Is inflation really necessary in a closed Universe?

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Please see also arXiv:1303.3203

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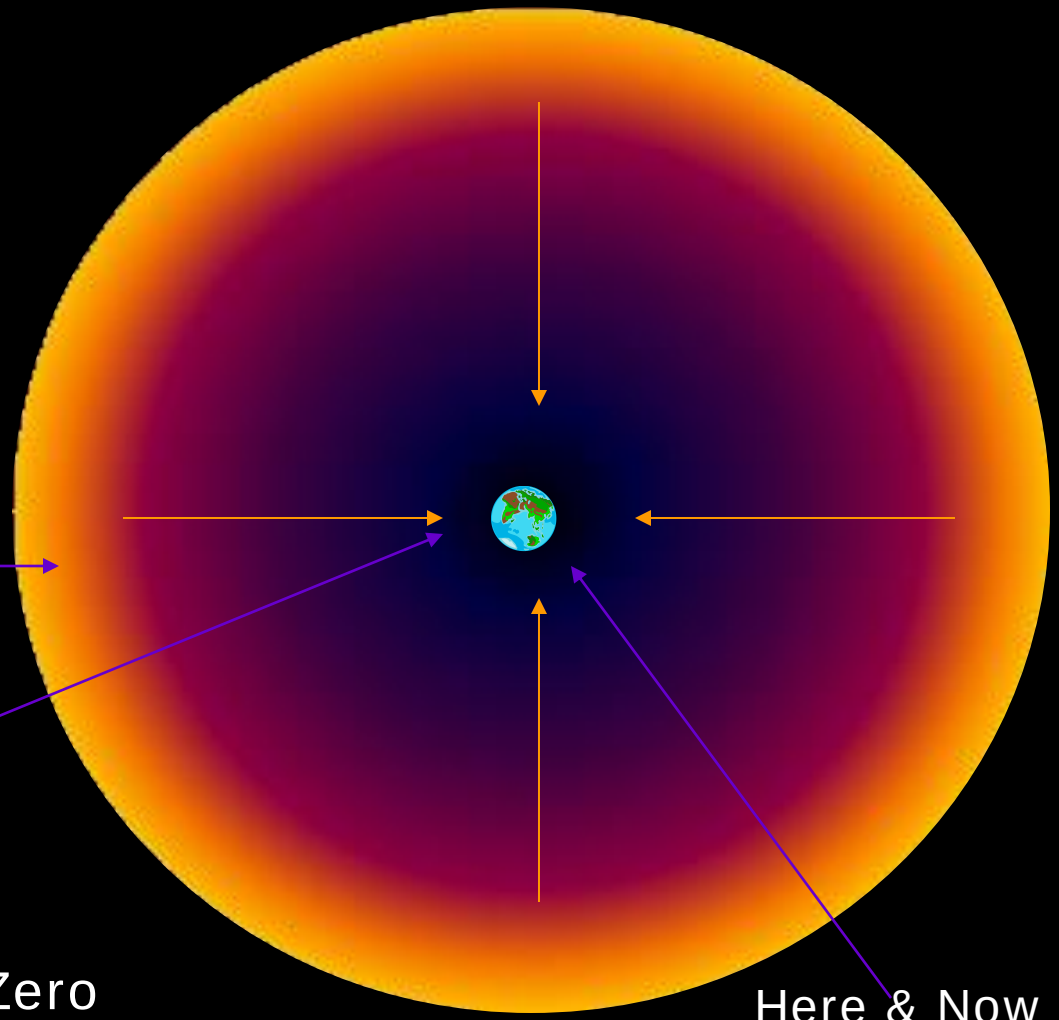
Summary

- Uniformity of CMB
- A Spherical shell model as a possible explanation for observed uniformity
- Compare two different models:

  a) Spherical Shell Model

  b) Friedmann-Robertson-Walker standard model
- Compare with observable data
- Arguments supporting the model

Properties of the Edge of the Observable Universe



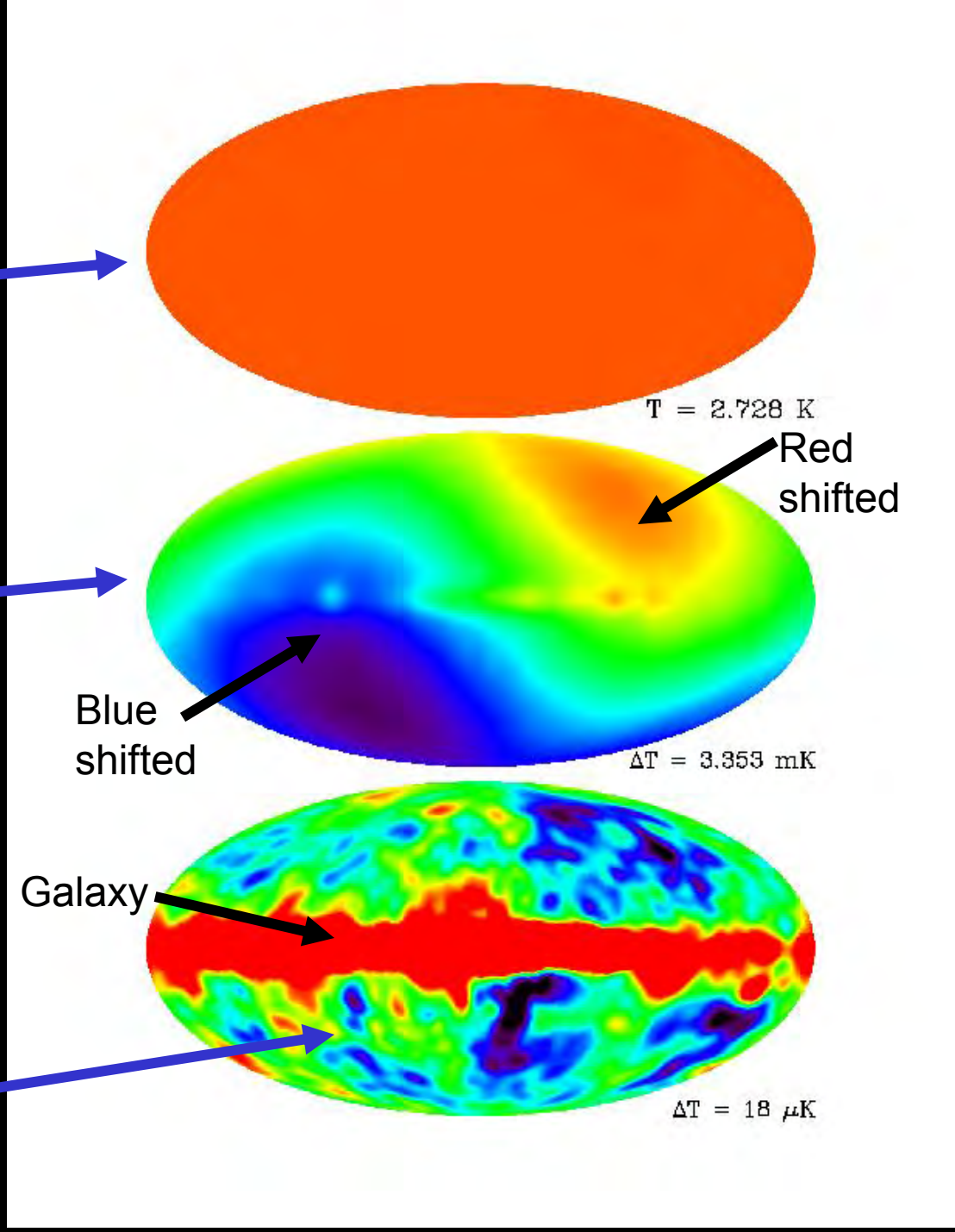
Today

- 2.726K above absolute Zero
- "Microwave Radiation" (The "Cosmic Microwave Background": CMB)
- 1,000,000 times weaker than ambient radiation in a pitch dark room.

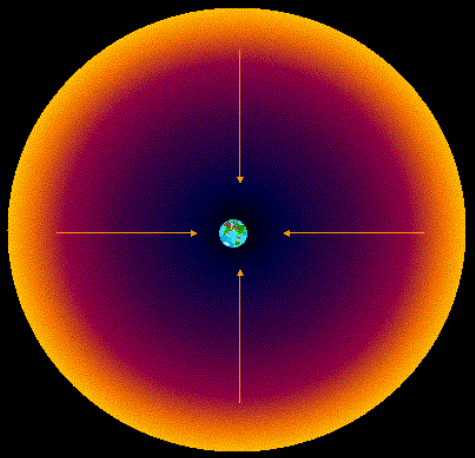
CMB Temperature smooth to 3 Decimal places

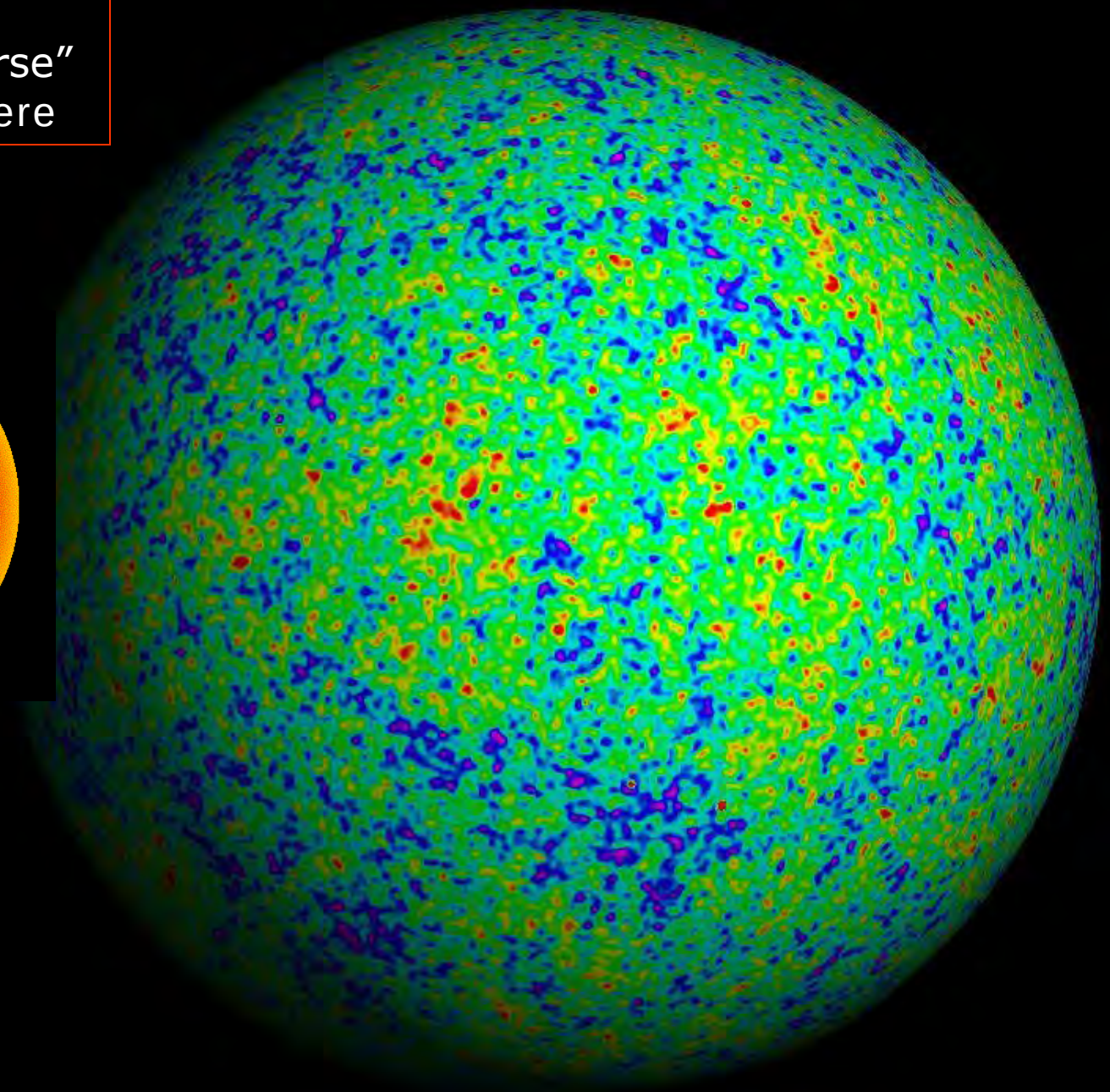
CMB “dipole” (at the 0.1% level)

Remove Galactic noise
Interpret CMB dipole as red/blue shifts due to our motion and remove to get “cosmic anisotropies”



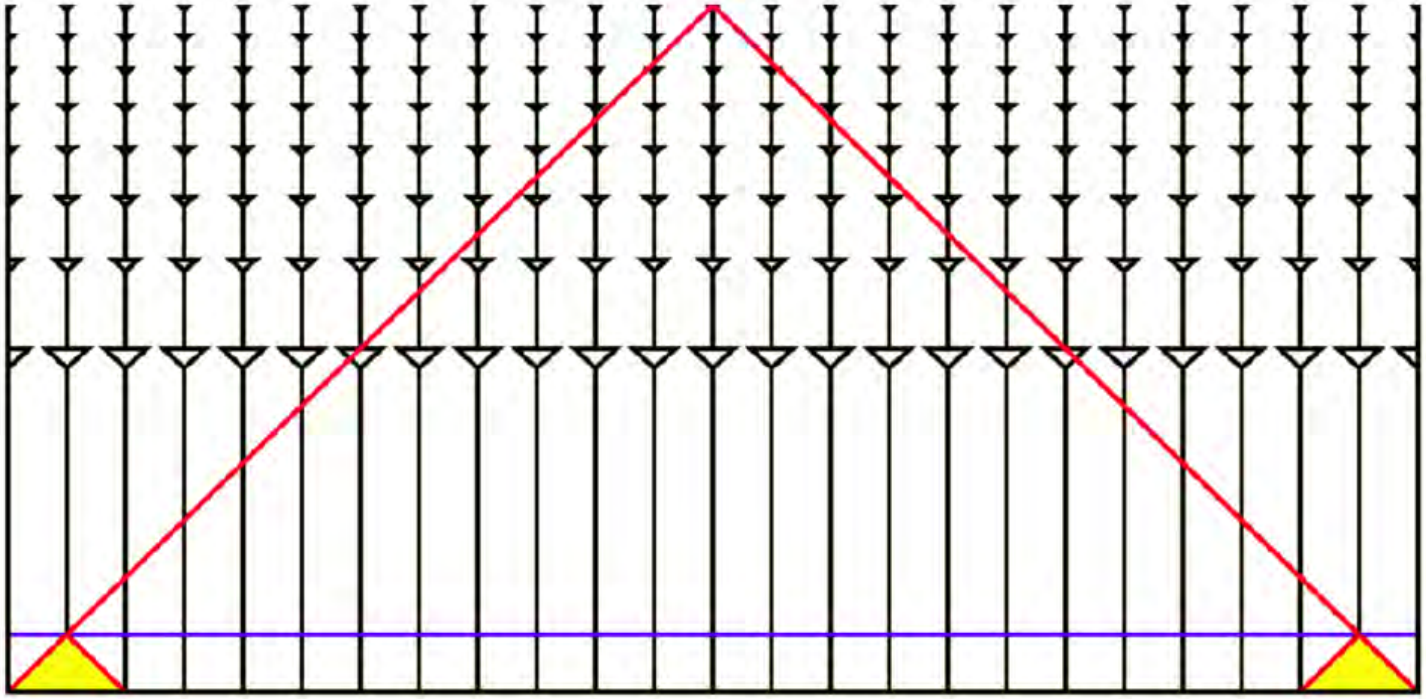
WMAP map of the "edge of the observable universe" plotted as a sphere



