Probing Dark Energy with Weak Lensing and Redshift Space Distortions

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Outline

Fun with Assumptions
   Constructing a metric theory of gravity w/o assuming GR
   What GR assumptions are we testing with WL+RSD?

Stacked Weak Lensing
   Measuring the potential followed by light

Redshift Space Distortions
   Measuring the potential followed by galaxies

Testing GR
   A null hypothesis test statistic
   Ideas for an estimator with minimal assumptions

Issues
   Potential pitfalls and caveats

Conclusions
Testing General Relativity

- General metric theory of gravity has two distinct potentials
- Newtonian potential $\Psi$, curvature potential $\Phi$. GR: $\Phi = \Psi$.

**Weak lensing**: sensitive to $\Phi + \Psi$  
**Peculiar velocities**: sensitive to $\Psi$

Hawkins et al. (2002), astro-ph/0212375
2dFGRS: $\beta = 0.49 \pm 0.09$
Goals

- Lay out assumptions of metric theory of gravity
- Understand where GR assumptions enter
- Define observables from stacked weak lensing (SWL) and redshift space distortions (RSD)
- Construct a test comparing SWL and RSD on same sky
- Null hypothesis test statistic $E_G$ for GR assumptions
- Construct an estimator for $E_G$ with minimal assumptions
- Figure out how well DESSpec could measure it
Perturbation Theory Equations

Perturbed FLRW metric in Newtonian gauge:

\[ ds^2 = - [1 + 2\Psi (t, x)] dt^2 + a(t)^2 [1 - 2\Phi (t, x)] \left[ d\chi^2 + r(\chi)^2 \, d\Omega^2 \right] \]

Assumptions:

- Vector and tensor modes can be neglected
- \( \Phi \) and \( \Psi \ll 1 \)
- At zeroth order \( (\Phi = \Psi = 0) \) the universe is homogenous and isotropic.
- Haven’t assumed GR
Conservation of Stress-Energy

\[ T_{\nu;\mu}^\mu (t, x) = 0 \] for a fluid characterized by density, pressure, anisotropic stress, and peculiar velocity leads to:

\[
\begin{align*}
\dot{\delta} &= - (1 + w) \frac{\theta}{a} - 3H \left( \frac{\delta P}{\bar{\rho}} + w\delta \right) \\
\dot{\theta} &= -H (1 - 3w) \theta - \frac{\dot{w}}{1 + w} \theta - \frac{1}{a} \nabla^2 \left( \frac{\delta P/\bar{\rho}}{1 + w} - \sigma + \Psi \right)
\end{align*}
\]

\( \delta \) is density perturbation, \( \delta P \) is pressure perturbation, \( \sigma \) is anisotropic stress, \( \theta \) is divergence of peculiar velocity \( v \), \( H \equiv \dot{a}/a, w \equiv P/\bar{\rho} \).

Applies to \( \delta_X \) and \( \theta_X \) for uncoupled fluid component \( X \).

**Assumptions:**

- Energy and momentum are locally conserved.
- \( v \ll c, \delta \) and \( \delta P \ll 1 \), no vorticity \( (\nabla \times v = 0) \).
- In quasi-static \( (\frac{\theta}{a} \gg \Phi) \), sub-horizon \( (k/aH \gg 1) \) regime.
- Still haven’t assumed GR
Okay, what happens when we finally add GR?

Apply Einstein Equations $G_{\mu\nu} = 8\pi G T_{\mu\nu}$

**Zeroth order:** Friedmann Equation

$$H^2 = \frac{8\pi G}{3} \bar{\rho}_t - \frac{K}{a^2}$$

**First order:** Poisson and Stress Equations

$$\left(\nabla^2 + 3K\right) \Phi = 4\pi G a^2 \bar{\rho}_t \left(\delta_t + 3 \left(1 + w_t\right) H a \nabla^{-2} \theta_t\right)$$

$$\approx 4\pi G a^2 \bar{\rho}_t \delta_t$$

$$- \left(\nabla^2 + 3K\right) \left(\Phi - \Psi\right) = 12\pi G a^2 \left(1 + w_t\right) \bar{\rho}_t \sigma_t$$

These apply to total fluid $\delta_t$, not to individual components $\delta_X$. 
General Case

Fully generalize:

**Zeroth order:** Friedmann Equation

\[ H^2 = \frac{8\pi G_F(a)}{3} \bar{\rho}_t - \frac{K}{a^2} \]

