Electronic LET for slow ions in dark matter detector media

Akira Hitachi  Kochi Medical School

The leading candidates for Dark Matter:
- Weakly Interacting Massive Particles (WIMPs)
- Low energy recoil ions may be detected.
  - Scintillation and/or ionization
    - Information on yield, decay shape, S/T ratio etc. are needed.

The electronic LET (linear energy transfer) is the specific electronic energy deposited along the charged particle track.

WIMP

\[ m \sim 100-1000 \text{ GeV} \]
\[ v = 230 \text{ km/s} \]

Recoil ion

\( < 100 \text{ keV} \)
Linear Energy Transfer (LET)

LET: The energy deposited per unit length

\[ \text{LET} \equiv \frac{-dE}{dx} \quad \text{for fast ions} \]

\[ S_T \approx S_e \]

The electronic LET

\[ \text{LET}_{el} = \frac{-d\eta}{dx} \]

should be introduced for slow ions

\[ S_T = S_e + S_n \]

The ionization density is given by \( \text{LET}_{el} \) in liquid and solid scintillators.

[not by the electronic SP, \( \frac{dE}{dx}_{el} \) ]

The Bragg-like curve for TPC practical & macroscopic

The quenching calc., S/T ratio etc.
Energy shearing in low energy

For slow ions, $\nu < \nu_0 = \frac{e^2}{\hbar}$, $S_e$ and $S_n$ are similar in magnitude. The secondaries, recoil atoms and electrons, may again go to the collision process and transfer the energy to new particles and so on. After this cascade process completes, the energy of the incident particle $E$ is given to atomic motion $\nu$ and electronic excitation $\eta$.

\[
q_{nc} = \frac{\eta}{E} = \frac{\eta}{E} \quad \text{(Lindhard factor)}
\]

integrated for individual recoil created in the cascade, the energy that went to electronic excitation.

\[
\text{RN/\gamma ratio} = \frac{(q_{nc} \cdot q_{el})}{L_{\gamma}}
\]
electronic Linear Energy Transfer (LET<sub>el</sub>)

Scintillation as well as its quenching are electronic processes. We introduce the electronic LET.

\[ \text{LET}_{el} \equiv -d\eta/dR = -\Delta\eta/\Delta R \quad R: \text{the range} \]
\[ = -(\eta_1-\eta_0)/(R_1-R_0) \]
for quenching calc. etc.

The true range \( R \) is given by the total stopping power
\[ R_T = \int (dE/dx)_{\text{total}}^{-1} \, dE \]

The Bragg-like curve for TPC

The projected range, \( R_{PRJ} \), may be used (depth)
Lindhard factor \( \eta/\varepsilon \)

The stopping powers contain only a part of the necessary information to obtain the quenching factor, \( \eta/\varepsilon \) ratio. The differential cross section in nuclear collisions is needed for the integral equations.

\[
\left( \frac{d\varepsilon}{d\rho} \right)_\varepsilon \cdot \nu'(\varepsilon) = \int_0^{\varepsilon^2} \frac{dt}{2t^{3/2}} \cdot f\left(t^{1/2}\right) \left\{ \nu\left(\varepsilon - \frac{t}{\varepsilon}\right) - \nu(\varepsilon) + \nu\left(\frac{t}{\varepsilon}\right) \right\}
\]

\[
\left( \frac{d\varepsilon}{d\rho} \right)_\varepsilon = k \varepsilon^{1/2}
\]

\( \varepsilon \): the dimension less reduced energy
\( \rho \): range

\( \eta \) as a fn of \( \varepsilon \) for \( k = 0.2, 0.15, 0.1 \)

For \( Z_1 = Z_2, \varepsilon > 0.01 \)
Stopping Power and electronic LET

Fig. The stopping power and the electronic LET as a function of the recoil energy for Xe in Xe, and Cs and I ions in CsI.

HMI & Lindhard
RN/γ ratios for LXe and CsI(Tl)

I, Xe and Cs are adjacent to each other in the periodic table.

\[ SP(I) \approx SP(Cs) \approx SP(Xe) \]
\[ q_{nc}(I) \approx q_{nc}(Cs) \approx q_{nc}(Xe) \]

Different energy dependences on RN/γ for recoil ions for CsI and LXe are due to \( q_{el} \).

\[
RN/\gamma = \frac{(q_{nc} \cdot q_{el})}{L_{\gamma}}
\]

Lindhard

\[ q_{el} \sim 1 \text{ for Gas, Si, Ge} \]
\[ q_{el} < 1 \text{ for CsI, NaI, LAr, LXe} \]

LET\(_{el}\)

Exciton-exciton collisions

Diffusion

実験値: DAMA, ICARUS, ZEPLIN, XENON, Coimbra.
$\alpha/\gamma$ ratio
Stopping power theory is not very good for $E < 10$ keV for Xe-Xe
Needs MO-theory which is quite complicated.
The energy partition between ionization, excitation and sub-excitation $e^{-}$ may be changed.
$W$-value measurements in gas gives information on $q_{nc}$. 
**Inorganic Scintillators**

**Fig.** Nuclear quenching factor $q_{nc}$ for recoil ions in NaI, CsI, CaF$_2$, and BaF$_2$ as a function of recoil ion energy.

