Overview of non-Gaussian models

Marilena Loverde
AMIAS Member, Institute for Advanced Study
Inflation as the origin of structure

- Quantum fluctuations
  - Inflation
  - Small matter & energy fluctuations
  - Curvature
  - \( \delta T_{\text{CMB}} \)
  - \( \delta \rho_{\text{matter}} \)
  - \( \delta n_{\text{galaxies}} \)

- Gravitational collapse

Sloan stars, galaxies, clusters of galaxies
Inflation as the origin of structure

quantum fluctuations

inflation

small matter & energy fluctuations

WMAP

gravitational collapse

stars, galaxies, clusters of galaxies

inflaton

curvature

T

CMB

matter

n
galaxies

statistics of these

probe of this era

$\delta \varphi_{\text{inflaton}} \rightarrow \Phi_{\text{curvature}} \sim \delta T_{\text{CMB}} \sim \delta \rho_{\text{matter}} \sim \delta n_{\text{galaxies}}$
reminder: simplest option, \textbf{Gaussian}

\[ \Phi_{\text{curvature}} \]

\[
\langle \Phi(x)\Phi(y) \rangle \leftrightarrow P_\Phi(k)
\]

\[ k^2 P_\Phi(k) \quad \text{power per mode} \]

probability distribution:

\[ \Phi \text{ value} \]
reminder: simplest option, **Gaussian**

\[ \Phi_{\text{curvature}} \]

**two-point function:**
\[ \langle \Phi(x)\Phi(y) \rangle \leftrightarrow P_\Phi(k) \]

\[ k^2P_\Phi(k) \quad \text{power per mode} \]

\[ \langle \Phi(x)\Phi(y)\Phi(z) \rangle = 0 \]
\[ \langle \Phi(x)\Phi(y)\Phi(z)\Phi(w) \rangle = \langle \Phi(x)\Phi(y) \rangle \langle \Phi(z)\Phi(w) \rangle \]
\[ + \langle \Phi(x)\Phi(z) \rangle \langle \Phi(y)\Phi(w) \rangle \]
\[ + \langle \Phi(x)\Phi(w) \rangle \langle \Phi(y)\Phi(z) \rangle \]

\[ \langle \Phi(x)\Phi(y)\Phi(z)\Phi(w)\Phi(s) \rangle = 0 \]

\[ \ldots \]
reminder: simplest option, Gaussian

\( \Phi_{\text{curvature}} \)

two-point function:
\[ \langle \Phi(x)\Phi(y) \rangle \leftrightarrow P_{\Phi}(k) \]

\[ k^2 P_{\Phi}(k) \quad \text{power per mode} \]

vanishing or trivially related to two-point

\[ \langle \Phi(x)\Phi(y)\Phi(z) \rangle = 0 \]
\[ \langle \Phi(x)\Phi(y)\Phi(z)\Phi(w) \rangle = \langle \Phi(x)\Phi(y) \rangle \langle \Phi(z)\Phi(w) \rangle \]
\[ + \langle \Phi(x)\Phi(z) \rangle \langle \Phi(y)\Phi(w) \rangle \]
\[ + \langle \Phi(x)\Phi(w) \rangle \langle \Phi(y)\Phi(z) \rangle \]
\[ \langle \Phi(x)\Phi(y)\Phi(z)\Phi(w)\Phi(s) \rangle = 0 \]
\[ \ldots \]
single-field, slow-roll inflation predicts this

observations suggest IC’s are nearly Gaussian

BUT small departures may exist and could provide one of few observational handles on physics of inflation
Example mildly non-Gaussian initial conditions
\( \Phi(x) \sim \delta \sigma(x) + f_{NL} \delta \sigma(x)^2 \)

Salopek and Bond 1990; Gangui, Lucchin, Matarrese, Mollerach 1994; Komatsu and Spergel 2001

\[ \text{skewness} \sim f_{NL} \]
\[ \text{kurtosis} \sim f_{NL}^2 \]
\[ \ldots \]
\[ \Phi(x) \sim \delta \sigma(x) + f_{\text{NL}} \delta \sigma(x)^2 \]

Here, \( \delta \sigma \) is a Gaussian field. The non-linear terms in \( \delta \sigma \) make \( \Phi \) non-Gaussian.