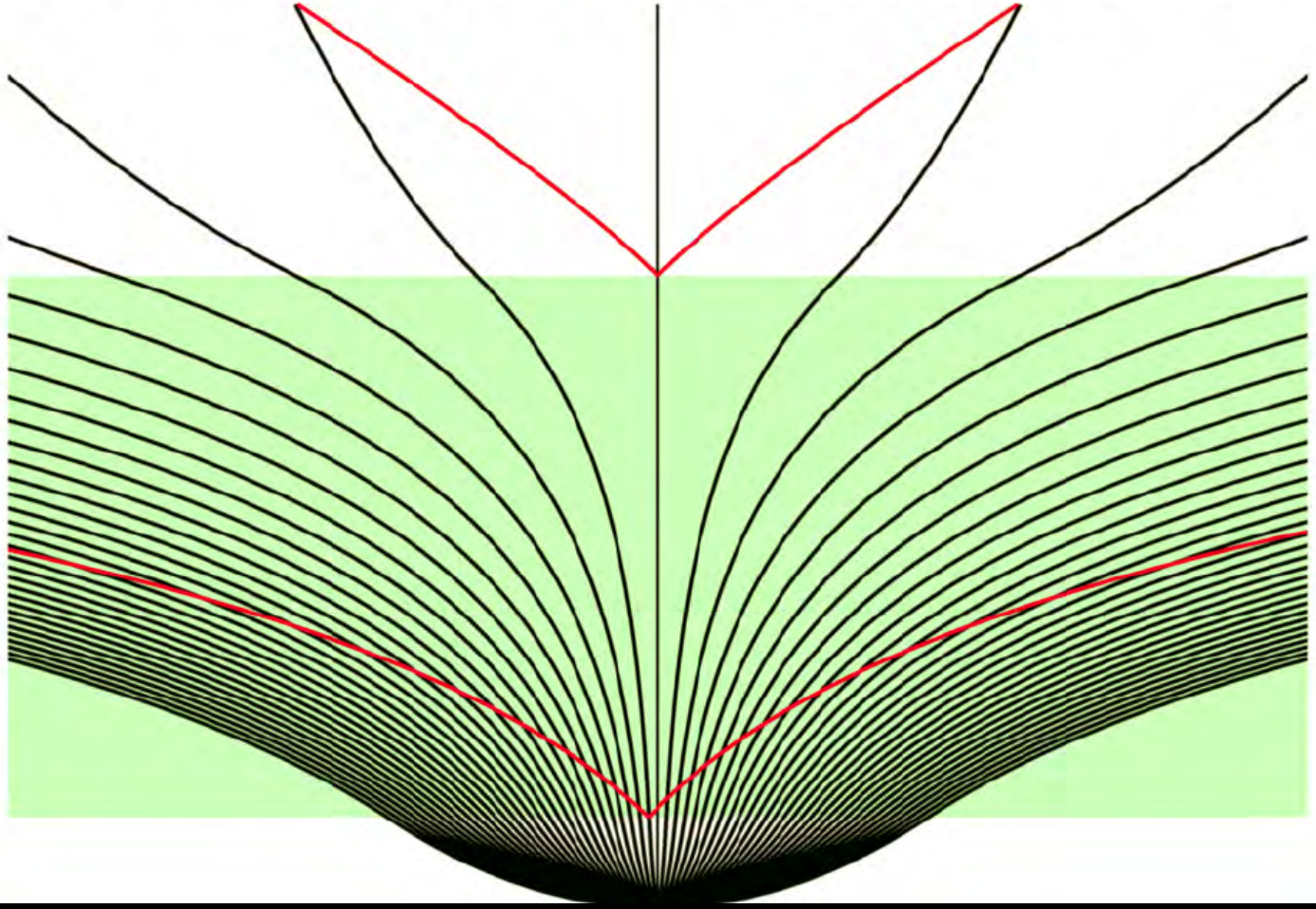


Horizon Problem

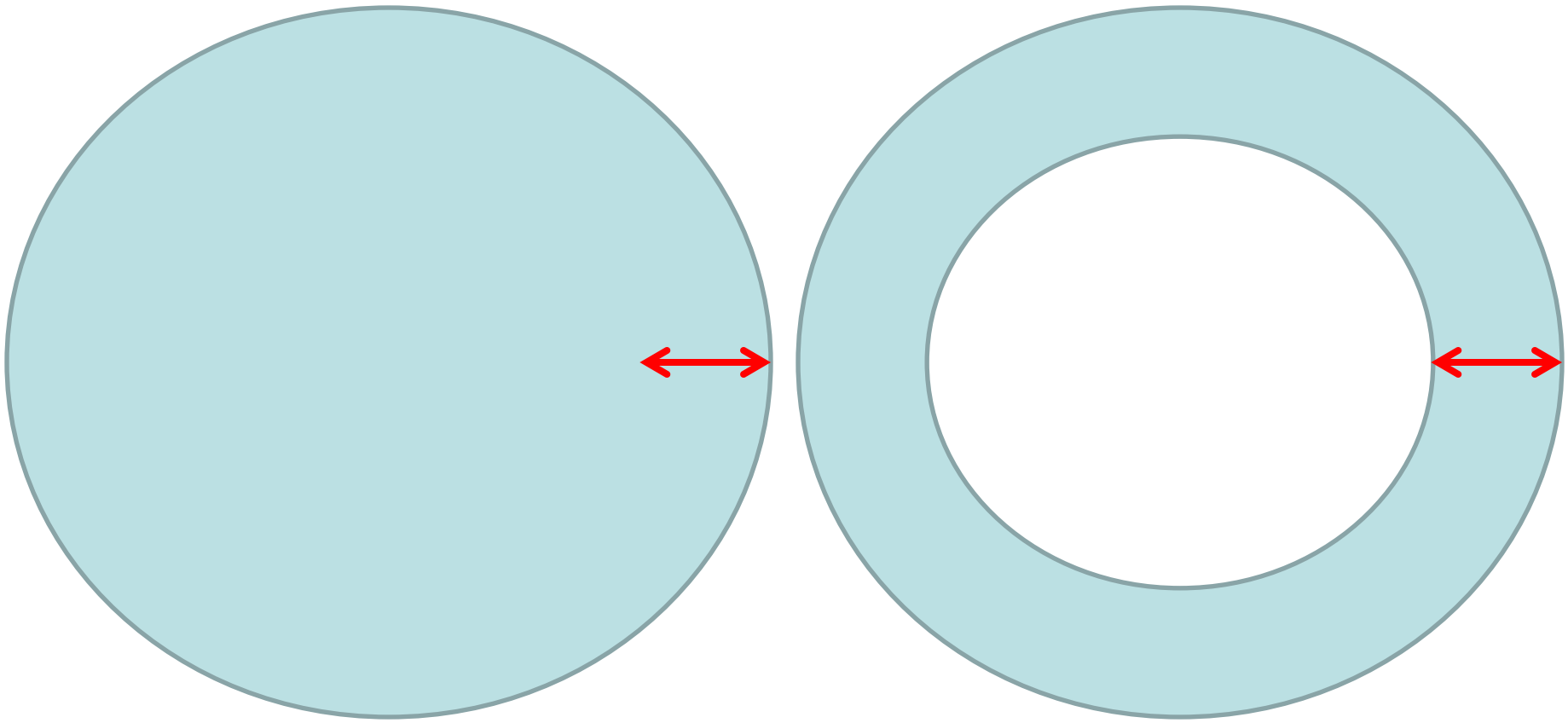
Regions seen on left and right of sky can only be influenced by the yellow areas in their past lightcones. These are disjoint, so why is the CMB T the same in both?



Inflation: The same parts of the early universe

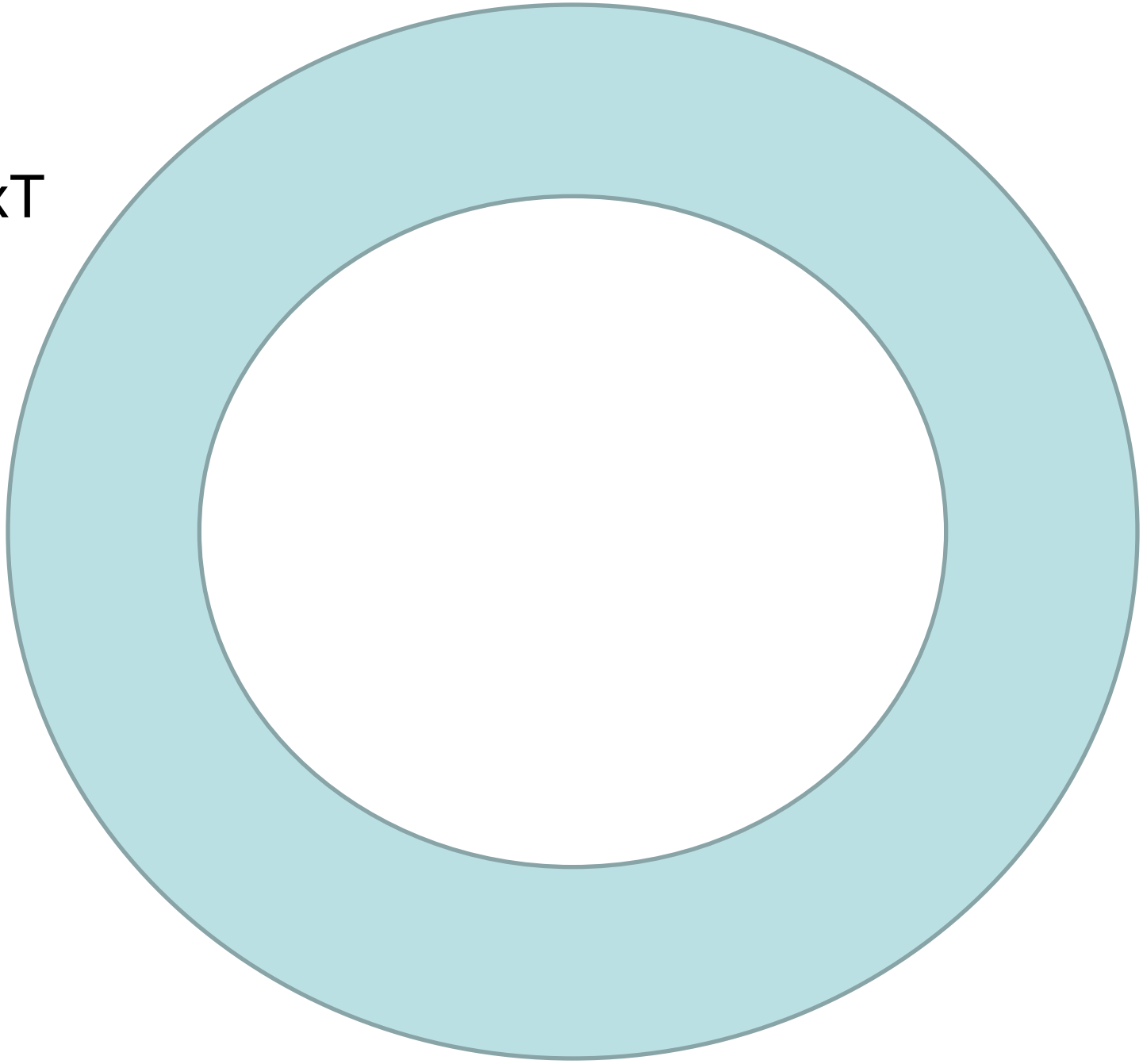


- **Hubble flow and isotropy of the universe can be used to justify spherical shell structure of the universe**

