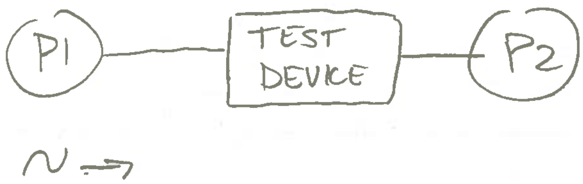
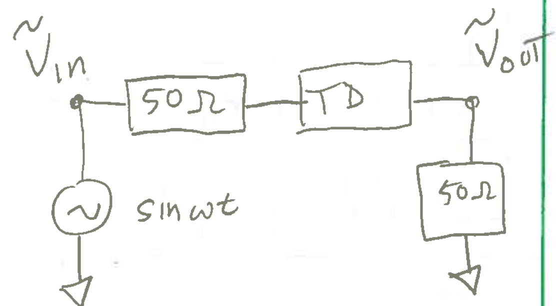
IN FREQUENCY SPACE, ANY SIGNAL CAN BE REPRESENTED AS $I \cdot \sin(\omega t) + Q \cdot \cos(\omega t)$

FOR LINEAR CIRCUITS, WE CAN CONSIDER EACH $\omega$ INDEPENDENTLY.

THE CIRCUITS WE'LL BE DISCUSSING LOOK LIKE THIS:

