Testing Gravity with the CMB: $E_G$ and Implications for CMB-S4

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Wednesday, Sept 21, 2016
Expansion and Growth of Structure distinguish dark energy and modified gravity

Type Ia Supernova

Credit: Union2.1 (SCP) Legacy Survey

Baryon Acoustic Oscillations

Credit: BOSS survey

Redshift-Space Distortions

Credit: BOSS survey

\[ f = \text{rate of structure growth} \]
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Probing 2 gravitational processes break degeneracy!
\[ E_G = \text{New Statistic to Probe Gravity} \]

\[ E_G = \frac{\nabla^2 (\psi - \phi)}{3H_0^2 (1 + z) f \delta} \]

Zhang et al. 2007
$E_G = \text{New Statistic to Probe Gravity}$

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- gravity potentials
- growth rate
- matter overdensity

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$$E_G = \frac{\nabla^2(\psi - \phi)}{3H_0^2(1 + z)f\delta}$$

$E_G[GR] = \frac{\Omega_{m,0}}{f(\tilde{z})}$

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gravity potentials

growth rate

matter

overdensity

Modified by
anisotropic stress
& weak gravity

$E_G[GR] = \frac{\Omega_{m,0}}{f(z)}$

Zhang et al. 2007
Modified gravity = scale-dependent $E_G$

General Relativity

Chameleoon Gravity

Pullen, Alam & Ho 2015
\[ E_G = \frac{\nabla^2 (\psi - \phi)}{3H_0^2 (1 + z) f \delta} \]
$E_G$ Estimator

$$E_G = \frac{\nabla^2 (\psi - \phi)}{3H_0^2(1 + z)f\delta} \times \frac{g}{g} \text{ per Fourier mode}$$

*Pullen, Alam & Ho 2015*
$$E_G = \frac{\nabla^2 (\psi - \phi)}{3H_0^2(1 + z)f \delta} \times \frac{g}{g} \text{ per Fourier mode}$$

$$E_G(\ell) = \Gamma \frac{C_{\ell}^{\kappa g}}{\beta C_{\ell}^{gg}}$$

Pullen, Alam & Ho 2015
Our measurement is consistent with other clustering analyses.

Figure 6. Measured the constraint on the growth rate but this analysis has a strongest constraint on the growth rate (of perturbation theory mocks). Our measurements of linear growth from the galaxy redshift survey. It has been used by assuming the General Relativity linear growth factor prediction for growth rate (CDM-GR prediction). Therefore, we have adopted a conservative fitting scale between the auto correlation function from data shows systematic error at large scales. At small scales due to non-linearity, and measurement of correlation function. The model used here does not work well at small scale either analytically or using fast simulation. Howlett et al. (2015) to measure the growth rate with fixed cosmology parameters of alternate theories of gravity and dark energy and growth from the galaxy redshift survey. It has been used by Reid et al. (2014) providing the CLPT-GSRSD code. We also thank Keisuke Ohashi for his support and guidance.

Credit: Alam et al. 2015 (BOSS)

Growth rate degenerate with bias

\[ \beta = \frac{f}{b_g} \]

clustering bias

RSD parameter

\[ f, b_g \]

\[ b\sigma_8, f\sigma_8 \]

Credit: Alam et al. 2015 (BOSS)
Growth rate degenerate with bias

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- Clustering bias relates galaxy and matter perturbations.
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- Bias and \( \sigma_8 \) must be marginalized over to get \( f \).
Growth rate degenerate with bias

\[ \beta = \frac{f}{b_g} \]

- Clustering bias relates galaxy and matter perturbations.
- Bias and \( \sigma_8 \) must be marginalized over to get \( f \).
- \( E_G \) is independent of clustering bias and \( \sigma_8 \)!

Credit: Alam et al. 2015 (BOSS)
First measured using galaxy lensing

Reyes et al. 2010

\( E_G \) measurements consistent with GR and f(R) gravity

Blake et al. 2015
CMB lensing has advantages over galaxy lensing

Image Credit: ESA

Pullen, Alam & Ho 2015
CMB lensing has advantages over galaxy lensing

- Probes the integrated matter distribution out to last-scattering surface of the CMB

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- Precise, well-defined source plane

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- High source redshift

Image Credit: ESA

Pullen, Alam & Ho 2015
CMB lensing has advantages over galaxy lensing

- Probes the integrated matter distribution out to last-scattering surface of the CMB
- Precise, well-defined source plane
- High source redshift
- No intrinsic alignments, astro systematics in CMB

Image Credit: ESA

Pullen, Alam & Ho 2015
Large Scales $E_G$ Measure

$L \ (\text{Mpc}/h)$

$E_G(z = 0.57) = 0.243 \pm 0.060$

Pullen, Alam, He & Ho 2015
Largest Scale $E_G$ Measure

- We estimate $E_G$ in 11 $l$-bins out to 150 Mpc/$h$!

