Primordial Magnetic Fields from Inflation and Beyond

Takeshi Kobayashi (SISSA)

based on arXiv:1403.5168
arXiv:1408.4141 w/ Niayesh Afshordi
arXiv:1511.08793 w/ Daniel Green

Theoretical Advances in Particle Cosmology
KICP, October 2016

OUR MAGNETIZED UNIVERSE

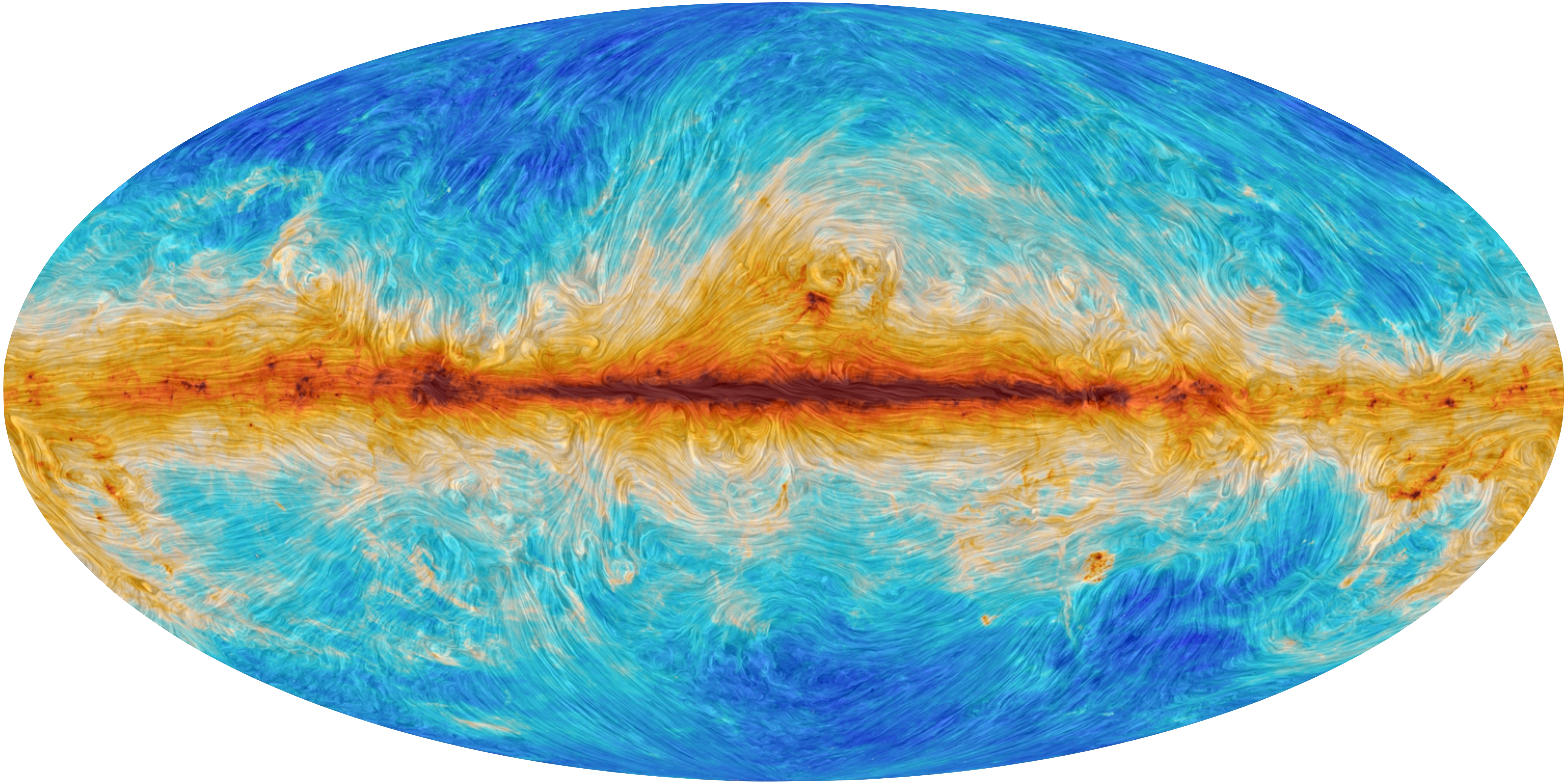

image : ESA and the Planck Collaboration

BLAZAR OBSERVATIONS

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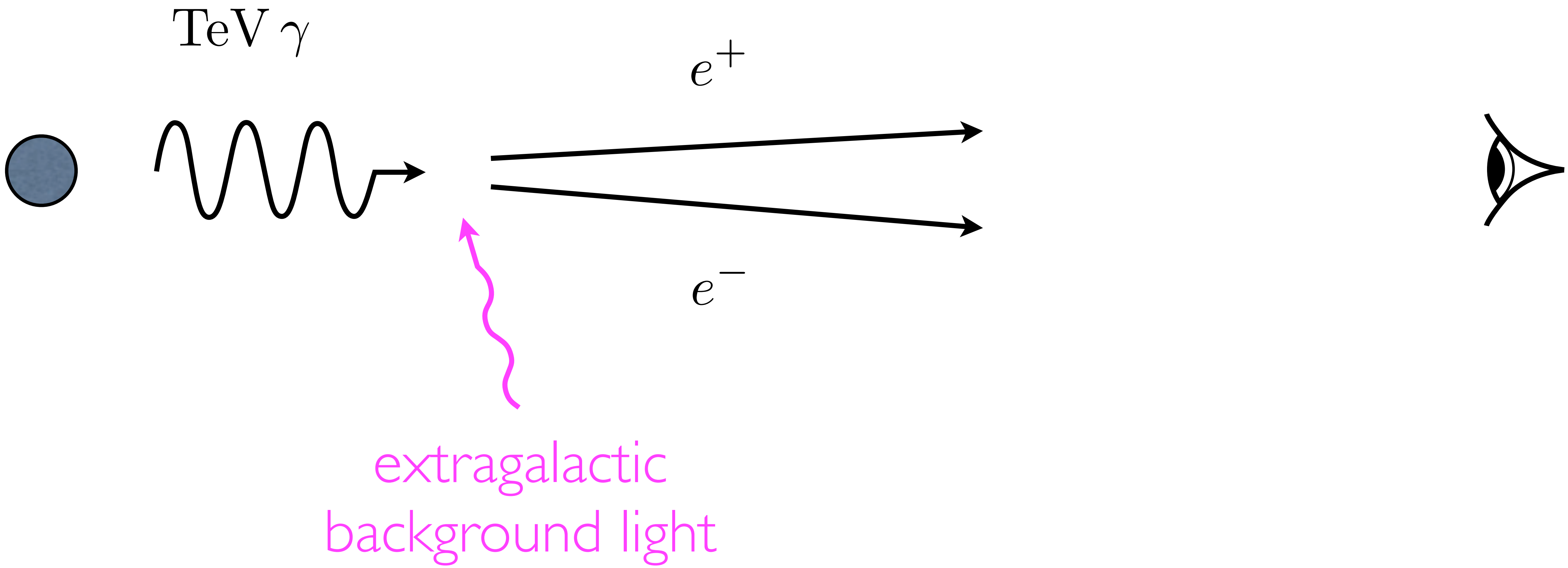

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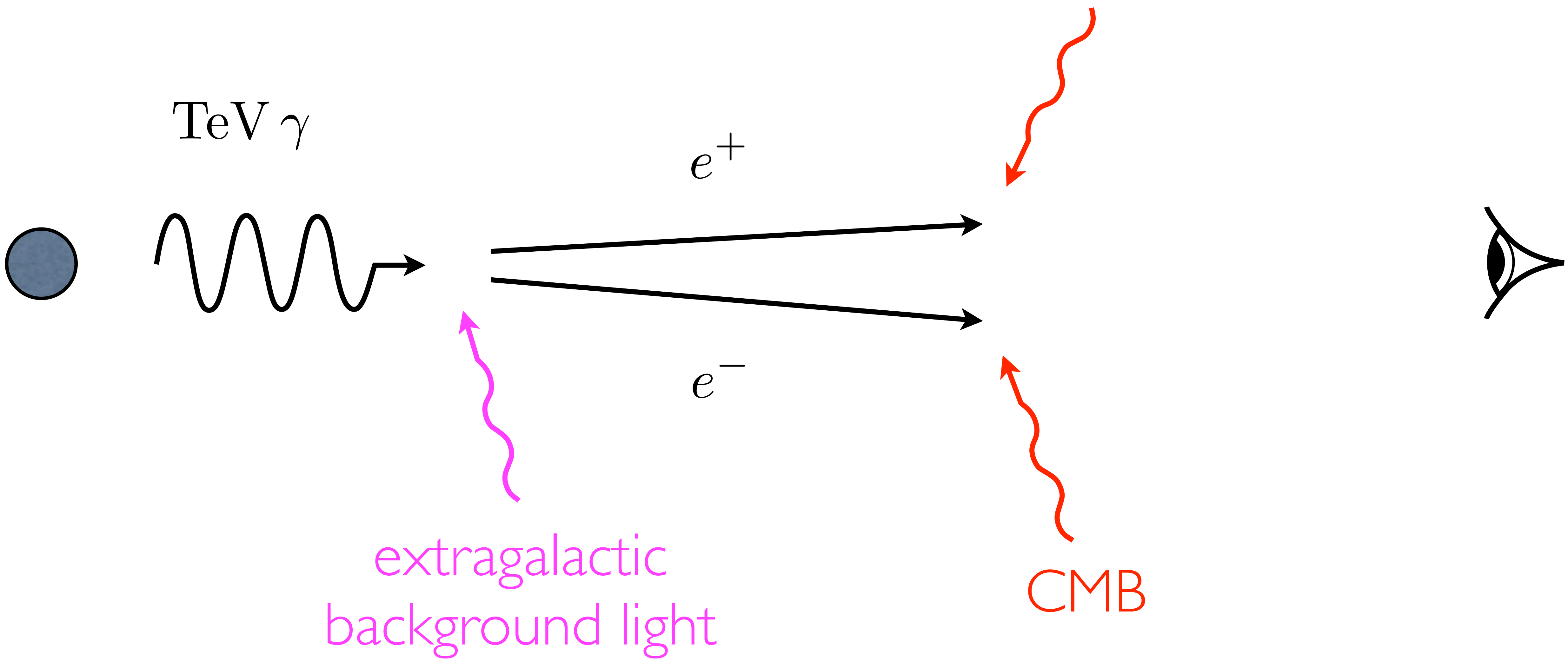

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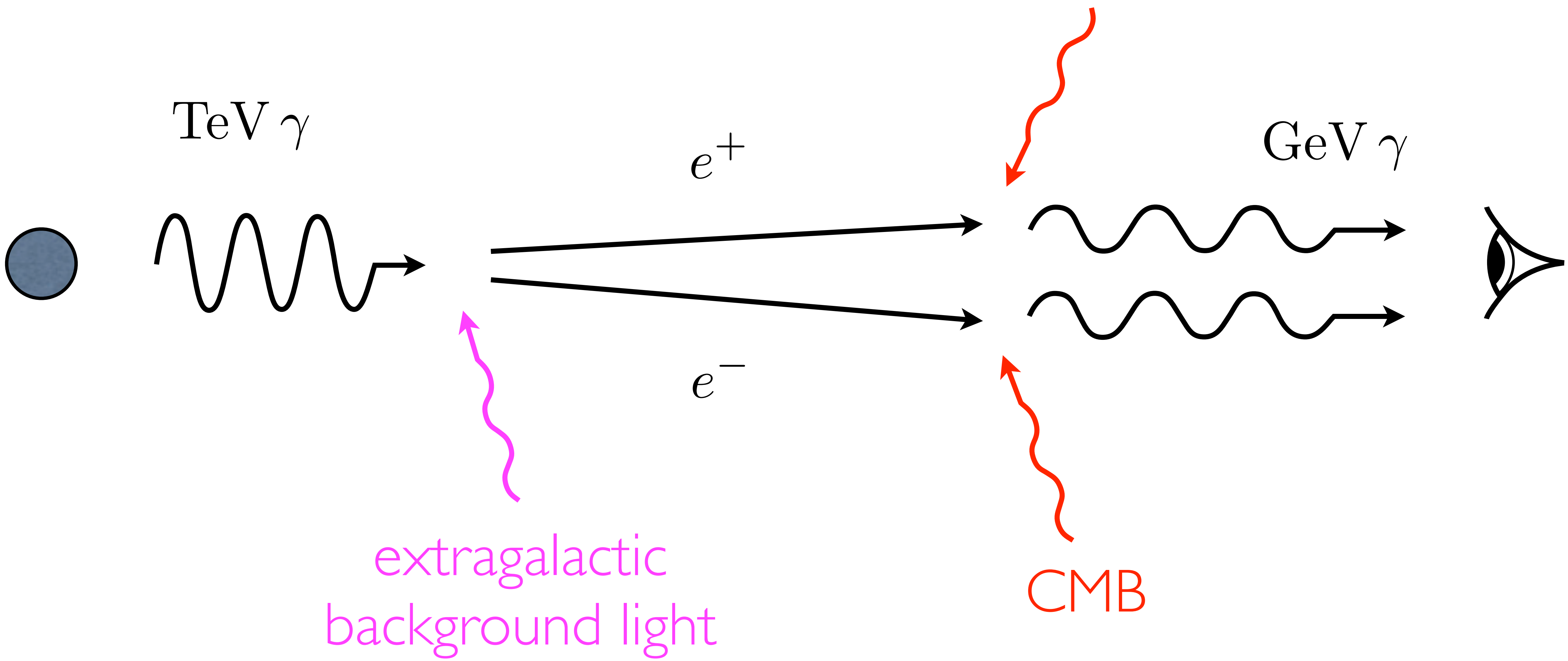