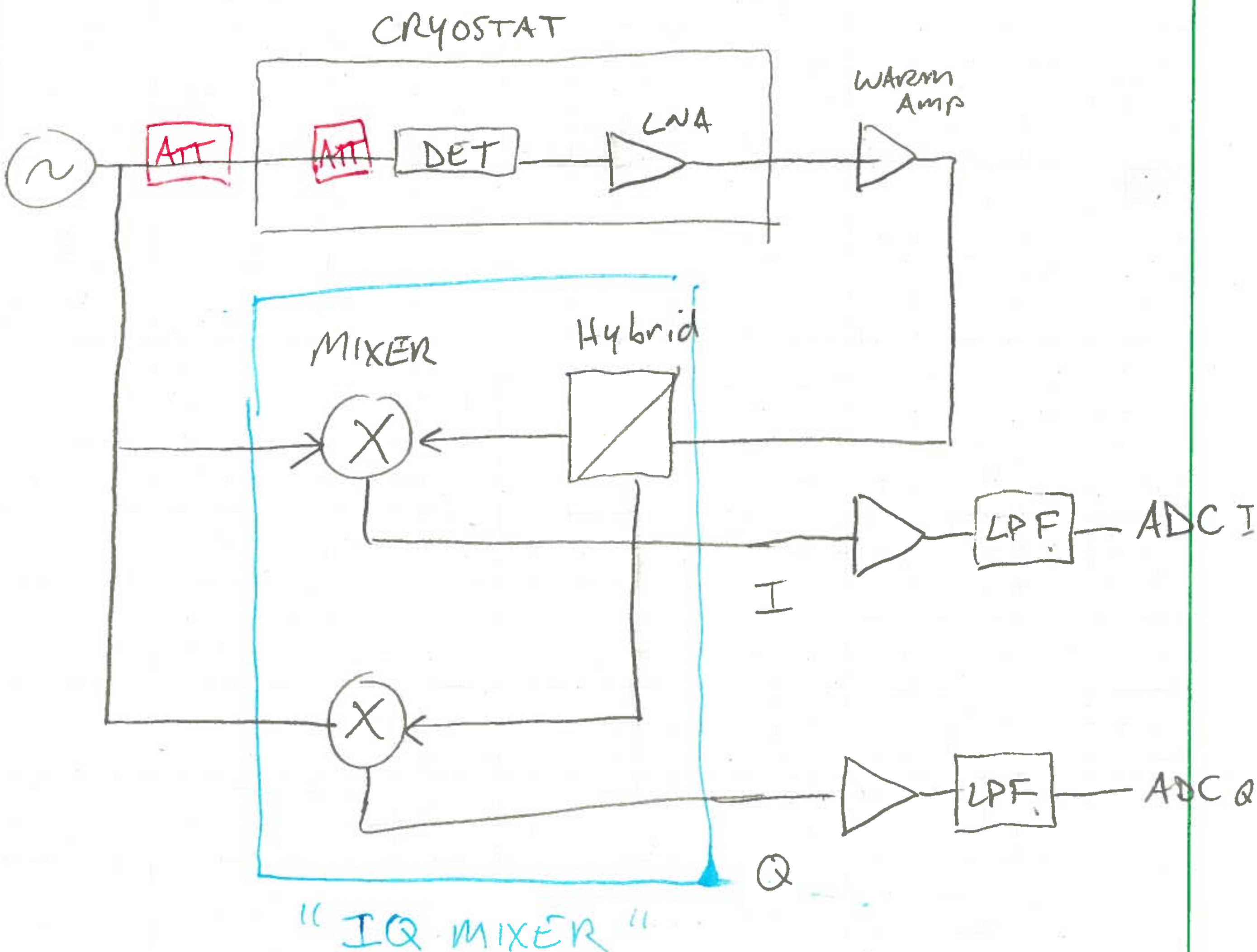



SEND IN sin

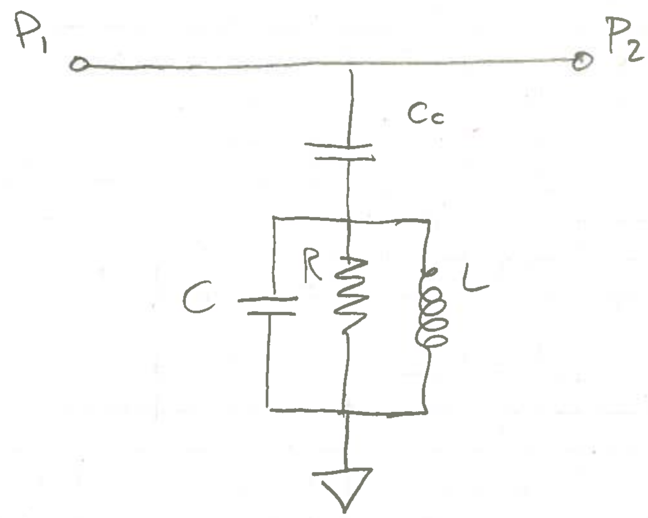



$$S21 = \underbrace{\text{Amplitude of sin comp. at P2.}}_{\text{"I"}} + i \times \underbrace{\text{Amplitude of cos. comp.}}_{\text{"Q"}}$$

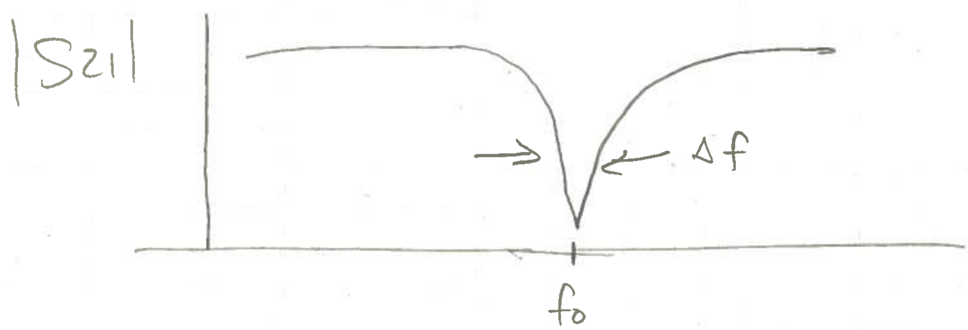
S11 = THE SAME, BUT W.R.T. POWER REFLECTED FROM THE DEVICE BACK TO P1 α1B


CRYOSTAT
ATT
ATT
DET
LNA
WARM
Amp
MIXER
Hybrid
LPF
ADC I
I
LPF
ADC Q
Q
"IQ MIXER"

RESONATOR CIRCUIT:




$$\omega_0 = 2\pi f_0 = \frac{1}{\sqrt{LC}}$$




$Q_r \equiv \dfrac{f_0}{\Delta f}$ = fraction of power lost per cycle when the resonator is momentarily excited.

