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Gravitational potentials and dynamics of galaxies against the cosmological background

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Outline

– the Universe deep inside of the cell of uniformity

– inhomogeneously distributed discrete structures (galaxies and their groups / clusters)

– hyperbolic, flat and spherical spaces

$$K = -1$$
$$K = 0$$
$$K = +1$$

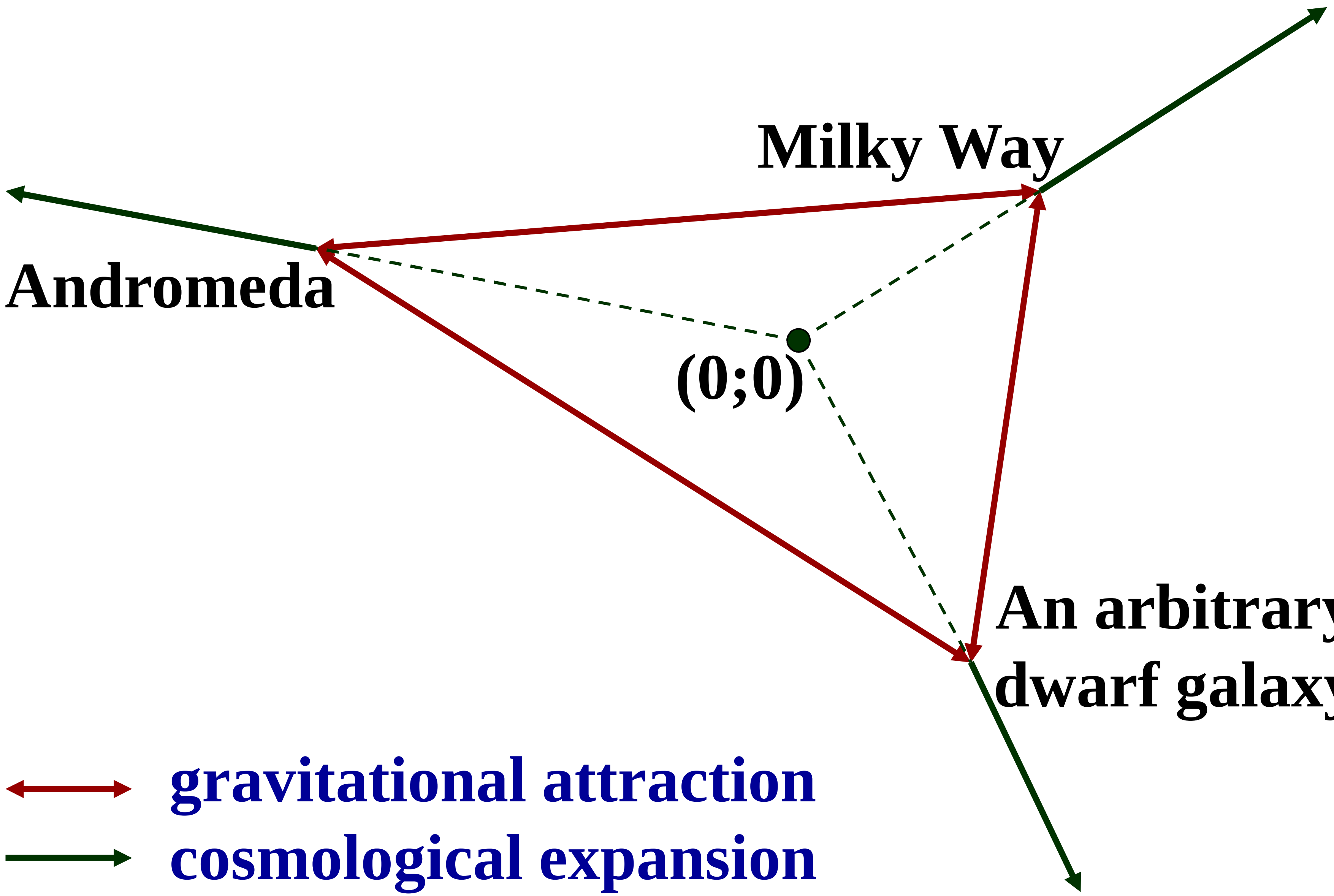
– gravitational potentials

– motion of test bodies (for example, dwarf galaxies)

– gravitational attraction vs. cosmological expansion

– the Hubble flow at a few Mpc for our Local Group

– Milky Way and Andromeda destiny


Milky Way
Andromeda
(0;0)
An arbitrary
dwarf galaxy
gravitational attraction
cosmological expansion

Introduction

[*Labini, CQG, 2011*]

less than 150 Mpc	more than 150 Mpc
?	the "ΛCDM" model
inhomogeneous and anisotropic Universe	homogeneous and isotropic Universe
?	hydrodynamical / continuous approach
Hubble's law	Hubble's law

Typical values of peculiar velocities
~ 200-400 km/sec

The present value of the Hubble parameter
~ 70 km/sec/Mpc

Distances at which the Hubble flow velocity is of the same order as peculiar velocities
~ 3-6 Mpc

Hence, the Hubble flow can be observed at distances greater than this rough estimate.

Data for our Local Group

It looks reasonable that Edwin Hubble discovered his law in 1929 after observing galaxies on scales less than ~ 20 Mpc.

However, recent observations indicate the presence of the Hubble flows at distances even less than 3 Mpc!

[*Sandage, Karachentsev, 1999-2012*]

Theoretical substantiation?

At which distances from gravitating masses does cosmological expansion (the Hubble flow) overcome gravitational attraction?

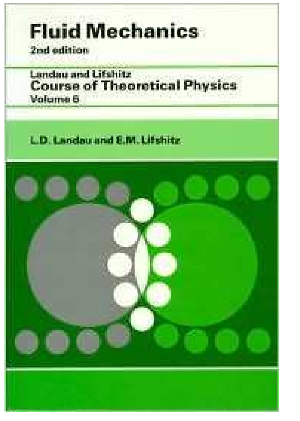
~ 1.4 Mpc

What is the size of the region of local gravity where attraction prevails over expansion?

At which points does a free falling cosmic body have the zero acceleration?

Hydrodynamical approach = myopia

L.D. Landau & E.M. Lifshitz "Fluid Mechanics"



CHAPTER I

IDEAL FLUIDS

§1. The equation of continuity

Fluid dynamics concerns itself with the study of the motion of fluids (liquids and gases). Since the phenomena considered in fluid dynamics are macroscopic, a fluid is regarded as a continuous medium. This means that any small volume element in the fluid is always supposed so large that it still contains a very great number of molecules. Accordingly, when we speak of infinitely small elements of volume, we shall always mean those which are "physically" infinitely small, i.e. very small compared with the volume of the body under consideration, but large compared with the distances between the molecules. The expressions *fluid particle* and *point in a fluid* are to be understood in a similar sense. If, for example, we speak of the displacement of some fluid particle, we mean not the displacement of an individual molecule, but that of a volume element containing many molecules, though still regarded as a point.

COSMOLOGICAL IDEAL FLUIDS

Any small volume element in the cosmological fluid is always supposed so large that it still contains a very great number of ~~molecules~~ galaxies.

"physically" infinitely small = very small compared with the ~~volume of the body~~ Universe, but large compared with the distances between the ~~molecules~~ galaxies

larger than
~150 Mpc

"a cosmological fluid particle" ,
"a point in a cosmological fluid"

Final and irrevocable conclusion

Hydrodynamical / continuous approach is inapplicable at distances less than the cell of uniformity size ~ 150 Mpc.

Hydrodynamics $\longrightarrow$ **Mechanics**

Energy-momentum tensor components:

$$T^{ik} = \sum_j \frac{m_j c^2}{(-g)^{1/2}[t]} \frac{dx^i}{ds} \frac{dx^k}{ds} \frac{ds}{cdt} \delta\left(\mathbf{r} - \mathbf{r}_j\right)$$

Rest mass density:

$$\rho = \frac{1}{\gamma^{1/2}} \sum_j m_j \delta\left(\mathbf{r} - \mathbf{r}_j\right)$$

The energy-momentum tensor choice

– The energy-momentum tensor of a system of point-like particles is acceptable if the distances between the bodies, modeled by them, are much greater than their sizes. This condition is satisfied very well in cosmology: cosmic bodies = galaxies.

– Exactly the same energy-momentum tensor choice in astrophysics underlies the prevalent weak gravitational field approximation and gives sure-fire results for the planets in the Solar system including prediction of relativistic effects.

Mechanical / discrete approach

Background: the "ΛCDM" model
(the homogeneous and isotropic Universe)

$$ds^2 = a^2\left(d\eta^2 - \gamma_{\alpha\beta}dx^\alpha dx^\beta\right) = a^2\left(d\eta^2 - \frac{\delta_{\alpha\beta}dx^\alpha dx^\beta}{\left[1+\frac{1}{4}K\left(x^2+y^2+z^2\right)\right]^2}\right)$$

hyperbolic	$K=-1$	**open**
flat	$K=0$	**flat**
spherical	$K=+1$	**closed**
spaces		**Universes**

The Friedmann equations

$$\frac{3\left(\mathrm{H}^2 + K\right)}{a^2} = \kappa \overline{T}_0^{\,0} + \Lambda \qquad \mathrm{H} = \frac{a'}{a} = \frac{1}{a}\frac{da}{d\eta}$$

$$\frac{2\mathrm{H}' + \mathrm{H}^2 + K}{a^2} = \Lambda \qquad \kappa = \frac{8\pi G_N}{c^4}$$

$\overline{T}^{ik}$ **are the average energy-momentum tensor components of dust (pressureless matter).**

The average energy density $\overline{T}_0^{\,0} = \overline{\rho} c^2 / a^3$ **is the only non-zero mixed component.**

quantity	symbol	dimension
scale factor	a	L (length)
rest mass density in comoving frame	ρ	M (mass)
average rest mass density in comoving frame	$\bar{\rho}$	M (mass)

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa\bar{\rho}c^4}{3a^3} + \frac{\Lambda c^2}{3} - \frac{Kc^2}{a^2} = H_0^2\left(\Omega_M \frac{a_0^3}{a^3} + \Omega_\Lambda + \Omega_K \frac{a_0^2}{a^2}\right)$$

$$\frac{\ddot{a}}{a} = -\frac{\kappa\bar{\rho}c^4}{6a^3} + \frac{\Lambda c^2}{3} = H_0^2\left(-\frac{1}{2}\Omega_M \frac{a_0^3}{a^3} + \Omega_\Lambda\right)$$

$$\Omega_M = \frac{\kappa\bar{\rho}c^4}{3H_0^2 a_0^3}, \quad \Omega_\Lambda = \frac{\Lambda c^2}{3H_0^2}, \quad \Omega_K = -\frac{Kc^2}{a_0^2 H_0^2}$$

$$\Omega_M + \Omega_\Lambda + \Omega_K = 1$$

Contribution of radiation is neglected.

$$H = \frac{\dot{a}}{a} = \frac{1}{a}\frac{da}{dt}$$

a_0 **and** H_0 **are the values of the scale factor** a **and the Hubble parameter** H **at present time** $t = 0$

The slightly perturbed "ΛCDM" model (the weakly inhomogeneous and anisotropic Universe)

The nonrelativistic limit in the comoving frame:

$$\frac{dx^\alpha}{d\eta} = a\frac{dx^\alpha}{dt}\frac{1}{c} \ll 1$$

$$|\,\delta T_\beta^0\,| \ll \delta T_0^0$$

Indeed, the typical present value ~ 300 km/sec of peculiar velocities is only 1/1000 (0.1 %) of the speed of light.