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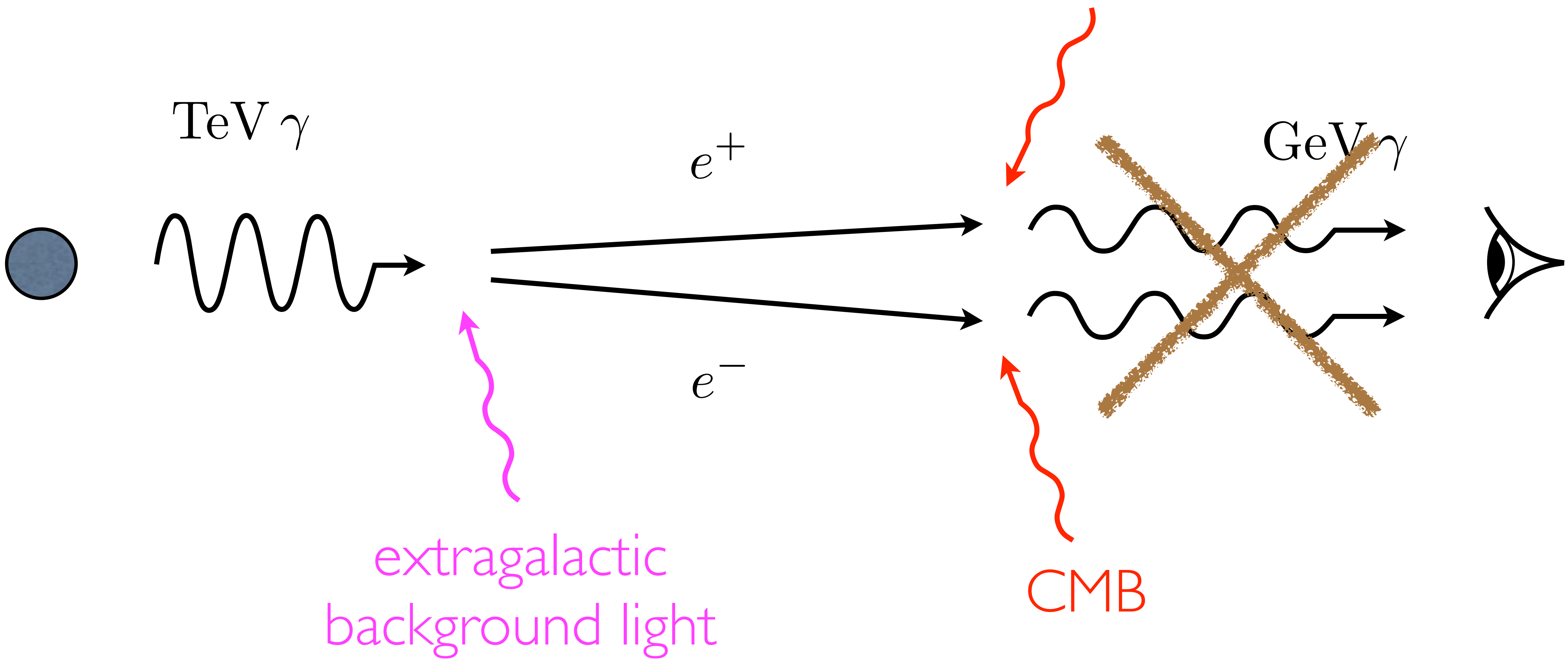

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With intergalactic magnetic fields...

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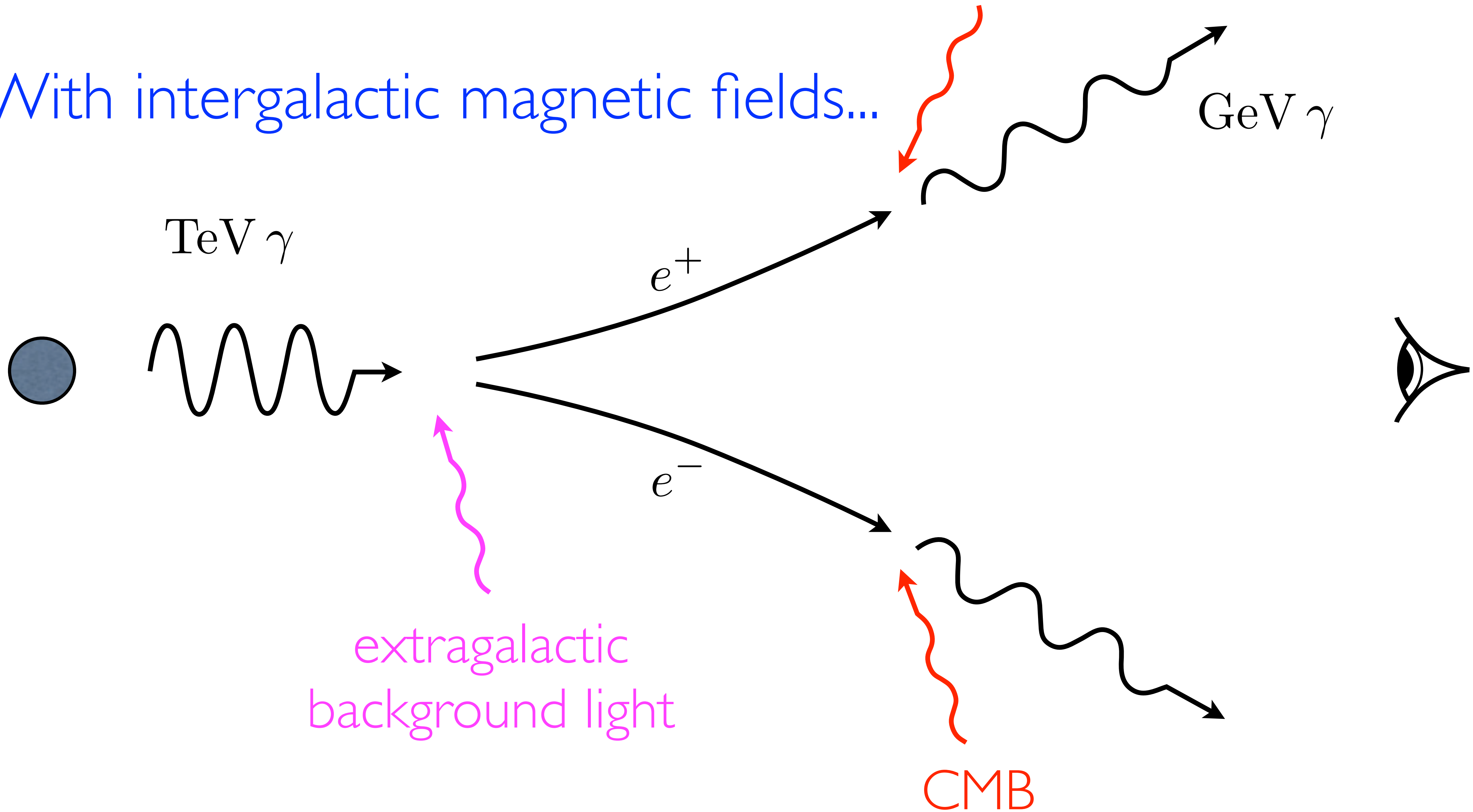

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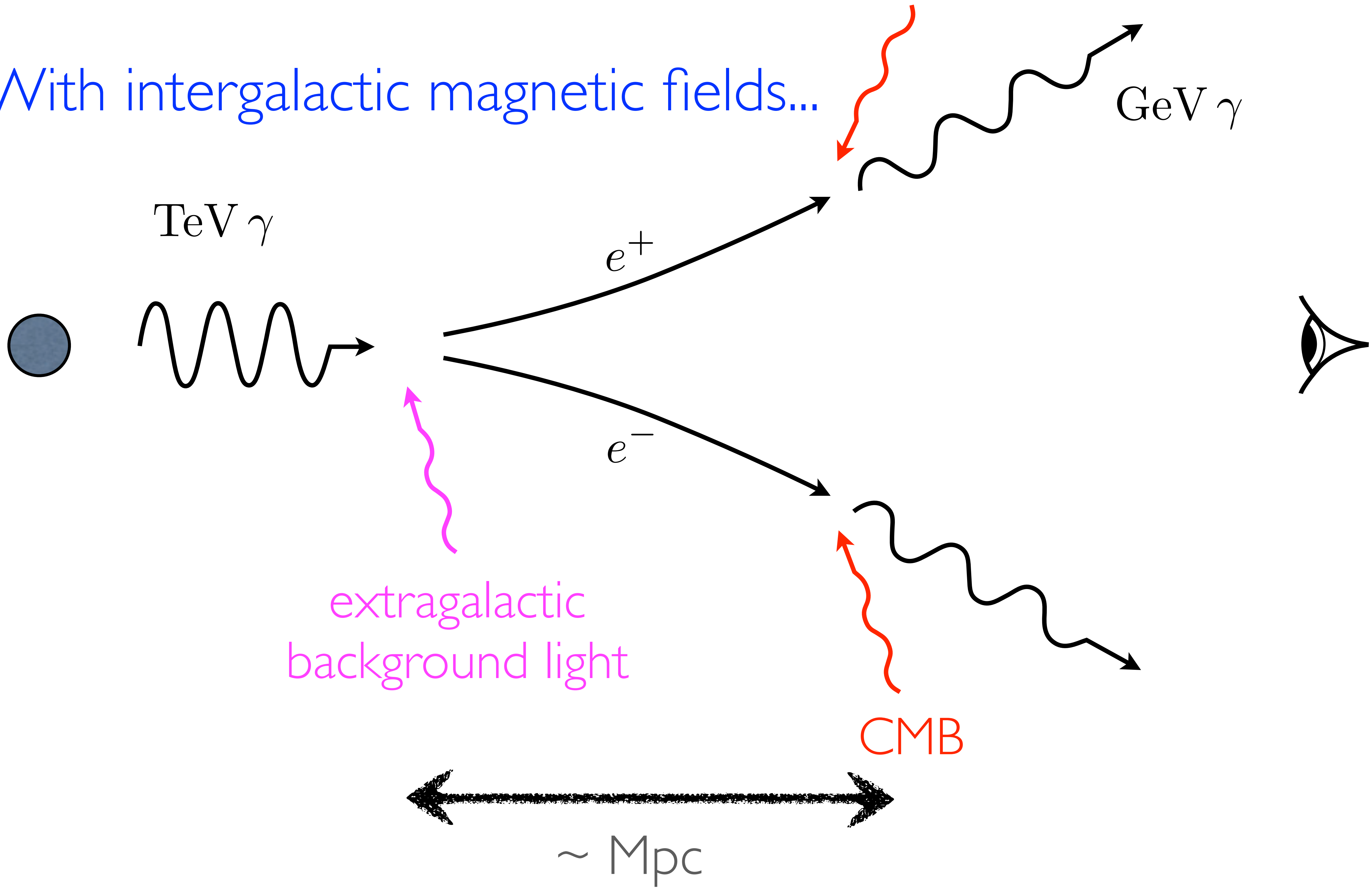