THIS COMES FROM TWO MECHANISMS:

1) Coupling Q: let $R \to 0$, find power lost to the feed line.

$\to$ algebra $\to$ $$Q_c = \frac{2C}{\omega_0 Z_0 C_c^2}$$

2) "internal Q": let $C_c \to 0$. POWER
Lost to THE RESISTOR GIVES:

$$Q_i = \omega_0 R C$$

WHAT is R?

- Q.P. resistance
- SURFACE + DIELECTRIC LOSSES.
- RADIATIVE LOSSES
- E&M COUPLING TO THE BOX

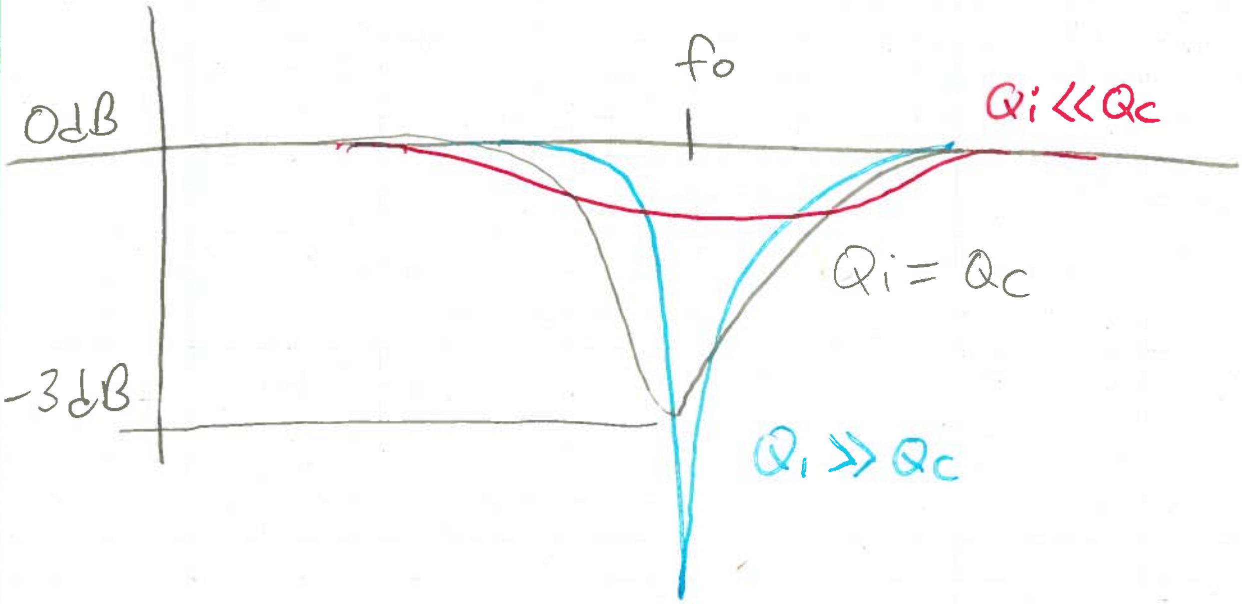
etc.

SOME PEOPLE BREAK THIS INTO $Q_i + Q_{LOSS}$
OR SIMILAR.

TOGETHER THESE DETERMINE THE TOTAL Q:

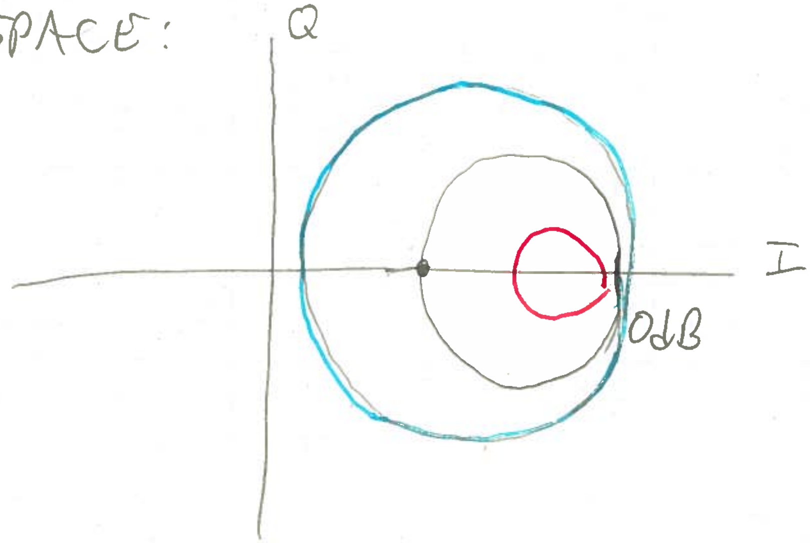
$$Q_R^{-1} = Q_c^{-1} + Q_i^{-1} + \ldots$$

WHAT DOES THIS LOOK LIKE IN PRACTICE?




ABOVE PLOT FOR A FIXED Qc

IN IQ SPACE:



FUNCTIONAL FORM IS APPROX:

$$S_{21} = 1 - \frac{Q_r}{Q_c}\left[\frac{1}{1+2jQ_i x}\right]$$

$$x \equiv \frac{\omega - \omega_0}{\omega_0} \qquad j \equiv \sqrt{-1}$$

COMPLICATIONS:
- CABLE DELAY
- NON-MATCHED IMPEDANCES
- AMPLITUDE, OFFSETS, Freq. dep gain.

MORE GENERAL:

$$Q_c \rightarrow \tilde{Q}_c \equiv Q_c^R + j\,Q_c^I$$

$$Q_c = |\tilde{Q}_c|$$

(Gao, etc use $Q_c e^{j\phi_0}$ notation)

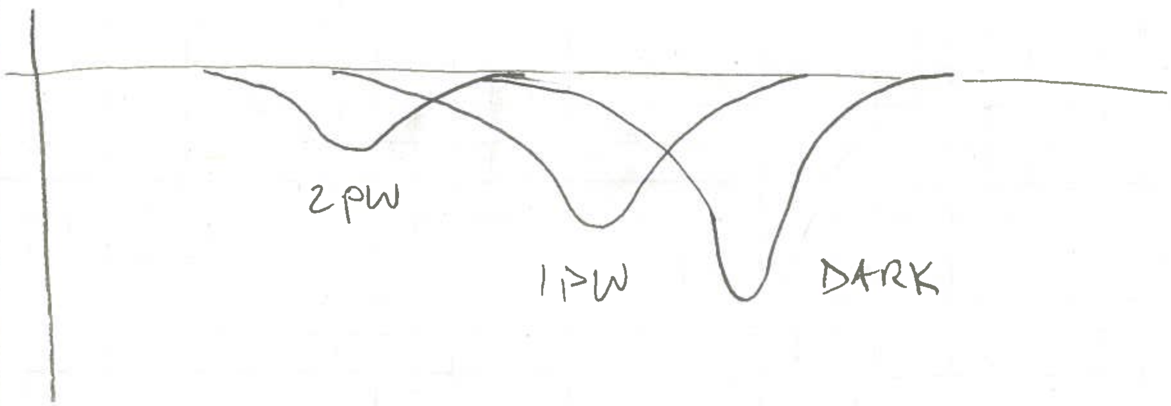
TO FIT CABLE DELAY ALSO, CAN ADD

$a e^{-j\omega\tau}$ multiplier to whole thing

$a$ ← normalization; $\tau$ ← cable delay

WHAT HAPPENS WHEN WE SHINE LIGHT ON THIS DEVICE?

$L_k$ and $R$ both change linearly in response to small amounts of optical power.