Skewness \( \sim f_{\text{NL}} \)  
Kurtosis \( \sim f_{\text{NL}}^2 \)

This map completely specifies \( \Phi \) statistics.
\[ \Phi(x) \sim \delta \sigma(x) + f_{NL} \delta \sigma(x)^2 \]

Salopek and Bond 1990; Gangui, Lucchin, Matarrese, Mollerach 1994; Komatsu and Spergel 2001

\[ \Phi(x) \sim \delta \sigma(x) + g_{NL} \delta \sigma(x)^3 + \ldots \]

(Okamoto and Hu 2002; Enqvist and Nurmi 2005)

skewness \sim f_{NL}
kurtosis \sim f_{NL}^2

\ldots

skewness \sim 0
kurtosis \sim g_{NL}

\ldots
\[ \Phi(x) \sim \delta \sigma(x) + f_{\text{NL}} \delta \sigma(x)^2 \]

Salopek and Bond 1990; Gangui, Lucchin, Matarrese, Mollerach 1994; Komatsu and Spergel 2001

\[ \Phi(x) \sim \delta \sigma(x) + g_{\text{NL}} \delta \sigma(x)^3 + \ldots \]

(Okamoto and Hu 2002; Enqvist and Nurmi 2005)

\[ \Phi(x) \sim \delta \varphi(x) + \delta \sigma(x) + \tilde{f}_{\text{NL}} \delta \sigma(x)^2 + \ldots \]

\[ \tilde{f}_{\text{NL}} = f_{\text{NL}}(1 + P_{\varphi \varphi}/P_{\sigma \sigma})^2 \]

skewness \sim f_{\text{NL}}

kurtosis \sim f_{\text{NL}}^2

\[ \tau_{\text{NL}} = f_{\text{NL}}^2(1 + P_{\varphi \varphi}/P_{\sigma \sigma}) \]

and \( P_{\varphi \sigma} = 0 \)

(\( \Phi = \text{primordial gravitational potential} \))

(Lyth and Wands 2002; Ichikawa, Suyama, Takahishi, Yamaguchi (2008); Tseliakhovich, Hirata, Slosar 2010)
more generally, non-Gaussianity introduces non-trivial multi-point correlation functions (or polyspectra)
Bispectrum:

\[
\langle \Phi(k)\Phi(k')\Phi(k'') \rangle = 2f_{NL} (P_\Phi(k) P_\Phi(k') + \ldots) (2\pi)^3 \delta(k+k'+k'')
\]

(\(\Phi=\)primordial gravitational potential)
Bispectrum:

\[ \langle \Phi(k)\Phi(k')\Phi(k'') \rangle = 2f_{NL} \left( P_{\Phi}(k) P_{\Phi}(k') + \ldots \right) (2\pi)^3 \delta(k+k'+k'') \]

function of triangle

largest in the “squeezed” limit

Trispectrum:

\[ \langle \Phi(k)\Phi(k')\Phi(k'')\Phi(k'''') \rangle_c = g_{NL} \left( P_{\Phi}(k) P_{\Phi}(k') P_{\Phi}(k'') + \ldots \right) (2\pi)^3 \delta(k+k'+k''+k'''') \]

+ 2 \[ T_{NL} \left( P_{\Phi}(k) P_{\Phi}(k') P_{\Phi}(|k+k'''|) + \ldots \right) (2\pi)^3 \delta(k+k'+k''+k'''') \]

function of a quadrilateral

g_{NL} term peaks in the limit

\[ T_{NL} \text{ term peaks in the squashed limit} \]

(\( \Phi = \text{primordial gravitational potential} \))
Bispectrum:

\[ \langle \Phi(k)\Phi(k')\Phi(k'') \rangle = 2f_{NL} (P_\Phi(k) P_\Phi(k') + \ldots) (2\pi)^3 \delta(k+k'+k'') \]

function of triangle

largest in the "squeezed" limit

Trispectrum:

\[ \langle \Phi(k)\Phi(k')\Phi(k'')\Phi(k''') \rangle_c = g_{NL} (P_\Phi(k) P_\Phi(k') P_\Phi(k'') + \ldots) (2\pi)^3 \delta(k+k'+k''+k''') \]