**Fig.** Electronic LET for recoil ions in NaI, CsI, and CaF$_2$ as a function of energy.

**Fig.** Scintillation efficiency for various particles as a function of LET in NaI(Tl). The horizontal bars show regions of electronic LET for recoil ions. Murray & Meyer, PR 122, 815 (1961).

**Fig.** Scintillation efficiency for protons and He ions (Zhang, 2008) as a function of differential LET in CaF$_2$(Eu). Horizontal bars show electronic LET ($=-d\eta/dx$) for recoil ions.
Quenching factor for Inorganic Scintillators

\[ \frac{R_N}{\gamma} = \frac{q_T}{L_g} = \frac{(q_{nc} \cdot q_{el})}{L_\gamma} \]

\[ q_{nc} \Rightarrow \text{LET}_{el} \Rightarrow q_{el} \]

Scintillation efficiency \( L_\gamma \) in NaI depends on LET and the velocity. We take values for slow ions.

![Graph showing nuclear quenching factor, \( q_{nc} \), and \( \frac{R_N}{\gamma} \) ratios for Ca and F ions in CaF\(_2\)(Eu) as a function of recoil ion energy. Solid curves are \( q_{nc} \) (p.w.). Broken curves are \( \frac{R_N}{\gamma} \) ratio (p.w.). Small dots represent \( q_{nc} \) using TRIM code (BPRS). The dot-dashed curves are \( \frac{R_N}{\gamma} \) ratio, fitting and extrapolated to the low energy region assuming that the \( q \)-value depends inversely on \( S_T \) (Osaka).](image)

\[ \text{Ca depends on sensitizer [TI] concentration.} \]
Head-Tail Detection

The directional detection of low-energy recoil ions can provide strong possibility for observation of dark matter in the Galaxy.

Detectors with directional capabilities can observe daily fluctuations.

gas TPC (Time Projection Chamber)

The Bragg-like curve
Bragg-like curve $d\eta/dR_{PRJ}$

57keV N in air

ions

N$_2$

Lindhard model

no straggling


136keV Ar in Ar

Evans, PR 90, 825 (1953)

$\Delta R_{\perp}$ \Rightarrow bell shape

$\Delta R_{\perp}$ & $\Delta R_{//}$ \Rightarrow drop-shape
No expressions for $q_{nc}$ in compounds are available. We use a simple approximation.

An independent element approach

$$q_{nc}(C/CS_2) \approx q_{nc}(C/C)$$
$$q_{nc}(S/CS_2) \approx q_{nc}(S/S)$$

The model should be tested in ionization measurements in gas. The simple approach is possible since $S_e/S_T$ ratios for collisions with homo- and hetero-atoms are close to each other.

--

Lindhard factor in compounds

Solid curves; the independent element approach


expt. & part simulation by Snowden-Ifft
**Head-Tail CS$_2$ CF$_4$**

Bragg-like curve $d\eta/dR_{PRJ}$ charge distribution in recoil direction

**CS$_2$**
- low diffusion CS$^-$ ions
- release $e^-$ at the anode and get electron multiplication

**CF$_4$**
- charge carrier is electron
- F is effective for WIMP search via spin-dependent interaction

---

Projected Range (μg/cm$^2$)

$d\eta/dR_{PRJ}$ (MeV cm$^2$/mg)

Recoil ions

---

Projected range (μg/cm$^2$)

$Δ\eta$ (ions cm$^2$/μg)