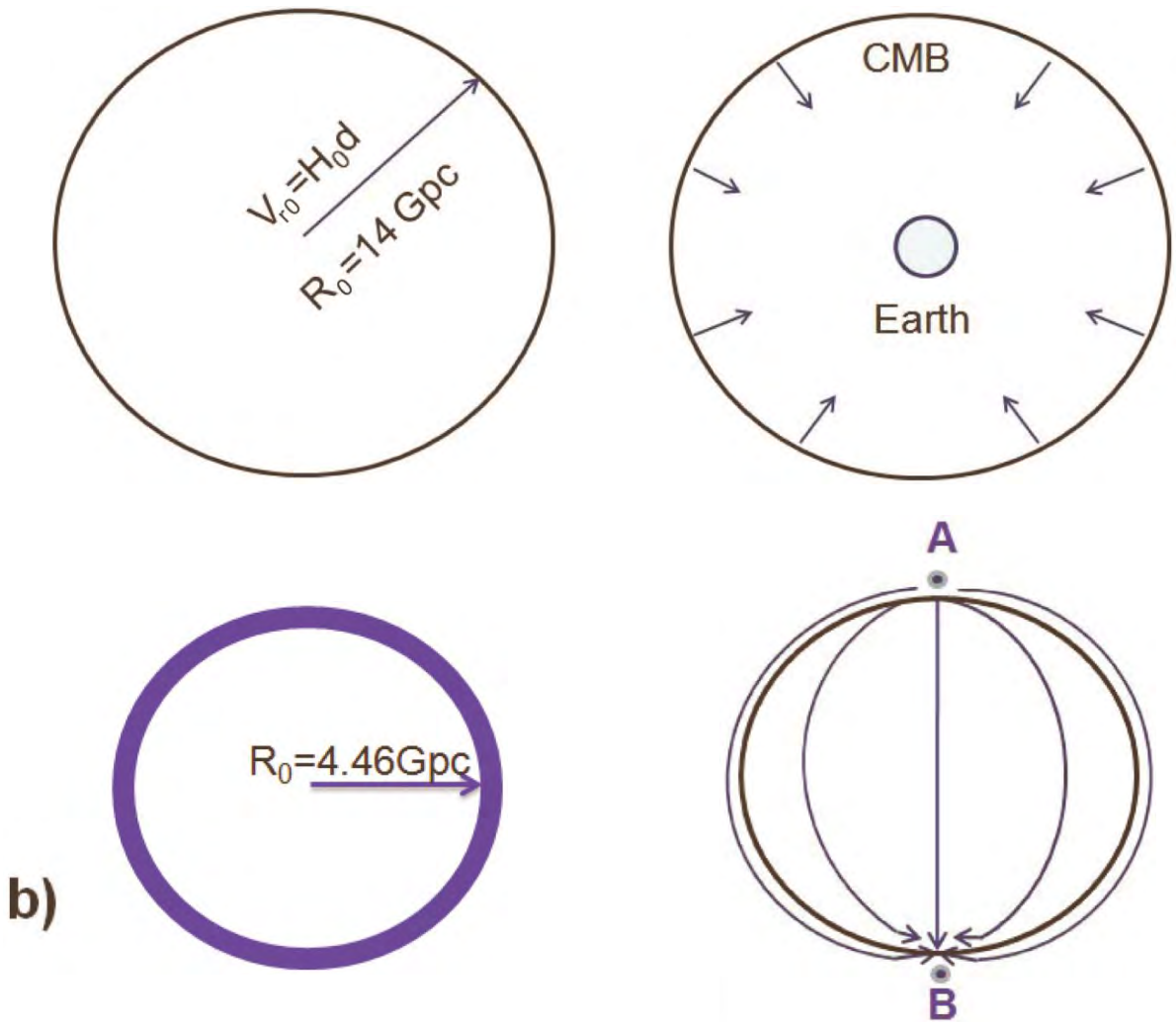


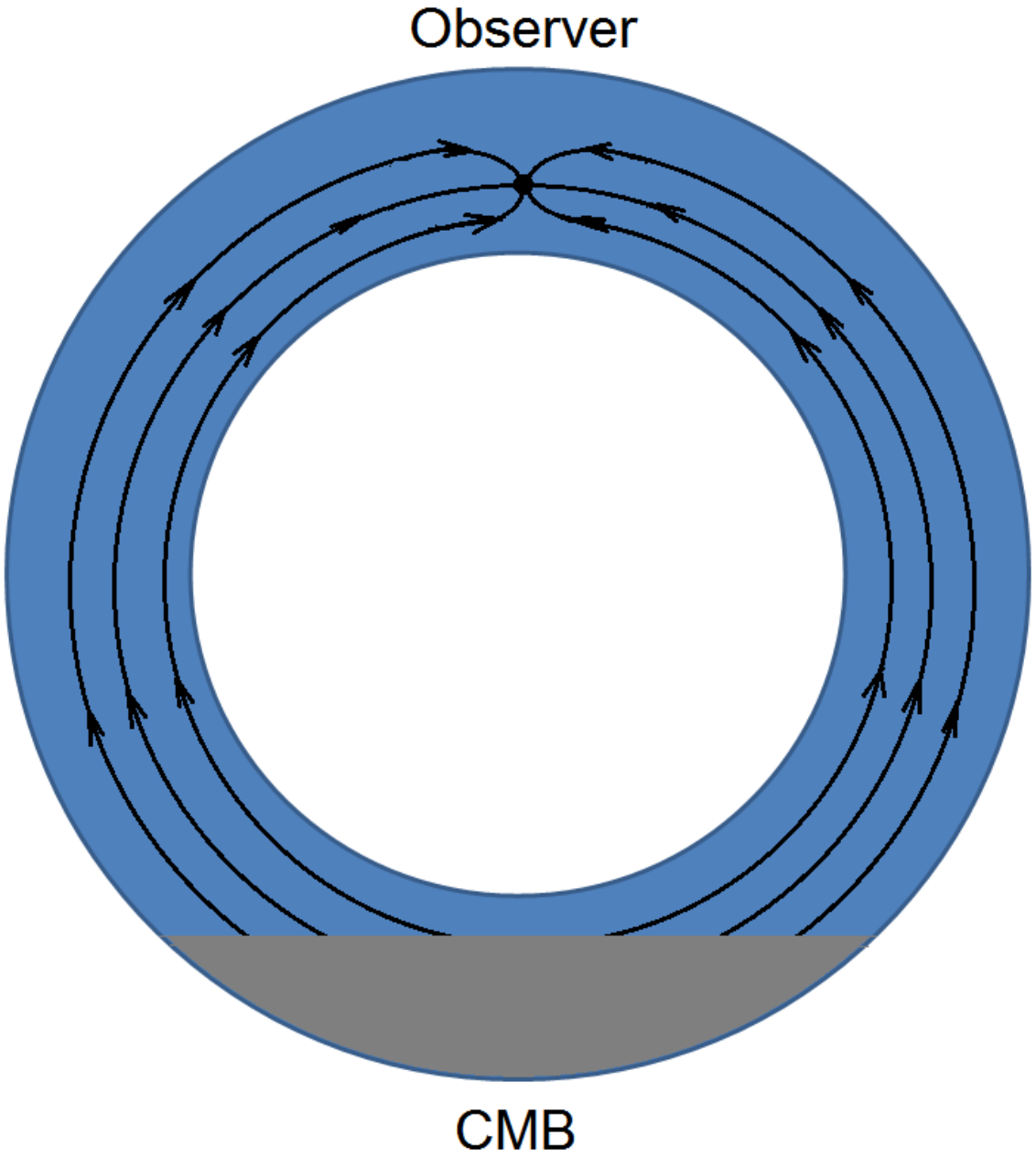
Thickness of the shell ~ 1 Gpc
Radius of the shell ~ 4.5 Gpc

S^3xT
S^2xRxT



Comparison of two models




Observer
CMB

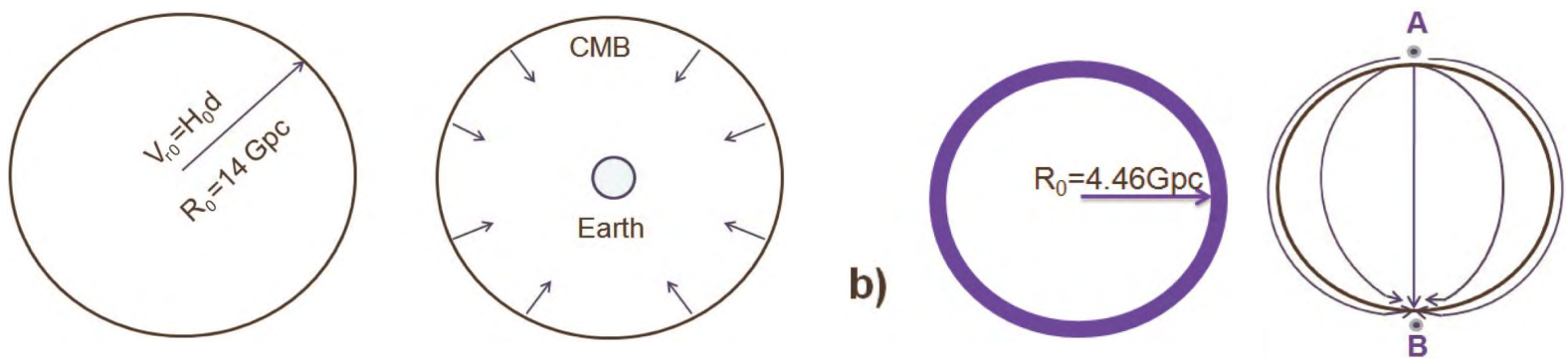
The visible universe is inside a photon sphere

$$\alpha = \frac{4GM}{c^2 b} \qquad R_{ps} = \frac{3GM}{c^2}$$

Since universe is homogenous and isotropic Gauss' or Birkhoff's flux theorem for gravity can be applied

$$\oint \mathbf{\Phi} d\mathbf{a} = -4\pi GM$$

$$M = 10^{23} M_\odot = 2\text{x}10^{53} \text{ kg, it gives } R_{ps} = 14.3 \text{ Gpc}$$



GR does not specify the topology of the space

FRW standard

Mass dominated
k=+1 Λ=0

$$R_{\rm obs} = a(t)\int_{t'=0}^{t}\frac{c}{a(t')}dt'$$

$$\frac{da}{dt} = \sqrt{\frac{\Omega_r}{a^2}+\frac{\Omega_m}{a}+\frac{\Omega_\Lambda}{a^{-2}}}$$