+ \[ 2 \tau_{NL} (P_\Phi(k) P_\Phi(k') P_\Phi(|k+k'''|) + \ldots) (2\pi)^3 \delta(k+k'+k''+k''') \]

function of a quadrilateral

g_{NL} term peaks in the limit

\[ \tau_{NL} \] term peaks in the squashed limit

so \( g_{NL} \) and \( \tau_{NL} \) different "shape" trispectra

\( \Phi=\text{primordial gravitational potential} \)
Helpful to consider how polyspectra couple different physical scales.
"$f_{\text{NL}}$" \quad $\Phi \sim \delta \sigma + f_{\text{NL}} \delta \sigma^2$

$$\langle \Phi_{\text{short}}^2 \rangle = \langle \sigma_{G,\text{short}}^2 \rangle \left(1 + 4 f_{\text{NL}} \sigma_{G,\text{long}}(x)\right)$$

small-scale power depends on large-scale fluctuations!

($\Phi =$ primordial gravitational potential)
\[ "f_{NL}\quad \phi \sim \delta \sigma + f_{NL} \delta \sigma^2 \]

\[ \langle \Phi_{\text{short}}^2 \rangle = \left( \sigma_{G,\text{short}}^2 \right) (1 + 4 f_{NL} \sigma_{G,\text{long}}(x)) \]

\[ "g_{NL}\quad \phi \sim \delta \sigma + g_{NL} \delta \sigma^3 + \ldots \]

\[ \langle \Phi_{\text{short}}^3 \rangle = 18 g_{NL} \left( \sigma_{G,\text{short}}^2 \right)^2 \sigma_{G,\text{long}}(x) \equiv f_{NL}^{\text{eff}}(x) \langle \sigma_{G,\text{short}}^2 \rangle^2 \]

(\Phi=\text{primordial gravitational potential})
"f_{NL}" \quad \Phi \sim \delta \sigma + f_{NL} \delta \sigma^2

\langle \Phi_{\text{short}}^2 \rangle = \langle \sigma_{G,\text{short}}^2 \rangle (1 + 4 f_{NL} \sigma_{G,\text{long}}(x))

"g_{NL}" \quad \Phi \sim \delta \sigma + g_{NL} \delta \sigma^3 + \ldots

\langle \Phi_{\text{short}}^3 \rangle = 18 g_{NL} \langle \sigma_{G,\text{short}}^2 \rangle^2 \sigma_{G,\text{long}}(x) \equiv f_{NL}^{\text{eff}}(x) \langle \sigma_{G,\text{short}}^2 \rangle^2

"T_{NL}" \quad \Phi \sim \delta \varphi + \delta \sigma + \tilde{f}_{NL} \delta \sigma^2 + \ldots

\langle \Phi_{s}^2 \rangle = \langle \Phi_{G,\text{short}}^2 \rangle (1 + 4 \tilde{f}_{NL} \sigma_{G,\text{long}}(x))

(\Phi=\text{primordial gravitational potential})
These are cartoon examples but these types of initial conditions can arise from real models.
For instance

\[ V(\varphi, \sigma) \]

Linde and Mukhanov 1997; Lyth and Wands 2002
For instance, the inflaton potential $V(\varphi, \sigma)$ determines the dynamics of the universe. Inflation occurs when the inflaton dominates energy density, driving exponential expansion through the equation $H^2 = \frac{8\pi G}{3} \left( \frac{1}{2} \dot{\varphi}^2 + V(\varphi) \right)$.

The curvaton is an additional light field $\sigma$ ($m_{\text{curv.}} \ll H$) that gets excited and eventually generates curvature perturbations $\Phi$, providing evidence for the inflationary universe model.

Linde and Mukhanov 1997; Lyth and Wands 2002
For instance

```latex
potential \sim V(\varphi, \sigma)
```

total energy dominated by inflaton:

```
H^2 = \frac{8\pi G}{3} \left( \frac{1}{2} \dot{\varphi}^2 + V(\varphi) \right)
```

curvature perturbations from curvaton can be non-Gaussian

```
\Phi \sim \delta \varphi + \delta \sigma + \delta \sigma^2 + \ldots
```

```
\Phi \sim \delta \sigma + \delta \sigma^3 + \ldots
```

```
\Phi \sim \delta \varphi + \delta \sigma + \delta \sigma^2 + \ldots
```