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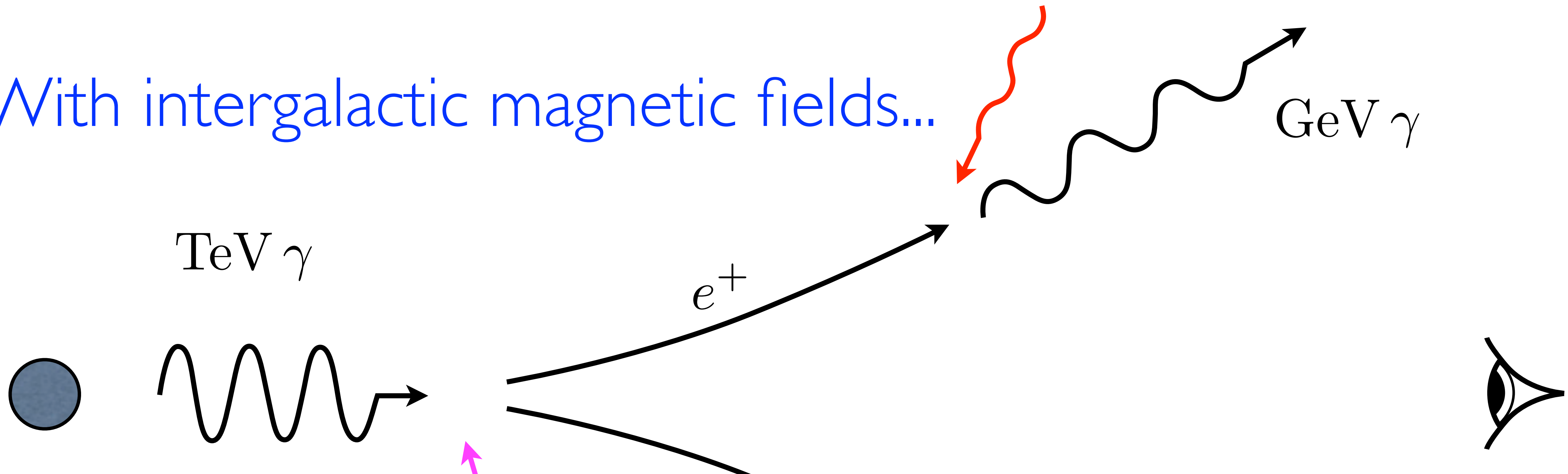
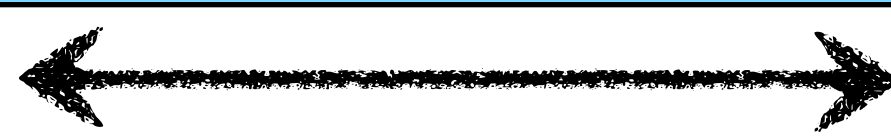


$B \gtrsim 10^{-15}\mathrm{G}$ with correlation length $\gtrsim \mathrm{Mpc}$

Tavecchio et al. '10 Neronov, Vovk '10 Takahashi et al. '13
Tashiro et al. '13 Chen et al. '14 ...



$\sim$ Mpc

INFLATIONARY MAGNETOGENESIS

Turner, Widrow '88   Ratra '92

$$\frac{\mathcal{L}}{\sqrt{-g}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

conformal symmetry : $g_{\mu\nu} \rightarrow \Omega^2 g_{\mu\nu}$

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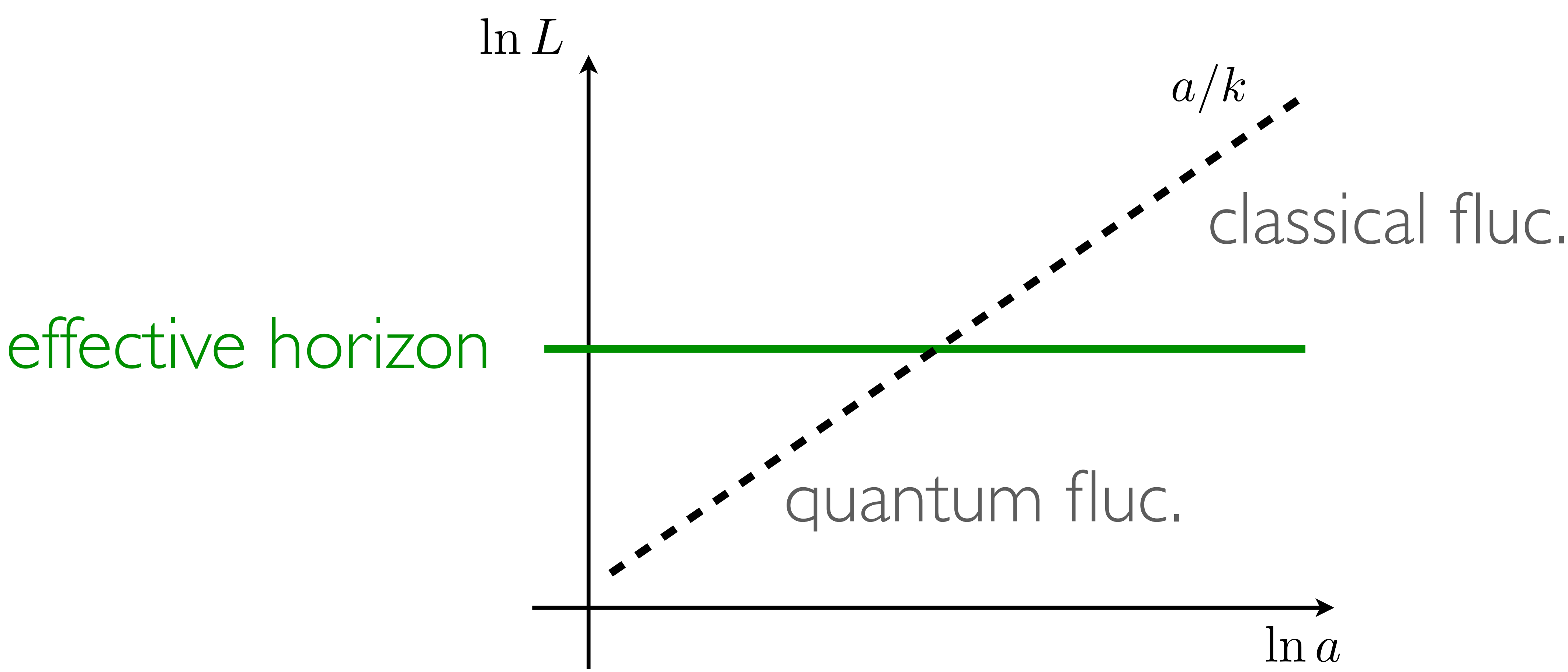
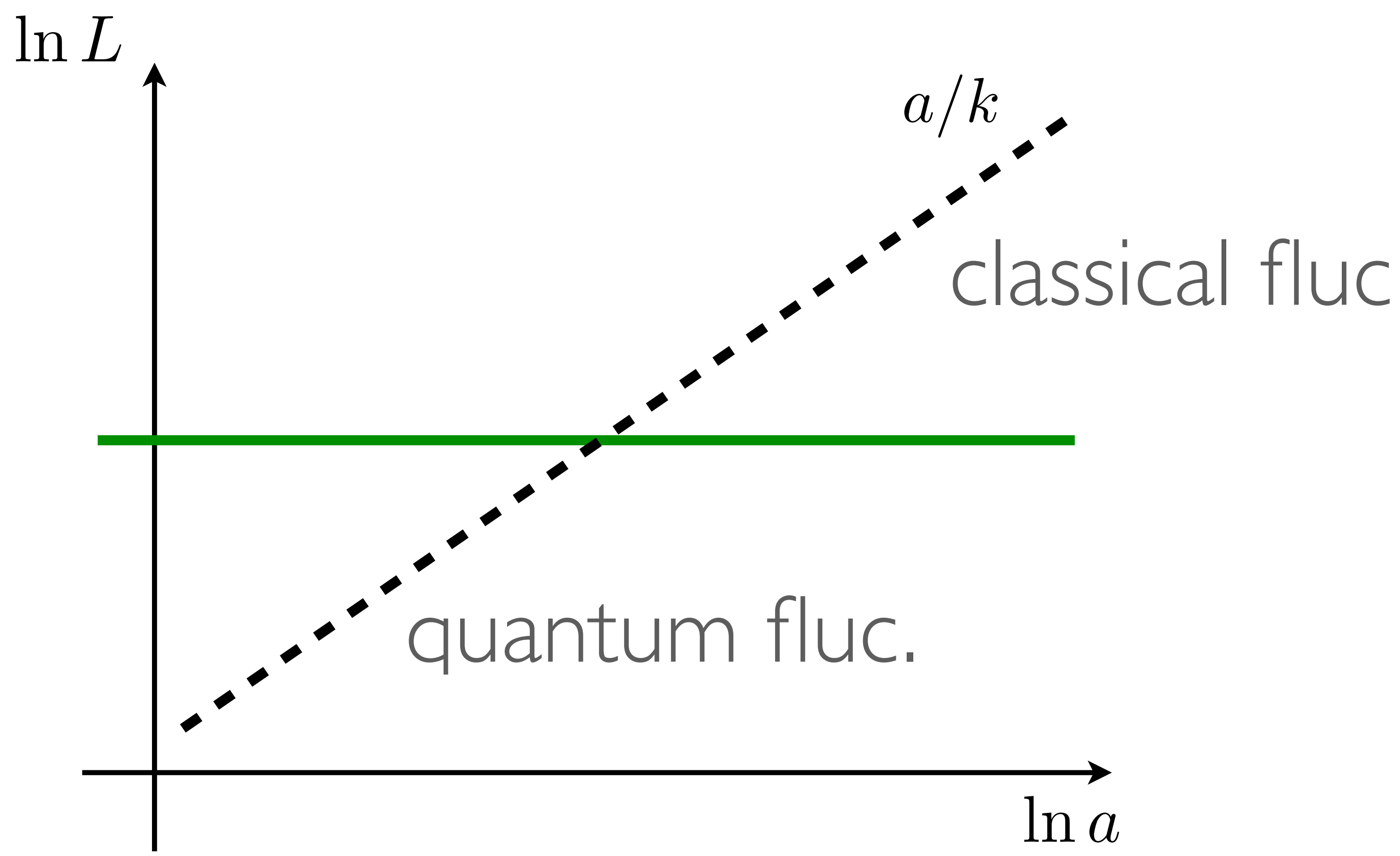


TWO WAYS TO PRODUCE LARGE B



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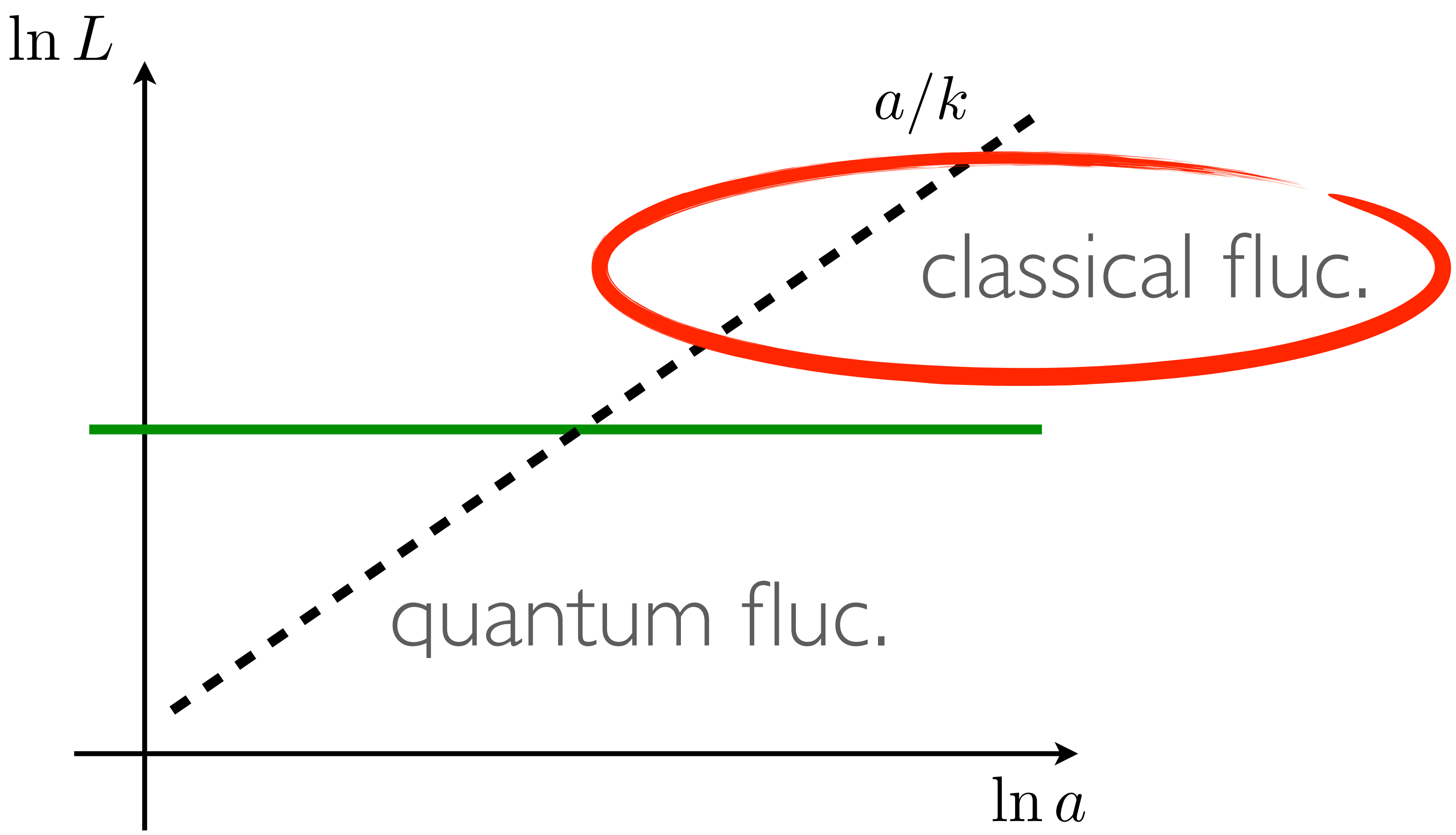