$R_\perp$ (Mpc/$h$)

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Pullen, Alam, He & Ho 2015
Spectroscopic Surveys

DESI x Planck/AdvACT Lensing

Credit: Pullen, Alam & Ho 2015

Dark Energy Spectroscopic Instrument
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Spectroscopic Surveys

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- Constrains chameleon gravity

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LSST, Ross et al. 2011, Asorey et al. 2014
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- Assumes photo RSD errors of $\sim 8\%$ over $\Delta z \sim 0.1$. 

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- Assumes photo RSD errors of \(~8\%\) over \(\Delta z \sim 0.1\).

- \(E_G\) errors of 1\% (Planck) or less (Adv. ACTPol, CMB-S4)

- Discriminates current \(f(R)\) by \(15\sigma\); can probe 100x lower!

LSST, Ross et al. 2011, Asorey et al. 2014

Credit: Pullen, Alam & Ho 2015
In Short…

Photometric surveys (number density) outperform spectroscopic surveys (precise redshifts), but both could yield useful gravity constraints!
Magnification Biases $E_G$
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- Number counts are distorted due to lensing field

Dizgah & Durrer 2016
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- Number counts are distorted due to lensing field
- Affects $C_{\ell}^{\kappa g}$ and $C_{\ell}^{gg}$ - makes $E_G$ bias and scale-dependent
- Biases low and high redshift measurements
- May be mitigated using galaxy-galaxy lensing to identify magnification bias

Credit: Dizgah & Durrer 2016

Not good!
Intensity Mapping for $E_G$

Credit: Pourtsidou 2016
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- Mapping the intensity of spectral lines will provide high sampling of LSS
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- **Ideal for $E_G$ measurements!**
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- Ideal for $E_G$ measurements!
- SKA would measure $E_G$ with similar errors as LSST without photo-z’s!

Pourtsidou 2016
Intensity Mapping for $E_G$

- Mapping the intensity of spectral lines will provide high sampling of LSS
- No magnification bias - surface brightness conserved
- *Ideal for $E_G$ measurements!*
- SKA would measure $E_G$ with similar errors as LSST *without photo-z’s!*

Pourtsidou 2016
**E\textsubscript{G} Preparation Program**

- Optimize CMB/LSS survey match for $E_G$ measurement
- Measure growth rate/$E_G$ using a photo-z survey (DES/LSST) or low-res spectroscopy
- Consider joint CMB lensing & galaxy lensing
- Predict $E_G$/constraints for modern modified gravity theories, e.g. massive gravity, galileons, Horndeski, etc.
- Plan for CMB lensing x intensity mapping studies
Extra Slides
What Modifies $E_G$?
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Anisotropic Stress

$$\phi = -\gamma(k, z)\psi$$
What Modifies $E_G$?

Anisotropic Stress

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Weak Newton’s Constant

$G(k, z) \neq G_N$

Also affects growth rate

Pullen, Alam & Ho 2015
What Modifies $E_G$?

Anisotropic Stress

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Weak Newton’s Constant

$G(k, z) \neq G_N$

Also affects growth rate

$$E_G(k, z) = \frac{\Omega_{m,0}}{f_{MG}(k, z)} \left( \frac{1 + \gamma}{2} \right) \left( \frac{G}{G_N} \right)$$

Pullen, Alam & Ho 2015
We measure $E_G$ using CMB lensing

\[
\hat{E}_G(k, z) = \frac{c^2 \hat{P}_{\nabla^2(\psi-\phi)g}(k)}{3H_0^2(1+z)f \hat{P}_{\delta g}(k)}
\]
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Angular Power Spectra (Variance on the sky)

$E_G(\ell) \approx \Gamma \frac{C^{\kappa g}_\ell}{\beta C^{gg}_\ell}$

(\ell \sim 2\pi/\theta)

Pullen, Alam & Ho 2015
We measure $E_G$ using CMB lensing
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- CMB photons are gravitationally lensed by LSS

Image Credit: ESA

Pullen, Alam & Ho 2015
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We measure $E_G$ using CMB lensing

- CMB photons are gravitationally lensed by LSS
- Lensing convergence is reconstructed from CMB maps
- Probes the integrated matter distribution out to last-scattering surface of the CMB
- Our paper first formalism/forecasts of CMB lensing as $E_G$ probe

Pullen, Alam & Ho 2015
With precise measurements, this lensing-induced noise can be characterized and removed in a procedure known as "delensing." Because B-mode polarization measurements from CMB-S4 are expected to be lensing-noise dominated, delensing will be critical to maximize the information we can infer about cosmic inflation; see the discussion in Section 5.4.