Linde and Mukhanov 1997; Lyth and Wands 2002
For instance

potential $\sim V(\varphi, \sigma)$

$H^2 = \frac{8\pi G}{3} \left( \frac{1}{2} \dot{\varphi}^2 + V(\varphi) \right)$

perturbations from inflaton Gaussian

curvature perturbations from curvaton can be non-Gaussian

non-linearities all "local" in position space

Linde and Mukhanov 1997; Lyth and Wands 2002
But local models (i.e. $\Phi_{NG}(x) = F(\sigma_G(x))$) of non-Gaussianity is not the only option
Single-field inflation with strong self-interactions can also generate detectable non-Gaussianity

\[ \langle \Phi^3 \rangle \quad \langle \Phi(k)\Phi(k')\Phi(k'') \rangle \propto \frac{1}{c_s^2} \]

skewness, bispectrum amplitude

BUT vanish in the "squeezed" limit

shape

largest in the "equilateral" limit

see Babich, Creminelli, Zaldarriaga 2004; Chen, Huang, Kachru, Shiu 2006; Senatore, Smith, Zaldarriaga 2011 (e.g. Dirac-Born-Infeld inflation, k-inflation, ghost inflation, inflation w dissipation)

Alishahiha, Silverstein, Tong 2004; Armendariz-Picon, Damour, Mukhanov 1999; Arkani-Hamed, Creminelli, Mukohyama, Zaldarriaga 2004; Nacir, Porto, Senatore, Zaldarriaga 2012
Single-field inflation with strong self-interactions can also generate detectable non-Gaussianity

skewness, bispectrum amplitude
\[ \langle \Phi^3 \rangle \quad \langle \Phi(k)\Phi(k')\Phi(k'') \rangle \propto \frac{1}{c_s^2} \]

BUT vanish in the "squeezed" limit

where, vanish means
\[ O+O(k_L^2/k_s^2) \]

see Babich, Creminelli, Zaldarriaga 2004; Chen, Huang, Zaldarriaga 2004; Armendariz-Picon, Damour, Mukhanov 1999; Arkani-Hamed, Creminelli, Mukohyama, Zaldarriaga 2004; Nacir, Porto, Senatore, Zaldarriaga 2012

(e.g. Dirac-Born-Infeld inflation, k-inflation, ghost inflation, inflation w dissipation)
a single-field that violates slow-roll can also generate observable non-Gaussianity

\[ V(\phi) \]

skewness, bispectrum amplitude

\[ \langle \Phi^3 \rangle \quad \langle \Phi(k)\Phi(k')\Phi(k'') \rangle \propto \text{local feature in } k \]

Chen, Easther, Lim 2006; Chen, Easther, Lim 2008; Flauger & Pajer 2010
(e.g. Axion monodromy: McAllister, Silverstein, Westphal 2008; Flauger, McAllister, Pajer, Westphal, Xu 2009)

still vanish in the "squeezed" limit

shape complicated!

\[ \frac{k_3}{k_1} \]

\[ \frac{k_2}{k_1} \]

\[ \frac{k'}{k} \]

\[ \frac{k''}{k} \]
single-field with modified initial vacuum state generates observable non-Gaussianity

skewness, bispectrum amplitude
\[ \langle \Phi^3 \rangle \]
\[ \langle \Phi(k)\Phi(k')\Phi(k'') \rangle \propto \beta_k \sim e^{-k^2/k_{\text{cut-off}}^2} \]

shape
largest in "flattened" configuration

still vanish in the "squeezed" limit

but, may have non-vanishing contributions in a limited, observable k-range

Holman and Tolley 2008
Agullo & Shandera 2012; Ganc & Komatsu 2012
In fact:

single-field inflation predicts

\[ \langle \Phi(k)\Phi(k')\Phi(k''\rightarrow0) \rangle \approx f_{NL} \]

\[ \approx \left(n_s - 1\right)(2\pi)^3 \delta(k+k') P_\Phi(k) P_\Phi(k') P_\Phi(k'') \]

where \( n_s = \frac{d\ln P_\Phi(k)}{d\ln k} + 4 \approx 1 \)

the so called “consistency relation”

so \( f_{NL} \gg \) few rules it out

Acquaviva, Bartolo, Matarrese, Riotto 2003; Maldacena 2003; Creminelli & Zaldarriaga 2004