h=H/100 kms^{-1}Mpc^{-1}=0.705 $\pm$0.013

$\Omega_m h^2$=0.136 $\pm$0.003

$$\Omega_r = \frac{8\pi G}{3H^2}\frac{\pi^2 k^4 T^4}{15c^5\hbar^3}$$

$$\Omega_\Lambda = 1-\Omega_r-\Omega_m$$

$$R_{\rm obs} = 46.9 \pm 0.4\,{\rm Glyr}$$

Shell model

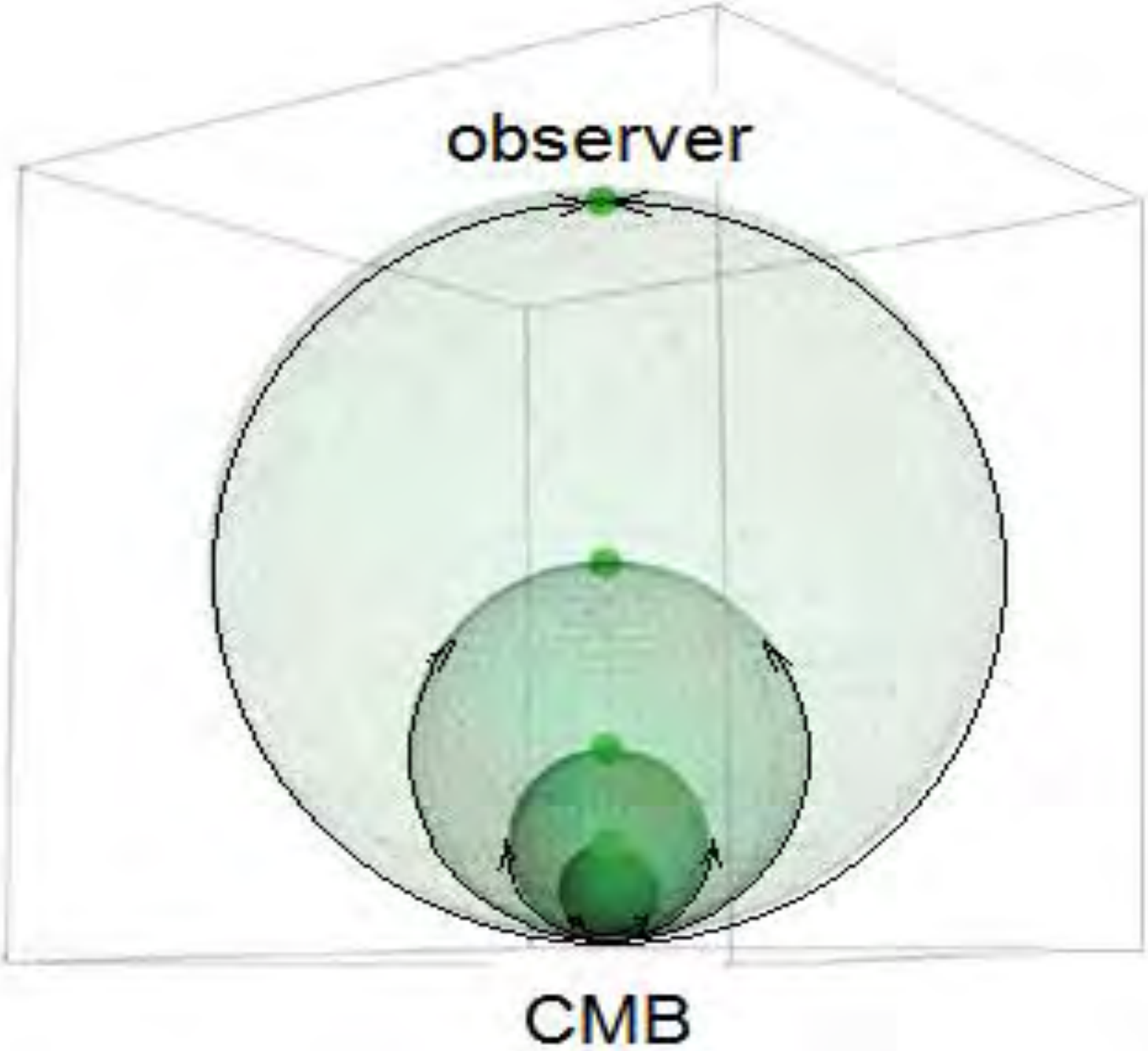
R= ct

t_0=13.75 $\pm$ 0.17Gyr

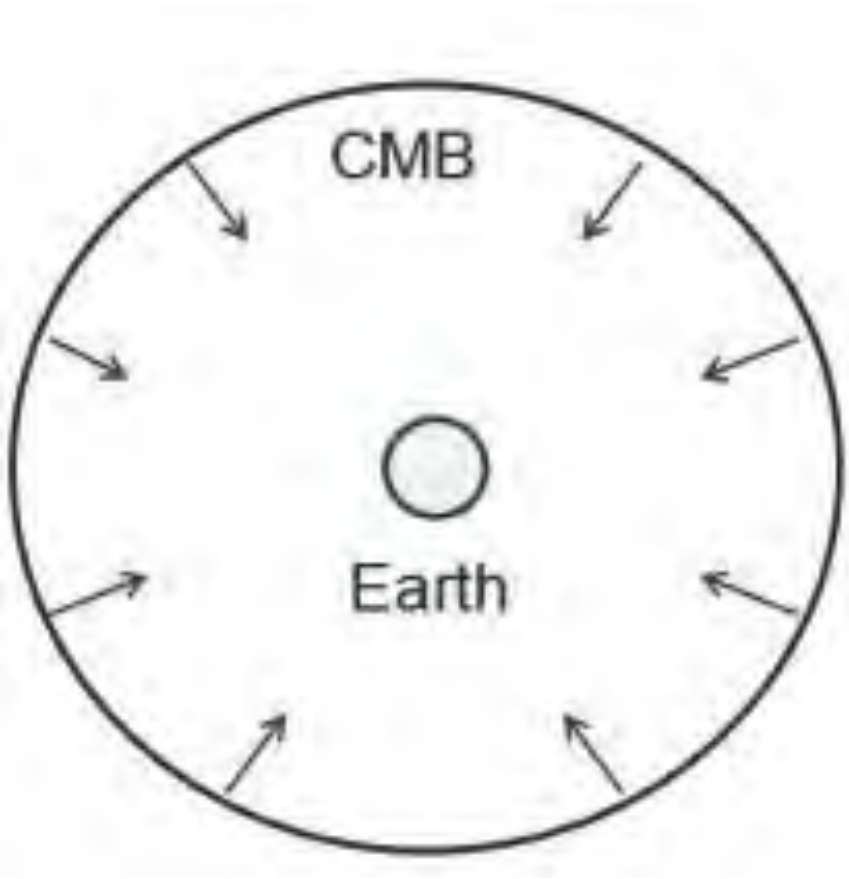
R_{obs}=14.51 Glyr

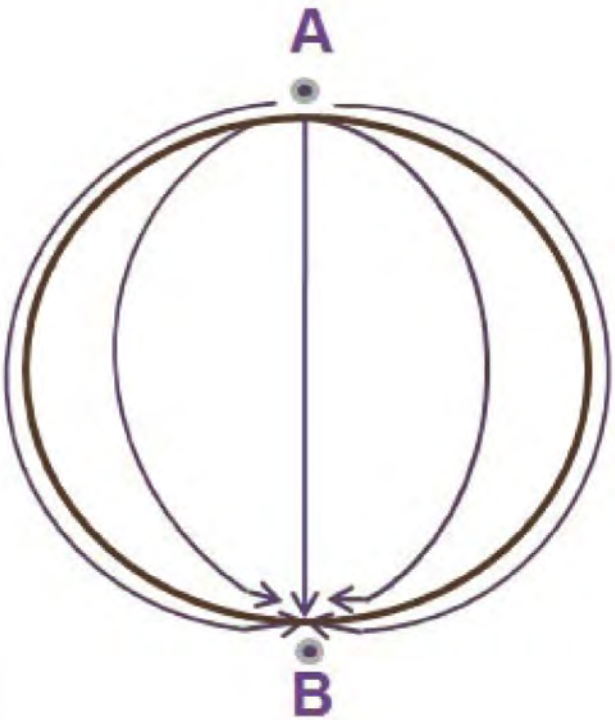
Particle horizon = 45.6 Glyr

$$d_A(Z^*) \equiv (1+Z^*)D_A(Z^*) = c\int_{t^*}^{t_0}\frac{dt}{a(t)} = 14.0 \pm 0.1 Gpc$$


observer
CMB

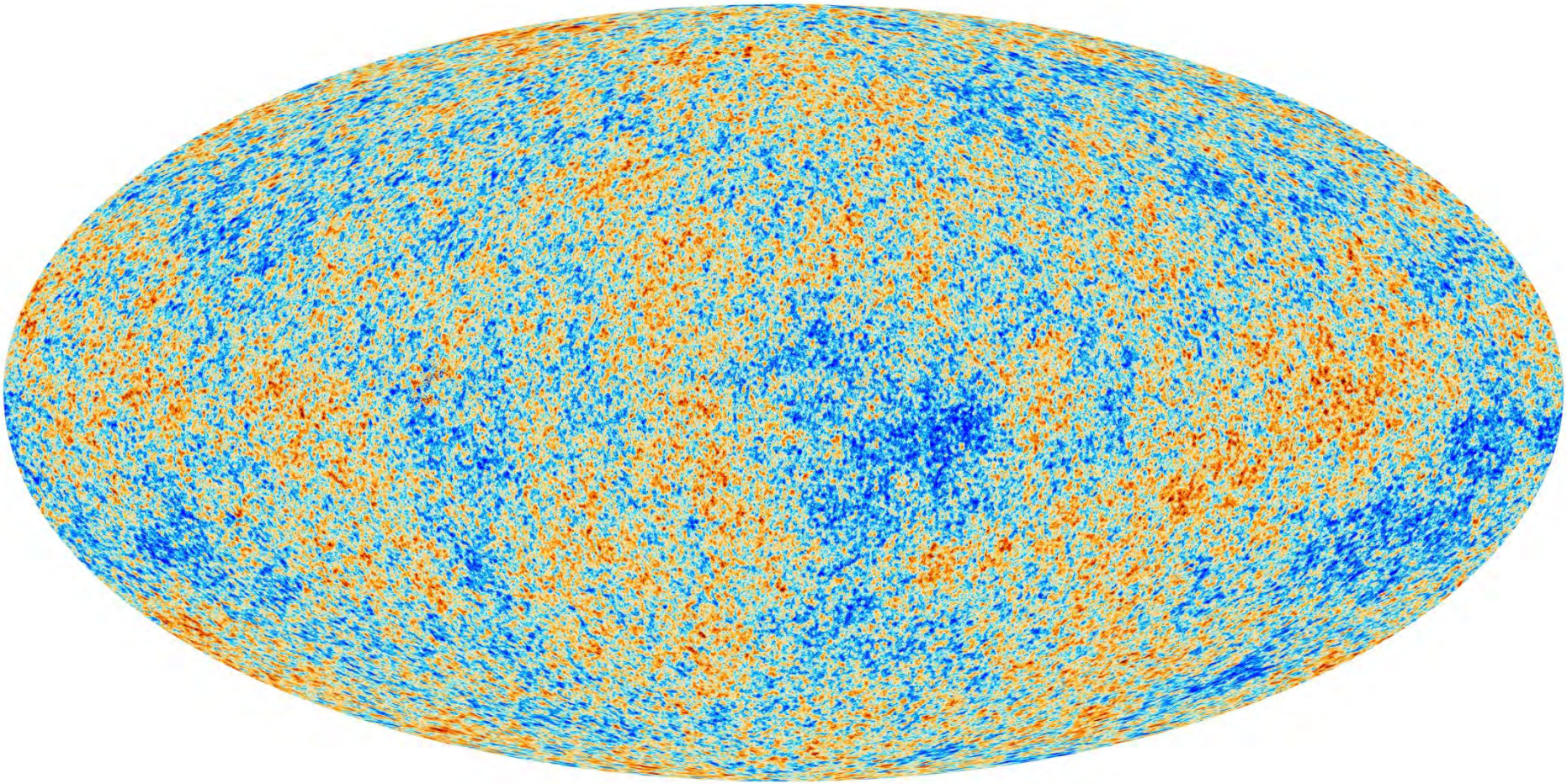
The model predicts uniformity of CMB





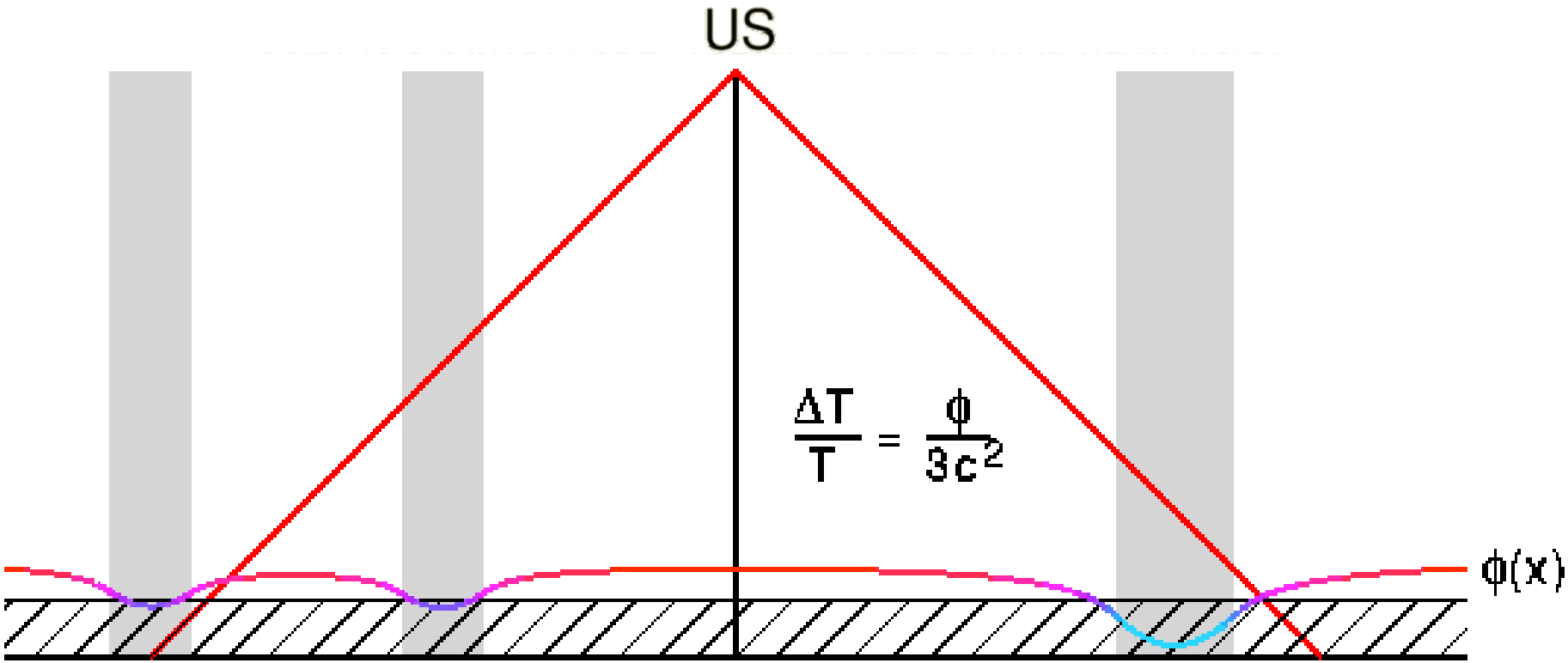
There is no requirement for inflation.
The CMB from different parts of the universe can be unlike.

Planck data



The observed fluctuations in the CMB are created by the integrated Sachs-Wolfe effect

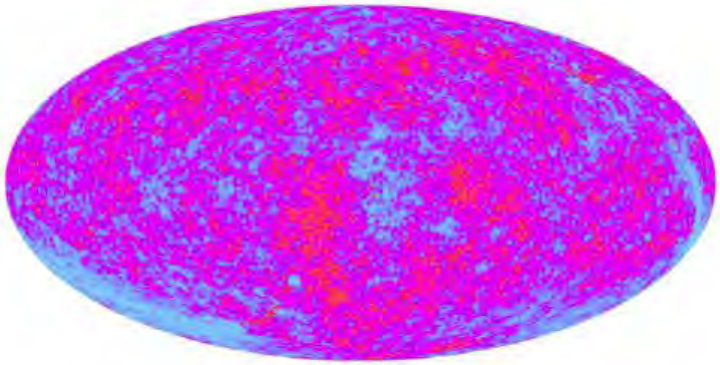
Gravitational Potential $\rightarrow \Delta T$

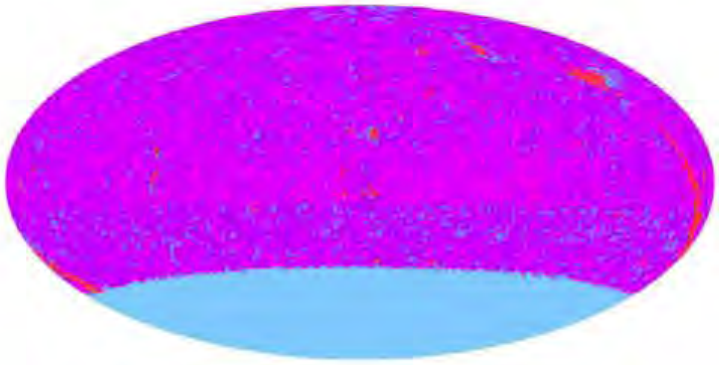


This potential also leads to large scale structure formation.

