TALK OUTLINE

- 1. Sampling variance cancellation
- 2. Forecast ingredients
- 3. Primordial non-Gaussianity
- 4. Neutrino mass
- 5. Measuring galaxy bias to get dark matter in 3D

All results are preliminary!
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SAMPLING

VARIANCE

CANCELLATION
**SAMPLING VARIANCE CANCELLATION**

- Toy model: Gaussian fluctuations around 1
- Primordial non-Gaussianity: Rescale galaxies by \( \sim [1 + (b - 1)f_{NL}/\ell^2] \)

\[ f_{NL} = 0 \]

\[ C_\ell \]

\[ 10^0 \]

\[ \ell \]

2 4 6 8 10 12 14 16 18 20

**Galaxies**

**DM**

*Dalal et al. (2008)*
**Sampling Variance Cancellation**

- Toy model: Gaussian fluctuations around 1
- Primordial non-Gaussianity: Rescale galaxies by $\sim \left[1 + (b - 1)f_{NL}/\ell^2\right]^{\frac{1}{2}}$

\[ f_{NL} = 5 \]

In principle, can measure ratio with infinite precision from single mode

Seljak (2009)
McDonald & Seljak (2009)
Fractional change of power spectra for $f_{\text{NL}}$=1: Up to 3x increase at low $l$
Compare to sampling variance noise: “By how many sigma do $C_l$’s change if $f_{\text{NL}}=1$?”

- $3\sigma$ for $f_{\text{NL}}=1$
- $0.3\sigma$ for $f_{\text{NL}}=1$
- but can cancel sampling variance by measuring ratios of $C_l$’s!
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FORECAST

INGREDIENTS
FORECAST INGREDIENTS

- CMB lensing reconstruction $\kappa$ from CMB-S4 (assume $\sigma_{\text{FWHM}} = 1'$, $N_{TT} = 1 \mu K'$)
- Various LSS samples: SDSS, DESI, LSST, [CIB]
- Limber $C_i$'s for galaxy-galaxy, galaxy-$\kappa$ and $\kappa$-$\kappa$:

\[
C^{gg}_\ell = \int dz \, W_g^2(z) P(\ell/\chi, z) \left\{ b^2(z) [1 + \alpha \beta(k = \ell/\chi, z)]^2 + [n_{\text{com}}(z) P(\ell/\chi, z)]^{-1} \right\}
\]

\[
C^{g\kappa}_\ell = \int dz \, W_g(z) W_\kappa(z) P(\ell/\chi, z) b(z) [1 + \alpha \beta(k = \ell/\chi, z)]
\]

\[
C^{\kappa\kappa}_\ell = \int dz \, W_\kappa^2(z) P(\ell/\chi, z) + N^{(0)}_\ell.
\]

- $\alpha = $ amplitude of scale-dependent bias: $f_{NL}$ or $m_\nu$
- $\beta(k, z) = $ scale-dependent bias:

\[
\beta(k, z) = \frac{\Delta b}{b} = 3 \left( \frac{b - 1}{b} \right) \frac{\Omega_{m,0} \delta_c}{k^2 T(k) D(z)} \left( \frac{H_0}{c} \right)^2 \quad \text{or} \quad \frac{1}{m_\nu^{\text{fid}}} \frac{P_{\text{tot, nov}}(k, z) - P_{\text{tot, tot}}(k, z)}{P_{\text{tot, tot}}(k, z)}
\]
FORECAST INGREDIENTS

- Fisher analysis for amplitude of scale-dependent bias ($f_{\text{NL}}$ or $m_\nu$)
- Split LSS samples into few redshift bins (typically 2-3)
- Marginalize over one bias parameter per redshift bin
- Marginalize over worst-case scenario where changes in total matter power spectrum (e.g. due to different cosmology) are perfectly degenerate with scale-dependent bias:
  \[
P(k = \ell/\chi, z) \rightarrow P(k = \ell/\chi, z) [1 + \alpha_{\text{fake}} \beta_{\text{fake}}(k = \ell/\chi, z)]^2
  \]
- For most results, exclude LSS auto-spectra to avoid potential systematics
- Assume all surveys overlap on the sky (though probe different volume if $z$ range does not overlap)
## LSS Samples