**First order:** Poisson and Stress Equations

\[
(\nabla^2 + 3K) \Phi = 4\pi G_P(a, x) a^2 \bar{\rho}_t (\delta_t + 3 (1 + w_t) Ha \nabla^{-2} \theta_t) \\
\approx 4\pi G_P(a, x) a^2 \bar{\rho}_t \delta_t
\]

\[
- (\nabla^2 + 3K) (\Phi - \eta(a, x) \Psi) = 12\pi G_S(a, x) a^2 (1 + w_t) \bar{\rho}_t \sigma_t
\]
Simplify General Case

Assume flatness \((K = 0)\) and make some simplifications:

**Zeroth order:** Friedmann Equation

\[
\text{Take } H, \, a \text{ to match } \Lambda CDM
\]

**First order:** Poisson and Stress Equations

\[
\nabla^2 \Phi = 4\pi G_{\text{eff}}(t, \mathbf{x}) \, a^2 \bar{\rho}_m \delta_m
\]

Dark energy clustering has been absorbed into \(G_{\text{eff}}\).

\[
-\nabla^2 (\Phi - \eta_{\text{eff}}(a, \mathbf{x}) \, \Psi) = 0
\]

Anisotropic stress has been absorbed into \(\eta_{\text{eff}}\).

Combine with conservation equations to get growth equation:

\[
\ddot{\delta}_m + 2H \dot{\delta}_m - 4\pi \bar{\rho}_m \frac{G_{\text{eff}}}{\eta_{\text{eff}}} \delta_m = 0
\]

Solution is \(\delta_m(a, \mathbf{x}) = D(a) \, \delta_0(\mathbf{x})\). Growth factor \(f \equiv \frac{d \ln D}{d \ln a}\).

Keep track of \(G_{\text{eff}}\), \(\eta_{\text{eff}}\), and \(f\) for the rest of the talk!
Stacked Weak Lensing
Stacked Weak Lensing in SDSS-II

Average cluster shear profile in richness bins

Model:
NFW profile
central BCG
neighboring halos
orange:
miscentering
dashed:
nonlinearity
TOTAL

see Johnston et al, Sheldon et al papers
Stacked Weak Lensing Observable

Observe average tangential shear in annulus:

$$\gamma_T (R) = \bar{\kappa} (< R) - \kappa (R)$$

Convergence $\kappa$:

$$\kappa = \frac{1}{2} \int_{0}^{z_s} \frac{dz}{H(z)} \frac{\chi (z) (\chi (z_s) - \chi (z))}{\chi (z_s)} \nabla^2_{2D} (\Phi + \Psi)$$

GR assumptions enter when we relate $\kappa$ to 2D density $\Sigma$:

$$\kappa = \Sigma (R) / \Sigma_{\text{crit}}$$

$$\Sigma_{\text{crit}} \equiv \frac{c^2}{4\pi \tilde{G}_{\text{eff}}} \frac{D_s}{D_l D_{ls}}, \quad \text{where} \quad \tilde{G}_{\text{eff}} \equiv \frac{1}{2} \left( 1 + \frac{1}{\eta_{\text{eff}}} \right) G_{\text{eff}}$$
Stacked density profile

Mean projected density $\Sigma_{lm}$ of mass around a lens population $l$ is projected lens-mass cross-correlation function:

$$\Sigma_{lm} (R) = \overline{\rho}_m \int_{-\infty}^{\infty} dx_3 \xi_{lm} (r) \equiv \overline{\rho}_m w_{lm} (R)$$

Tangential shear observable is $\Delta \Sigma_{lm} (R)$:

$$\Delta \Sigma_{lm} (R) = \Sigma_{lm} (< R) - \Sigma_{lm} (R)$$

Expected value of observable:

$$\langle \Delta \Sigma_{lm} (R) \rangle = \frac{\tilde{G}_{\text{eff}}}{G} \overline{\rho}_m [\bar{w}_{lm} (< R) - w_{lm} (R)]$$
Redshift Space Distortions
Recent RSD measurements with BOSS

small scales

large scales

Reid et al 2012 1203.6641

Reid et al 2012 1203.6641
2D galaxy power spectrum:

\[ P_{gg}^s (k, \mu_k) = P_{gg} (k) + 2\mu_k^2 P_g \Theta (k) + \mu_k^4 P_{\Theta \Theta} (k) \]

where \( \mu_k \equiv \hat{k} \cdot \hat{n} \) is cosine of angle between \( k \) and the line of sight, \( \Theta \equiv -\frac{\theta_g}{aH} \) is rescaled velocity divergence.