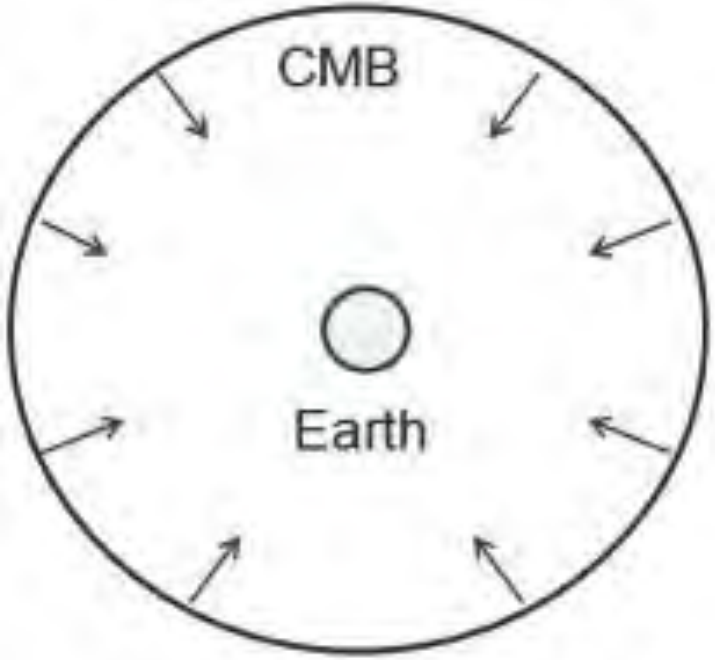
Structure CMB & IR maps

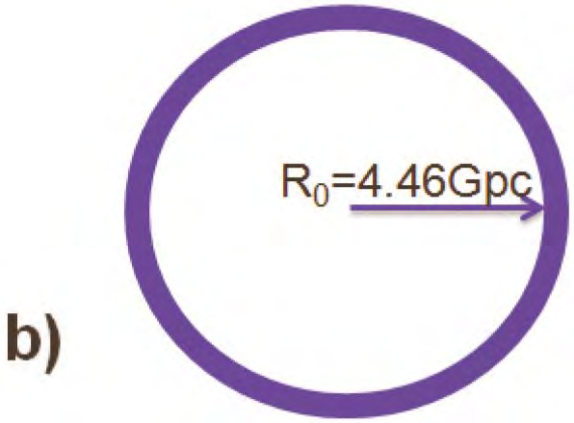
- This late Integrated Sachs-Wolfe effect occurs on our past light cone so the CMB ΔT we see is due to structures we also see.
- Correlation between WMAP and large-scale structure seen by:
  - Boughn & Crittenden at 99.7% confidence with hard X-ray background
  - Nolta at 98% confidence with the NRAO VLA Sky Survey
  - Ashfordi at 99.4% with the 2MASS 2 micron all sky survey



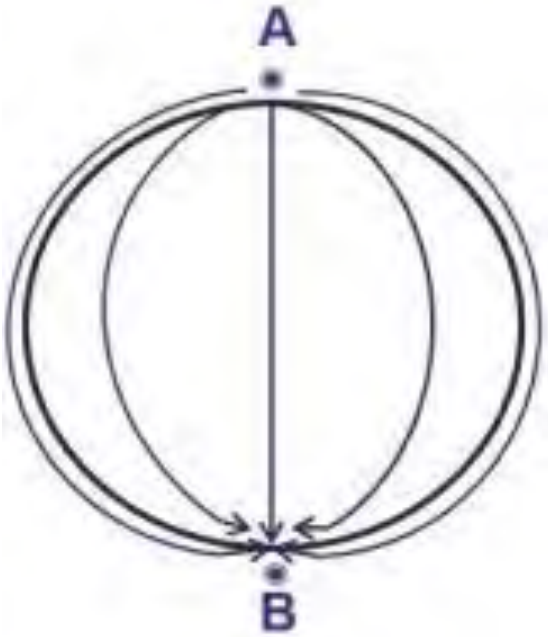


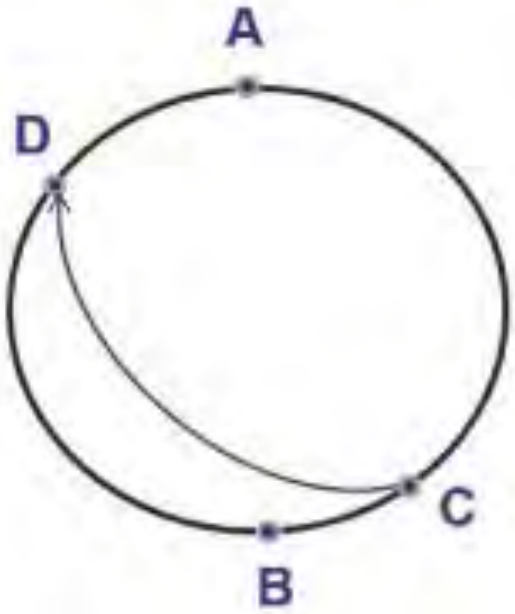
Origin of the fluctuations does not require inflation

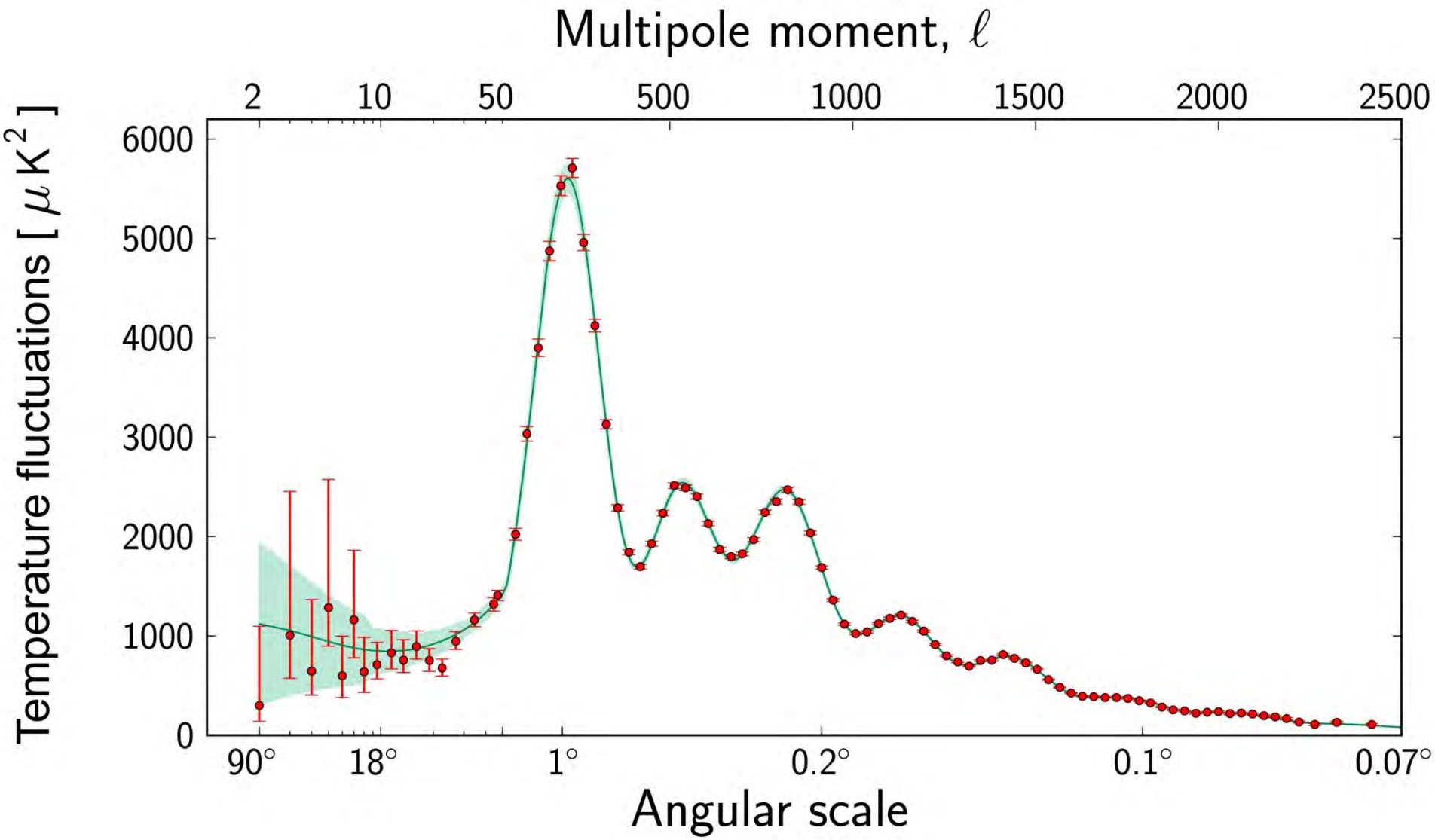


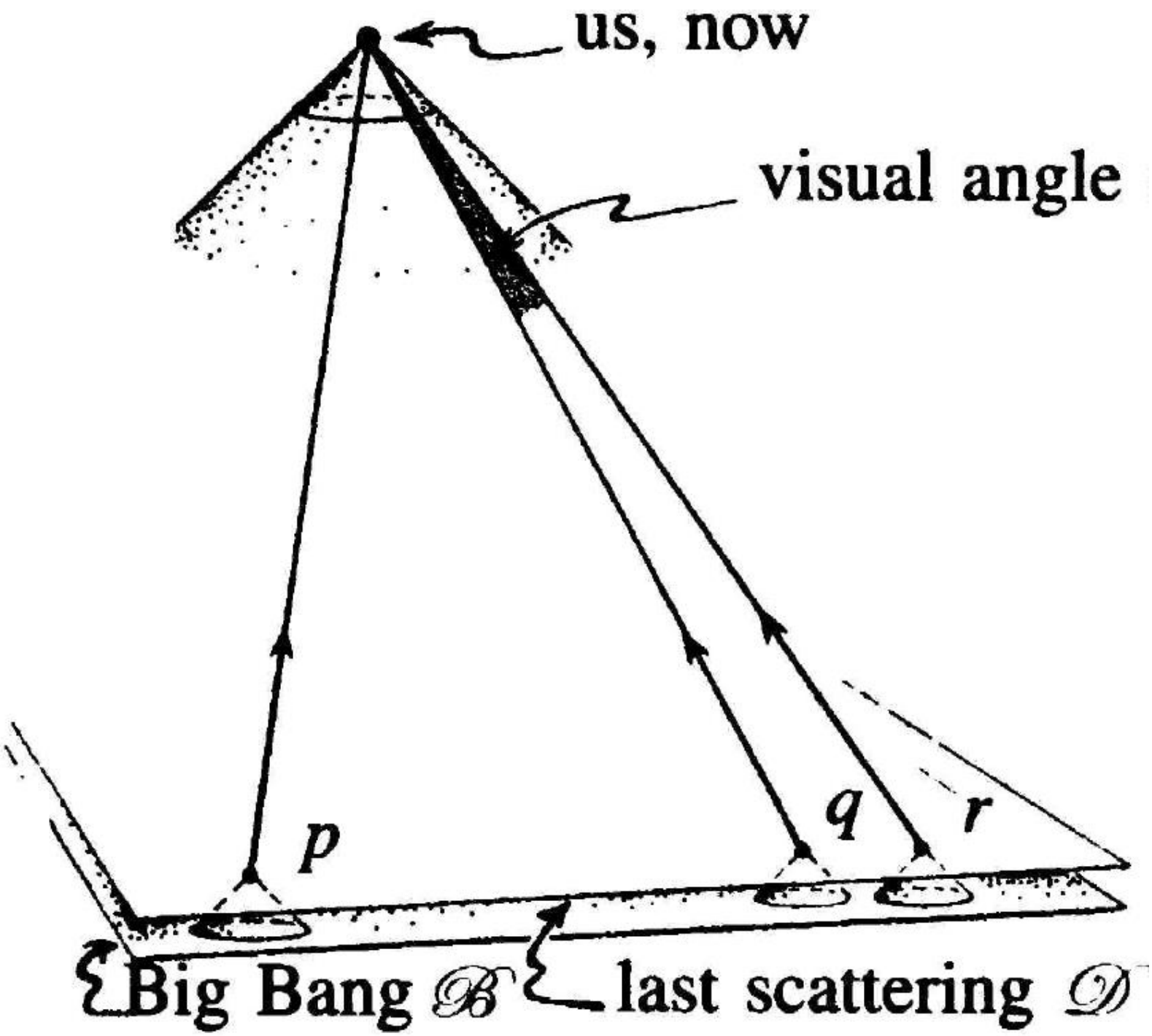


b)






Multipole moment, ℓ
2
10
50
500
1000
1500
2000
2500
Temperature fluctuations [μK^2]
6000
5000
4000
3000
2000
1000
0
90°
18°
1°
0.2°
0.1°
0.07°
Angular scale