(see also Tanaka, Urakawa 2011)
Note:

single-field consistency relation

\[ f_{\text{NL}} \approx \frac{\partial \ln k^3 P_\phi}{\partial \ln k} = (n_s - 1) \]

also applies to \( g_{\text{NL}} \) and \( \tau_{\text{NL}} \)

\[ g_{\text{NL}} \approx \frac{\partial \ln k^6 B_\phi}{\partial \ln k} = n_{\text{NG}} \]

\[ \tau_{\text{NL}} \approx (n_s - 1)^2 \]

e.g. Chen, Huang, Shiu 2008; Leblond & Pajer 2011

(see also Tanaka, Urakawa 2011)

in terms of physical observables these are strictly zero

also have,

\[ \tau_{\text{NL}} \gtrsim f_{\text{NL}}^2 \]

Suyama & Yamaguchi 2008; Sugiyama, Komatsu, Futamase 2011; Smith, ML, Zaldarriaga 2011
Single-field models do not generate such extreme couplings of perturbations on short and long length scales.

\[ \langle \Phi(k_S)\Phi(-k_S-k_L)\Phi(k_L) \rangle \sim \langle p_{\Phi}(k_S)\Phi(k_L) \rangle \sim f_{NL} k_L^{-3} \]

"squeezed" limit

\[ k_S \quad k_{S-k_L} \quad k_L \]
mother fields $\gg H \longrightarrow$ single-field $\langle \Phi(k_S)\Phi(-k_S-k_L)\Phi(k_L) \rangle \sim 0$

mother fields $\ll H \longrightarrow$ other fields relevant

can get $\langle \Phi(k_S)\Phi(-k_S-k_L)\Phi(k_L) \rangle \sim f_{NL}k_s^{-3}k_L^{-3}$

mother fields $\sim H ? \longrightarrow$ quasi single-field

Chen & Wang 2010; Baumann & Green 2011
\[ m_{\text{other fields}} \gg H \longrightarrow \text{single-field} \quad \langle \Phi(k_S)\Phi(-k_S-k_L)\Phi(k_L) \rangle = 0 \]

\[ m_{\text{other fields}} \ll H \longrightarrow \text{other fields relevant} \quad \langle \Phi(k_S)\Phi(-k_S-k_L)\Phi(k_L) \rangle = f_{NL} k_S^{-3} k_L^{-3} \]

\[ m_{\text{other fields}} \sim H \quad \text{?} \longrightarrow \text{quasi single-field} \quad \text{Chen & Wang 2010; Baumann & Green 2011} \]

\[ \langle \Phi(k_S)\Phi(-k_S-k_L)\Phi(k_L) \rangle \sim \langle P_\Phi(k_S)\Phi(k_L) \rangle \sim f_{NL} k_S^{-3} k_L^{-3} \left( \frac{k_L}{k_S} \right)^{3/2-v} \]

intermediate scalings possible!

\[ v \sim \sqrt{9/4 - m^2/H^2} \]
How does primordial non-Gaussianity show up in large-scale structure?
HALO ABUNDANCE

dark matter halos form in peaks of the density field

non-Gaussianity changes the number density of peaks

Gaussian  positive skewness  no skewness, positive kurtosis

Lucchin & Matarrese 1988; Chiu, Ostriker, Strauss 1998; Robinson, Gawiser, Silk 2000
HALO ABUNDANCE

\[ \frac{dn_{NG}}{dM} / \frac{dn_G}{dM} \]

seems to work in comparison to N-body!

(with caveats about how you approximate the PDF)

see also Dalal, Dore, Huterer, Shirokov 2007; Grossi et al 2009; Kang, Norberg, Silk 2009; Pillepich, Porciani, Hahn 2009; Desjacques and Seljak 2010; Wagner, Verde, Boubekeur 2010
HALO ABUNDANCE

Pros: ingredients just $\langle \delta_M^2 \rangle$, $\langle \delta_M^3 \rangle$, $\langle \delta_M^4 \rangle$ -- insensitive to “shape” of bispectrum trispectrum. In principle $\langle \delta_M^3 \rangle$, $\langle \delta_M^4 \rangle$ effects not degenerate in $dn/dM$

Cons: cosmology with cluster abundance is really hard (mass–observable, degeneracy with $\sigma_8$ etc)
SCALE-DEPENDENT HALO-BIAS

A dark matter halo forms when $\delta \rho / \rho$ is larger than the collapse threshold $\delta_c$. 