Scalar perturbations

$$ds^2 \approx a^2\left\{(1+2\Phi)d\eta^2-(1-2\Psi)\gamma_{\alpha\beta}dx^\alpha dx^\beta\right\}$$

$$\Delta\Psi-3\mathrm{H}(\Psi'+\mathrm{H}\Phi)+3K\Psi=\frac{1}{2}\kappa a^2\delta T_0^0$$

$$\frac{\partial}{\partial x^\beta}(\Psi'+\mathrm{H}\Phi)=\frac{1}{2}\kappa a^2\delta T_\beta^0=0$$

$$\left[\Psi''+\mathrm{H}(2\Psi+\Phi)'+\left(2\mathrm{H}'+\mathrm{H}^2\right)\Phi+\frac{1}{2}\Delta(\Phi-\Psi)-K\Psi\right]\delta_\beta^\alpha-$$

$$-\frac{1}{2}\gamma^{\alpha\sigma}\left(\Phi-\Psi\right)_{;\sigma;\beta}=-\frac{1}{2}\kappa a^2\delta T_\beta^\alpha$$

Laplace operator

$\Phi=\Psi$ **[*Mukhanov, Rubakov*]**

$$\frac{\partial}{\partial x^\beta}(\Phi' + \mathrm{H}\Phi) = 0 \longrightarrow \Phi(\eta, \mathbf{r}) = \frac{\varphi(\mathbf{r})}{c^2 a(\eta)}$$

$$\Delta\Phi - 3\mathrm{H}(\Phi' + \mathrm{H}\Phi) + 3K\Phi = \frac{1}{2}\kappa a^2 \delta T_0^0$$

$$\delta T_0^0 = \frac{\delta\rho c^2}{a^3} + \frac{3\bar{\rho}c^2\Phi}{a^3} + \delta\varepsilon_{\mathrm{rad}} = \frac{\delta\rho c^2}{a^3} + \frac{3\bar{\rho}\varphi}{a^4} + \delta\varepsilon_{\mathrm{rad}}$$

$$\Phi'' + 3\mathrm{H}\Phi' + \left(2\mathrm{H}' + \mathrm{H}^2\right)\Phi - K\Phi = \frac{1}{2}\kappa a^2 \delta p_{\mathrm{rad}} = \frac{1}{6}\kappa a^2 \delta\varepsilon_{\mathrm{rad}}$$

$\delta\rho = \rho - \bar{\rho}$ **(the difference between real and average rest mass densities)**

$$\boxed{\Delta\varphi + 3K\varphi = 4\pi G_N(\rho - \bar{\rho})}$$

$$\boxed{\delta\varepsilon_{\mathrm{rad}} = -\frac{3\bar{\rho}\varphi}{a^4}}$$

The nonrelativistic gravitational potential is defined by the positions of the galaxies, but not by their velocities!

[*Landau*]

I. Flat space / Universe $K = 0$

$$\Delta\varphi = 4\pi G_N(\rho - \underline{\bar{\rho}})$$

The role of the term $-\bar{\rho}$: the gravitational (Neumann-Seeliger) paradox is absent but spatial distribution of inhomogeneities is not arbitrary! It is an essential drawback!

The nonzero spatial curvature case

$$dl^2 = \gamma_{\alpha\beta} dx^\alpha dx^\beta = d\chi^2 + \Sigma^2(\chi) d\Omega_2^2$$

$K \neq 0$

$$\Sigma(\chi) = \begin{cases} \sin\chi, & \chi \in [0,\pi], \quad K = +1 \\ \chi, & \chi \in [0,+\infty), \quad K = 0 \\ \sinh\chi, & \chi \in [0,+\infty), \quad K = -1 \end{cases}$$

The Helmholtz equation:

$$\phi = \varphi + \frac{4\pi G_N \bar{\rho}}{3K}$$

$$\Delta\phi + 3K\phi = 4\pi G_N \rho$$

Spatial distribution of inhomogeneities

is absolutely arbitrary!

One gravitating mass in the origin of coordinates: $$\frac{1}{\Sigma^2(\chi)}\frac{d}{d\chi}\left(\Sigma^2(\chi)\frac{d\phi}{d\chi}\right)+3K\phi=0$$

II. The spherical space = the closed Universe $\quad K=+1$

$$\Sigma(\chi)=\sin\chi$$

$$\underbrace{\phi=2C_1\cos\chi-G_N m_0\left(\frac{1}{\sin\chi}-2\sin\chi\right)}_{\text{divergent at } \chi\to\pi}\ \xrightarrow[\chi\to 0]{}\ \underbrace{-\frac{G_N m_0}{\chi}}_{\text{the Newtonian limit at } \chi\to 0}$$

divergent at $\chi\to\pi$

the Newtonian limit at $\chi\to 0$

III. The hyperbolic space = the open Universe $K = -1$

$$\Sigma(\chi) = \sinh \chi$$

No gravitational paradox!

$$\boxed{\phi = -G_N m_0 \frac{\exp(-2\chi)}{\sinh \chi} \underset{\chi \to +\infty}{\longrightarrow} 0}$$

An arbitrary number of gravitating masses:

$$\varphi = -G_N \sum_i m_{0i} \frac{\exp(-2l_i)}{\sinh l_i} + \frac{4\pi G_N \bar{\rho}}{3}$$

l_i **denotes the geodesic distance between the i-th mass** m_{0i} **and the point of observation**

The physical distance $R = a\chi$

$$R \ll a \;\;\Rightarrow\;\; \chi \ll 1$$

Experimental data:

$$\Omega_{K=-1} = \frac{c^2}{a_0^2 H_0^2} < 8.4 \times 10^{-3}$$ ***[Komatsu, 2011]***

$$\Omega_{\mathcal{K}=-1} \approx 10^{-4} \;\;\Rightarrow\;\; a_0 \approx 4 \times 10^5 \;\; Mpc$$

$$R \ll 10^5 \;\; Mpc \;\;\Rightarrow\;\; \chi \ll 1$$

$$\phi \approx -\frac{G_N m_0}{\chi}$$

$$H_0 \approx 70 \; km/sec/Mpc \approx 2.3 \times 10^{-18} sec^{-1}$$

$$\Omega_\Lambda \approx 0.73 \qquad\qquad \Omega_M \approx 0.27$$

Motion of a nonrelativistic test body with the mass m

The Lagrange function: $$L = -\frac{m\varphi}{a} + \frac{ma^2v^2}{2}$$

The Lagrange equation: $$\frac{d}{dt}\left(a^2\mathbf{v}\right) = -\frac{1}{a}\frac{\partial\varphi}{\partial\mathbf{r}}$$

The physical distance $\mathbf{R} = a\mathbf{r}$

The physical velocity $$\mathbf{V} = \frac{d\mathbf{R}}{dt} = \frac{d\left(a\mathbf{r}\right)}{dt} = \frac{\dot{a}}{a}\mathbf{R} + a\mathbf{v}$$

the physical peculiar velocity

The case of the negligible gravitational potential

$$\mathbf{v} = \frac{\mathbf{c}_1}{a^2}, \quad \mathbf{r} = \mathbf{c}_1 \int_{t_0}^{t} \frac{1}{a^2}\, dt + \mathbf{c}_2$$

$$\mathbf{V} = \underbrace{\frac{\dot{a}}{a}\mathbf{R}}_{\text{Hubble velocity}} + \underbrace{\frac{\mathbf{c}_1}{a}}_{\text{physical peculiar velocity}}, \quad \mathbf{R} = a\mathbf{c}_1 \int_{t_0}^{t} \frac{1}{a^2}\, dt + a\mathbf{c}_2$$

The reason for the Hubble flow is the global cosmological expansion of the Universe.

The case of spherical symmetry of the problem

The Lagrange function in spherical coordinates $r, \theta = \pi/2, \psi$:

$$L = -\frac{m\varphi}{a} + \frac{ma^2}{2}\left(\dot{r}^2 + r^2\dot{\psi}^2\right)$$

Lagrange equations:

$$\frac{d}{dt}\left(ma^2r^2\dot{\psi}\right) = 0 \quad \Rightarrow \quad \dot{\psi} = \frac{M}{ma^2r^2}$$

$$\frac{d}{dt}\left(a^2\dot{r}\right) = -\frac{1}{a}\frac{\partial\varphi}{\partial r} + \frac{M^2}{m^2a^2r^3}$$

$$\ddot{R} = \frac{\ddot{a}}{a}R - \frac{G_N m_0}{R^2} + \frac{M^2}{m^2 R^3} = \underbrace{H_0^2\left(-\frac{1}{2}\Omega_M \frac{a_0^3}{a^3} + \Omega_\Lambda\right)R}\underbrace{- \frac{G_N m_0}{R^2} + \frac{M^2}{m^2 R^3}}$$

cosmological expansion, ? gravitational
responsible for the Hubble flow = attraction

<u>The zero-acceleration sphere</u>

$$\left|\frac{\ddot{a}}{a}\right| R = \frac{G_N m_0}{R^2} \quad \Rightarrow \quad R^3\Big|_{a=a_0} = \overline{R}_H^{\,3} = \frac{G_N m_0}{|\ddot{a}/a|}\Bigg|_{a=a_0} = \frac{G_N m_0}{H_0^2\,|\Omega_\Lambda - \Omega_M/2|}$$

$$\overline{R}_H \approx 1.4\,Mpc$$

This value is close to the observed one ~ 1 Mpc!

