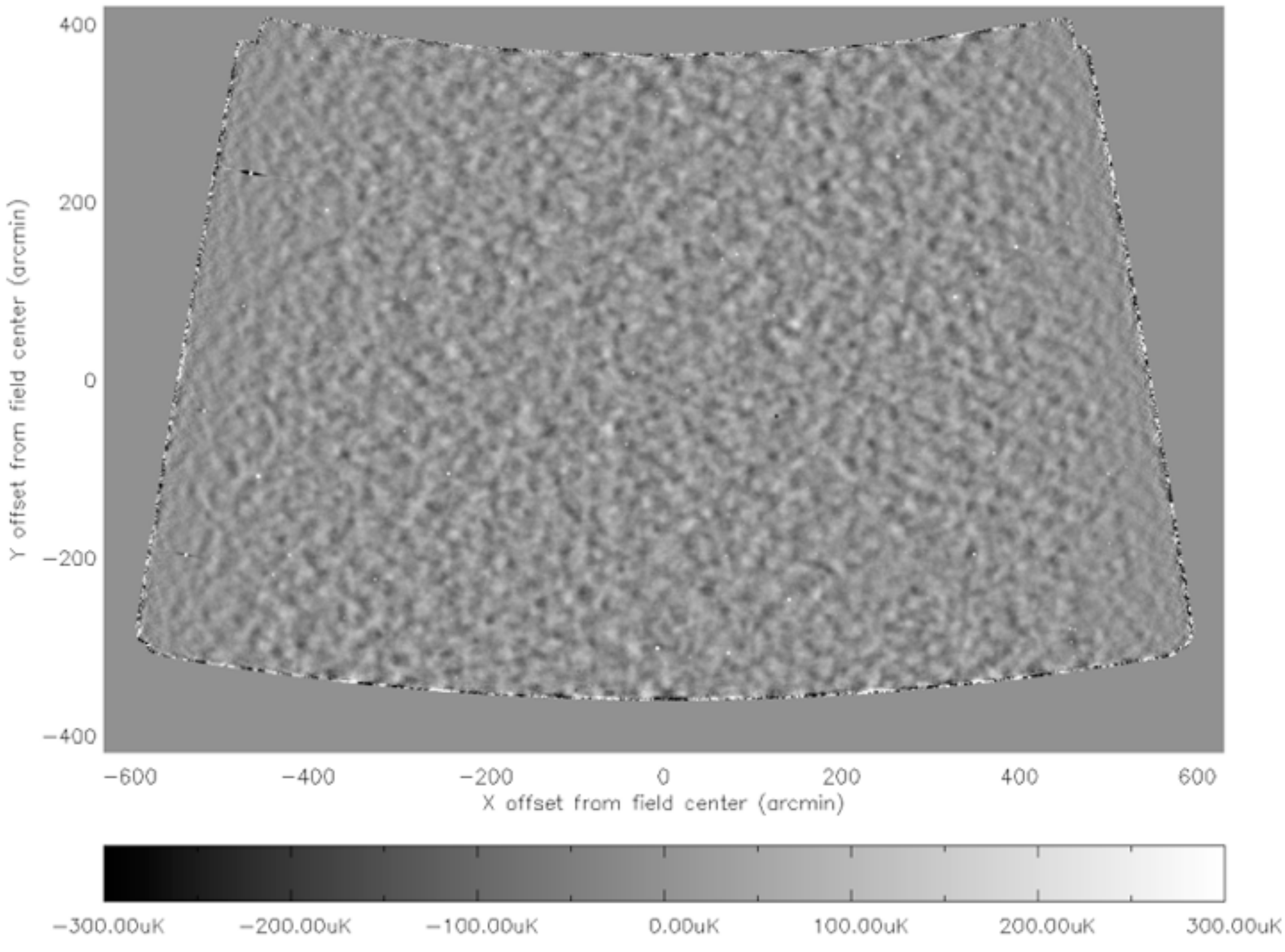
Single-observation Map



20 Observations

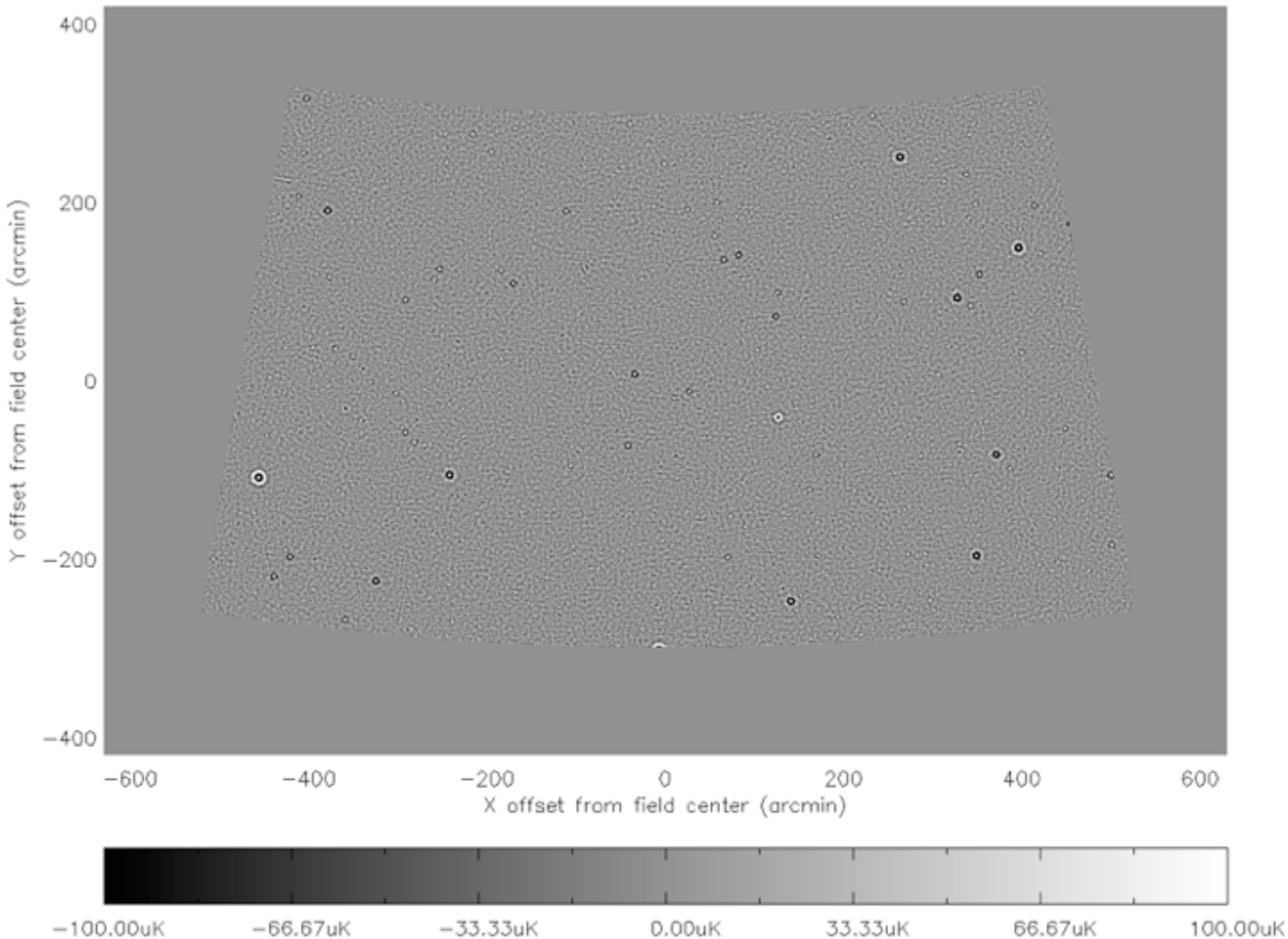


Full Coadd

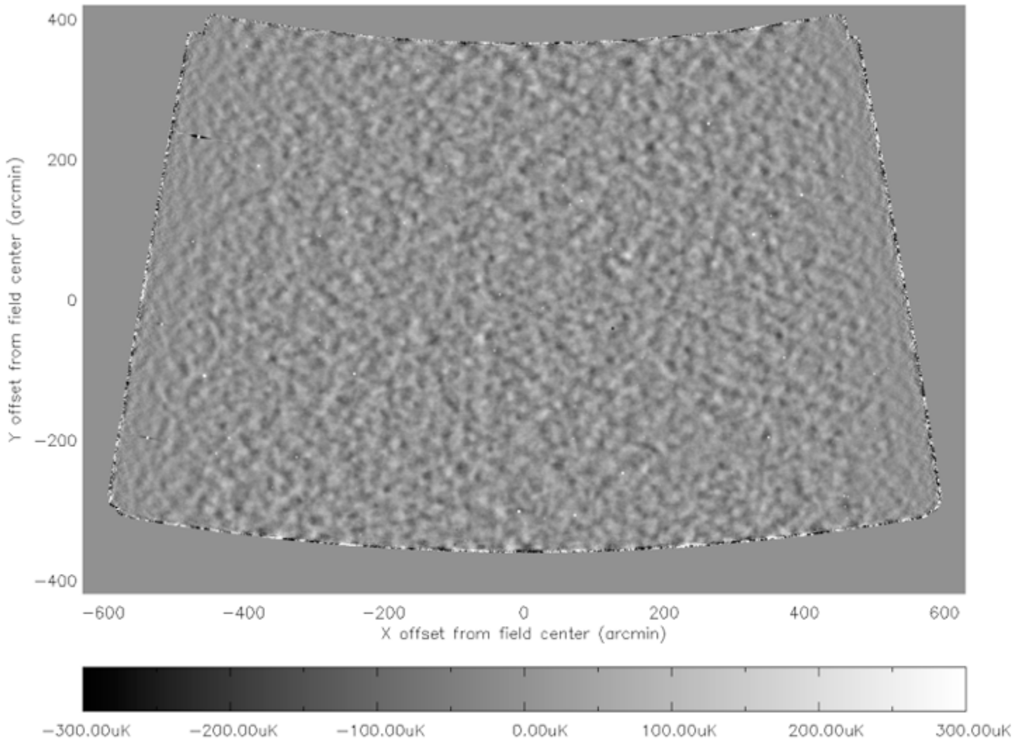