us, now
visual angle
p
q
r
Big Bang $\mathscr{B}$
last scattering $\mathscr{D}$

During the matter dominated era the scaling factor a(t) increases as $t^{2/3}$,

Since the time of last scattering the horizon size was of the order

$$d_H \approx H_0^{-1}(1 + z_L)^{-3/2}$$

The angular diameter distance d_A to the surface of last scattering is of order

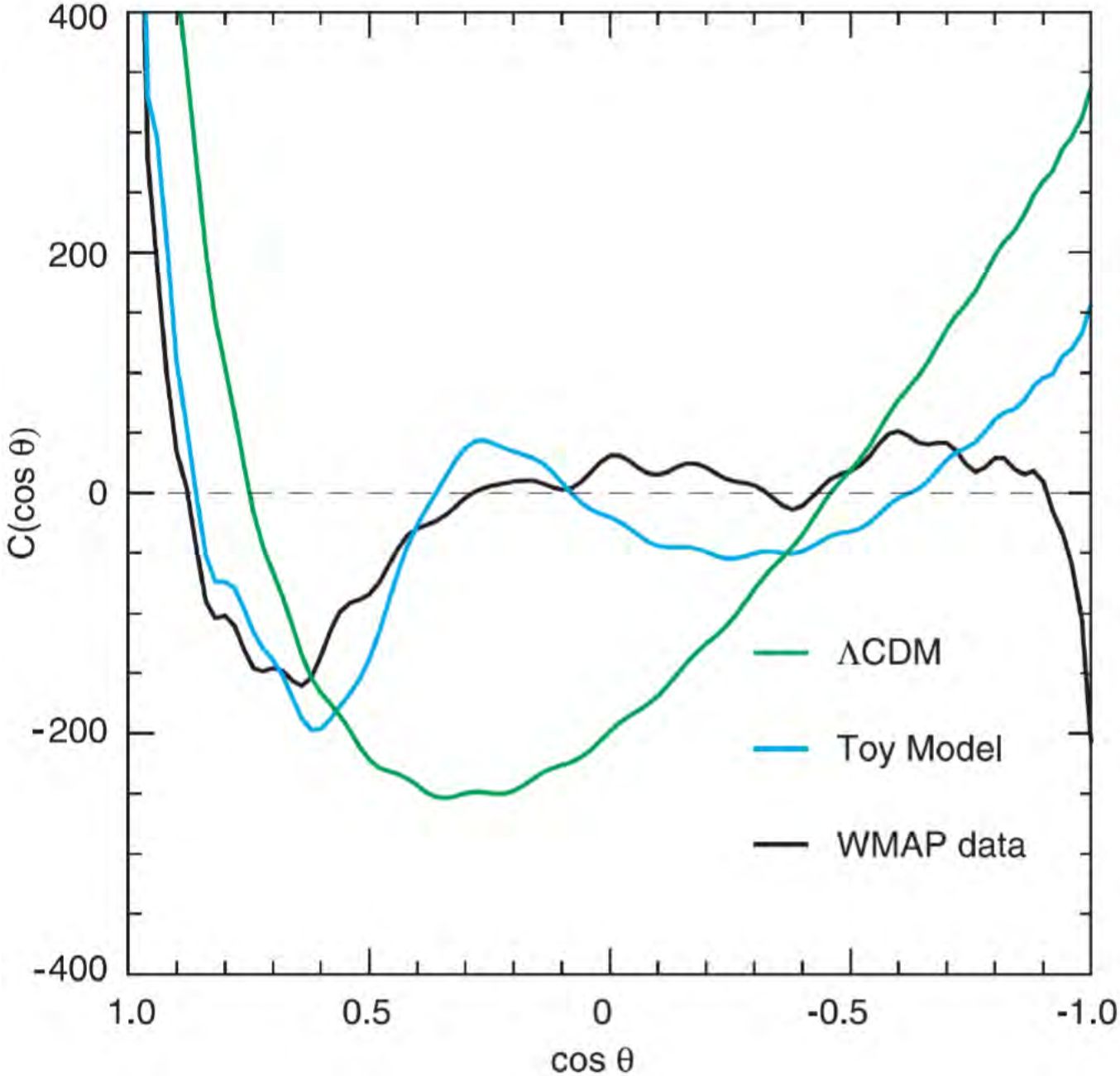
$$d_A \approx H_0^{-1}(1 + z_L)^{-1}$$

$$d_H/d_A \approx (1 + z_L)^{-1/2}$$

which is for $z_L \simeq 1100$ about 1.7°

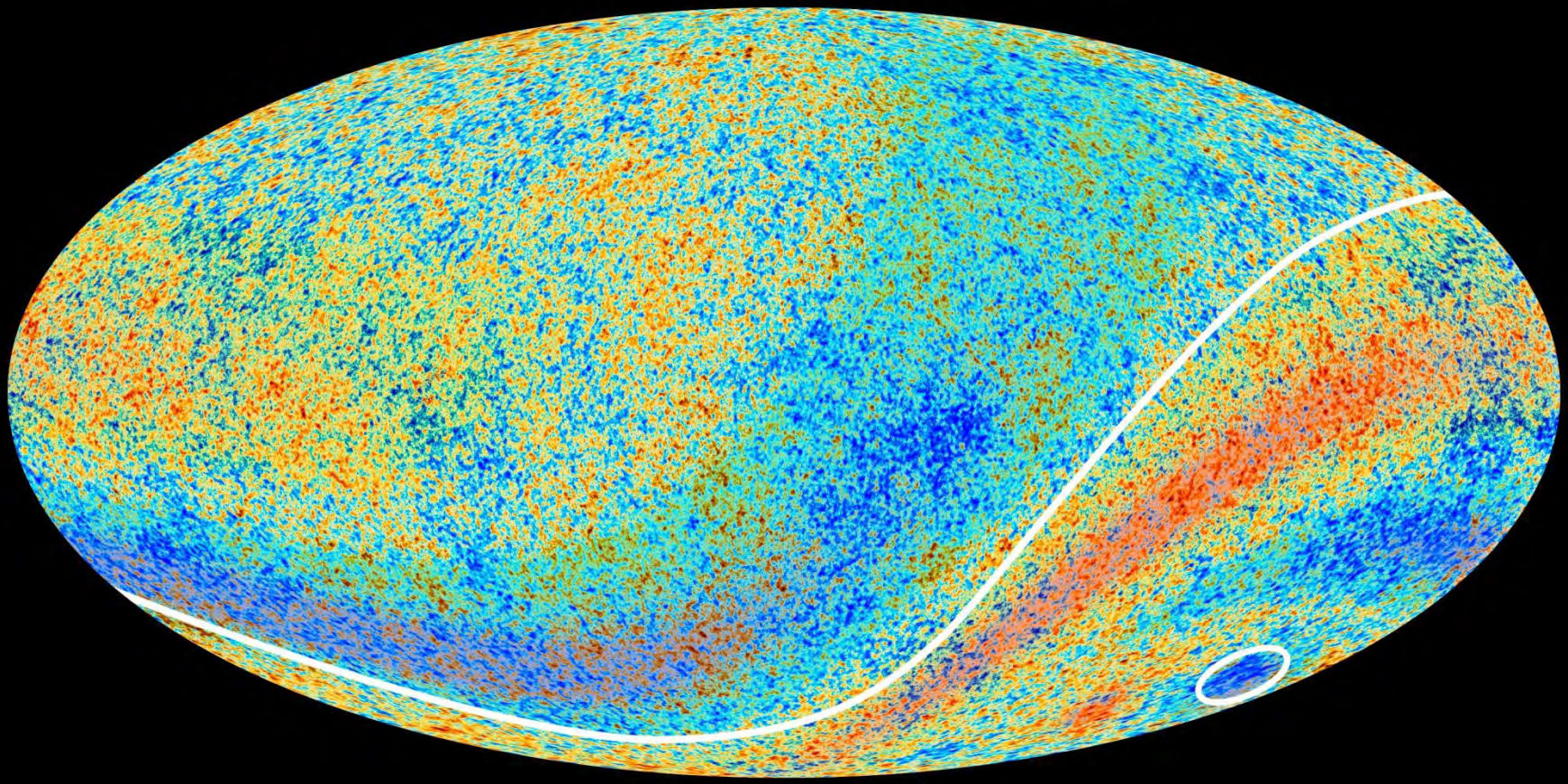
In CMB spectrum points further apart than several degrees should be not correlated, but correlations up to 60° are observed

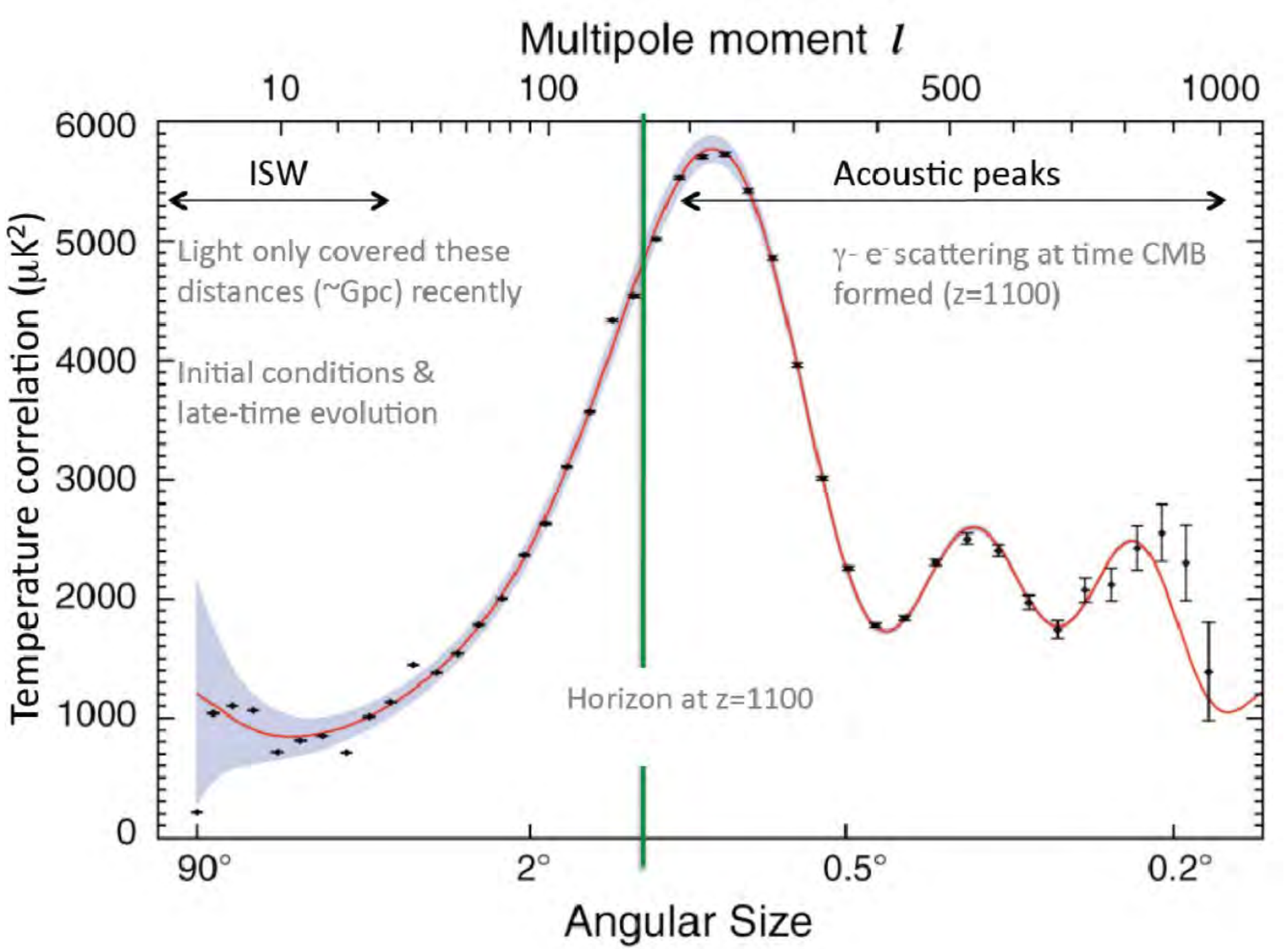
Inflation models require angular correlation at all angles, not only at angles up to 60°, because inflation occurred at all scales



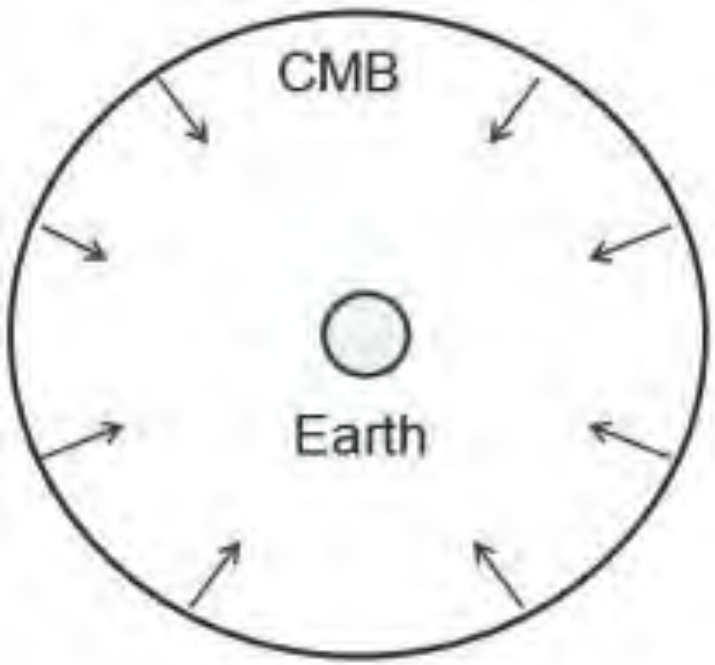
Angular correlation function of the best-fit ΛCDM model with WMAP data on large angular scales. (Adopted from C.L. Bennett et al. 2003)

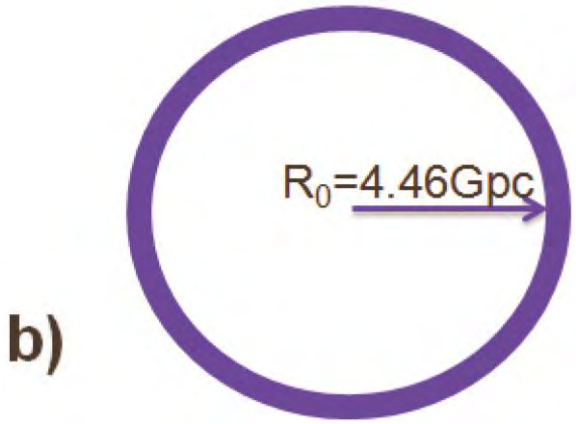
Planck anomalies




Multipole moment l
10
100
500
1000
6000
5000
4000
3000
2000
1000
0
Temperature correlation (μK^2)
ISW
Acoustic peaks
Light only covered these distances (~Gpc) recently
Initial conditions & late-time evolution
γ- e⁻ scattering at time CMB formed (z=1100)
Horizon at z=1100
90°
2°
0.5°
0.2°
Angular Size

The uniformity of the observed CMB requires a new interpretation





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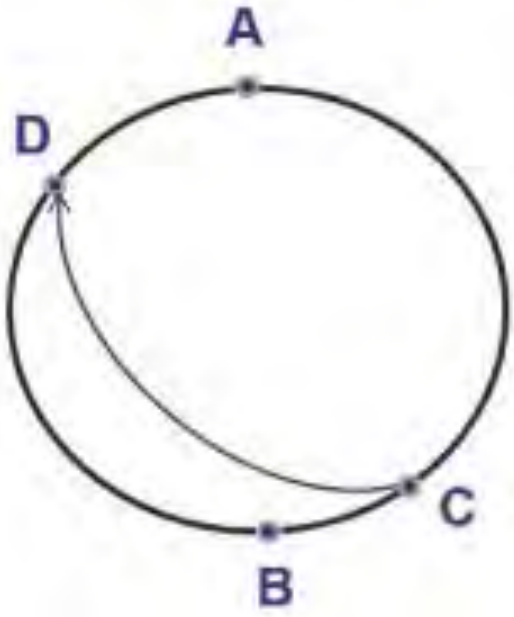