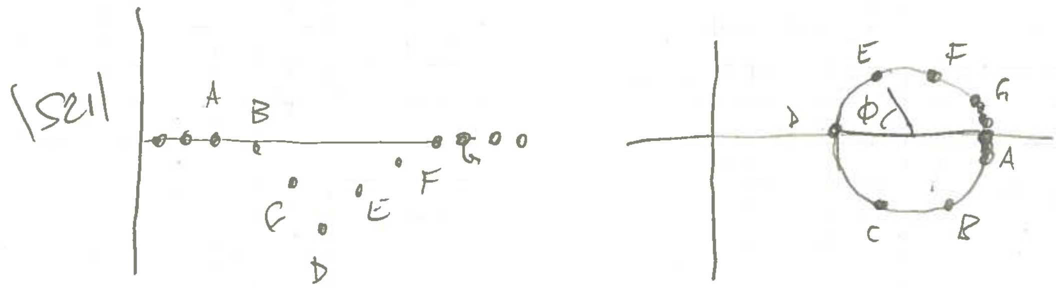

RESONATOR MOVES TO THE LEFT AND DEPTH DECREASES. ($Q_i$ goes down.)

IN MOST CASES, THE FREQUENCY SIGNAL IS MORE SENSITIVE.

WE WANT TO RECORD $\frac{\Delta f}{f_0}$ vs. time.

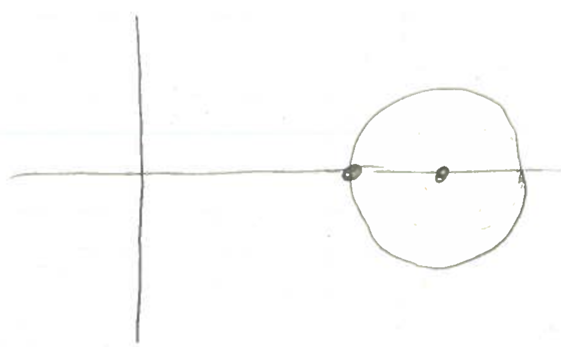
SO - HOW DO WE ACTUALLY TAKE DATA?

1- DO A SWEEP TO FIND $f_0$, $\phi(\Delta f)$



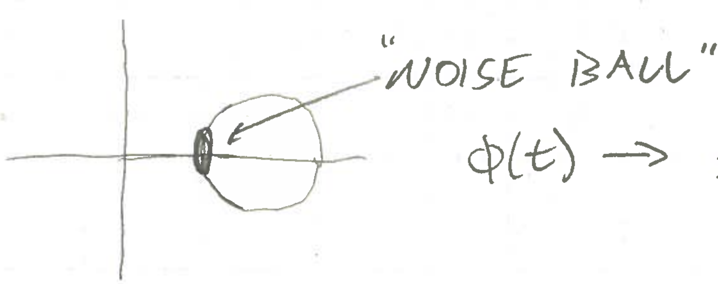

NOTE THE POINTS GET FARTHER APART IN $|\Delta I^2 + \Delta Q^2|$

2- PLACE A TONE AT $F_0$, record $I, Q\,(t)$ $\rightarrow \phi(t)$.



WE KNOW $\phi(f - f_0)$ from sweep.

now $f$ = fixed, $f_0$ changes with power on the KID.




$\phi(t) \rightarrow \frac{\Delta f}{f}(t)$

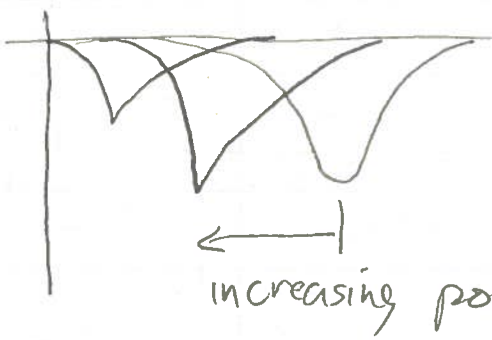
CONSIDERATIONS:

1. BIFURCATION.

KINETIC INDUCTANCE IS NON-LINEAR TO 1st ORDER.

$$L_k(I) = L_k(0)\left[1 + I^2/I_*^2\right]$$

SINCE THE CURRENT IN THE RESONATOR IS A FUNCTION OF $f - f_0$, this makes $S21(f)$ complicated and power dependent.




EVENTUALLY, $S21(f)$ has two solutions

$I^*$ SCALES WITH: $V_c$, $T_c$, $N_0$