Aside: Kaiser limit: if \( \Theta = f \delta_m \) and \( \delta_g = b \delta_m \),

\[ P_{gg}^s (k, \mu_k) = (b + \mu_k^2 f)^2 P_{mm} (k). \]

Translating back into position space (Hamilton 1992) gives

\[ \xi_{gg}^s (r, \mu) = \xi_0 (r) P_0 (\mu) + \xi_2 (r) P_2 (\mu) + \xi_4 (r) P_4 (\mu) \]

\( P_\ell (\mu) \) are Legendre polynomials and \( \xi_\ell (r) \) are moments of \( \xi_{gg}^s (r, \mu) \). (monopole, quadrupole, hexadecapole)
Measuring Legendre Polynomial moments

Relate moments to $g$, $\Theta$ correlation functions:

\[
\xi_0 (r) = \xi_{gg} (r) + \frac{2}{3} \xi_{g\Theta} (r) + \frac{1}{5} \xi_{\Theta\Theta} (r)
\]

\[
\xi_2 (r) = \frac{4}{3} [\xi_{g\Theta} (r) - \bar{\xi}_{g\Theta} (r)] + \frac{4}{7} [\xi_{\Theta\Theta} (r) - \bar{\xi}_{\Theta\Theta} (r)]
\]

\[
\xi_4 (r) = \frac{8}{35} \left[ \xi_{\Theta\Theta} (r) + \frac{5}{2} \bar{\xi}_{\Theta\Theta} (r) - \frac{7}{12} \bar{\bar{\xi}}_{\Theta\Theta} (r) \right]
\]

\[
\bar{\xi} (r) \equiv 3r^{-3} \int_0^r \xi (r') r'^2 dr'
\]

\[
\bar{\bar{\xi}} (r) \equiv 5r^{-5} \int_0^r \xi (r') r'^4 dr'
\]

Reid et al 2012 measure monopole and quadrupole directly.
Recast RSD into SWL-type expression

Multipoles can be combined into estimator $\hat{\xi}_{g\Theta}(r)$ for galaxy-velocity cross-correlation. Project into 2D plane:

$$\hat{w}_{g\Theta}(R) = 2 \int_{R}^{\infty} \hat{\xi}_{g\Theta}(r) (r^2 - R^2)^{-1/2} r \, dr$$

and define $\Delta w_{g\Theta}$ in analogy with $\Delta \Sigma_{lm}$:

$$\Delta w_{g\Theta} \equiv \tilde{w}_{g\Theta}(< R) - w_{g\Theta}(R)$$

Haven’t assumed GR yet! GR comes in when we relate $\Theta$ back to mass via $\Theta = f \delta_m$. $f$ comes from growth equation - can be different under modified gravity.

Plugging this in: [assuming galaxy velocity traces DM velocity]

$$\langle \Delta w_{g\Theta} \rangle = f \left[ \bar{w}_{gm}(< R) - w_{gm}(R) \right]$$
Null Hypothesis Test Statistic

Zhang et al 2007 (0704.1932) define a test of GR by comparing these 2 methods:

- Estimate $P_{\nabla^2(\Phi+\Psi)g} \equiv \langle \nabla^2 (\Phi + \Psi) \delta_g \rangle$ from lensing
  \[ \nabla^2 (\Phi + \Psi) = \left( \frac{\tilde{G}_{\text{eff}}}{G} \right) 3H_0^2 a^{-1} \Omega_{m0} \delta_m \]

- Estimate $P_{g\Theta} \equiv \langle \delta_g \Theta \rangle$ from redshift space distortions
  \[ \Theta = f \delta_m \]

- Define ratio $E_G$:
  \[ E_G \equiv \frac{P_{\nabla^2(\Phi+\Psi)g}}{3H_0^2 a^{-1} P_{g\Theta}} \sim \frac{\tilde{G}_{\text{eff}} \Omega_{m0}}{Gf} \]

- $\delta_g$ - and thus all galaxy bias ugliness - cancels out
Forecasts

Null hypothesis:

\[ E_G = \Omega_{m0} \Lambda_{\text{CDM}} / f(z) \Lambda_{\text{CDM}} \]
If not, then GR is wrong!

OR...

- DE is clustered or has anisotropic stress
- DE and DM are coupled
- There is velocity bias
- ...