---

$Δx$ (ions cm$^2$/μg)
Recoil ions in $\alpha$-decay

A power law approximation for $Z_1 \neq Z_2$ at very low energy (Lindhard 1963),

$$\eta = CE^{3/2}, \quad E < E_{1c}, E_{2c}$$

$$C = \frac{2}{3}\gamma^{-1/2} + \frac{1}{2} \gamma^{-1/2} E_c^{-1/2},$$

$$\gamma = 4A_1 A_2 / (A_1 + A_2)^2$$

$$E_c = \gamma E_{2c}.$$  

Energy Straggling

Power law approximation

$$\sigma^2 = \frac{\Omega^2}{\eta^2}$$

$$\sigma \sim 14\% \quad \text{LAr}$$

Recoil Pb ions in $\alpha$-decay in LAr

110 keV  $\sigma \sim 12\%$

144 keV  $\sigma \sim 11\%$

Daughter nuclei are produced into an ionized state

Recoil energy = 110 keV

Recoil energy = 144 keV

WARP 2007
### Lindhard factor for recoil ions in $\alpha$-decay

<table>
<thead>
<tr>
<th>Recoil ion</th>
<th>$^{206}$Pb</th>
<th>$^{208}$Tl</th>
<th>$^{208}$Pb</th>
<th>$E_{2c}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy keV</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>gas</td>
<td>103</td>
<td>117</td>
<td>168</td>
<td></td>
</tr>
<tr>
<td>expt calc</td>
<td>expt calc expt calc</td>
<td>keV</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ar</td>
<td>0.221a</td>
<td>0.196</td>
<td>0.263</td>
<td>0.205</td>
</tr>
<tr>
<td>Xe</td>
<td>0.139 a</td>
<td>0.124</td>
<td>0.132</td>
<td>0.158</td>
</tr>
<tr>
<td>H$_2$</td>
<td>0.73</td>
<td>0.457</td>
<td>0.78</td>
<td>0.544</td>
</tr>
<tr>
<td>He</td>
<td>0.53</td>
<td>0.500b</td>
<td>0.56</td>
<td>0.546 b</td>
</tr>
<tr>
<td>CH$_4$</td>
<td>0.250</td>
<td>0.265</td>
<td></td>
<td>0.307</td>
</tr>
<tr>
<td>C$_2$H$_4$</td>
<td>0.236</td>
<td>0.269</td>
<td></td>
<td>0.321</td>
</tr>
<tr>
<td>C$_3$H$_6$</td>
<td>0.272</td>
<td></td>
<td></td>
<td>0.281</td>
</tr>
<tr>
<td>CO$_2$</td>
<td></td>
<td>0.336</td>
<td></td>
<td>0.347</td>
</tr>
<tr>
<td>C + 2O</td>
<td>0.297</td>
<td>0.316</td>
<td></td>
<td>0.378</td>
</tr>
<tr>
<td>C</td>
<td>0.323</td>
<td>0.344</td>
<td></td>
<td>0.411</td>
</tr>
<tr>
<td>O</td>
<td>0.284</td>
<td>0.302</td>
<td></td>
<td>0.361</td>
</tr>
<tr>
<td>N$_2$</td>
<td>0.319</td>
<td>0.302</td>
<td>0.320</td>
<td>0.384</td>
</tr>
<tr>
<td>Dry air</td>
<td>0.296</td>
<td></td>
<td>0.317</td>
<td>0.379</td>
</tr>
<tr>
<td>4N + O</td>
<td>0.298</td>
<td></td>
<td>0.317</td>
<td>0.379</td>
</tr>
<tr>
<td>CS$_2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C + 2S</td>
<td>0.246</td>
<td>0.262</td>
<td></td>
<td>0.314</td>
</tr>
<tr>
<td>S</td>
<td>0.208</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Al (for CS$_2$)</td>
<td>0.228</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Recoil ion</th>
<th>$^{214}$Pb</th>
<th>$^{210}$Pb</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy keV</td>
<td>112</td>
<td>147</td>
</tr>
<tr>
<td>CS$_2$</td>
<td>0.346</td>
<td>0.397</td>
</tr>
<tr>
<td>C + 2S</td>
<td>0.253</td>
<td>0.292</td>
</tr>
<tr>
<td>Al (for CS$_2$)</td>
<td>0.237</td>
<td>0.272</td>
</tr>
</tbody>
</table>

**calc**: the power law approximation by Lindhard which is only good at $E < E_{2c}$.

**expt**: $W(\alpha)/W(\text{recoil})$ from:

Summary

a) The Lindhard model looks inconsistent with experimental results in inorganic crystals in the first sight. However, the inclusion of electronic (scintillation) quenching, the model still gives the best results.

b) The electronic LET (=dη/dx) plays important role in the evaluation of scintillation quenching.

c) The electronic LET for recoil ions is close to LET for α-particles. Therefore, the scintillation characteristics, such as decay shape, S/T ratio and the ionization yield due to recoil ions are expected to be similar to those due to α-particles.

d) The Bragg-like curve is useful for estimation of head-tail detection.

The scintillation efficiency for γ-rays, protons, α-particles and fission fragments are needed to evaluate the quenching factor for recoil ions.

Refinement of stopping theory for low energy ions is required.

Refs.