Data for our Local Group

$$m_0 \approx 1.9\times 10^{12} M_{Sun} \approx 3.8\times 10^{45} g$$

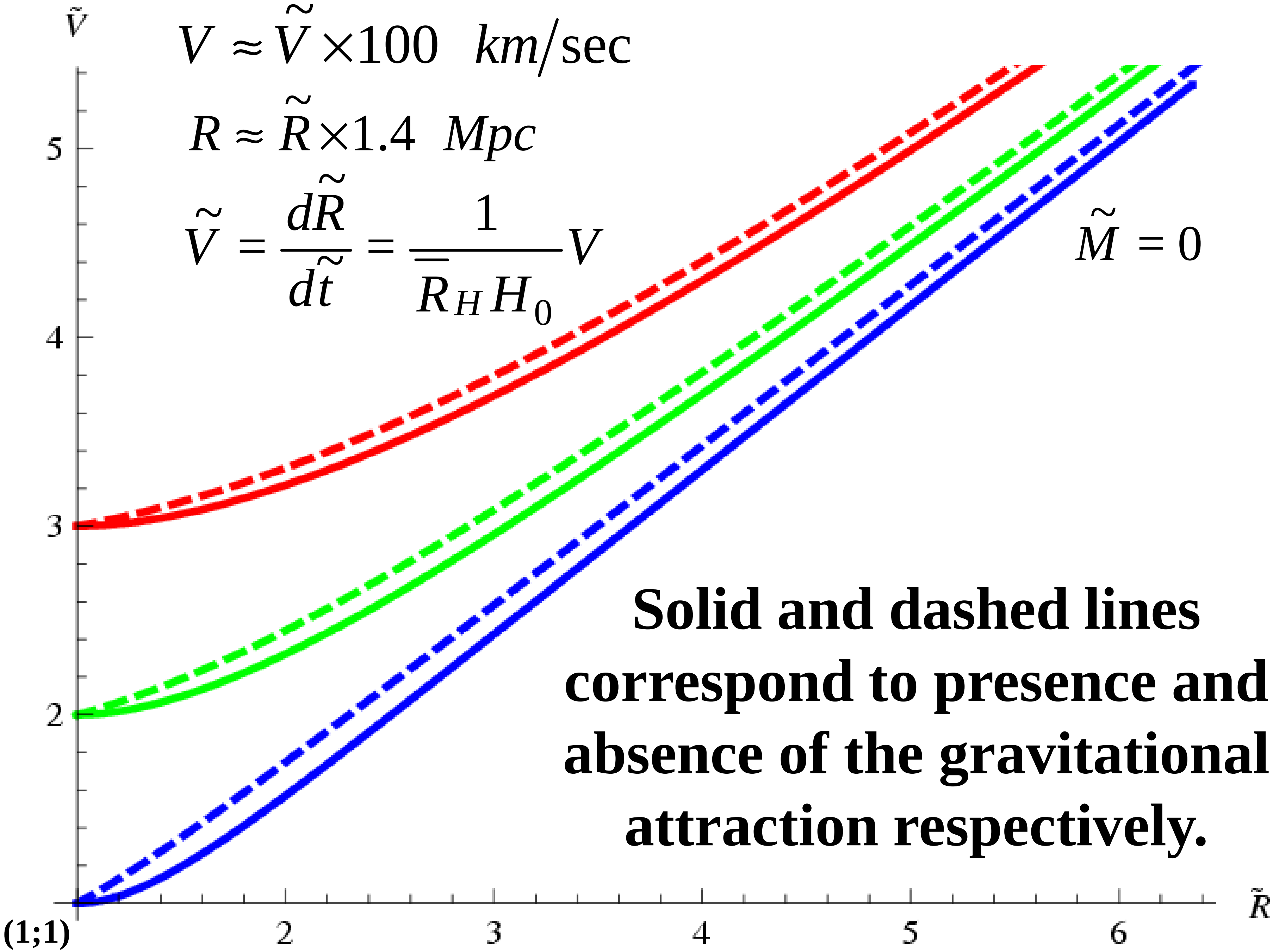
[*Karachentsev, 2012*]

Dimensionless units:

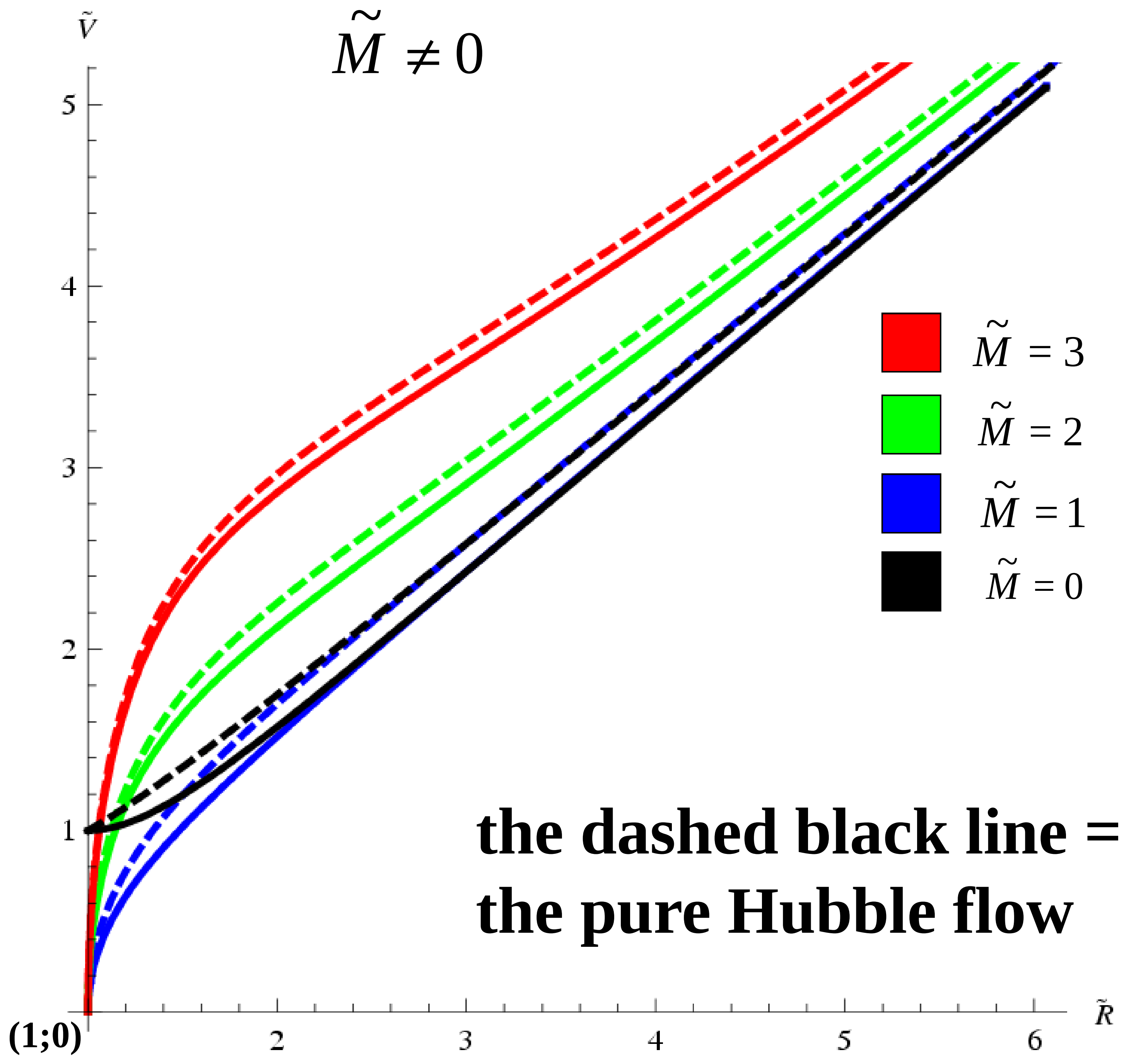
$$\tilde{a} = \frac{a}{a_0}, \quad \tilde{t} = H_0 t, \quad \tilde{R} = \frac{R}{\overline{R}_H}, \quad \tilde{M}^2 = \frac{M^2}{H_0^2 m^2 \overline{R}_H^4}$$

$$\frac{d^2\tilde{R}}{d\tilde{t}^2} = \left(-\frac{\Omega_M}{2\tilde{a}^3} + \Omega_\Lambda\right)\tilde{R} - \frac{\Omega_\Lambda - \Omega_M/2}{\tilde{R}^2} + \frac{\tilde{M}^2}{\tilde{R}^3}$$

$$\boxed{\tilde{a} = \left(\frac{\Omega_M}{\Omega_\Lambda}\right)^{1/3}\left[\left(1+\frac{\Omega_\Lambda}{\Omega_M}\right)^{1/2} \sinh\left(\frac{3}{2}\Omega_\Lambda^{1/2}\tilde{t}\right) + \left(\frac{\Omega_\Lambda}{\Omega_M}\right)^{1/2} \cosh\left(\frac{3}{2}\Omega_\Lambda^{1/2}\tilde{t}\right)\right]^{2/3}}$$



$\tilde{V}$
$V \approx \tilde{V} \times 100 \quad km/\sec$
$R \approx \tilde{R} \times 1.4 \quad Mpc$
$\tilde{V} = \frac{d\tilde{R}}{d\tilde{t}} = \frac{1}{\bar{R}_H H_0} V$
$\tilde{M} = 0$
Solid and dashed lines correspond to presence and absence of the gravitational attraction respectively.
5
4
3
2
(1;1)
2
3
4
5
6
$\tilde{R}$



$\tilde{V}$
$\tilde{M} \neq 0$
5
4
3
2
1
(1;0)
2
3
4
5
6
$\tilde{R}$
$\tilde{M} = 3$
$\tilde{M} = 2$
$\tilde{M} = 1$
$\tilde{M} = 0$
the dashed black line =
the pure Hubble flow

The average gravitational potential

$$\varphi = -G_N \sum_i m_{0i} \frac{\exp(-2l_i)}{\sinh l_i} + \underbrace{\frac{4\pi G_N \bar{\rho}}{3}}$$

this constant term does not affect motion of nonrelativistic bodies, but it contributes to the metrics, and at present time this contribution is

$$\frac{8\pi G_N \bar{\rho}}{3a_0 c^2} = \frac{\Omega_M H_0^2 a_0^2}{c^2} \sim 2\times 10^3$$

There is no any contradiction, because

$$\bar{\phi}_{\text{total}} = -\frac{4\pi G_N \bar{\rho}}{3}$$

$$\bar{\varphi} = 0$$

The average gravitational potential is zero, as it should be!