Black line: \( \Lambda_{\text{CDM}} \)
Dotted: flat DGP
Dashed: \( f(r) \)
Colored: TEVES

To do: DESpec forecast

Zhang et al 2007 0704.1932
Application to SDSS-II

Reyes et al 2010 (1003.2185) applied this to SDSS-II data with an annulus method:

\[
\hat{E}_G^{Reyes} (R) \equiv \frac{1}{\beta} \frac{\gamma_{gm} (R)}{\gamma_{gg} (R)}
\]

\(\gamma_{gm} (R)\) is \(\Delta \Sigma_{gm} (R)\) from SWL with scales \(R < R_0\) excised:

\[
\gamma_{gm} (R) \equiv \Delta \Sigma_{gm} (R) - \left( \frac{R_0}{R} \right)^2 \Delta \Sigma_{gm} (R_0)
\]

\(\gamma_{gg} (R)\) is defined similarly using \(gg\) projected corrfunc:

\[
\gamma_{gg} (R) \equiv \Delta w_{gg} (R) - \left( \frac{R_0}{R} \right)^2 \Delta w_{gg} (R_0)
\]

\(\beta\) is RSD parameter \(f/b\) from LRG \(P(k)\) (Tegmark et al 2006)
Results for $E_G$ from SDSS-II

$E_G^{\Lambda CDM} = 0.408 \pm 0.029$

at $z = 0.32$

(mean survey redshift)

$\hat{E}_G^{Reyes}(R) = 0.392 \pm 0.065$

on scales of $10 - 50 \, h^{-1} \, \text{Mpc}$

Consistent with GR!

Reyes et al 2010 1003.2185
Building from Reyes et al method

How can we improve upon the Reyes et al measurement for applying this test to DESpec data?

\( \hat{E}_G^{\text{Reyes}} (R) \) bundles several assumptions into \( \beta = f/b \).

\( \beta w_{gg} (R) = w_{g\Theta} (R) \) if:

- \( w_{gg} (R) \) and \( \beta \) are measured for the same galaxies. (Reyes et al use similarly selected LRGs for both.)
- galaxy bias \( b \) is not stochastic or scale-dependent.
- galaxy bias \( b \) measured from \( P(k) \) analysis cancels perfectly with \( b \) from \( w_{gg} (R) \) (not true if \( b \) is scale-dependent).
- all of the assumptions in the \( P(k) \) analysis are valid.

These are \(< 5 - 10\% \) effects vs. Reyes et al 15\% error bars.
Minimal assumption statistic

Can we do better? Try to build an \( \hat{E}_G (R) \) without making these assumptions. (Work in progress)

Combine multipoles of \( \xi_{gg}^s (r, \mu) \) to get estimator for \( \xi_g \Theta (r) \):

\[
\hat{\xi}_g \Theta (r) = \frac{3}{4} \xi_2 (r) - \frac{15}{8} \xi_4 (r) \\
- \frac{3}{16} \int_0^\infty \left[ 12 \xi_2 (r') - 175 \left( \frac{r}{r'} \right)^2 \xi_4 (r') + 75 \xi_4 (r') \right] \frac{dr'}{r'}
\]

Project along line of sight to get \( \hat{w}_g \Theta (R) \) and integrate within radius to match SWL:

\[
\Delta w_g \Theta \equiv \tilde{\hat{w}}_g \Theta (< R) - \hat{w}_g \Theta (R)
\]

Now we can construct \( \hat{E}_G (R) \equiv \frac{\gamma_{gm}(R)}{\gamma_{g \Theta}(R)} \) without \( \beta \).
Potential pitfalls and caveats

• How feasible would this be with DESpec data?
  • Can we measure $\xi_4 (r)$ well enough to integrate over it 4 times?
  • Can we get around this using cleverly-weighted sums of pairs? (e.g. Reid et al 2012’s technique for $\xi_1 (r)$ and $\xi_2 (r)$)
  • Can SWL lensing measurements go to large enough scales to be in linear regime?

• Can we use clusters as the lens population?
  • $\langle \hat{\xi}_{g\Theta} \rangle = \frac{1}{2} (\xi_{g\Theta} + \xi_{c\Theta})$
    doesn’t cancel elegantly with $\Delta \Sigma_{cm}$.

• Degeneracies with Alcock-Paczynski or magnification bias?
• Velocity bias! Can’t get rid of it. Test independently?
Conclusions

• Combining stacked weak lensing and redshift space distortions provides powerful test of GR
• Nice results from SDSS-II already
• DESpec would be an excellent dataset to do such a test
• Promising ideas for generating observables with minimal assumptions about galaxy bias
• ... but can’t get rid of velocity bias assumption
• Still a lot of work to to!