(1) classically evolve initally tiny fields

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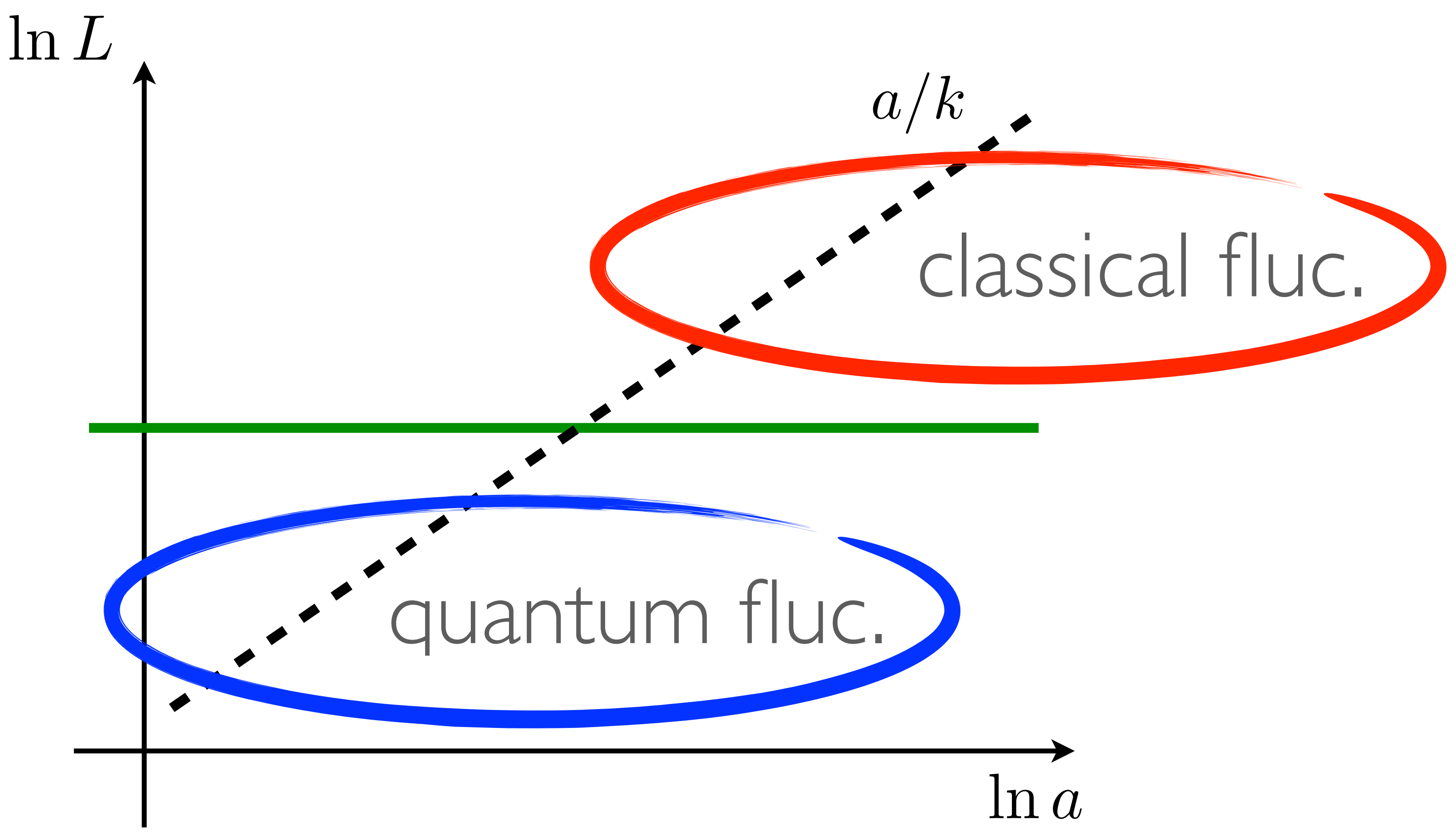

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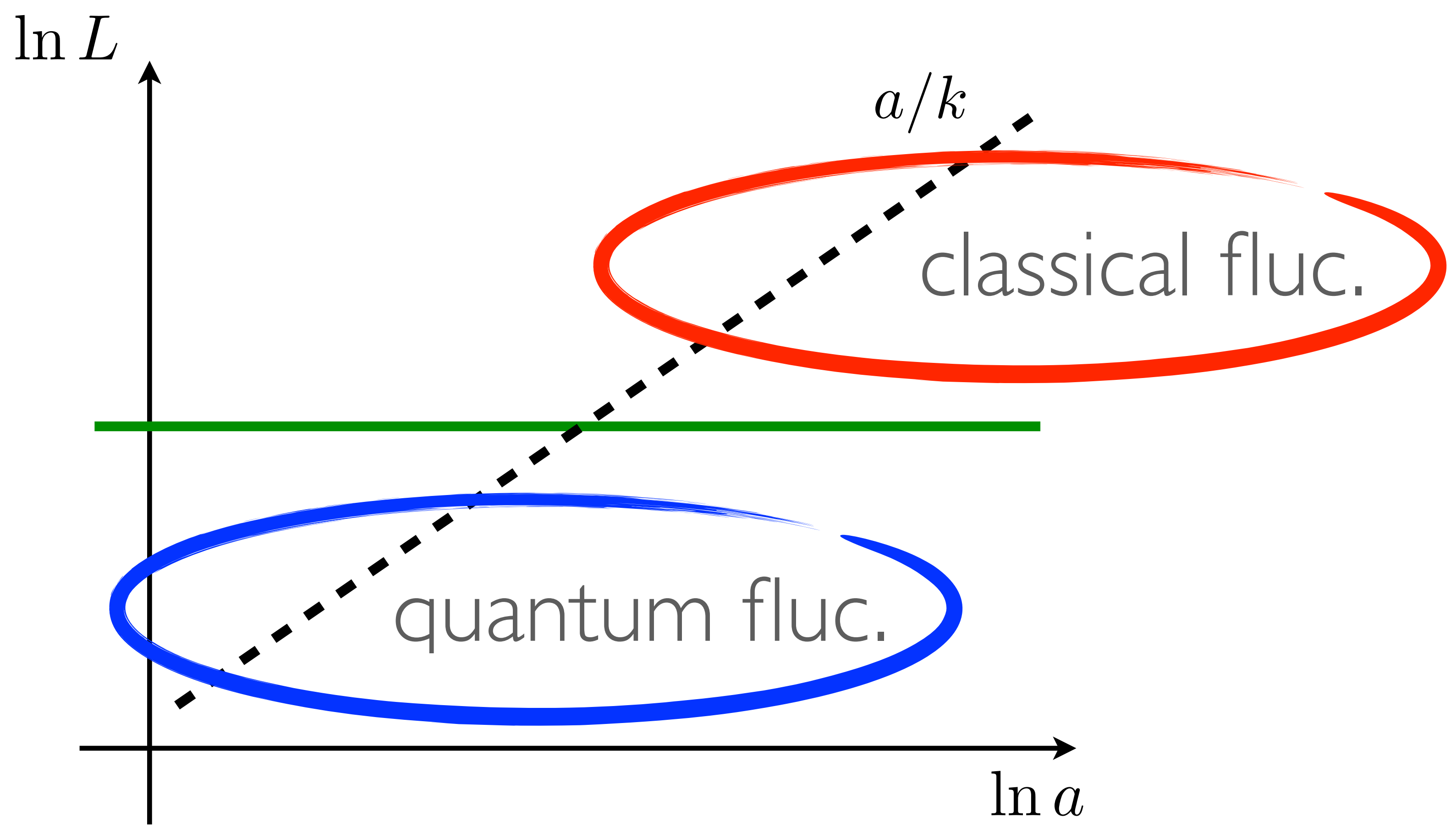

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Can inflationary magnetogenesis produce
$B \geq 10^{-15}$ G on Mpc scales?

OUTLINE

- Constraints on Inflationary Magnetogenesis
  - ✓ Classical Scenarios and Schwinger Effect
  - ✓ Quantum Mechanical Scenarios
- New Idea for Magnetic Field Generation: Post-Inflationary Magnetogenesis

Classical Scenarios and Constraints from Schwinger Effect

arXiv:1408.4141 w/N. Afshordi
arXiv:1511.08793 w/D. Green

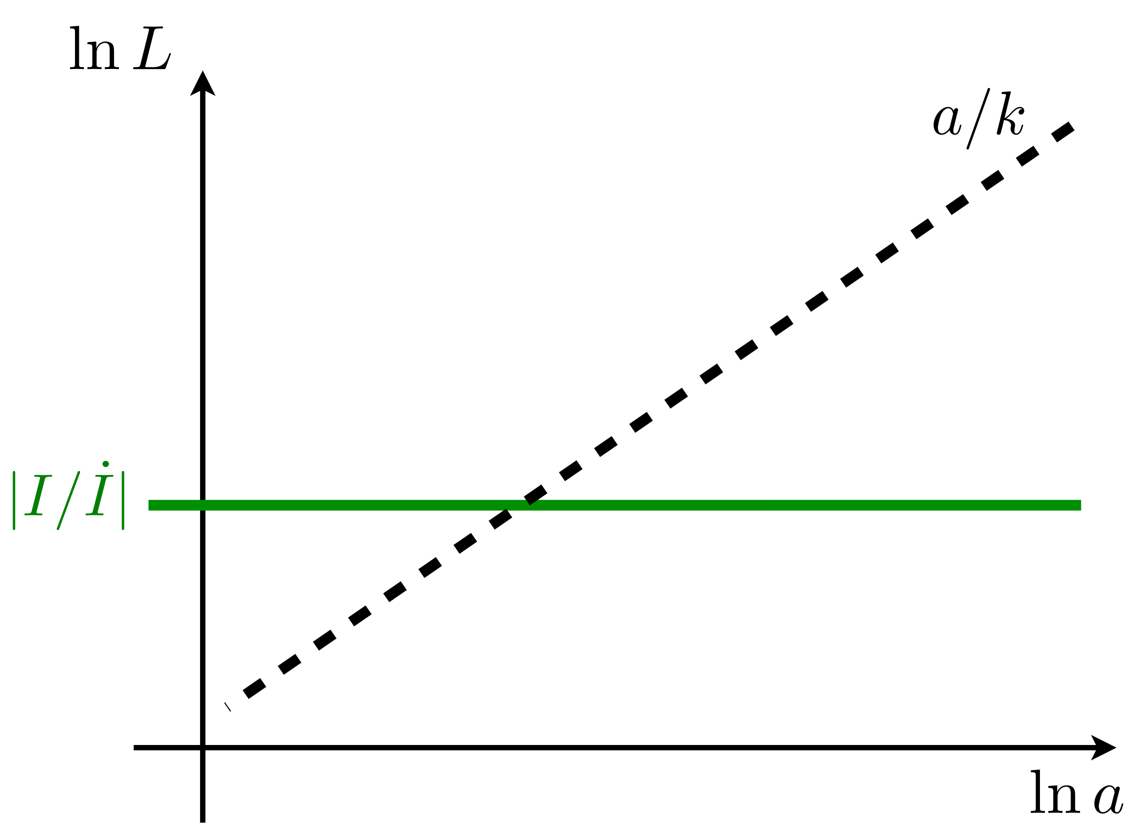
EXAMPLE OF A CLASSICAL MODEL

$$\frac{\mathcal{L}}{\sqrt{-g}} = -\frac{I(\sigma)^2}{4} F_{\mu\nu} F^{\mu\nu}$$

Ratra '92

modified Maxwell's equations : $A_{\boldsymbol{k}}^{\prime\prime(p)} + 2\frac{I'}{I} A_{\boldsymbol{k}}^{\prime(p)} + k^2 A_{\boldsymbol{k}}^{(p)} = 0$