\[
\delta \rho / \rho
\]
a dark matter halo forms when $\delta \rho / \rho$ is larger than the collapse threshold $\delta_c$ which is easier to reach on top of a long wavelength density perturbation

\[ \delta_{c-l} \]
a dark matter halo forms when $\delta \rho/\rho$ is larger than the collapse threshold $\delta_c$ which is easier to reach on top of a long wavelength density perturbation.

so the number of halos fluctuates depending on $\delta_1$

$\delta n = \frac{\partial \delta n}{\partial \delta} \delta_1 \ldots$
the number of halos fluctuates depending on $\delta_l$

BUT with $f_{\text{NL}}$, the small-scale power fluctuates also depending on $\Phi_l$
the number of halos fluctuates depending on $\delta_l$

BUT with $f_{NL}$, the small-scale power fluctuates also depending on $\Phi_l$

$$\delta n = \frac{\partial n}{\partial \delta} \delta_l + 4f_{NL} \frac{\partial n}{\partial P_s} \Phi_l \ldots$$
the number of halos fluctuates depending on $\delta_1$

$\delta n = \frac{\partial n}{\partial \delta} \delta_1 + 4f_{NL} \frac{\partial n}{\partial \rho_s} \Phi_1 \ldots$

Poisson's

$\nabla^2 \Phi_1 \sim 4\pi G \delta_1$

$\delta n \sim \left( \frac{\partial n}{\partial \delta} + \frac{4f_{NL}}{k^2} \frac{\partial n}{\partial \rho_s} \right) \delta_1$

BUT with $f_{NL}$, the small-scale power fluctuates also depending on $\Phi_1$

Dalal, Doré, Huterer, Shirokov 2007

Matarrese & Verde 2008; Slosar, Hirata, Seljak, Ho, Padmanabhan 2008; Afshordi & Tolley 2008; McDonald 2008
a dark matter halo forms when $\delta \rho / \rho$ is larger than the collapse threshold

with $g_{NL}$ non-Gaussianity, the small-scale skewness fluctuates with $\Phi_1$

so the number of halos fluctuates depending on $\delta_1$ and $\Phi$

$\delta n = \frac{\partial n}{\partial \delta} \delta_1 + 18 g_{NL} \frac{\partial n}{\partial S_3} \Phi_1 \ldots$

Desjacques & Seljak 2009; Smith, Ferraro, ML 2011
a dark matter halo forms when $\delta \rho / \rho$ is larger than the collapse threshold

with $g_{\text{NL}}$ non-Gaussianity, the small-scale skewness fluctuates with $\Phi_1$

so the number of halos fluctuates depending on $\delta_1$ and $\phi$

$\nabla^2 \Phi_1 \sim 4\pi G \delta_1$

$\frac{\delta n}{\delta \delta} \delta_1 + 18g_{\text{NL}} \frac{\delta n}{\delta S_3} \Phi_1 \ldots$

$\approx \left( \frac{\delta n}{\delta \delta} + 18g_{\text{NL}} \frac{\delta n}{\delta S_3} / k^2 \right) \delta_1(k) \ldots$

bias depends on Fourier scale $k$

Desjacques & Seljak 2009; Smith, Ferraro, ML 2011
**SCALE-DEPENDENT HALO-BIAS**

**local** non-Gaussianity

\[ \Phi(x) = \Phi_G(x) + f_{NL} (\Phi_G(x)^2 - \langle \Phi_G^2 \rangle) + g_{NL} (\Phi_G(x)^3 - \Phi_G \langle \Phi_G^2 \rangle) \]

\[ \rightarrow \text{scale dependent halo bias} \]

\[ b_{f_{NL},g_{NL}}(k) \sim b + \frac{f_{NL},g_{NL}}{k^2} \times \text{constant} \]

impossible to generate with single field inflation!

e.g. Creminell, D’Amico, Musso, Noreña 2011

Smith, Ferraro, ML 2011

(Desjacques and Seljak 2010; Desjacques, Jeong, Schmidt 2011; Scoccimarro et al 2012)
**SCALE-DEPENDENT HALO-BIAS**

**local non-Gaussianity**

\[ \Phi(x) = \Phi_G(x) + f_{NL} (\Phi_G(x)^2 - \langle \Phi_G^2 \rangle) + g_{NL} (\Phi_G(x)^3 - \Phi_G \langle \Phi_G^2 \rangle) \]

\[ b_{f_{NL},g_{NL}}(k) \sim b + \frac{f_{NL},g_{NL} \times \text{constant}}{k^2} \]

impossible to generate with single field inflation!