For the sake of revolution

less than 150 Mpc

the slightly perturbed "ΛCDM" model

weakly inhomogeneous and anisotropic Universe

mechanical /discrete approach

Hubble's law

Physical coordinates: $X_i = ax_i, \quad Y_i = ay_i, \quad Z_i = az_i$

The Lagrange equations for the i-th mass:

$$-G_N \sum_{j \neq i} \frac{m_j \left(X_i - X_j \right)}{\left[\left(X_i - X_j \right)^2 + \left(Y_i - Y_j \right)^2 + \left(Z_i - Z_j \right)^2 \right]^{3/2}} = \frac{1}{a} \left(\ddot{X}_i a - \ddot{a} X_i \right)$$

$$-G_N \sum_{j \neq i} \frac{m_j \left(Y_i - Y_j \right)}{\left[\left(X_i - X_j \right)^2 + \left(Y_i - Y_j \right)^2 + \left(Z_i - Z_j \right)^2 \right]^{3/2}} = \frac{1}{a} \left(\ddot{Y}_i a - \ddot{a} Y_i \right)$$

$$-G_N \sum_{j \neq i} \frac{m_j \left(Z_i - Z_j \right)}{\left[\left(X_i - X_j \right)^2 + \left(Y_i - Y_j \right)^2 + \left(Z_i - Z_j \right)^2 \right]^{3/2}} = \frac{1}{a} \left(\ddot{Z}_i a - \ddot{a} Z_i \right)$$

Motion on the plane

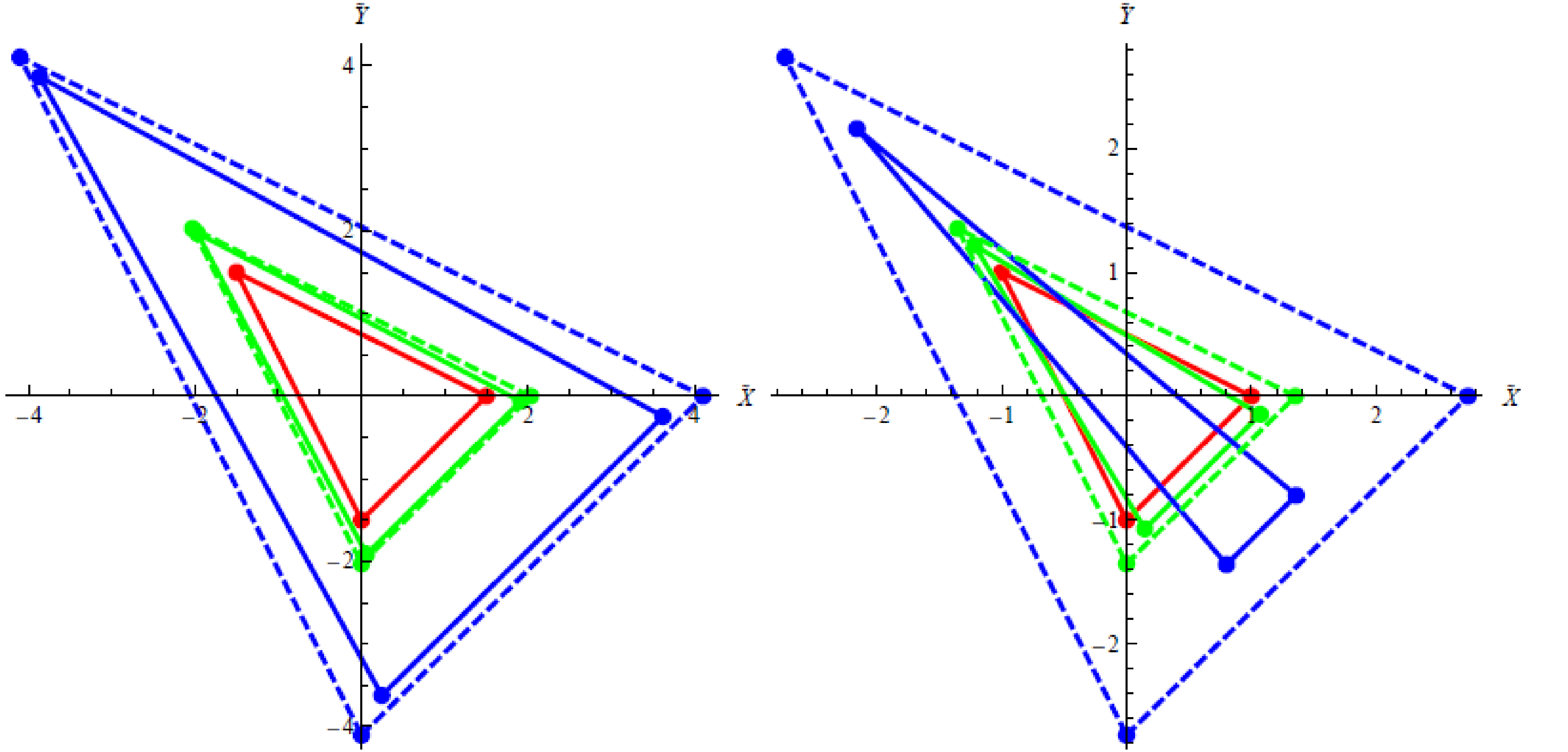
$$Z = 0 \qquad \bar{m} = \frac{1}{N}\sum_{i=1}^{N} m_i$$

$$\frac{d^2\tilde{X}_i}{d\tilde{t}^2} = -\frac{1}{\bar{m}}\sum_{j\neq i}\frac{m_j\left(\tilde{X}_i - \tilde{X}_j\right)}{\left[\left(\tilde{X}_i - \tilde{X}_j\right)^2 + \left(\tilde{Y}_i - \tilde{Y}_j\right)^2\right]^{3/2}} + \frac{1}{\tilde{a}}\frac{d^2\tilde{a}}{d\tilde{t}^2}\tilde{X}_i, \quad i, j = 1,\ldots,N$$

$$\frac{d^2\tilde{Y}_i}{d\tilde{t}^2} = -\frac{1}{\bar{m}}\sum_{j\neq i}\frac{m_j\left(\tilde{Y}_i - \tilde{Y}_j\right)}{\left[\left(\tilde{X}_i - \tilde{X}_j\right)^2 + \left(\tilde{Y}_i - \tilde{Y}_j\right)^2\right]^{3/2}} + \frac{1}{\tilde{a}}\frac{d^2\tilde{a}}{d\tilde{t}^2}\tilde{Y}_i, \quad i, j = 1,\ldots,N$$

$$\tilde{X}_i = X_i\left(\frac{H_0^2}{G_N\bar{m}}\right)^{1/3} = \frac{X_i}{0.95\ \mathrm{Mpc}}\left(\frac{10^{12}M_\odot}{\bar{m}}\right)^{1/3}$$

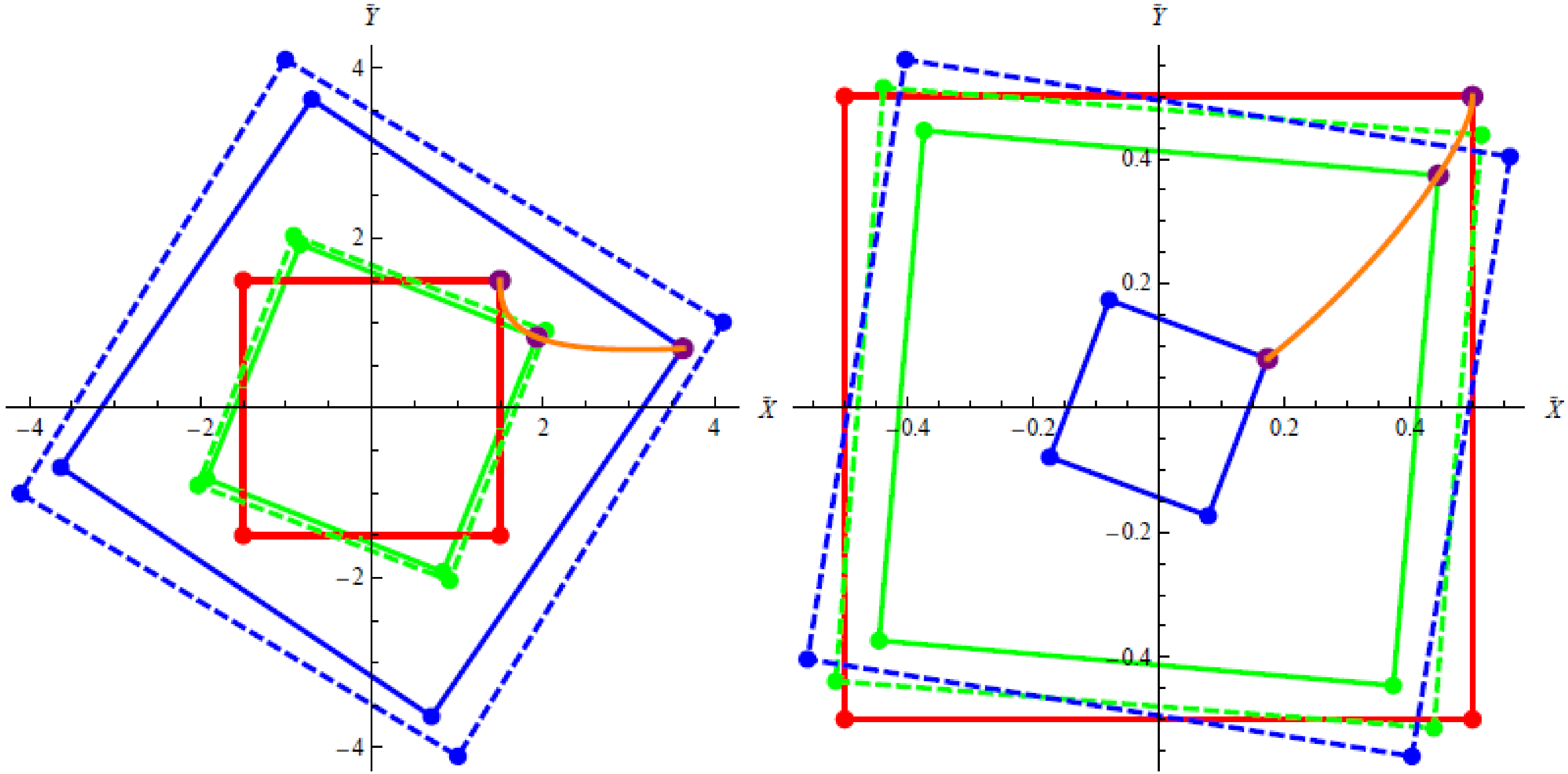
$$\tilde{Y}_i = Y_i\left(\frac{H_0^2}{G_N\bar{m}}\right)^{1/3} = \frac{Y_i}{0.95\ \mathrm{Mpc}}\left(\frac{10^{12}M_\odot}{\bar{m}}\right)^{1/3}$$



Dynamics of three gravitating masses with zero initial velocities

$\triangle$ $\tilde{t} = 0$ $\triangle$ $\tilde{t} = 1$ $\triangle$ $\tilde{t} = 2$