(in Coulomb gauge)



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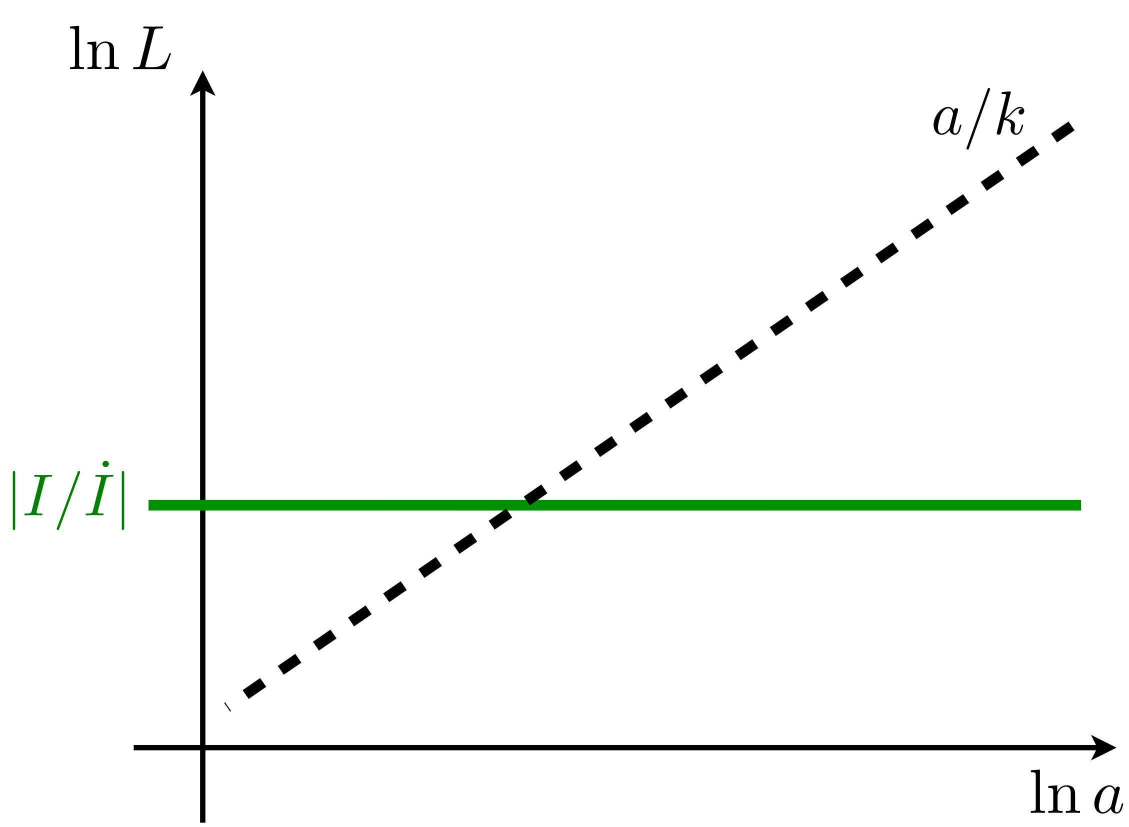

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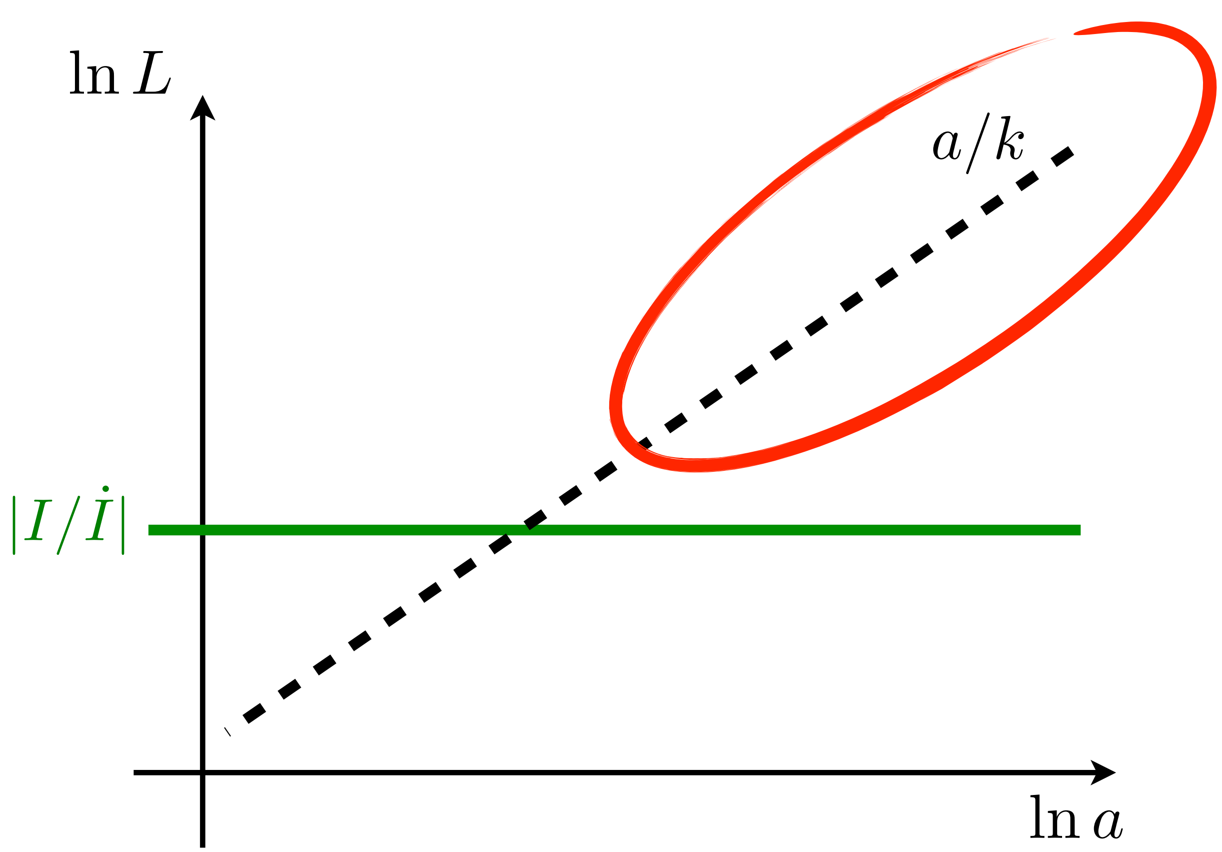

classical growth of magnetic fields outside the horizon

LARGE B COMES WITH LARGER E

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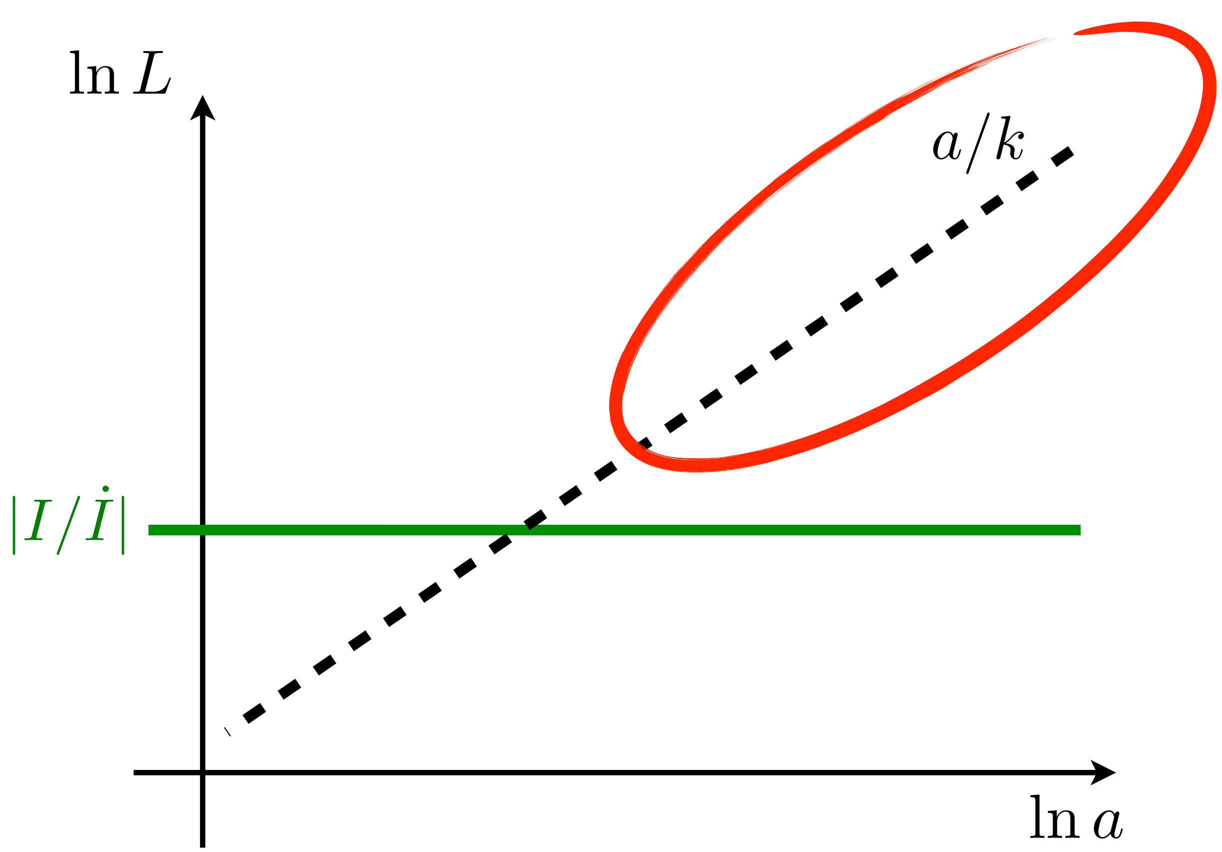

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Worries :

- backreaction from electric fields

  Bamba, Yokoyama '03, Demozzi, Mukhanov, Rubinstein, '09

- cosmological density perturbations

  Barnaby, Namba, Peloso '12, Bartolo, Matarrese, Peloso, Ricciardone '12

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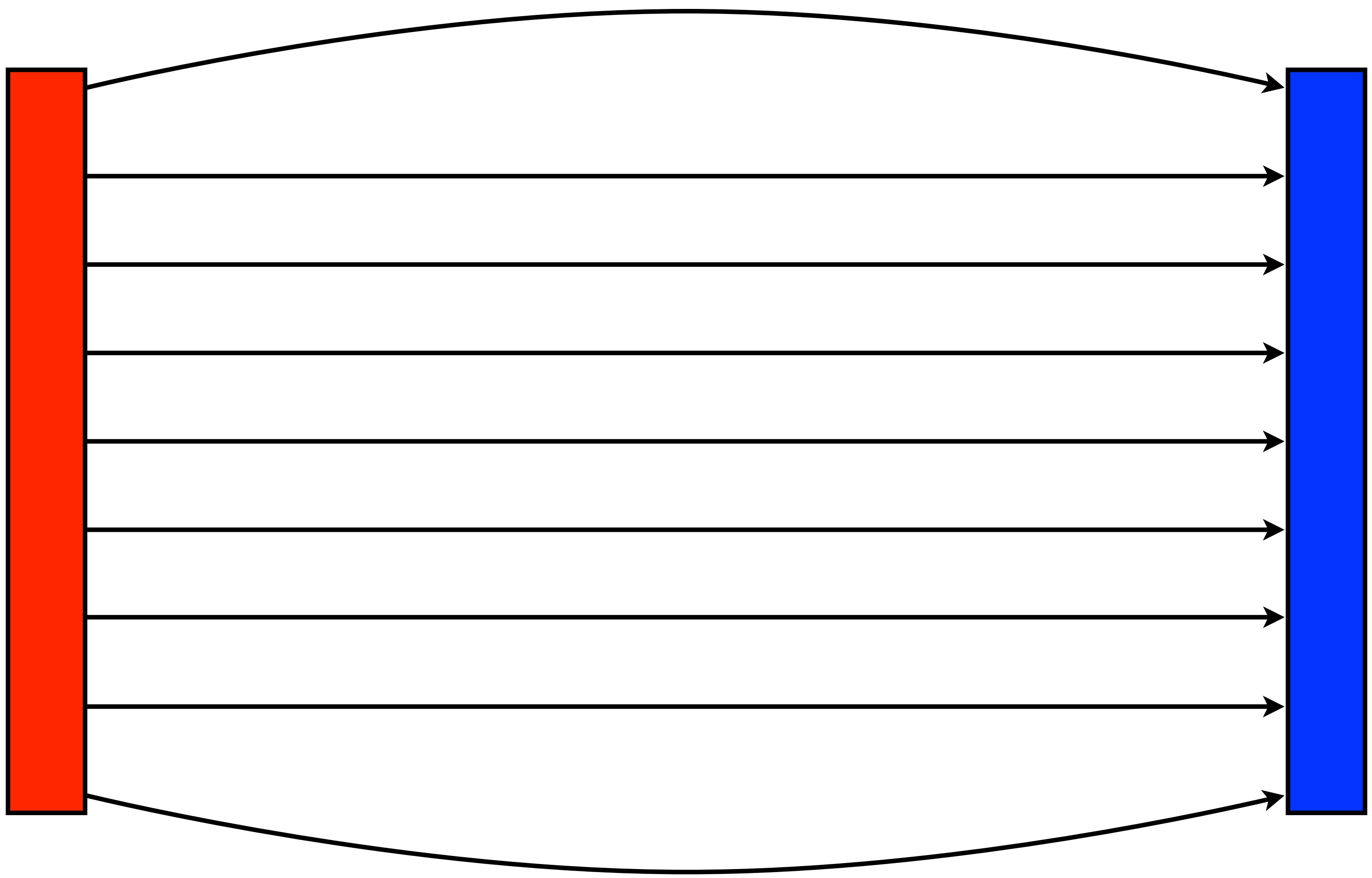
- Schwinger effect TK, Afshordi '14