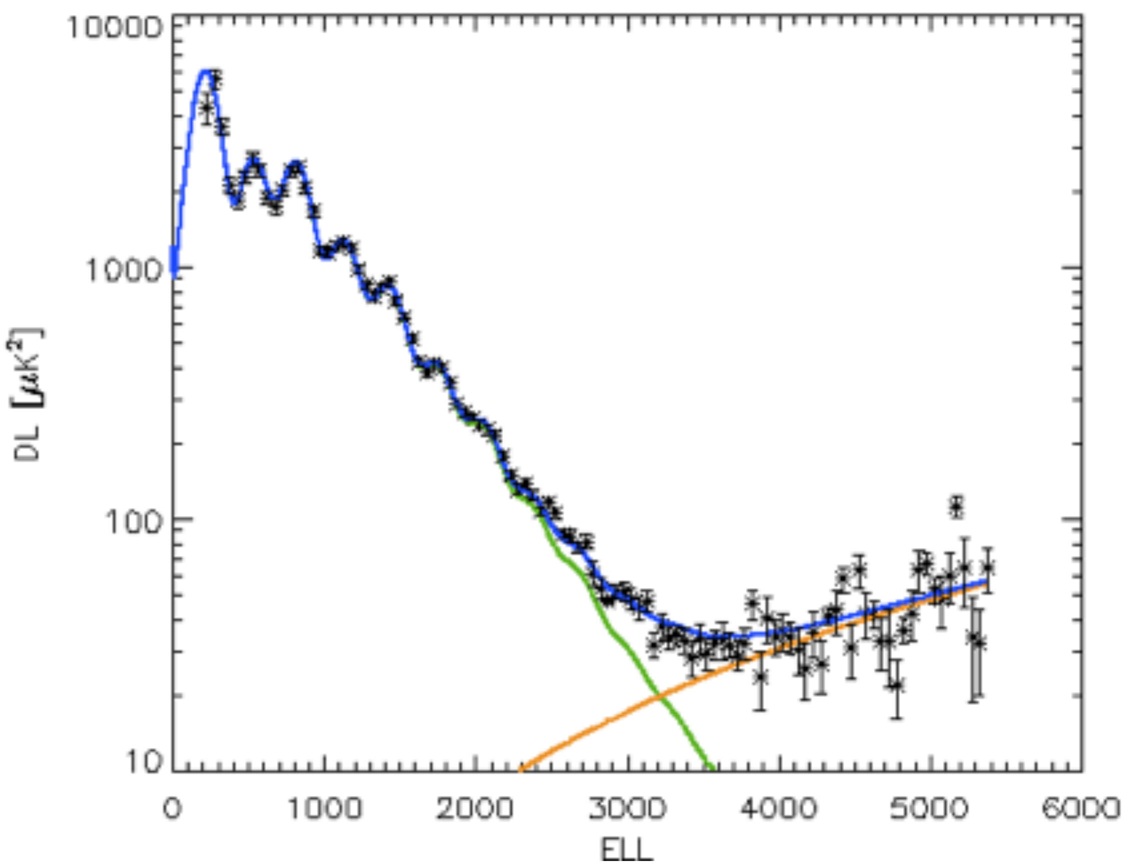
400
200
0
−200
−400
Y offset from field center (arcmin)
−600
−400
−200
0
200
400
600
X offset from field center (arcmin)