We discuss systematics from astrophysical and instrumental effects that can impact the lensing signal as well as ways to mitigate them in Section 5.5. Section 5.6 describes forecasted parameter constraints when including CMB lensing measurements as well as the instrument requirements for CMB-S4 to maximize the science gain from CMB lensing.

5.2 Measuring CMB Lensing

5.2.1 Constructing a Lensing Map

A map of the CMB lensing deflection field is a direct probe of the projected matter distribution that exists in the observable Universe. This lensing map is a fundamental object for nearly all areas of CMB lensing science: it is used to measure the lensing power spectrum, measure cross correlations between CMB lensing and external data sets, and to de-lens maps of the B-mode polarization.

Figure 14. Signal and noise-per-mode curves for three experiments. "Stage 2" is meant to represent a current-generation survey like SPTpol or ACTPol and has $T = 9 \mu K$; "Stage 3" is an imminent survey like SPT-3G or AdvACT, with $T = 5 \mu K$; and "Stage 4" has a nominal noise level of $T = 1 \mu K$. These noise-per-mode curves do not depend on the area of sky surveyed. All experiments assume a 1.4' beam.
Table 2. Forecasts of the SNR and $\chi_{\text{rms}} = \sqrt{\chi^2}$ between GR and $f(R)$ or chameleon gravity for $E_G$ measurements from various current and upcoming surveys. For $f(R)$ gravity, we assume $B_0 = 5.65 \times 10^{-5}$. For chameleon gravity, the first column assumes $B_0 = 3.2 \times 10^{-4}$ with $\beta_1$ and $s$ set to the base model, and the second column assumes $\beta_1 = 1.1$ with $B_0$ and $s$ set to the base model (see the beginning of Section 4).