$\tilde{t} = 0;\ 1;\ 2$ $\qquad\qquad$ $\tilde{t} = 0;\ 0.3;\ 0.6$



Four masses, nonzero initial velocities

——— both attraction and expansion

- - - - - - expansion only

<u>The collision between Milky Way and Andromeda</u>

Separation distance ~ 0.78 Mpc

Longitudinal velocity ~ 120 km/s

Transversal velocity ~ 100 km/s

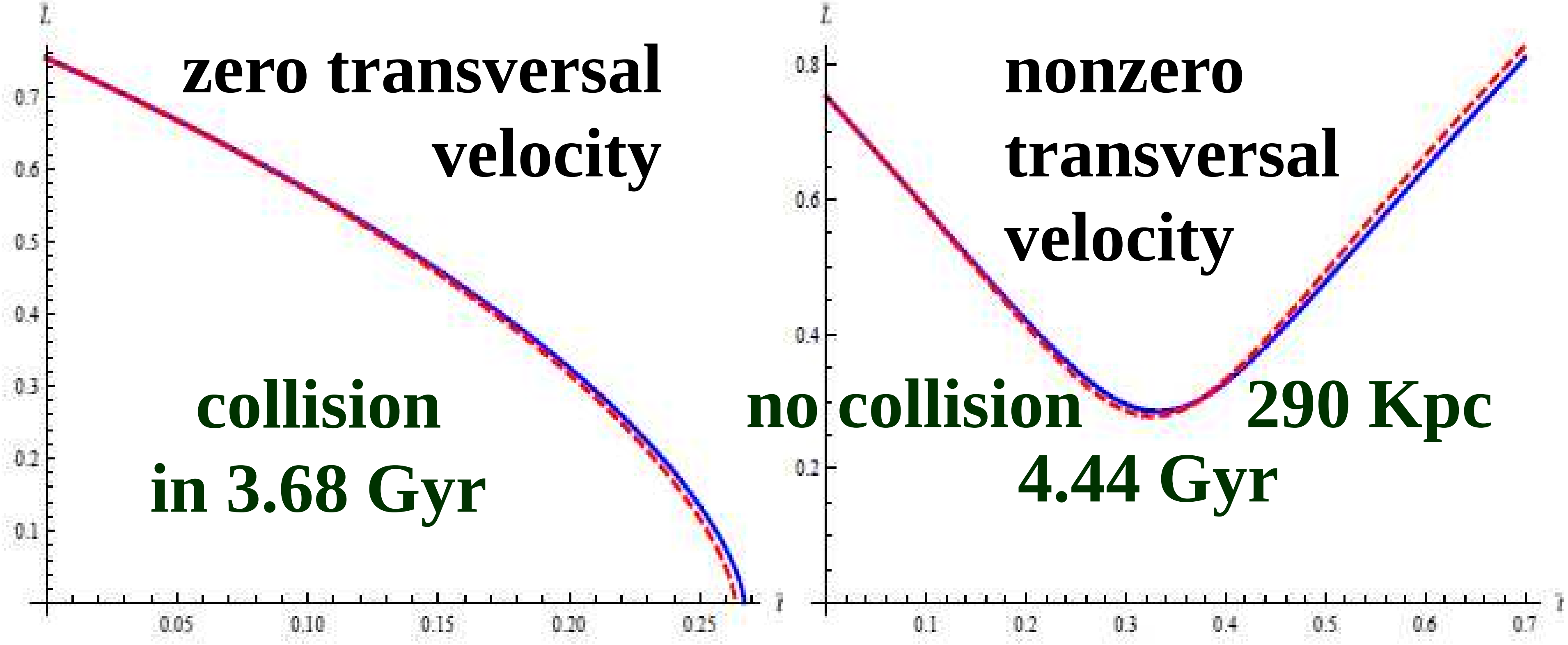
Merger distance ~ 100 Kpc

Mass of MW ~ 10^{12} $M_{\odot}$

Mass of M31 ~ 1.6×10^{12} $M_{\odot}$

[*Cox and Loeb, 2008*]

The separation distance as a function of time in the free fall approximation without dynamical friction



both attraction and expansion

attraction only

Chandrasekhar's dynamical friction

$$\frac{d\mathbf{V}_M}{dt} = -\frac{4\pi Q G_N^2 M \rho_{\mathrm{ph},m}}{V_M^3}\left[\mathrm{erf}\left(\chi\right) - \frac{2\chi}{\sqrt{\pi}}\exp\left(-\chi^2\right)\right]\mathbf{V}_M$$

$\mathbf{V}_M$ **is the physical velocity of the mass** M

$\rho_{\mathrm{ph},m}$ **is the physical rest mass density of the intragroup media**

$$\chi = \frac{V_M}{\sqrt{2}\sigma}$$

$$Q = \frac{1}{2}\ln\left(1+\lambda^2\right)$$

is the Coulomb logarithm defined by the largest impact parameter b_{max}**, the initial relative velocity** V_0 **and the masses** m **and** M

m **is the mass of the intragroup media particle**

$$\tilde{\chi}_i = \frac{\tilde{v}_{\mathrm{pec},i}}{\sqrt{2}\tilde{\sigma}} \qquad \rho_{\mathrm{ph},m} = \alpha\rho_{\mathrm{cr}} = \alpha\frac{3H_0^2}{8\pi G_N}, \quad \alpha \sim 10$$

$$\tilde{\sigma} = \left(\frac{\sigma}{H_0}\right)\left(\frac{H_0^2}{G_N \bar{m}}\right)^{1/3} \qquad \sigma = \sqrt{\frac{kT}{m}} \qquad T \sim 10^5\,\mathrm{K}$$

The physical rest mass density $\rho_{\mathrm{ph},m}$ begins to decrease at scales where cosmological expansion dominates over gravitational attraction, that is approximately at 1 Mpc. Here $\alpha \sim 5$. Therefore, $b_{\max} \sim 1\ \mathrm{Mpc}$.

$$\lambda = \frac{b_{\max}V_0^2}{G_N\left(M+m\right)} \approx \frac{b_{\max}V_0^2}{G_N M} \qquad Q \sim 1$$

$$\frac{d^2\tilde{X}_i}{d\tilde{t}^2} = -\frac{1}{\bar{m}}\frac{m_j\left(\tilde{X}_i-\tilde{X}_j\right)}{\left[\left(\tilde{X}_i-\tilde{X}_j\right)^2+\left(\tilde{Y}_i-\tilde{Y}_j\right)^2\right]^{3/2}}+\frac{1}{\tilde{a}}\frac{d^2\tilde{a}}{d\tilde{t}^2}\tilde{X}_i$$

$$-\frac{3Qm_i\alpha}{2\bar{m}\tilde{v}^3_{\mathrm{pec},i}}\left[\mathrm{erf}\left(\tilde{\chi}_i\right)-\frac{2\tilde{\chi}_i}{\sqrt{\pi}}\exp\left(-\tilde{\chi}_i^2\right)\right]\left(\frac{d\tilde{X}_i}{d\tilde{t}}-\frac{1}{\tilde{a}}\frac{d\tilde{a}}{d\tilde{t}}\tilde{X}_i\right), \quad i,j=A,B; \;\; i\neq j$$

$$\frac{d^2\tilde{Y}_i}{d\tilde{t}^2} = -\frac{1}{\bar{m}}\frac{m_j\left(\tilde{Y}_i-\tilde{Y}_j\right)}{\left[\left(\tilde{X}_i-\tilde{X}_j\right)^2+\left(\tilde{Y}_i-\tilde{Y}_j\right)^2\right]^{3/2}}+\frac{1}{\tilde{a}}\frac{d^2\tilde{a}}{d\tilde{t}^2}\tilde{Y}_i$$

$$-\frac{3Qm_i\alpha}{2\bar{m}\tilde{v}^3_{\mathrm{pec},i}}\left[\mathrm{erf}\left(\tilde{\chi}_i\right)-\frac{2\tilde{\chi}_i}{\sqrt{\pi}}\exp\left(-\tilde{\chi}_i^2\right)\right]\left(\frac{d\tilde{Y}_i}{d\tilde{t}}-\frac{1}{\tilde{a}}\frac{d\tilde{a}}{d\tilde{t}}\tilde{Y}_i\right), \quad i,j=A,B; \;\; i\neq j$$

$$\mathrm{erf}\left(x\right)=\frac{2}{\sqrt{\pi}}\int_0^x\exp\left(-\xi^2\right)d\xi$$

$$\tilde{\mathbf{v}}_{\mathrm{pec},i}=\left(\frac{d\tilde{X}_i}{d\tilde{t}}-\frac{1}{\tilde{a}}\frac{d\tilde{a}}{d\tilde{t}}\tilde{X}_i,\;\frac{d\tilde{Y}_i}{d\tilde{t}}-\frac{1}{\tilde{a}}\frac{d\tilde{a}}{d\tilde{t}}\tilde{Y}_i\right), \quad i=A,B$$