Comparison of two models

- No difference in the size of the visible universe
- Both definitions predict the same value for the particle horizon

  $$14.0 \pm 0.1 \text{ Gpc} = 45.7 \pm 0.3 \text{ Gly}$$

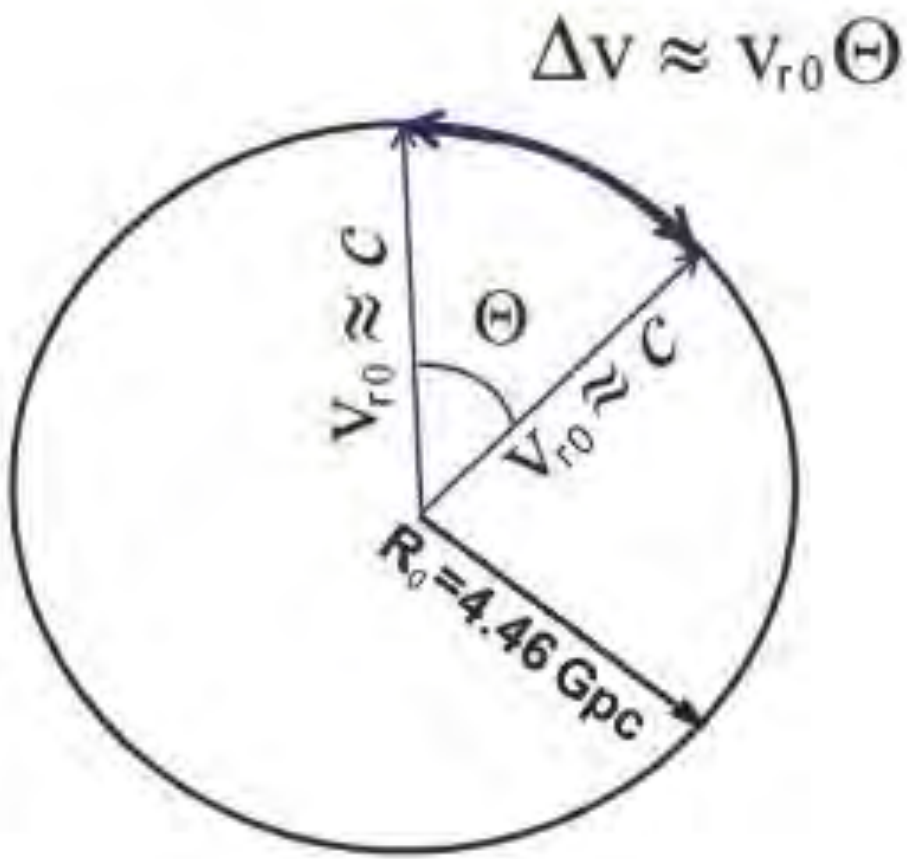
- Current definition – this is the radius

  $R_o = 14.0 \pm 0.1$ Gpc = 45.7 ± 0.3 Gly
- New definition – this is the length of the arc on the sphere

  $R_o = 4.5 \pm 0.05$ Gpc = 14.46 ± 0.15 Gly
- Different volume for at least 31 times

  Different density

The visible universe as expanding sphere or thin shell



$$v = H_0 \times distance$$

$$v = v_{r0}\Theta = H_0 R_0 \Theta$$

$$H = \frac{v_r}{R} \approx \frac{c}{R}$$

H_0 = 67.26 ± 0.90 km/s/Mpc

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2$$

$$H(0.5)/H_0 = 1.6,\ H(1.0)/H_0 = 2.3,\ \text{and}$$
$$H(1.4)/H_0 = 3.0$$

which agrees with observations

In the standard FLRW model

$$R_o = a(t)\int_{t'=0}^{t}\frac{c}{a(t')}dt' \qquad \frac{da}{dt} = \sqrt{\frac{\Omega_r}{a^2}+\frac{\Omega_m}{a}+\frac{\Omega_\Lambda}{a^{-2}}}$$

$$R_0 = 14.0 \pm 0.2 \text{ Gpc}$$

For closed universe in FRW metric

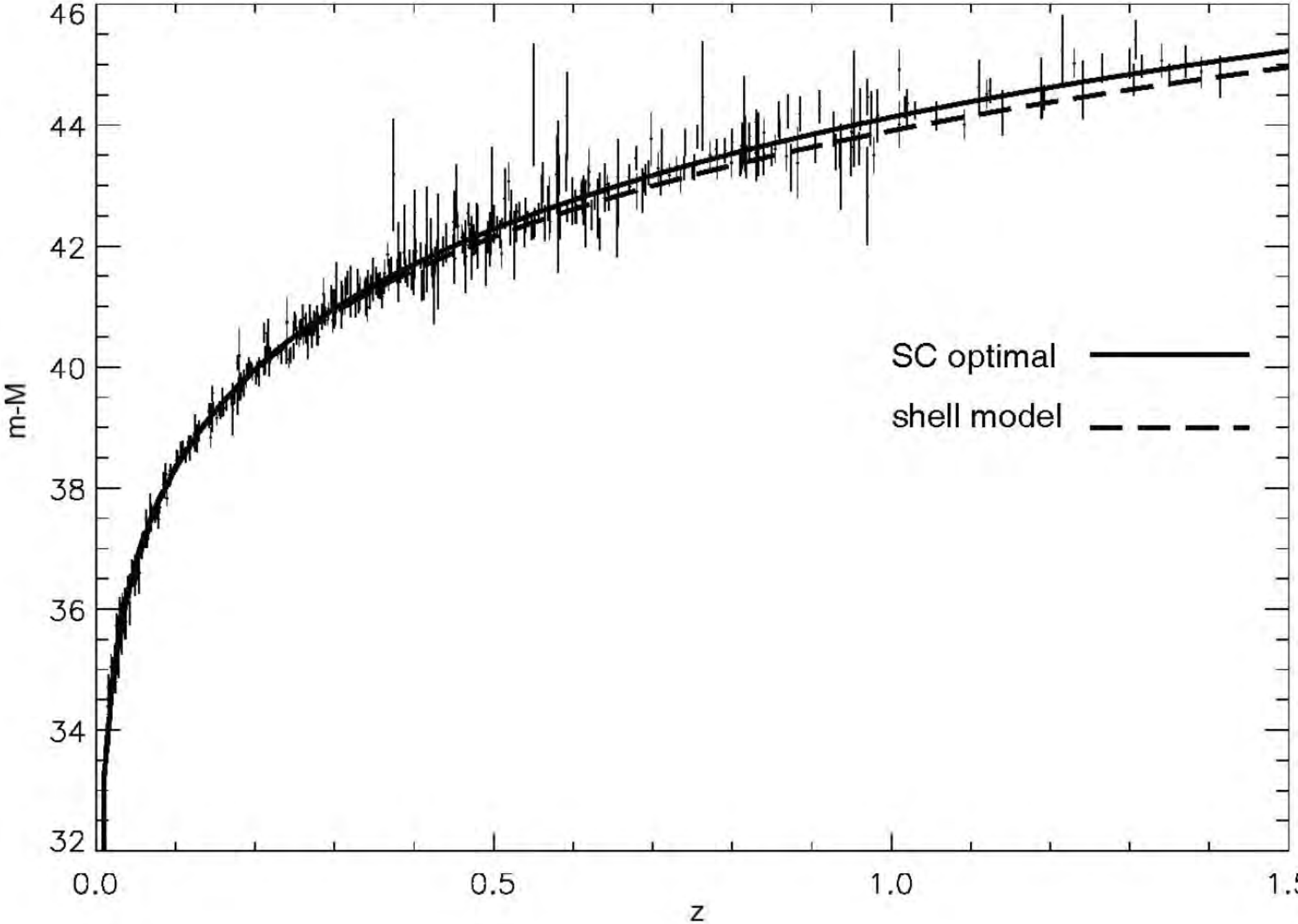
$$ds^2 = c^2dt^2 - a(t)^2[d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta d\phi^2)] \qquad \text{where } \chi = \int cdt/a(t)$$

a photon traveling on a hypersphere of radius a(t) covers an arc $d\eta = dt/a(t)$

$$d_L = r\sin(r/R_0)(1+z) \qquad c = H(z)R(z)$$

$$d_L = r\sin(r/R_0)(1+z) = R_0(1+z)\int_0^z \frac{\sin(r/R_0)}{H(z')}dz'$$

$$= \frac{c}{H_0}(1+z)\int_0^z \frac{\sin(\pi/[2(1+z/\pi)])}{1+z}dz'.$$



Suzuki et al. Union2.1 data 2012.

$$d_L = \frac{c}{H_0}(1+z)\int_{1/(1+z)}^{1} \frac{dx}{\sqrt{\Omega_r + x\Omega_m + x^4\Omega_\Lambda}}$$

where $x = a/a_0$

$$d_L = r\sin(r/R_0)(1+z) = R_0(1+z)\int_0^z \frac{\sin(r/R_0)}{H(z')}dz'$$

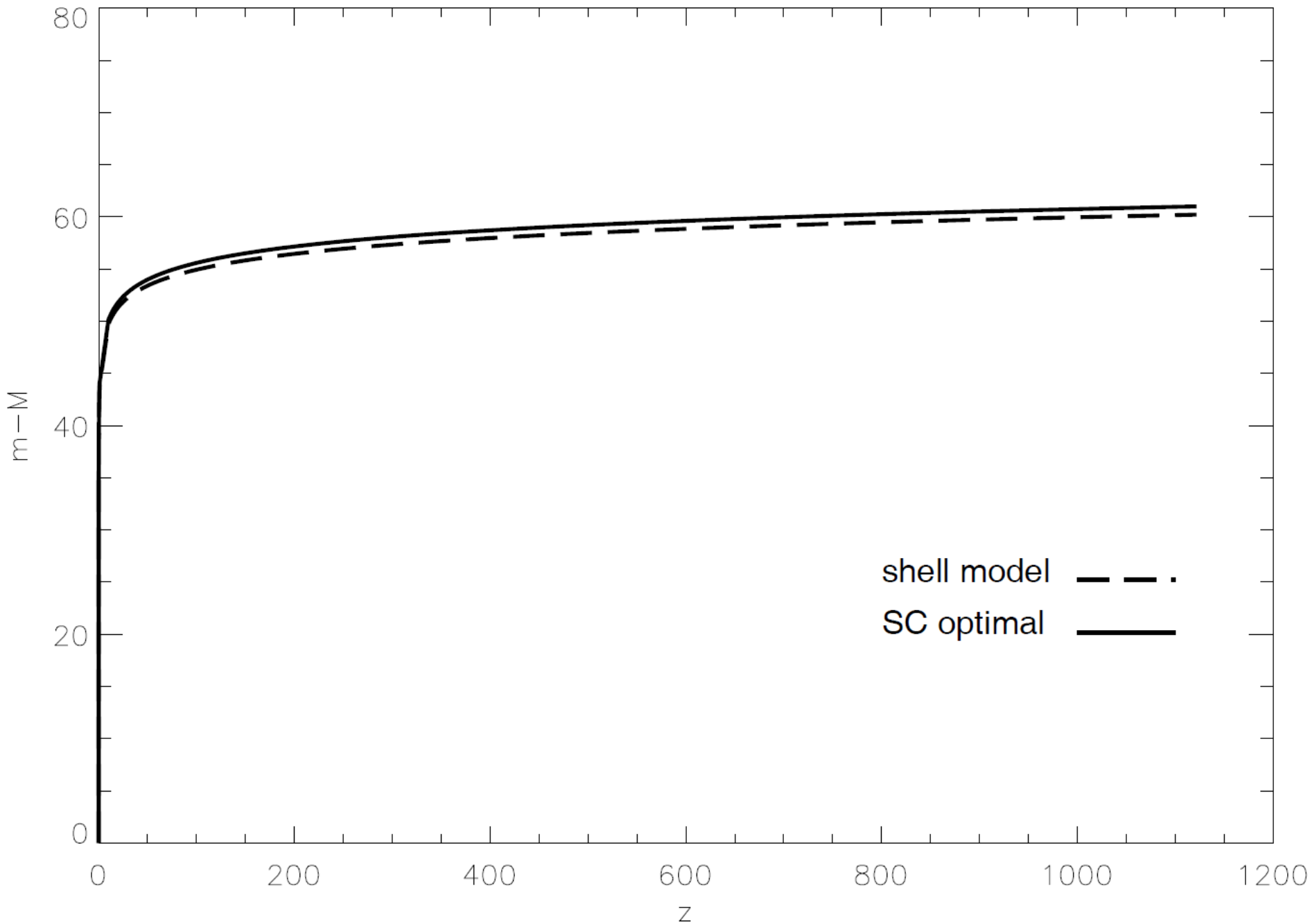
$$= \frac{c}{H_0}(1+z)\int_0^z \frac{\sin(\pi/[2(1+z/\pi)])}{1+z}dz'.$$

$$\chi^2 = \sum_i \frac{[\mu_i - 5log10(d_L(z_i)/10pc)]^2}{\sigma_i^2}$$

$\chi^2 = 563.81$ for the ΛCDM

$\chi^2 = 734.19$ for the simple shell model that expands with speed c

Suzuki et al. Union2.01 data 2012.


80
60
40
20
0
m−M
0
200
400
600
800
1000
1200
z
shell model
SC optimal

Is there a possibility from the mathematical point of view that the last scattering surface is approximately point-like in the closed FLRW Universe?

$$ds^2 = c^2 dt^2 - a^2(t)\left[d\chi^2 + \sin^2\chi\left(d\theta^2 + \sin^2\theta d\psi^2\right)\right]$$

Hyperspherical coordinates:

$$\chi \in [0,\pi]; \quad \theta \in [0,\pi]; \quad \psi \in [0,2\pi)$$

I. The pure ΛCDM model

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa\bar{\rho}c^4}{3a^3} + \frac{\Lambda c^2}{3} - \frac{c^2}{a^2} = H_0^2\left(\Omega_M \frac{a_0^3}{a^3} + \Omega_\Lambda + \Omega_K \frac{a_0^2}{a^2}\right)$$

$$\Omega_M = \frac{\kappa\bar{\rho}c^4}{3H_0^2 a_0^3}, \quad \Omega_\Lambda = \frac{\Lambda c^2}{3H_0^2}, \quad \Omega_K = -\frac{c^2}{a_0^2 H_0^2}, \quad \kappa = \frac{8\pi G_N}{c^4}$$

0.31 **0.69** **–0.001** **[~ PLANCK]**

0.535

The deceleration parameter

$$-q = \frac{1}{aH^2}\frac{d^2a}{dt^2} = \left(\frac{H_0}{H}\right)^2 \left\{-\frac{\Omega_M}{2}\left(\frac{a_0}{a}\right)^3 + \Omega_\Lambda\right\}$$

Dimensionless quantities:

$$\tilde{a} = \frac{a}{a_0}, \quad \tilde{t} = H_0 t, \quad \tilde{a}(0) = 1, \quad \tilde{a}(-\tilde{t}_0) = 0$$

The age of the Universe:

$$\tilde{t}_0 = \int_0^1 \frac{\sqrt{\tilde{a}}\, d\tilde{a}}{\sqrt{\Omega_M + \Omega_K \tilde{a} + \Omega_\Lambda \tilde{a}^3}}$$

$$\downarrow$$

0.96 $\longrightarrow$ **13.9 billions of years**

$$H_0 \approx 2.2 \times 10^{-18}\,\mathrm{s}^{-1}$$

The approximate condition of light traveling between the antipodal points during the age of the Universe

$$\int_{-t_0}^{0} \frac{cdt}{a(t)} = \pi$$

$$\sqrt{-\Omega_K} \int_{0}^{1} \frac{d\tilde{a}}{\sqrt{\Omega_M \tilde{a} + \Omega_K \tilde{a}^2 + \Omega_\Lambda \tilde{a}^4}} \approx 0.1 \neq \pi$$

Thus, this condition is not satisfied in the standard cosmological model.