e.g. Creminell, D’Amico, Musso, Noreña 2011

observational systematics may be hard!

precise values of \(f_{NL}, g_{NL}\) will require care -- but seeing \(1/k^2\) is the most exciting part

Smith, Ferraro, ML 2011

(Desjacques and Seljak 2010; Desjacques, Jeong, Schmidt 2011; Scoccimarro et al 2012)
SCALE-DEPENDENT HALO-BIAS

Figure 1: Halo bias $b_m(k)$ for selected redshifts and halo mass bins, estimated from N-body simulations as described in Appendix A. The curves are the predicted form in Eq. (27), with $b_0$ treated as a free parameter which is fit from data.

$scales \text{ as } 1/k^2$

Dalal, Doré, Huterer, Shirokov 2007
Smith, ML 2010 Smith, Ferraro, ML 2011
Pillepich, Porciani, Hahn 2008; Desjacques, Seljak, Iliev 2008; Grossi et al 2009
Shandera, Dalal, Huterer 2010
(Desjacques and Seljak 2010; Desjacques, Jeong, Schmidt 2011; Scoccimarro et al 2012)
bias coefficient for $g_{NL}$ in terms of mass

$$b_{gNL}(k) = b + \frac{3g_{NL} \partial \ln \nbar(M)}{k^2} \frac{\partial f_{NL}}{\partial f_{NL}}$$

contrast w/$f_{NL}$ where coefficient in terms of bias

$$b_{fNL}(k) = b + \frac{2 \delta_c f_{NL} (b-1)}{k^2}$$
bias coefficient for $g_{NL}$ in terms of mass

$$b_{gNL}(k) = b + \frac{3g_{NL} \partial \ln n(M)}{k^2} \frac{\partial f_{NL}}{\partial f_{NL}}$$

contrast w/f$_{NL}$ where coefficient in terms of bias

$$b_{fNL}(k) = b + \frac{2 \delta_c f_{NL} (b-1)}{k^2}$$

we have a fit for $g_{NL}$ in terms of bias:

$$b_{gNL}(k) \sim b + g_{NL} \frac{\text{non-linear function}(b)}{k^2}$$

form will depend on selection of population in $M$, $z$
bias coefficient for $g_{NL}$ in terms of mass

contrast $w/f$ NL where coefficient in terms of bias $b_{gNL}(k) = b + g_{NL}$

we have a fit for $g_{NL}$ in terms of bias:

\[ b_{gNL}(k) \sim b + g_{NL} \quad \text{non-linear function}(b) \quad k^2 \]

form will depend on selection of population in $M, z$

but! exact $1/k^2$ not necessarily expected!

Smith, Ferraro, ML 2011
SCALE-DEPENDENT HALO-BIASES

generalized local ansatz

\[ \langle \Phi(k_S)\Phi(-k_S-k_L)\Phi(k_L) \rangle \sim \xi_\sigma(k_S)k_S^{-3}k_L^{-3} \]

\[ \langle \Phi(k_S)\Phi(-k_S-k_L)\Phi(k_L) \rangle \sim \xi_{\sigma\varphi}(k_S)\xi_{\sigma\varphi}(k_L)k_S^{-3}k_L^{-3} \]

Shandera, Dalal, Huterer 2010
More precisely, in terms of the Gaussian Eulerian bias

\[ \langle \Phi(k_S)\Phi(-k_S-k_L)\Phi(k_L) \rangle \sim \xi_\sigma(k_S)k_S^{-3}k_L^{-3} \]

\[ \langle \Phi(k_S)\Phi(-k_S-k_L)\Phi(k_L) \rangle \sim \xi_{\sigma\varphi}(k_S)\xi_{\sigma\varphi}(k_L)k_S^{-3}k_L^{-3} \]

\[ \xi(k) \sim k^{\text{slow-roll}} \]

\[ f_{\text{NL}}(M) \sim \xi_{\sigma\varphi}(k_S)\xi_\sigma(k_S) \]

\[ b(k) \sim \xi_{\sigma\varphi}(k)k^{-2} \sim k^{-2+\text{slow-roll}} \]