<table>
<thead>
<tr>
<th>Sample</th>
<th>$N_{\text{objects}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BOSS LRG $z=0-1$</td>
<td>$1.31 \times 10^6$</td>
</tr>
<tr>
<td>SDSS $r &lt; 22$ $z=0-0.5$</td>
<td>$8.98 \times 10^7$</td>
</tr>
<tr>
<td>SDSS $r &lt; 22$ $z=0.5-0.8$</td>
<td>$1.82 \times 10^7$</td>
</tr>
<tr>
<td>DESI BGS $z=0-0.4$</td>
<td>$8.71 \times 10^6$</td>
</tr>
<tr>
<td>DESI BGS $z=0.4-0.5$</td>
<td>$1.95 \times 10^5$</td>
</tr>
<tr>
<td>DESI LRG $z=0.6-1.3$</td>
<td>$3.34 \times 10^6$</td>
</tr>
<tr>
<td>DESI ELG $z=0.6-0.8$</td>
<td>$3.34 \times 10^6$</td>
</tr>
<tr>
<td>DESI ELG $z=0.8-2$</td>
<td>$1.35 \times 10^7$</td>
</tr>
<tr>
<td>DESI QSO $z=0.6-2$</td>
<td>$1.28 \times 10^6$</td>
</tr>
<tr>
<td>LSST $i &lt; 25$ $z=0-1$</td>
<td>$1.4 \times 10^9$</td>
</tr>
<tr>
<td>LSST $i &lt; 25$ $z=1-2$</td>
<td>$4.75 \times 10^8$</td>
</tr>
<tr>
<td>LSST $i &lt; 25$ $z=2-3$</td>
<td>$2.95 \times 10^7$</td>
</tr>
<tr>
<td>LSST $i &lt; 25$ $z=3-4$</td>
<td>$7.31 \times 10^6$</td>
</tr>
</tbody>
</table>
REDSHIFT KERNELS

LOD kernel $W(z)$ [arbitrary units]

- $\kappa_{\text{CMB}}$
- BOSS LRG $z=0-1$
- SDSS r22 $z=0-0.5$
- SDSS r22 $z=0.5-0.8$
- DESI BGS $z=0-0.4$
- DESI BGS $z=0.4-0.5$
- DESI LRG $z=0.6-1.3$
- DESI ELG $z=0.6-0.8$
- DESI ELG $z=0.8-2$
- DESI QSO $z=0.6-2$
- LSST i25 $z=0-1$
- LSST i25 $z=1-2$
- LSST i25 $z=2-3$
CORRELATION COEFFICIENT WITH CMB LENSING SIGNAL

\[ \rho_\ell (\kappa_{\text{CMB}}, \text{tracer}) \]

- BOSS LRG \( z=0-1 \)
- DESI LRG \( z=0.6-1.3 \)
- LSST i<25 \( z=1-2 \)
- SDSS r<22 \( z=0-0.5 \)
- DESI ELG \( z=0.6-0.8 \)
- LSST i<25 \( z=2-3 \)
- SDSS r<22 \( z=0.5-0.8 \)
- DESI ELG \( z=0.8-2 \)
- LSST i<25 \( z=3-4 \)
- DESI BGS \( z=0-0.4 \)
- DESI QSO \( z=0.6-2 \)
- LSST i<25 \( z=0-1 \)

\[ \ell \]

10^0 10^1 10^2 10^3
Correlation $\rho_\ell$ of combined LSS sample and CMB lensing $\kappa$ is $\sim 93\%$

Sampling variance cancellation improves signal-to-noise of e.g. $f_{NL}$ as

$$\frac{S}{N} \propto \frac{1}{\sqrt{1 - \rho_\ell^2}}$$

- Expect $\sim 3x$ improvement from sampling variance cancellation if using only combined LSS sample and CMB lensing $\kappa$

- Gain even more if using all cross-spectra

Seljak (2009)
McDonald & Seljak (2009)
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PRIMORDIAL
NON-GAUSSIANITY
All LSS auto- and cross-spectra. Marginalize over bias parameters and fake $f_{\text{NL}}$. 

Precision needed to separate multi-/single-field inflation.