−300.00uK
−200.00uK
−100.00uK
0.00uK
100.00uK
200.00uK
300.00uK

Post-map Analyses: Object Detection



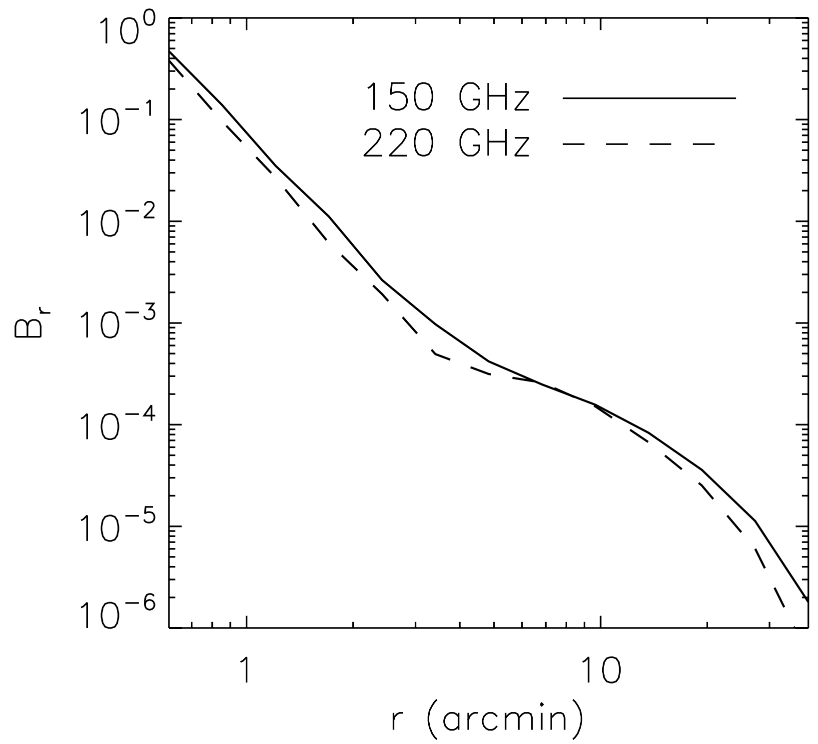
Post-map Analyses: Power Spectra





Post-map Analyses: Auxiliary Data Products

- ❖ Beams
- ❖ "Transfer functions"
- ❖ Noise estimates
- ❖ Masks



- ❖ All of these require extensive simulations