<table>
<thead>
<tr>
<th>Survey (Galaxy × CMB lensing)</th>
<th>$z$</th>
<th>SNR</th>
<th>$\chi_{\text{rms}}[f(R)]$</th>
<th>$\chi_{\text{rms}}[\text{Cham}, B_0]$</th>
<th>$\chi_{\text{rms}}[\text{Cham}, \beta_1]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BOSS CMASS × Planck (current)</td>
<td>0.43–0.7</td>
<td>9.3</td>
<td>0.40</td>
<td>0.53</td>
<td>0.52</td>
</tr>
<tr>
<td>BOSS LOWZ × Planck (current)</td>
<td>0.15–0.43</td>
<td>5.2</td>
<td>0.42</td>
<td>0.42</td>
<td>0.30</td>
</tr>
<tr>
<td>BOSS QSOs × Planck (current)</td>
<td>2.1–3.5</td>
<td>6.8</td>
<td>0.051</td>
<td>0.042</td>
<td>0.26</td>
</tr>
<tr>
<td>BOSS (CMASS+LOWZ+QSOs) × Planck (current)</td>
<td>–</td>
<td>13</td>
<td>0.58</td>
<td>0.68</td>
<td>0.65</td>
</tr>
<tr>
<td>DESI ELGs × Planck (full)</td>
<td>0.6–1.7</td>
<td>31</td>
<td>0.51</td>
<td>0.84</td>
<td>1.5</td>
</tr>
<tr>
<td>DESI LRGs × Planck (full)</td>
<td>0.6–1.2</td>
<td>23</td>
<td>0.55</td>
<td>0.83</td>
<td>1.1</td>
</tr>
<tr>
<td>DESI QSOs × Planck (full)</td>
<td>0.6–1.9</td>
<td>25</td>
<td>0.29</td>
<td>0.52</td>
<td>1.2</td>
</tr>
<tr>
<td>DESI (ELG+LRG+QSO) × Planck (full)</td>
<td>–</td>
<td>46</td>
<td>0.80</td>
<td>1.3</td>
<td>2.2</td>
</tr>
<tr>
<td>DESI ELGs × Advanced ACTPol</td>
<td>0.6–1.7</td>
<td>73</td>
<td>1.4</td>
<td>2.3</td>
<td>3.6</td>
</tr>
<tr>
<td>DESI LRGs × Advanced ACTPol</td>
<td>0.6–1.2</td>
<td>56</td>
<td>1.8</td>
<td>2.5</td>
<td>2.9</td>
</tr>
<tr>
<td>DESI QSOs × Advanced ACTPol</td>
<td>0.6–1.9</td>
<td>50</td>
<td>0.66</td>
<td>1.1</td>
<td>2.4</td>
</tr>
<tr>
<td>DESI (ELG+LRG+QSO) × Advanced ACTPol</td>
<td>–</td>
<td>105</td>
<td>2.4</td>
<td>3.6</td>
<td>5.2</td>
</tr>
<tr>
<td>Euclid (spectro) × Planck (full)</td>
<td>0.5–2.0</td>
<td>41</td>
<td>0.96</td>
<td>1.4</td>
<td>2.1</td>
</tr>
<tr>
<td>Euclid (spectro) × Advanced ACTPol</td>
<td>0.5–2.0</td>
<td>83</td>
<td>2.4</td>
<td>3.2</td>
<td>4.1</td>
</tr>
<tr>
<td>WFIRST × Planck (full)</td>
<td>1.05–2.9</td>
<td>20</td>
<td>0.12</td>
<td>0.21</td>
<td>0.91</td>
</tr>
<tr>
<td>WFIRST × Advanced ACTPol</td>
<td>1.05–2.9</td>
<td>44</td>
<td>0.28</td>
<td>0.55</td>
<td>2.0</td>
</tr>
<tr>
<td>DES × Planck (full)</td>
<td>0.0–2.0</td>
<td>35</td>
<td>1.2</td>
<td>1.3</td>
<td>1.7</td>
</tr>
<tr>
<td>DES × Advanced ACTPol</td>
<td>0.0–2.0</td>
<td>78</td>
<td>3.0</td>
<td>3.3</td>
<td>3.9</td>
</tr>
<tr>
<td>LSST × Planck (full)</td>
<td>0.0–2.5</td>
<td>84</td>
<td>5.1</td>
<td>5.2</td>
<td>6.0</td>
</tr>
<tr>
<td>LSST × Advanced ACTPol</td>
<td>0.0–2.5</td>
<td>189</td>
<td>15</td>
<td>15</td>
<td>16</td>
</tr>
<tr>
<td>Euclid (photo) × Planck (full)</td>
<td>0.0–3.7</td>
<td>90</td>
<td>4.9</td>
<td>5.1</td>
<td>5.9</td>
</tr>
<tr>
<td>Euclid (photo) × Advanced ACTPol</td>
<td>0.0–3.7</td>
<td>205</td>
<td>15</td>
<td>15</td>
<td>16</td>
</tr>
</tbody>
</table>

Pullen, Alam, & Ho 2015
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Table 1. Forecasts of the SNR and $\chi_{\text{rms}} = \sqrt{\chi^2}$ between GR and the MG models under consideration for the various survey combinations we consider. For the chameleon gravity model we set $(B_0, s, \beta_1) = (0.4, 4, 1.2)$, while for the modified growth model we use $\gamma_L = 0.65$ (see text for further details).