SCHWINGER EFFECT

Sauter '31   Heisenberg, Euler '36   Schwinger '51

creation of charged particle pairs under strong electric fields



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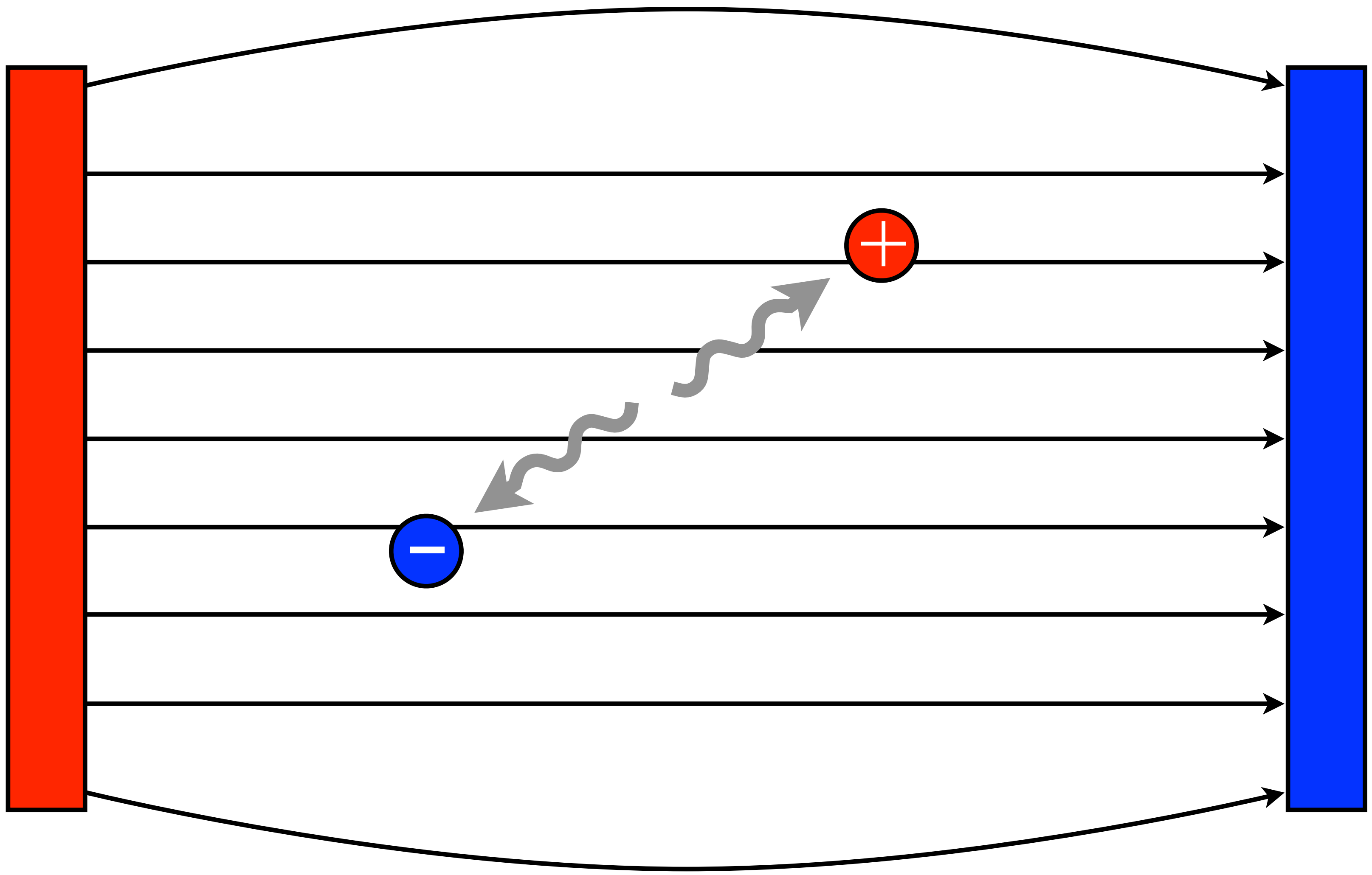

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$$\dot{B} \longrightarrow E \xrightarrow{\text{Schwinger effect}} \text{conductivity in inflating universe}$$

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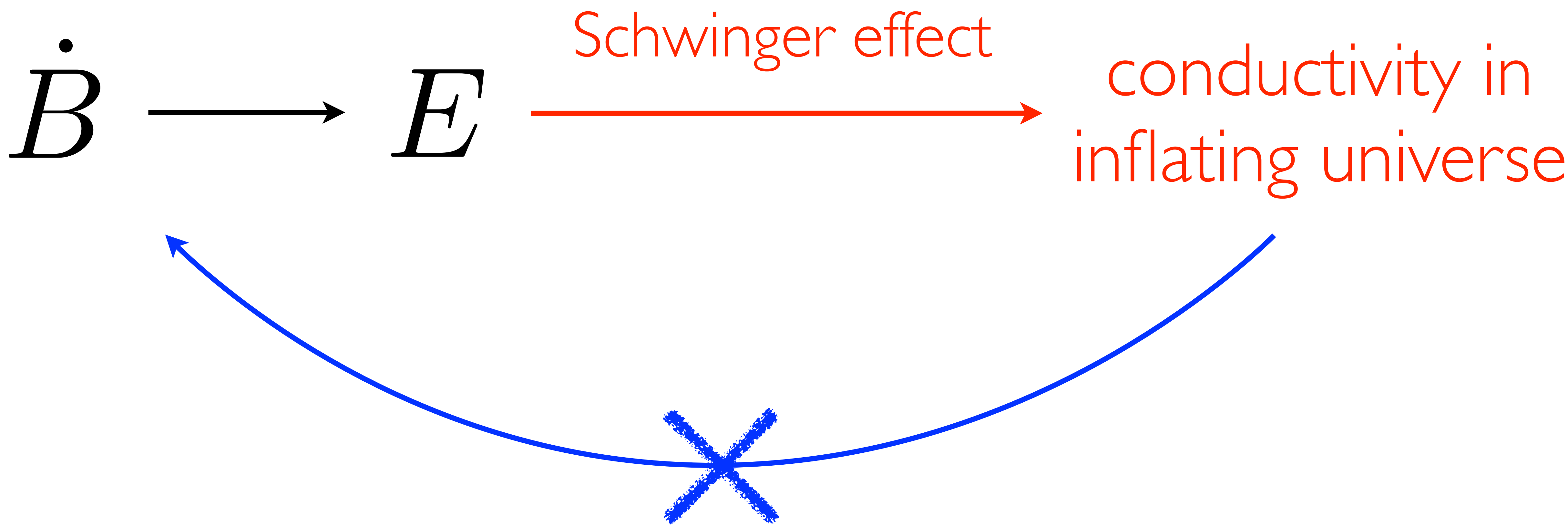
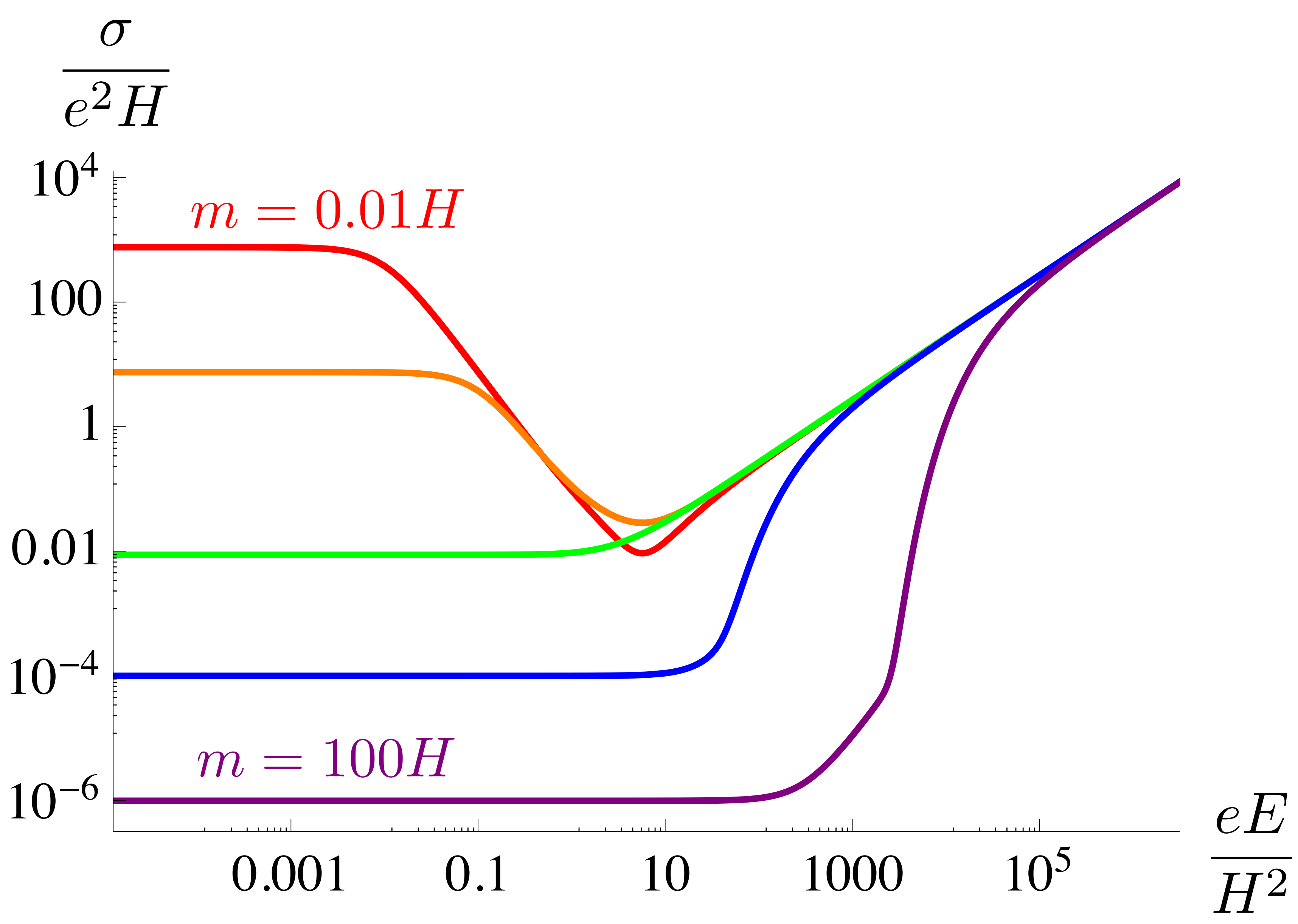


Magnetic field generation eventually saturates!