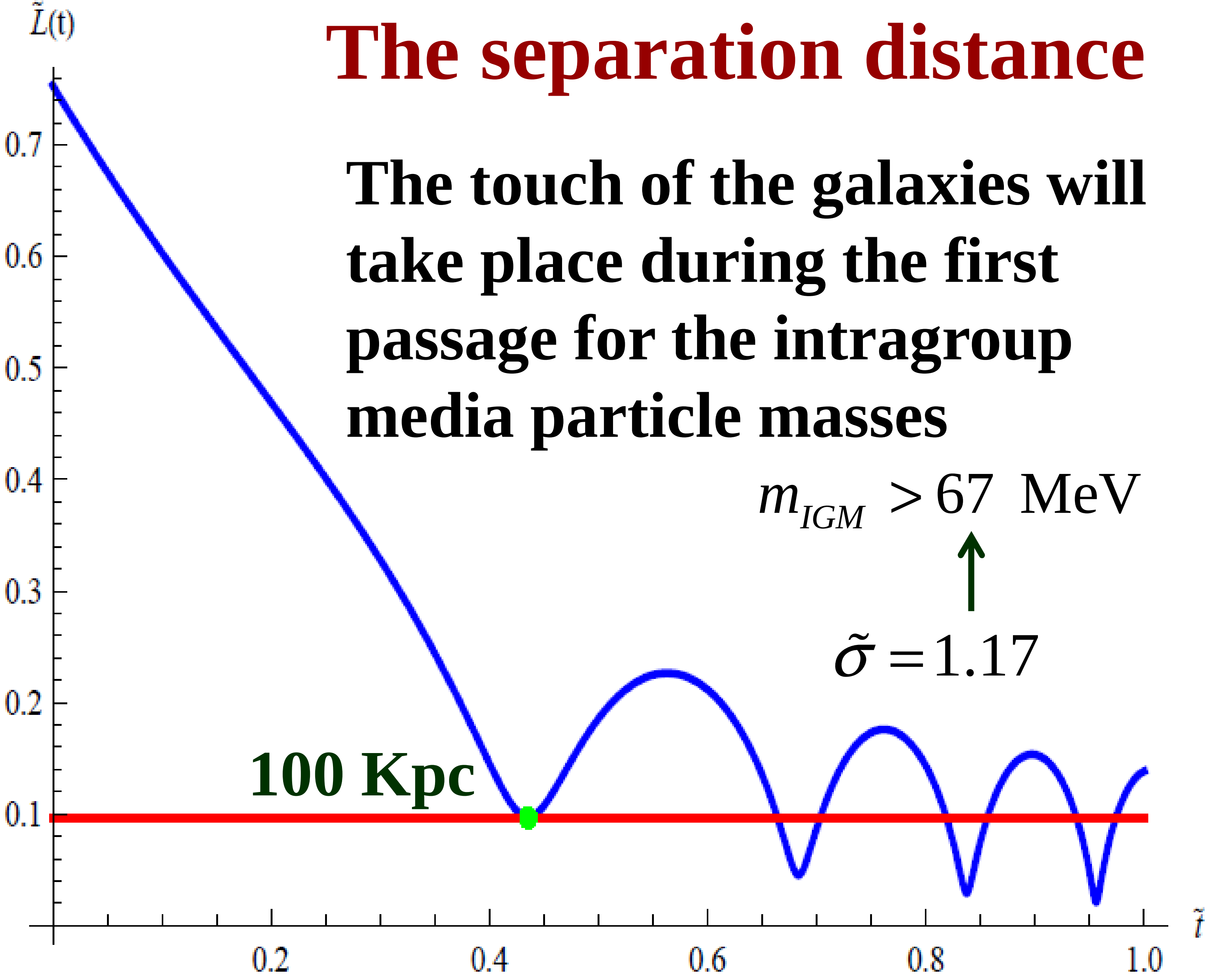
$$\tilde{v}_{\mathrm{pec},i}=\left[\left(\frac{d\tilde{X}_i}{d\tilde{t}}-\frac{1}{\tilde{a}}\frac{d\tilde{a}}{d\tilde{t}}\tilde{X}_i\right)^2+\left(\frac{d\tilde{Y}_i}{d\tilde{t}}-\frac{1}{\tilde{a}}\frac{d\tilde{a}}{d\tilde{t}}\tilde{Y}\right)^2\right]^{1/2}$$

The separation distance

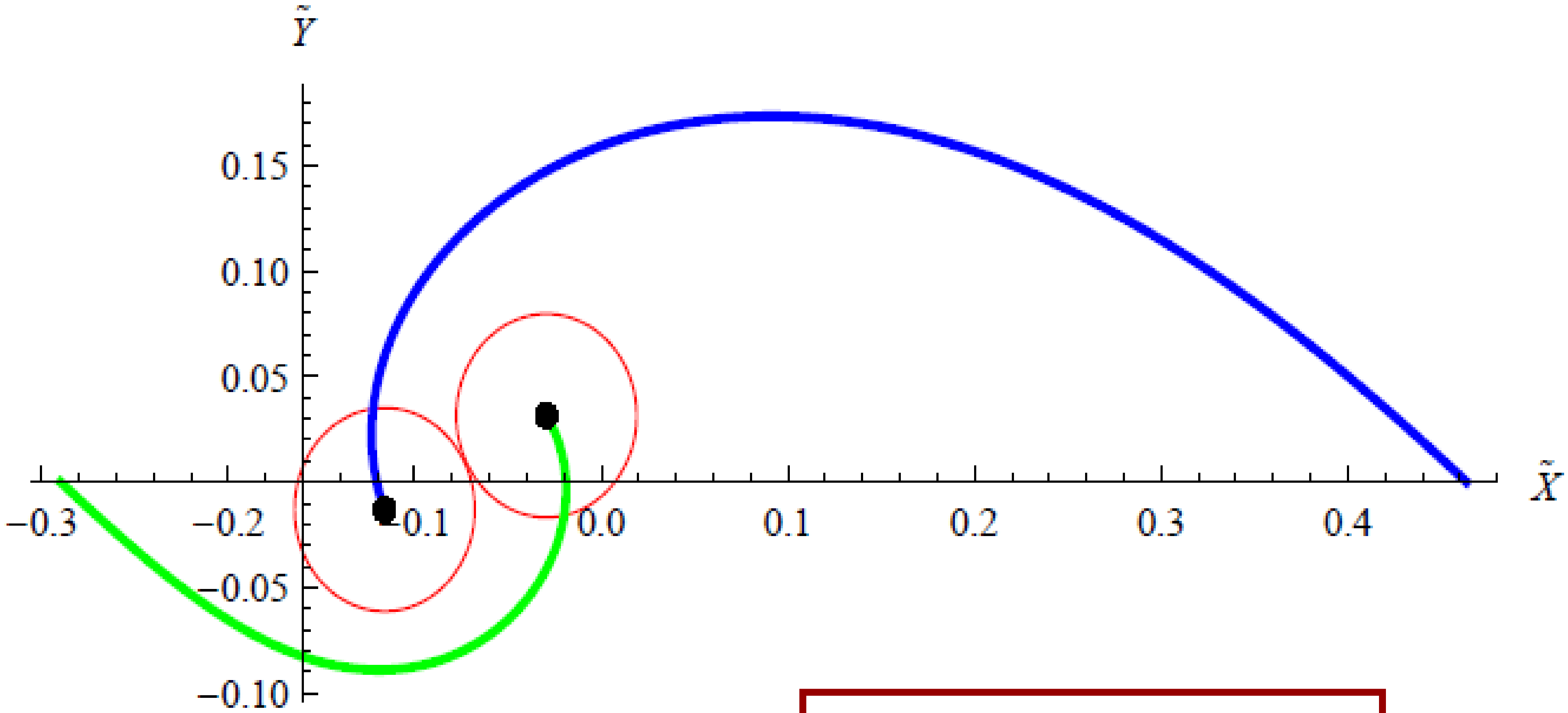
The touch of the galaxies will take place during the first passage for the intragroup media particle masses