**Left panel**

- Blue short dashed: single-field, \( n_f^{(s)} = +0.6 \)
- Blue long dashed: single-field, \( n_f^{(s)} = -0.6 \)
- Red short dashed: multi-field, \( n_f^{(m)} = +0.3 \)
- Red long dashed: multi-field, \( n_f^{(m)} = -0.3 \)

**Right panel**

- Black: Local ansatz
- Blue dashed: single-field, \( n_f^{(s)} = +0.6 \)
- Red dashed: multi-field, \( n_f^{(m)} = +0.3 \)
- Pink dashed: multi-field, \( n_f^{(m)} = -0.3 \)

**Figure 1**

- Case of constant local non-Gaussianity

For example, Giannantonio and Porciani

There is an additional factor multiplying the expression above for the correction to the bias of objects of mass \( x \) that increases on small scales with a function of the object's mass for two modifications of the local ansatz.

The blue short dashed curve is the single-field model with the multipfield scenario with.

These combine into the mass-dependent coefficient

\[ k \sim (\rho/M)^{1/3} \]
SCALE-DEPENDENT HALO-BIAS

quasi-single field models?

\[ \langle \Phi(k_{S})\Phi(-k_{S}-k_{L})\Phi(k_{L}) \rangle \sim \langle P_{\Phi}(k_{S})\Phi(k_{L}) \rangle \sim f_{NL} \, k_{S}^{-3} \, k_{L}^{-3} \left( \frac{k_{L}}{k_{S}} \right)^{3/2-v} \]

\[ f_{NL}(M) \sim f_{NL} \, k_{S}^{-3/2+v} \]

\[ k_{S} \sim (\rho/M)^{1/3} \]

\[ b(k) \sim k^{-2+3/2-v} \]

\[ 0 \leq v \leq 3/2 \]
MORE:

scale-dep bias only probes a particular configuration of bispectrum (or higher)

"squeezed" limit

\( k_s \)

\( k_s - k_L \)

\( k_L \)

and, it’s one that vanishes in single-field models
MORE:

scale-dep bias only probes a particular configuration of bispectrum (or higher)

"squeezed" limit

and, it's one that vanishes in single-field models

FULL bispectrum, trispectrum sensitive to more general models, contains more information

\[ B(k_1, k_2, k_3) \quad T(k_1, k_2, k_3, k_4) \]
Summary

- Lots of different kinds of non-Gaussian initial conditions
- Qualitatively different shapes & scalings of non-Gaussianity from qualitatively different models
- Halo abundance sensitive to local statistics of $\delta_M$
- Halo clustering (scale-dep bias) probes squeezed limits of bispectrum, trispectrum -- power to rule out single-field inflation
- Analytic description for the halo mass function looks good compared with N-body so far
- Analytic descriptions of halo bias agree well with sims
First theory breakout session summary:

scale-dep bias only probes a particular configuration of bispectrum (or higher)

Every bispectrum has a squeezed limit
It just might be very small.....

Seeing anything in scale-dep. bias/squeezed limit is indicative of new physics incredibly exciting

the current limits are already interesting
First theory breakout session summary:

Since every bispectrum (i.e. models other than $f_{NL}$ local) has a squeezed limit, scale dependent bias constrains a broad space of theories.

However, scale-dependent bias in other theories will not have the usual form: $b_0 + 2\delta_c \ f_{NL}(b_0-1)/k^2$

more powerful to fit:

$$b(k) = b_0 + f(M)/k^\alpha$$

where $f(M)$ is a function of mass (that can be calculated from a non-Gaussian model) that is proportional to the amplitude of non-Gaussianity (e.g. $f_{NL}$, $g_{NL}$) and it’s probably safe to assume $0 \leq \alpha \leq 3$

special values of $\alpha$:

$0 \leq \alpha \leq 2$: quasi-single-field
$\alpha = 2$: exact local model ($f_{NL}$, $g_{NL}$)
$\alpha = 2 \pm \varepsilon$: two fields contributing to primordial perturbations
$\alpha = 3$: modified initial state
First theory breakout session summary:

AGAIN, seeing anything in scale-dep. bias/squeezed limit is indicative of new physics incredibly exciting

A detection would mean there are other signatures to go after and help distinguish between models

the current limits are already interesting

There are non-Gaussian models that have vanishingly small squeezed limits (and therefore vanishingly small scale-dep bias) BUT detectably large signals in other, non-squeezed configurations. SO we should continue to explore other observables (e.g. galaxy bispectrum in non-squeezed configurations)