Not bad, but also not great.
Adding CMB lensing gives
- 2x improvement for SDSS/DESI
- 10x improvement with LSST
- Driven by SV cancellation
Excluding g-g auto spectra gives

- 30% degradation for SDSS/DESI
- 2.5x degradation with LSST

No LSS auto spectra, all LSS cross spectra. Marginalize over bias parameters and fake $f_{\text{NL}}$. CMB lensing: $\sigma_{\text{FWHM}} = 1'$, $N_{TT} = 1 \mu K'$. 

**EXCLUDING GALAXY-GALAXY AUTO SPECTRA**
So far, used $i < 25$ "gold" sample at $0 < z < 4$

- Extrapolate to $4 < z < 6$
- Use $i < 27$ 10 years
EXTRAPOLATING LSST TO 4<z<6

No LSS auto spectra, all LSS cross spectra. Marginalize over bias parameters and fake $f_{NL}$. CMB lensing: $\sigma_{\text{FWHM}} = 1'$, $N_{TT} = 1 \mu K$.

- Extrapolating LSST to 4<z<6: 15% better
- i<27, 10 year sample: 2.2x better
ADDING PERFECT "FAKE" CMB LENSING TRACER

(dn/dz matched to CMB lensing kernel, no shot noise, bias $b=1+z$)

Base = $\kappa_{\text{CMB+SDSS}} + \text{DESI} + \text{LSST}^{z<6}_i < 27, 10^y$

- Base + $\kappa_{\text{fake}}(0 < z < 0.5)$
- Base + $\kappa_{\text{fake}}(0.5 < z < 2)$
- Base + $\kappa_{\text{fake}}(2 < z < 6)$
- Base + $\kappa_{\text{fake}}(6 < z < 10)$

No galaxy-galaxy auto

- Base + $\kappa_{\text{fake}}(2 < z < 6)$

With galaxy-galaxy auto

No galaxy-galaxy auto

No galaxy-galaxy auto

$\sigma_{\text{FWHM}} = 1'$, $N_{TT} = 1 \mu K$

- 2x better if more galaxies at 2$<z<$6
- Only 10% better with g-g auto

No LSS auto spectra, all LSS cross spectra. Marginalize over bias parameters and fake $f_{NL}$. CMB lensing: $\sigma_{\text{FWHM}} = 1'$, $N_{TT} = 1 \mu K$. 

22
No LSS auto spectra, all LSS cross spectra. Marginalize over bias parameters and fake $f_{\text{NL}}$. CMB lensing: $\sigma_{\text{FWHM}} = 1'$, $N_{TT} = 1 \mu\text{K}'$. 

\[ 2 \leq \ell < 2000, \quad f_{\text{sky}} = 0.5, \quad \text{no } I_i \times I_i \]
No LSS auto spectra, all LSS cross spectra. Marginalize over bias parameters and fake $f_{NL}$. CMB lensing: $\sigma_{\text{FWHM}} = 1'$, $N_{TT} = 1 \, \mu\text{K}'$.

CIB: 4 Planck frequencies, assume their best-fit model and 1% residual dust.
PRIMORDIAL NON-GAUSSIANITY

- LSS probes $f_{\text{NL}} \sim 1$. Multi-field inflation would be ruled out if $f_{\text{NL}} > 1$!

- **Adding CMB lensing to LSS helps (10x improvement with LSST)**

- Can exclude galaxy-galaxy auto spectra. Then, unknown auto systematics
  - do not bias expectation values $\langle C_{\ell}^{AB} \rangle$
  - only affect error bars as $\text{var}(C_{\ell}^{AB}) = [(C_{\ell}^{AB})^2 + C_{\ell}^{AA} C_{\ell}^{BB}] / (2\ell + 1)$

- Alternatively, go to space and use galaxy-galaxy auto: SPHEREx

- Adding more galaxies at $2 < z < 6$ helps

- Very sensitive to $l_{\text{min}}$, i.e. need large area
  Also sensitive to shot noise

- CIB could also help, but worry about residual dust contamination at low $l$
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NEUTRINO MASS
**Ignore main signal** from shape of total matter power spectrum