$$m_{IGM} > 67\ \text{MeV}$$

$$\tilde{\sigma} = 1.17$$

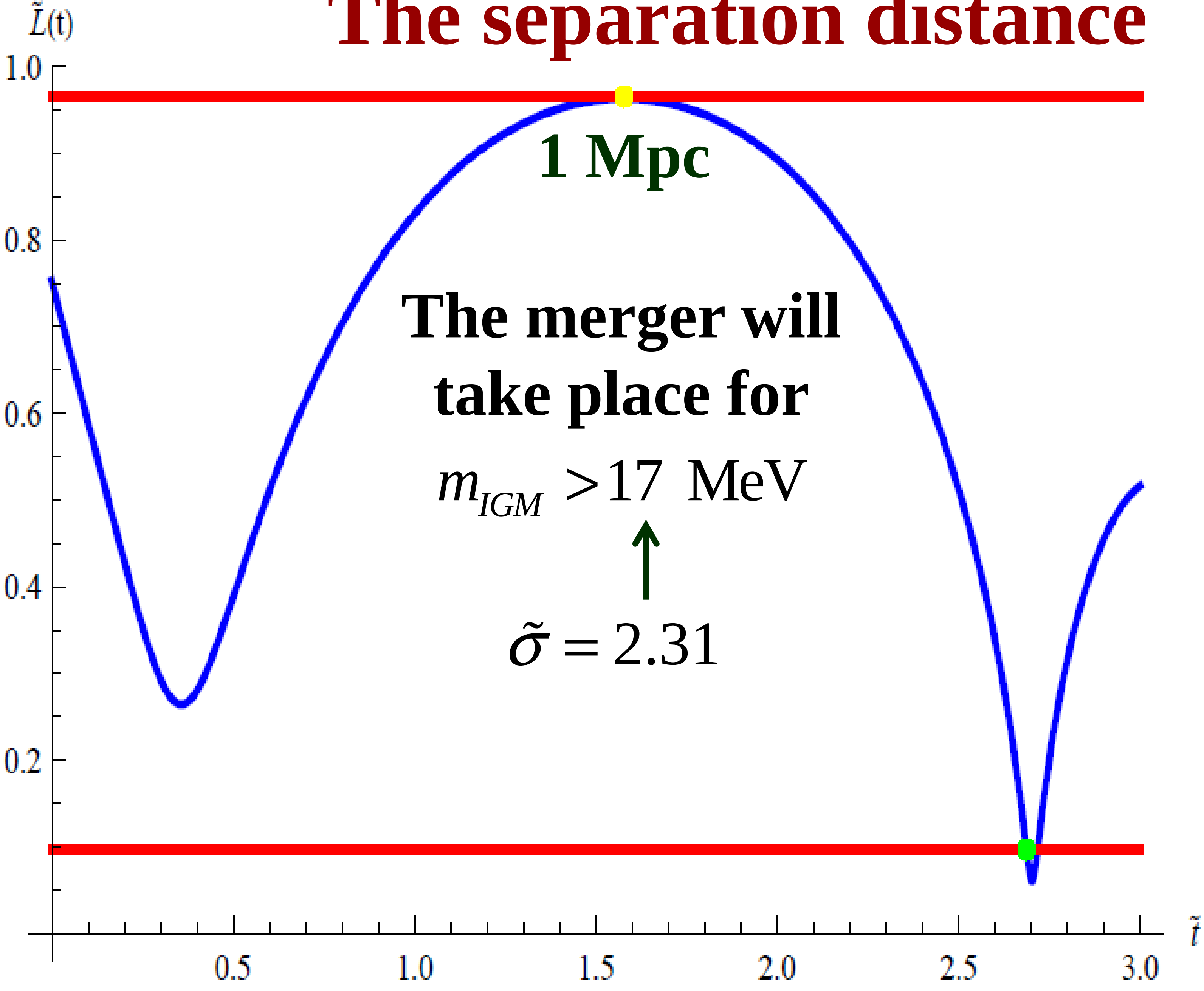


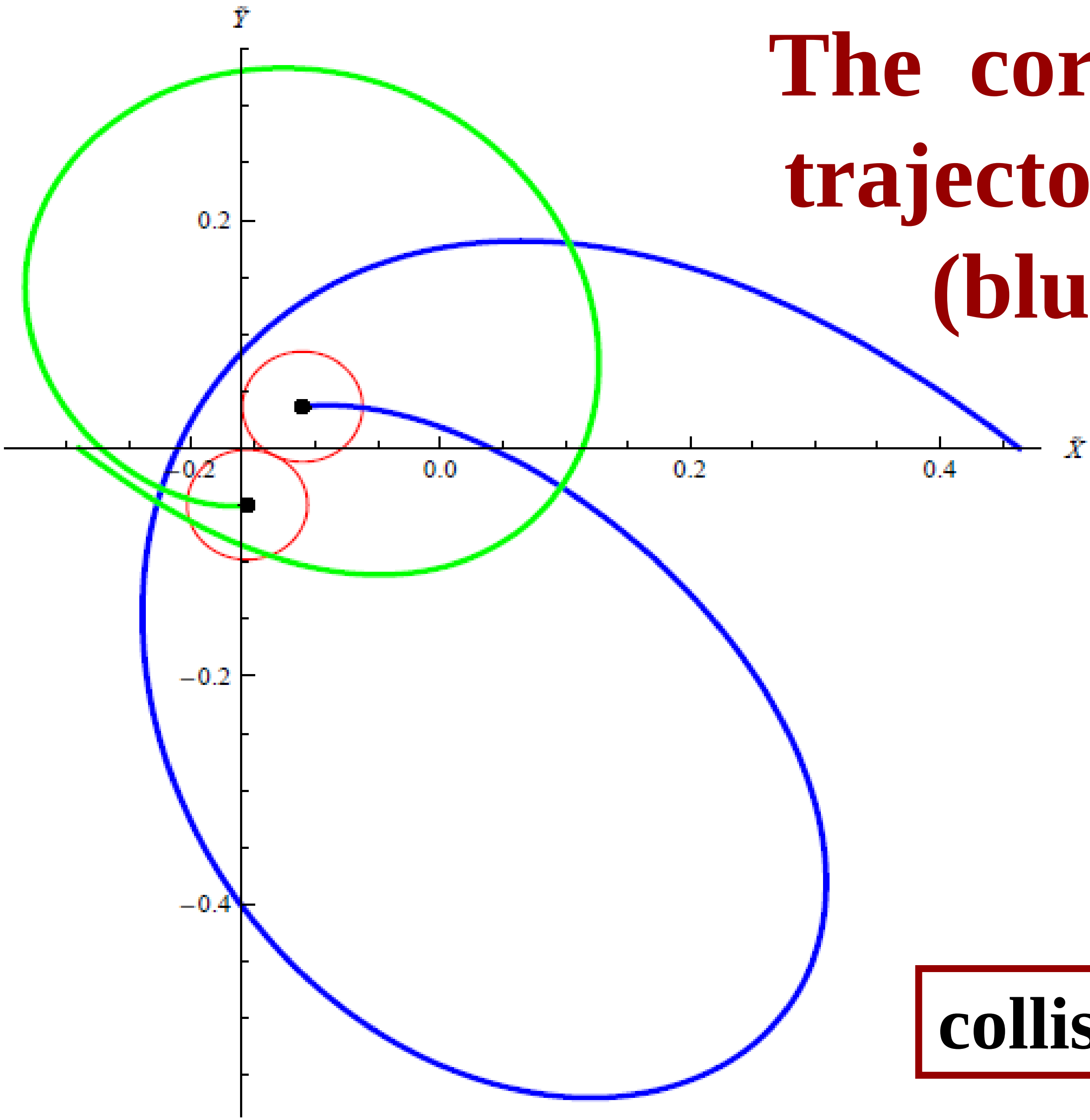
The corresponding trajectories of MW (blue) and M31 (green)



collision in 6 Gyr

The separation distance





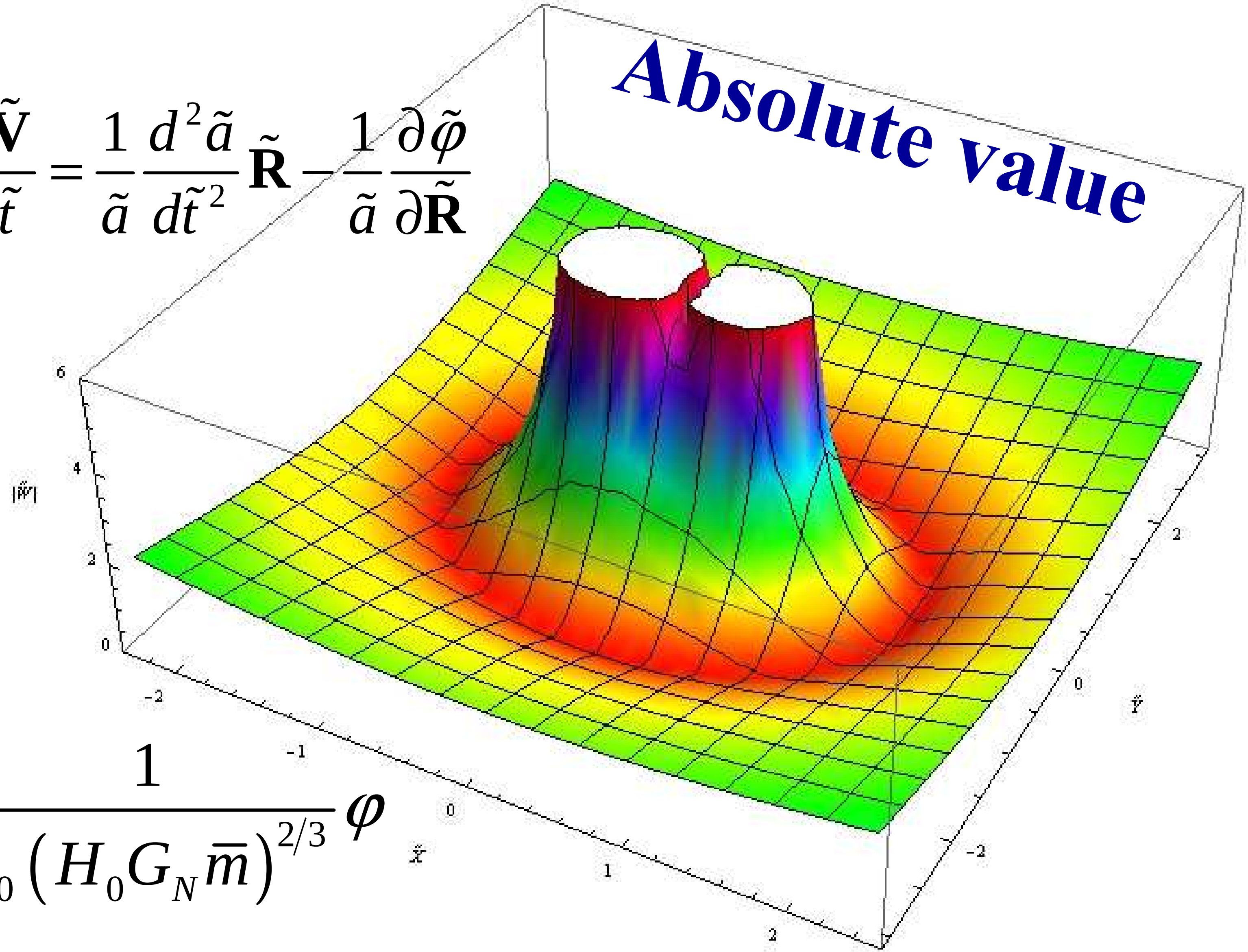
The corresponding trajectories of MW (blue) and M31 (green)

collision in 37 Gyr

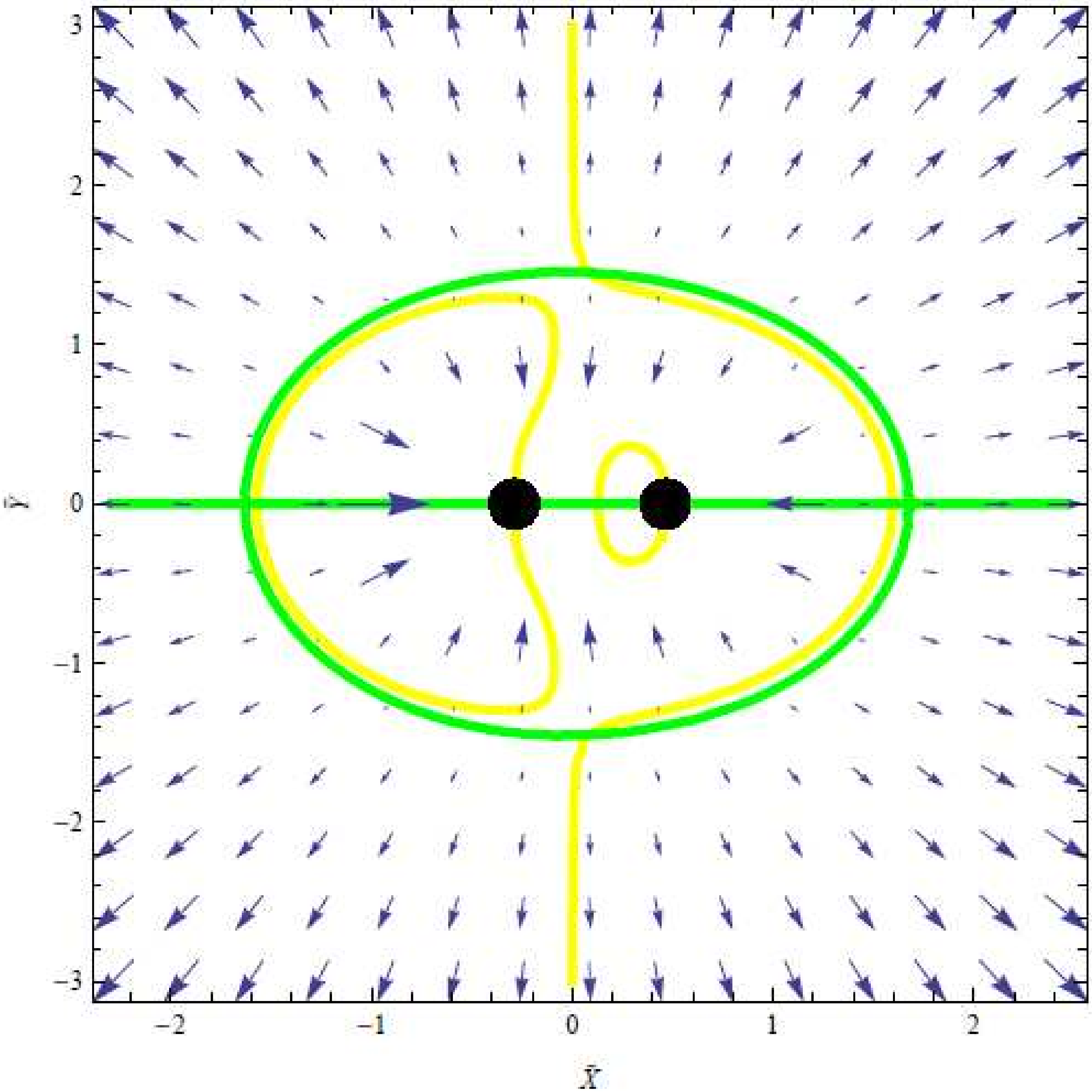
The dwarf galaxy acceleration

$$\tilde{\mathbf{W}} = \frac{d\tilde{\mathbf{V}}}{d\tilde{t}} = \frac{1}{\tilde{a}}\frac{d^2\tilde{a}}{d\tilde{t}^2}\tilde{\mathbf{R}} - \frac{1}{\tilde{a}}\frac{\partial\tilde{\varphi}}{\partial\tilde{\mathbf{R}}}$$