SCHWINGER EFFECT IN DE SITTER SPACE

TK, Afshordi '14 (2D dS : Fröb et al. '14)

conductivity of dS



Schwinger production of fields with charge e and mass m

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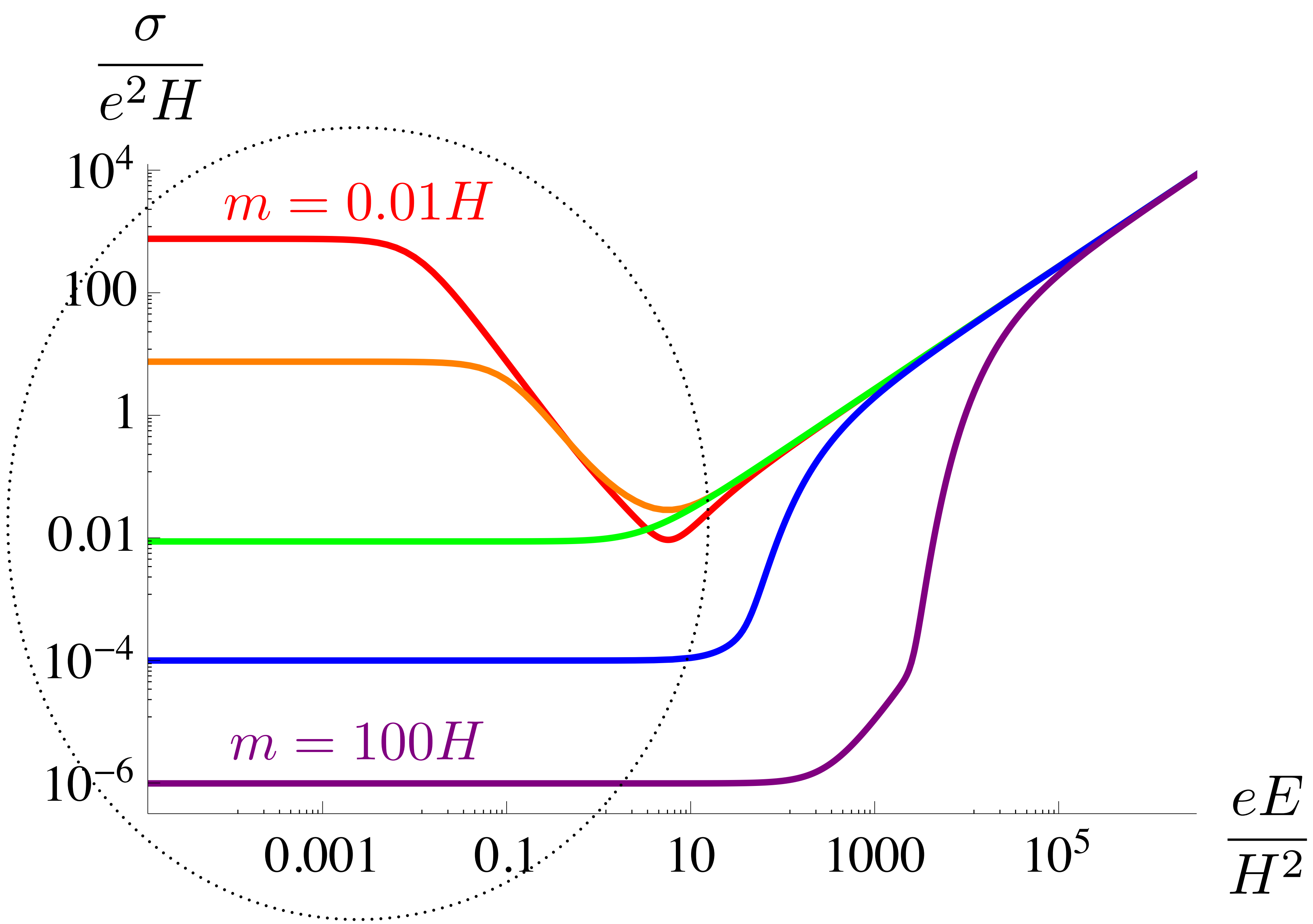
$$\sigma \sim \frac{e^2 H^3}{m^2}$$



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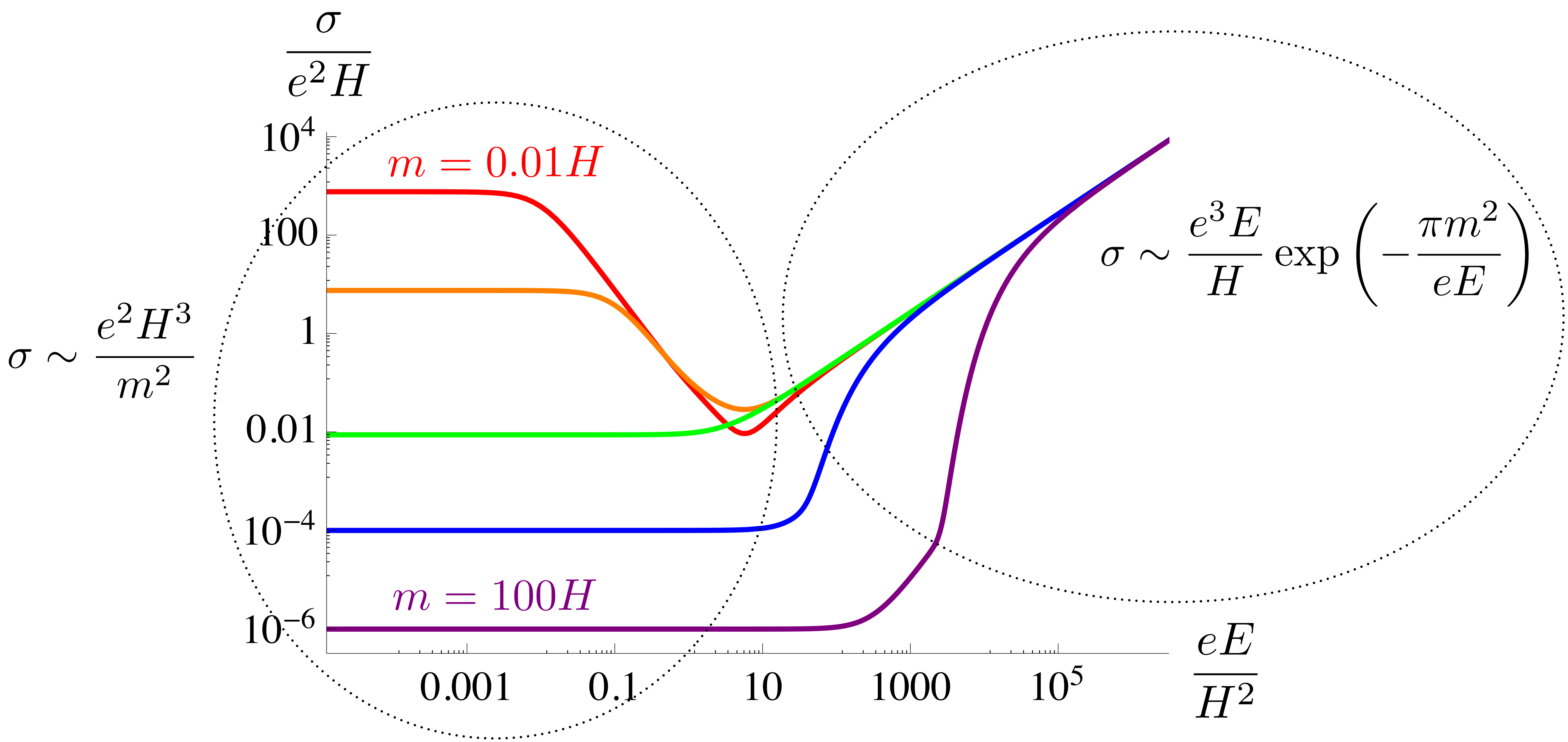

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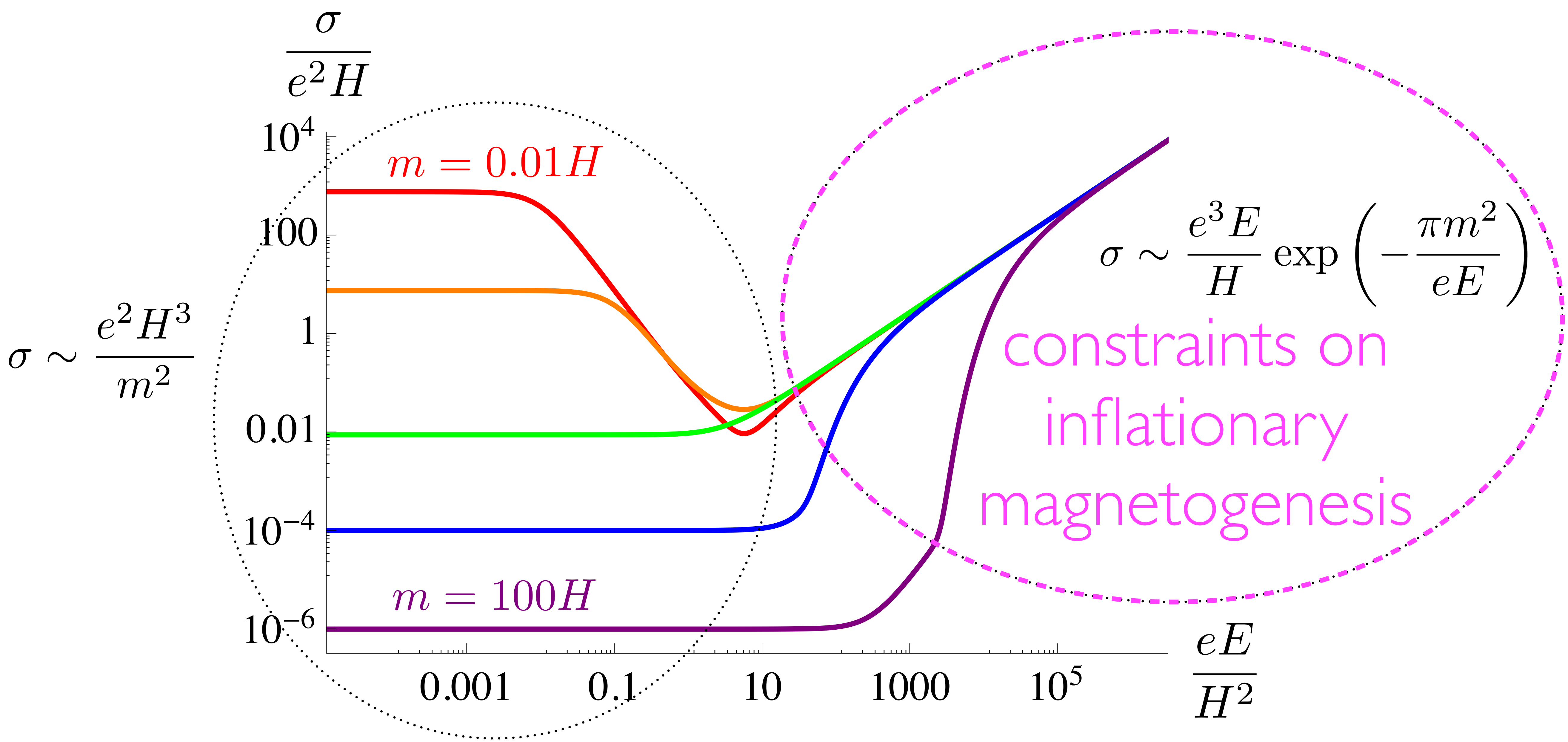