- Only use signal from scale-dependent bias
  \[
  \frac{\Delta b}{b} = \frac{1}{m_\nu^{\text{fid}}} \frac{P_{\text{tot, no}\nu}(k, z) - P_{\text{tot, tot}}(k, z)}{P_{\text{tot, tot}}(k, z)}
  \]

- This adds to main signal from shape of total matter power spectrum (independent information)

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**Neutrino Mass**

Villaescusa-Navarro et al (2014)  
LoVerde (2016)
All LSS auto- and cross-spectra. Marginalize over bias parameters but not fake $m_\nu$. CMB lensing: $\sigma_{\text{FWHM}} = 1'$, $N_{TT} = 1 \mu K'$. Constraints can be pretty good if using all auto- and cross-spectra (need g-g auto spectra)
Promising constraints from CMB lensing x LSS

Need galaxy-galaxy auto

Sensitive to CMB-S4 specifications: 2x worse CMB beam and noise degrades $\sigma(m_\nu)$ from scale-dependent bias by $\sim\text{30\%}$

Depends on $l_{\text{max}}$ (e.g. 1.5-2x worse for $l_{\text{max}}=1000$ instead of 2000)

Degeneracies with changes of total matter power spectrum can degrade constraints somewhat (1.5x-2x)

No $P_{\text{mm}}(k)$ shape info
5
MEASURING BIAS /
3D DARK MATTER
With signal-dominated CMB lensing maps, can constrain bias

Error should go like $1/N_{\text{modes}}$, so sub-% level achievable

Error even smaller with sampling variance cancellation (by $1/(1-r^2)$)

Once bias parameters all measured:

- Divide each sample by its bias to get dark matter density at each redshift
- Get 3D dark matter modes of the universe, including their amplitude!
CONCLUSIONS

- CMB lensing x LSS is useful for primordial non-Gaussianity, neutrino mass, and measuring 3D dark matter modes

- Relies on sampling variance cancellation to measure scale-dependent bias

- Especially CMB lensing x LSST very promising for $f_{\text{NL}}$

- $f_{\text{NL}}$ forecasts promising even if galaxy-galaxy auto-spectra are excluded (avoiding unknown systematics)

- Neutrino mass constraints from scale-dependent bias need galaxy-galaxy auto-spectra

- All preliminary! Comments welcome :)

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BONUS SLIDES
FORECAST INGREDIENTS: FISHER ANALYSIS

- Fisher analysis at the field level

\[ F_{ij} = \sum_{\ell} \frac{2\ell + 1}{2} \sum_{abcd=0}^{1} \frac{\partial C_{\ell}^{ab}}{\partial \theta_i} (C^{-1})_{\ell}^{bc} \frac{\partial C_{\ell}^{cd}}{\partial \theta_j} (C^{-1})_{\ell}^{da} \]

- Fisher analysis at the power spectrum level

\[ F_{ij} = \sum_{\ell} \frac{\partial d_\ell}{\partial \theta_i} [\text{cov}(d_\ell, d_\ell)]^{-1} \frac{\partial d_\ell}{\partial \theta_j} . \]

where \( d = (C_{\ell_{\text{min}}}^{00}, C_{\ell_{\text{min}}}^{01}, \ldots, C_{\ell_{\text{min}}}^{0,N-1}, C_{\ell_{\text{min}}}^{11}, C_{\ell_{\text{min}}}^{12}, \ldots, C_{\ell_{\text{min}}}^{1,N-1}, \ldots, C_{\ell_{\text{max}}}^{N-1,N-1}) \),

\[ \text{cov}(\hat{C}_{\ell_1}^{UV}, \hat{C}_{\ell_2}^{U'V'}) = \delta_{\ell_1\ell_2} \frac{1}{2\ell_1 + 1} \left( \hat{C}_{\ell_1}^{UU'} \hat{C}_{\ell_1}^{VV'} + \hat{C}_{\ell_1}^{UV'} \hat{C}_{\ell_1}^{VU'} \right) \]

- Agree if all power spectra and fields are included
Fractional change of power spectra for $m_\nu=1$eV

\[ (P_{\nu\nu} - P_{\text{tot}})/P_{\text{tot}} \]

$k \, [h \, \text{Mpc}^{-1}]$