Absolute value



$$\tilde{\varphi} = \frac{1}{a_0\left(H_0 G_N \overline{m}\right)^{2/3}}\varphi$$



Vector field

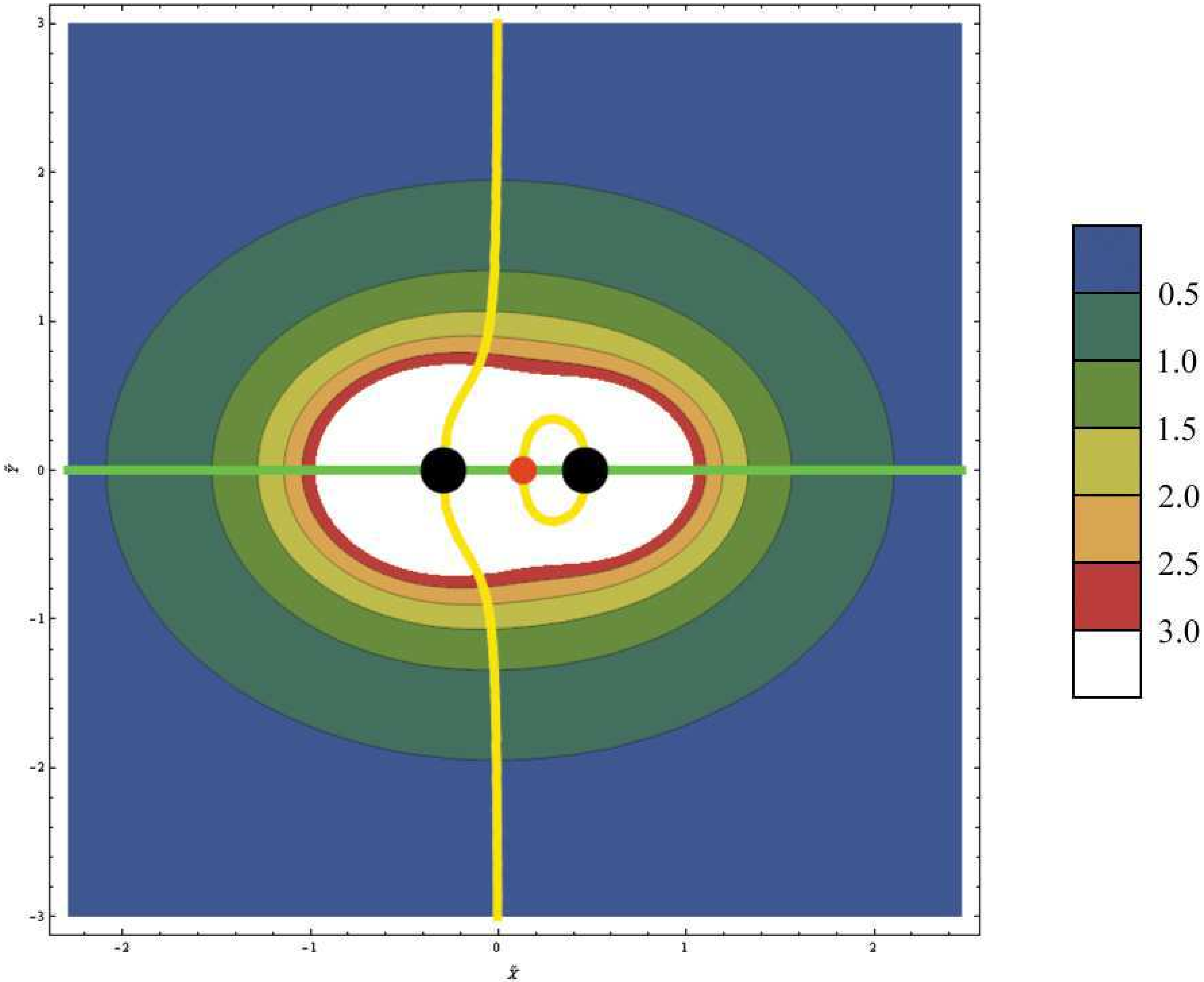
Left ● = Andromeda

Right ● = Milky Way

$\tilde{W}_x = 0$

$\tilde{W}_y = 0$

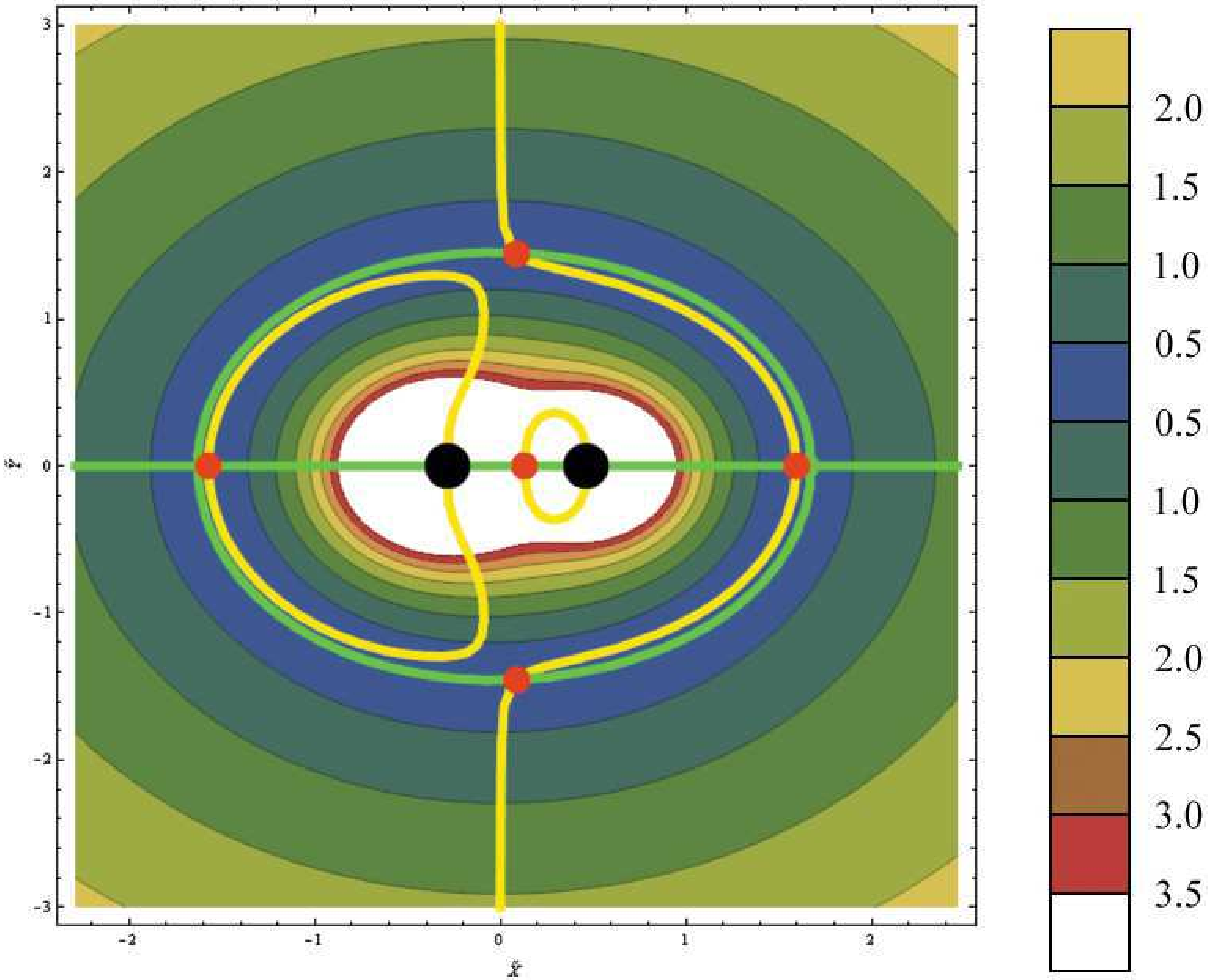
The contour plot of the acceleration absolute value, if the cosmological expansion is disregarded



$$\tilde{W}_x = \tilde{W}_y = 0$$

$$\left|\tilde{\mathbf{W}}\right| = 0$$

The real contour plot



The exact zero acceleration surface is absent.

Results and conclusions

The mechanical approach in modern cosmology is consistently developed and substantiated.

The late Universe inside the cell of uniformity (less than 150 Mpc) is described by the slightly perturbed “ΛCDM” model:

– **the metrics** $ds^2 \approx a^2\left\{(1+2\Phi)d\eta^2 - (1-2\Phi)\gamma_{\alpha\beta}dx^\alpha dx^\beta\right\}$

– **the gravitational potential** $\Phi(\eta,\mathbf{r}) = \dfrac{\varphi(\mathbf{r})}{c^2 a(\eta)}$

$$\Delta\varphi + 3K\varphi = 4\pi G_N(\rho - \overline{\rho})$$

In the case $K = -1$ (the hyperbolic space = the open Universe):

– inhomogeneities may be distributed completely randomly;

– the gravitational potential is finite at any vacuum point and its average value is zero;

– at present time the distance from our Local Group of galaxies, at which the cosmological expansion '=' the gravitational attraction, or the zero acceleration sphere radius, ~ 1.4 Mpc, and this theoretical value is close to the observed one, namely 1 Mpc.

The mechanical approach gives a possibility to simulate the dynamical behavior of an arbitrary number of randomly distributed inhomogeneities inside the cell of uniformity, including the Milky Way and Andromeda galaxies together with the dwarf galaxies, taking into account gravitational interaction between them as well as cosmological expansion of the Universe.

The observations of the Hubble flows even at a few Mpc may reveal presence of dark energy in the Universe.

almost the end

THANK YOU FOR ATTENTION !