The disadvantage of the spherical space

The gravitational potential

$$\phi = 2C_1 \cos\chi - G_N m_0 \left(\frac{1}{\sin\chi} - 2\sin\chi \right)$$

of a point-like mass m_0 **resting at** $\chi = 0$
has the Newtonian limit at this point

$$\phi \underset{\chi \to 0}{\to} -\frac{G_N m_0}{\chi}$$

but is divergent at the antipodal one

$$\chi = \pi$$

II. ΛCDM + Quintessence with w = -1/3

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = H_0^2\left(\Omega_M \frac{a_0^3}{a^3} + \Omega_\Lambda + \Omega_Q \frac{a_0^2}{a^2} + \Omega_K \frac{a_0^2}{a^2}\right)$$

$$\begin{cases} \Omega_M + \Omega_\Lambda + \Omega_Q + \Omega_K = 1 \\ -\dfrac{\Omega_M}{2} + \Omega_\Lambda = -q = 0.535 \end{cases}$$

$$\underline{\Omega_M} = \frac{2}{3}\left(1 - \Omega_Q - \Omega_K + q\right) \qquad \underline{\Omega_\Lambda} = \frac{2}{3}\left(\frac{1}{2} - \frac{\Omega_Q}{2} - \frac{\Omega_K}{2} - q\right)$$

$$\sqrt{-\Omega_K}\int_0^1 \frac{d\tilde{a}}{\sqrt{\Omega_M \tilde{a} + \Omega_Q \tilde{a}^2 + \Omega_K \tilde{a}^2 + \Omega_\Lambda \tilde{a}^4}} = \pi$$

Two examples

A) Exact compensation: $-\Omega_K = \Omega_Q = 0.93$

$$\Omega_M = 0.31, \quad \Omega_\Lambda = 0.69$$

$$\varphi = \frac{G_N m_0}{2\pi} - G_N m_0 \frac{\cos\chi}{\sin\chi}\left(1 - \frac{\chi}{\pi}\right), \quad 0 < \chi \le \pi$$

B) Visible matter: $\Omega_M = 0.040, \quad \Omega_\Lambda = 0.555$

$$\Omega_Q = 0.721, \quad \Omega_K = -0.316$$

$$d_{L(\mathrm{I,II})}(z) = \frac{c}{H_0}(1+z)\sqrt{-1/\Omega_K}\,\sin\left[\sqrt{-\Omega_K}\int_{\frac{1}{1+z}}^{1}\frac{da}{\sqrt{a\Omega_M + a^4\Omega_\Lambda + a^2\left(\Omega_Q + \Omega_K\right)}}\right]$$

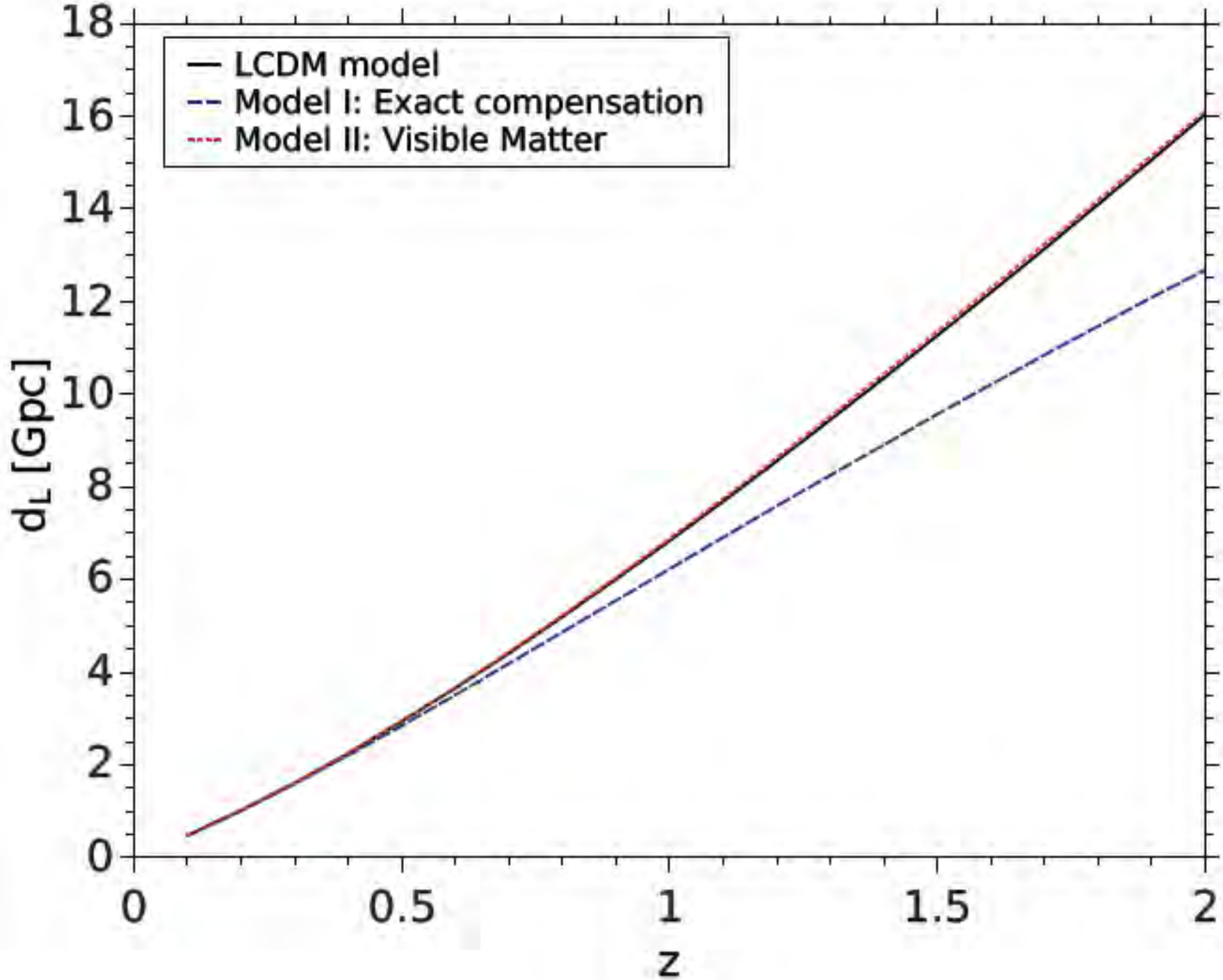
$$d_{L(\Lambda CDM)}(z) = \frac{c}{H_0}(1+z)\int_{\frac{1}{1+z}}^{1}\frac{da}{\sqrt{a\Omega_M + a^4\Omega_\Lambda + a^2\Omega_K}}$$

A) Exact compensation

z	a/a_0	-t, Gyr	-ct, Gpc	d_L, Gpc	$d_{L(\Lambda CDM)}$, Gpc
0.1	0.91	1.34	0.41	0.48	0.48
0.5	0.67	5.17	1.59	2.85	2.94
1.0	0.50	7.99	2.45	6.23	6.84
1.5	0.40	9.58	2.94	9.57	11.27
2.0	0.33	10.63	3.26	12.69	16.04
1000	0.001	13.89	4.26	489.4	14046

B) Visible matter

z	a/a_0	-t, Gyr	-ct, Gpc	d_L, Gpc	$d_{L(\Lambda CDM)}$, Gpc
0.1	0.91	1.34	0.41	0.48	0.48
0.5	0.67	5.23	1.61	2.95	2.94
1.0	0.50	8.26	2.54	6.89	6.84
1.5	0.40	10.13	3.11	11.36	11.27
2.0	0.33	11.45	3.51	16.11	16.04
1000	0.001	16.86	5.17	1385.7	14046


LCDM model
Model I: Exact compensation
Model II: Visible Matter
d_L [Gpc]
z
0
2
4
6
8
10
12
14
16
18
0
0.5
1
1.5
2

Tests of the size – holographic principle

Frampton - by the holographic principle the entropy S/k has an upper limit equal to that of a black hole:

$$\left(\frac{S}{k}\right)_{Uni} \leq \left(\frac{S}{k}\right)_{BH} = \frac{4\pi R_S^2}{l_P^2}$$

$$\frac{\left(\frac{S}{k}\right)_{Uni}}{\left(\frac{S}{k}\right)_{BH}} = R_{BET}^4 \leq 1$$

$$R_{BET} = \frac{\sqrt{\Omega_m H_0^2}}{c} R_0$$

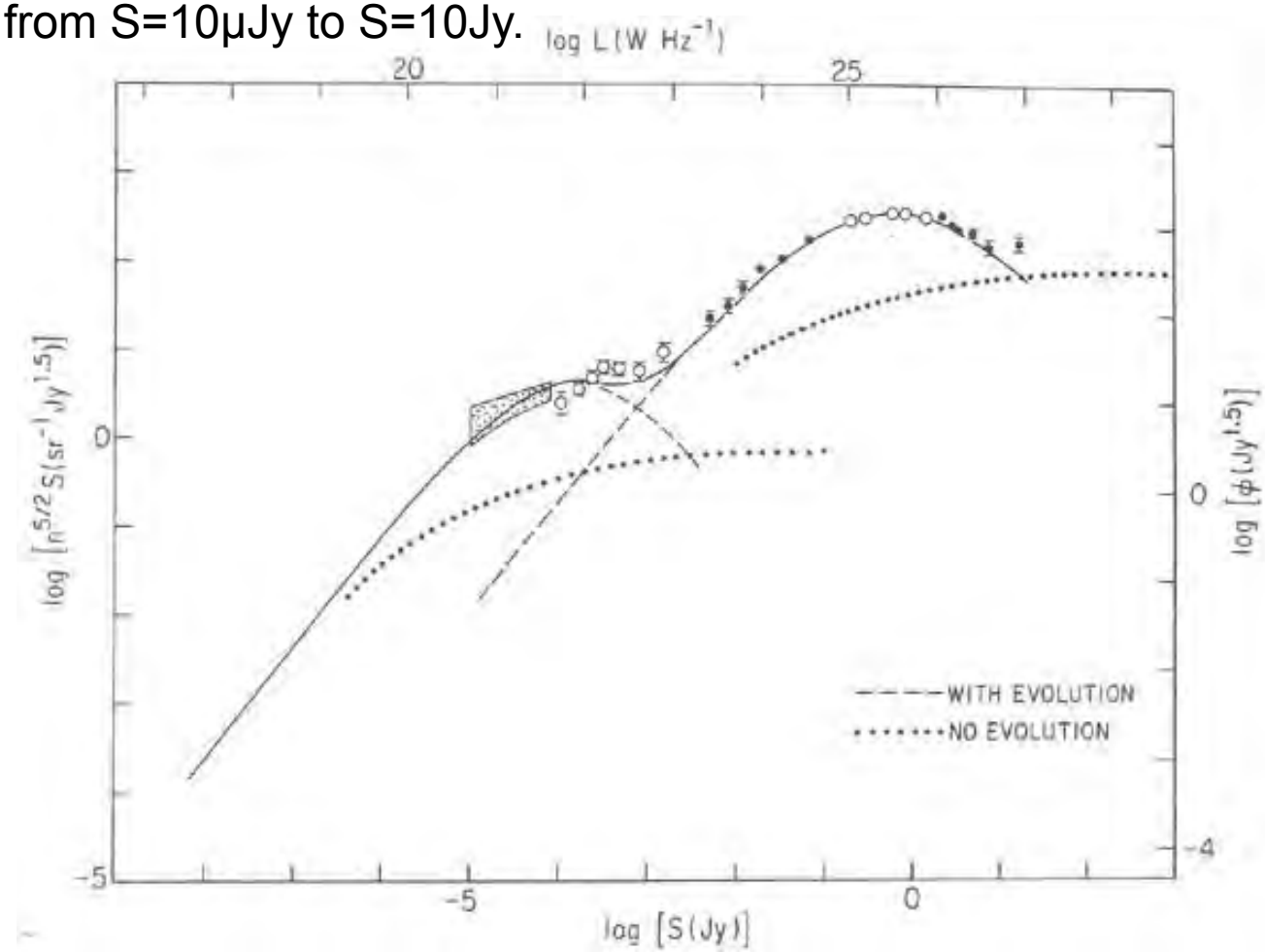
$$R_{BET}^4 = 8.85 \pm 0.1$$

$$R \leq 8.4 \pm 0.1 Gpc$$

Distribution of radio sources supports the model

The ratio of flux density to luminosity and angular size to projected linear size at different redshifts can be used to test geometrical properties of the model.

A hollow shell model reproduces the entire observed weighted radio sources count from S=10μJy to S=10Jy.



Superposition of the weighted source count at 1.4 GHz (data points) and the hyperbolic fits for radio sources in spiral and elliptic galaxies (dashed lines). The solid curve plots the weighted source count predicted by shell model. J. J. Condon in "Galactic and extragalactic radio astronomy" 1988.

Most sources are confined to a thin shell of thickness Δz_s at redshift z_s.

Conclusion

- In the ΛCDM model supplemented by the quintessence with w = -1/3 in the spherical space there is an elegant solution of the horizon problem without inflation: under the proper choice of the parameters light travels between the antipodal points during the age of the Universe. In the constructed model the gravitational potential of each mass is convergent.
- Explains uniformity of CMB, originated at the single point or region at antipode, without inflation
- Model predicts R_0=14 Glyr, particle horizon 45.6 Glyr, H_0 = 67.26 ± 0.90 km/s/Mpc, and expansion rate $H(z)$ in agreement with observations.
- In agreement with observable data – SN Ia distance – redshift relation

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