<table>
<thead>
<tr>
<th>Survey</th>
<th>$z_c$</th>
<th>$z_s$</th>
<th>SNR</th>
<th>$\chi_{\text{rms}}[\text{Cham}]$</th>
<th>$\chi_{\text{rms}}[\gamma_L]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>DES × Planck (full)</td>
<td>0.0–2.0</td>
<td>$z_{\text{cmb}}$</td>
<td>41</td>
<td>4.3</td>
<td>1.5</td>
</tr>
<tr>
<td>DES × CoRE-like</td>
<td>0.0–2.0</td>
<td>$z_{\text{cmb}}$</td>
<td>85</td>
<td>8.9</td>
<td>3.0</td>
</tr>
<tr>
<td>LSST × Planck (full)</td>
<td>0.0–2.5</td>
<td>$z_{\text{cmb}}$</td>
<td>95</td>
<td>10.1</td>
<td>3.1</td>
</tr>
<tr>
<td>LSST × CoRE-like</td>
<td>0.0–2.5</td>
<td>$z_{\text{cmb}}$</td>
<td>198</td>
<td>21.1</td>
<td>6.4</td>
</tr>
<tr>
<td>LSST × SKA_Low-like</td>
<td>0.0–2.5</td>
<td>$z_{\text{EoR}} = 7$</td>
<td>238</td>
<td>25.0</td>
<td>8.9</td>
</tr>
<tr>
<td>LSST × SKA1_Mid</td>
<td>0.0–2.5</td>
<td>3</td>
<td>47</td>
<td>4.8</td>
<td>2.1</td>
</tr>
<tr>
<td>LSST × SKA2_Mid</td>
<td>0.0–2.5</td>
<td>3</td>
<td>127</td>
<td>12.9</td>
<td>5.8</td>
</tr>
<tr>
<td>SKA1_Mid$^{(\text{sd})}$ × Planck (full)</td>
<td>0.35–3.0</td>
<td>$z_{\text{cmb}}$</td>
<td>34</td>
<td>3.5</td>
<td>1.3</td>
</tr>
<tr>
<td>SKA1_Mid × Planck (full)</td>
<td>0.35–3.0</td>
<td>$z_{\text{cmb}}$</td>
<td>92</td>
<td>10.6</td>
<td>2.0</td>
</tr>
<tr>
<td>SKA1_Mid × CoRE-like</td>
<td>0.35–3.0</td>
<td>$z_{\text{cmb}}$</td>
<td>200</td>
<td>23.1</td>
<td>4.6</td>
</tr>
<tr>
<td>SKA1_Mid × SKA_Low-like</td>
<td>0.35–3.0</td>
<td>$z_{\text{EoR}} = 7$</td>
<td>227</td>
<td>25.3</td>
<td>6.0</td>
</tr>
</tbody>
</table>
The kernels of the galaxy number count and the CMB lensing are given by

\[
W^\kappa_\ell(k, z_*) = \frac{3 \Omega_m H_0^2}{2} \int_0^{z_\star} \frac{dz}{H(z)} \frac{\chi(z) - \chi}{\chi}(z) D(z) j_\ell(k \chi(z))
\]

(2.16)

\[
W^g_\ell(k, z) = \int_0^{z_\star} dz \, w(z) \frac{dN}{dz} T^g_\ell(z, k)
\]

(2.17)

where \(w(z)\) is the window function describing the redshift bin in a given survey and \(\chi_\star = \chi(z_\star)\). \(D(z)\) is the growth function defined by

\[
\delta_m(z, k) = \frac{D(z)}{1 + z} \delta_0(k),
\]

where \(\delta_0(k)\) is the (linear) density fluctuation today and \(P(k)\) is its power spectrum. \(dN/dz\) is the redshift distribution of the galaxies considered. The transfer function \(T^g_\ell(z, k)\) is given by

\[
T^g_\ell(z, k) = \left[ j_\ell(k \chi(z)) b \frac{D(z)}{1 + z} + 2W^\kappa_\ell(k, z) \right],
\]

where the galaxy bias, \(b\) in general depends on redshift and on scale. This includes the terms of Eq. (2.13), assuming \(\Phi = \Psi\). It is a good approximation when the redshift slice is
Redshift-space GPS probes growth!

\[ P_g(k, \mu) = b_g^2 (1 + \beta \mu^2)^2 P_m(k) \]

- Line-of-sight velocities induce anisotropies in power spectrum.
- Velocities determined by growth rate \( f \), influenced by gravity.
- These redshift-space distortions (RSD) appear in correlation measurements.
$$\Gamma = \frac{W(\chi) \beta(z)(1 + z)}{2f_g(\chi)}$$
BOSS systematic errors are small

4.5% systematic error due to galaxy sample contamination

Pullen, Alam, He & Ho 2015
No evidence of point sources contamination

2.7% systematic error due to lensing-galaxy bias

Pullen, Alam, He & Ho 2015

Lensing-galaxy bias

$$\Delta C^{\kappa g}_{\ell, i} = \frac{C^{\kappa M_i} C^{g M_i}}{C^{M_i M_i}}$$
Galaxy Surveys

- BOSS [0.15-0.7] [2.1-3.5]
- WFIRST [1-2.9]
- DES [0-2]
- DESI [0.6-1.9]
- LSST [0-2.5]
- Euclid (photo) [0-3.7]
- Euclid (spectro) [0.5-2]

redshifts

Areal Density \( [\text{deg}^{-2}] \)

\( f_{\text{sky}} \)
CMB Surveys

![Graph showing beam size and detector noise for CMB surveys, with points labeled Adv. ACTPol and Planck.](image)