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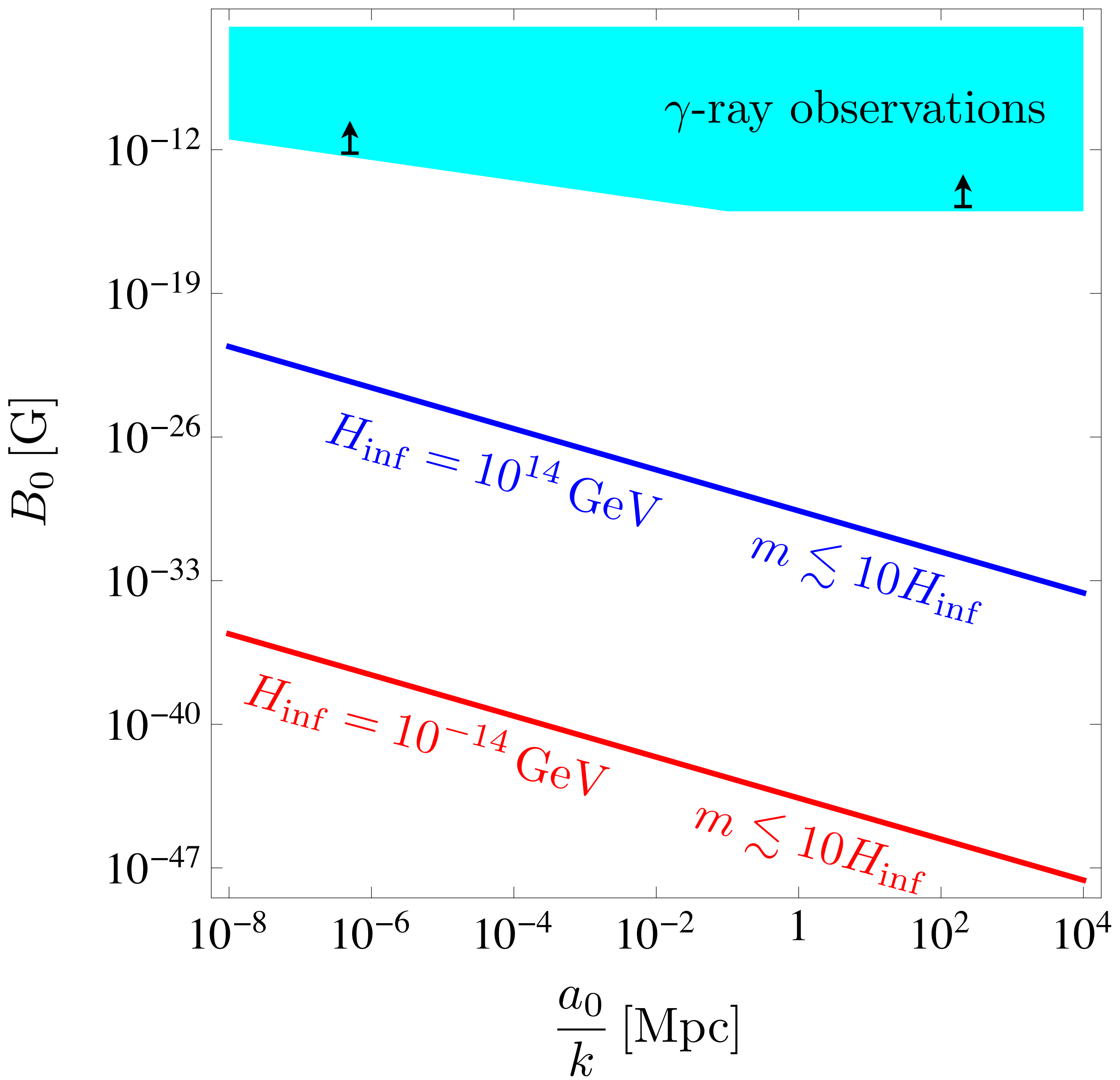
SCHWINGER CONSTRAINT ON I^2FF MODELS

$$|B_0| \lesssim 10^{-28}\mathrm{G}\left(\frac{k}{a_0}\,\mathrm{Mpc}\right)\left(\frac{H_{\rm inf}}{M_p}\right)^{1/2}\left(\frac{\sqrt{4\pi\alpha}}{e}\right)^3 I_{\rm end}^2 \exp\left\{W\left(10^{-3}\,\frac{e^2}{4\pi\alpha}\,\frac{1}{sI_{\rm end}^2}\,\frac{m^2}{H_{\rm inf}^2}\right)\right\}$$



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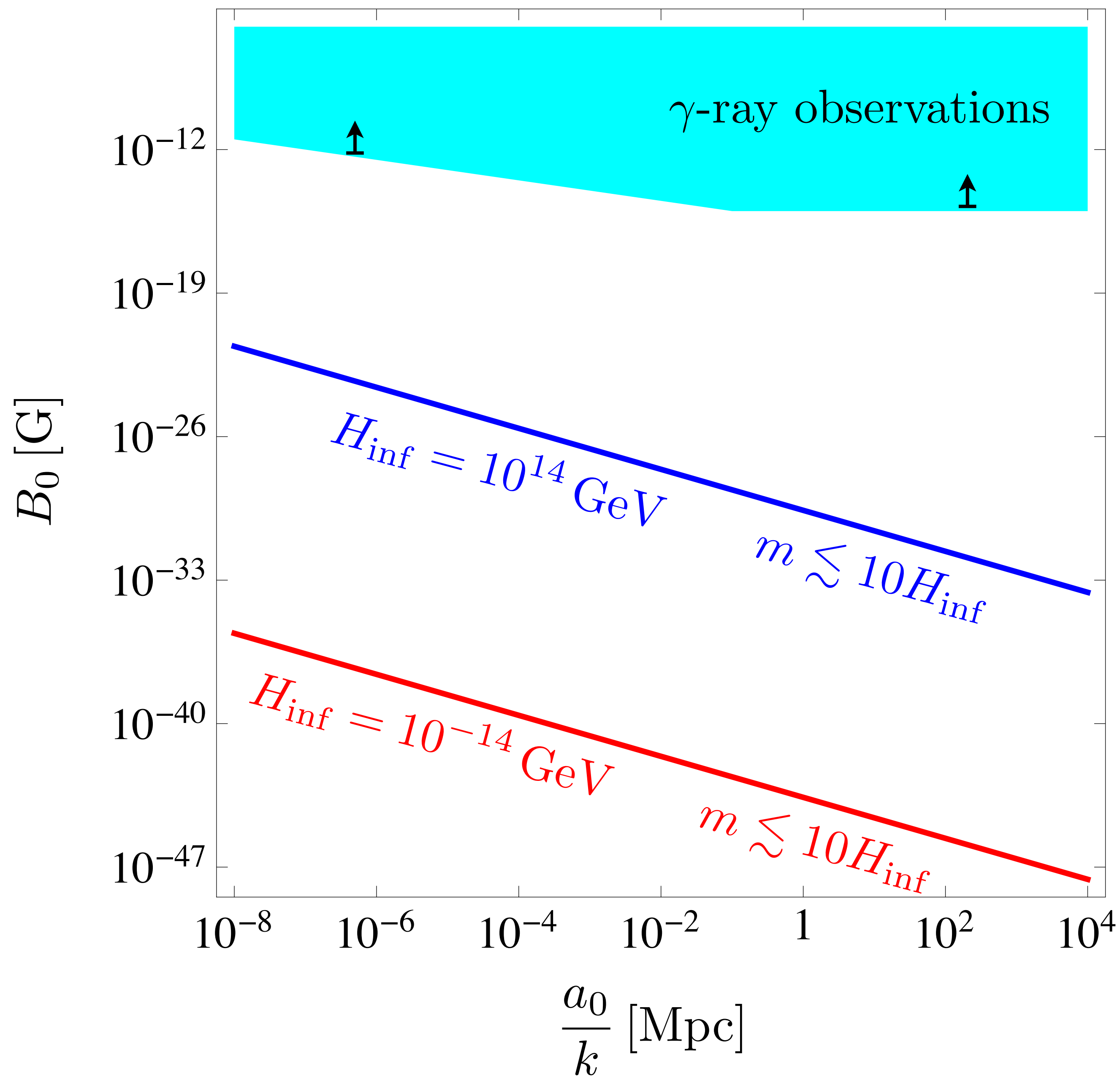

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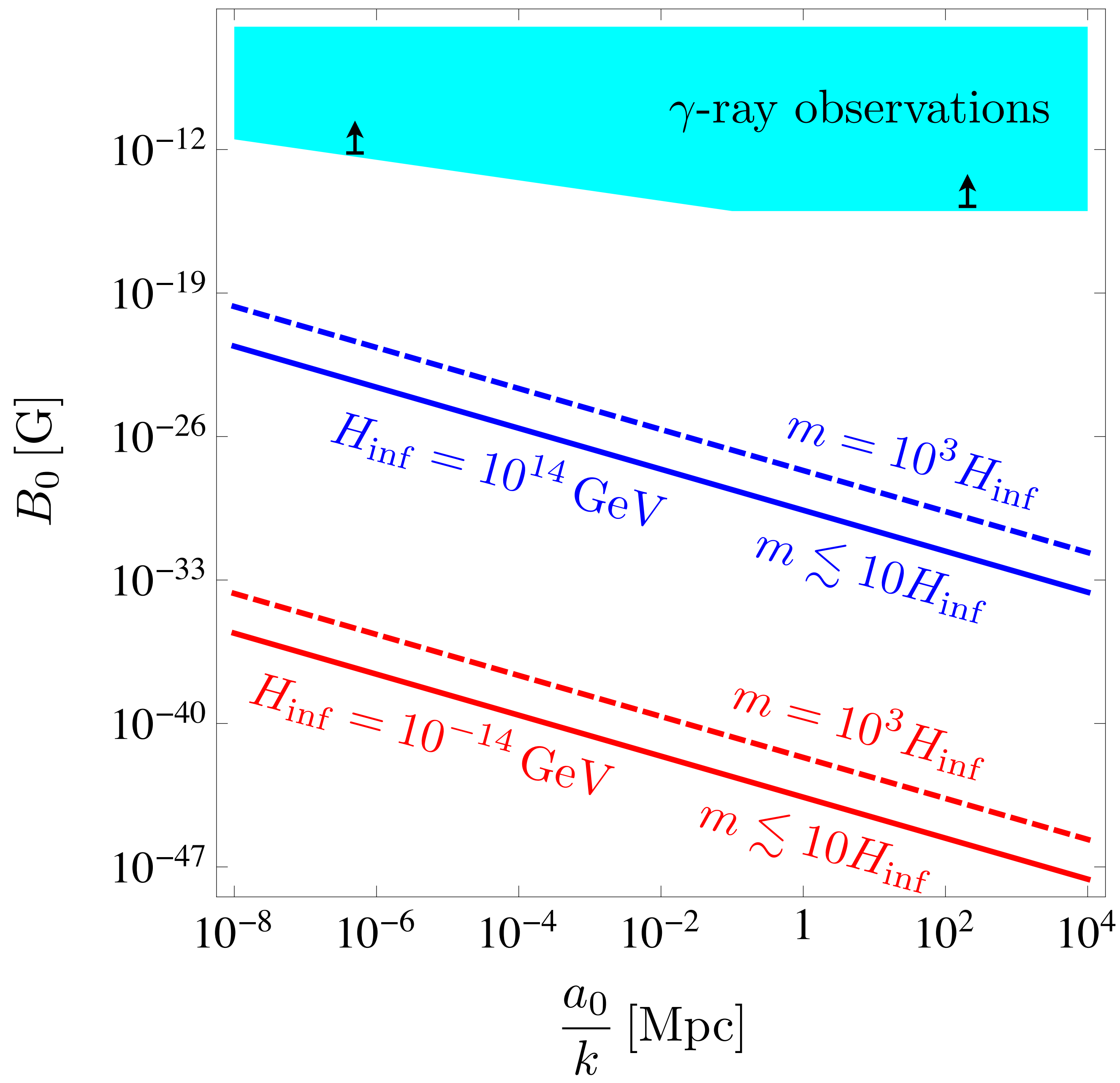

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Schwinger constraint on I^2FF models:

$B \lesssim 10^{-30}\mathrm{G}$ on Mpc scales

unless...

- all charged fields have heavy mass ($\ggg H_{inf}$)
- all charged fields have tiny charges
- all charged fields absent from action during inflation

CONSTRAINTS ON GENERAL CLASSICAL SCENARIOS

For general vector theories with
two-derivative time kinetic terms,

$$S = \int d\tau d^3x \left(\frac{I^2}{2} A_i' A_i' + \cdots \right)$$

by requiring the absence of strong couplings and
the vector not to spoil cosmological perturbations,

$$\mathcal{P}_{B0}(k) \lesssim (10^{-15}\,\mathrm{G})^2 \left(\frac{k}{a_0}\,\mathrm{Mpc} \right)^2 \frac{10^{-19}\,\mathrm{GeV}}{H_{\mathrm{inf}}}$$

Green, TK '15

→ $T_{\mathrm{reh}} \lesssim 10^2\,\mathrm{MeV}$ for 10^{-15} G on Mpc scales or larger

SUMMARY OF CLASSICAL SCENARIOS

- Classical scenarios of inflationary magnetogenesis with a two-derivative time kinetic term generally require $T_{\rm reh} \lesssim 10^2$ MeV for producing 10^{-15} G on Mpc scales.

- Individual models are further restricted by Schwinger effect.

Quantum Mechanical Production and General Constraints

arXiv:1511.08793 w/D. Green

VECTOR THEORY WITH SPONTANEOUSLY BROKEN TIME DIFFS

action in unitary gauge :

$$S = \int d^4x \sqrt{-g} \left\{ \mathcal{J}(\tau) F_{\mu\nu} F^{\mu\nu} + \mathcal{K}(\tau) F^0{}_\mu F^{0\mu} + \cdots \right\}$$

(cf. EFT of inflation)

In an FRW universe, and in Coulomb gauge,

$$S = \int d^4x \, \frac{I(\tau)^2}{2} \left\{ A_i' A_i' - c(\tau)^2 \partial_i A_j \partial_i A_j + \cdots \right\}$$

$$I(\tau)^2 \equiv 2 \frac{\mathcal{K}(\tau)}{a(\tau)^2} - 4 \mathcal{J}(\tau) \qquad c(\tau)^2 \equiv \left(1 - \frac{\mathcal{K}(\tau)}{2 a(\tau)^2 \mathcal{J}(\tau)} \right)^{-1}$$

QUANTUM MECHANICAL PRODUCTION FROM A VARIABLE LIGHT SPEED

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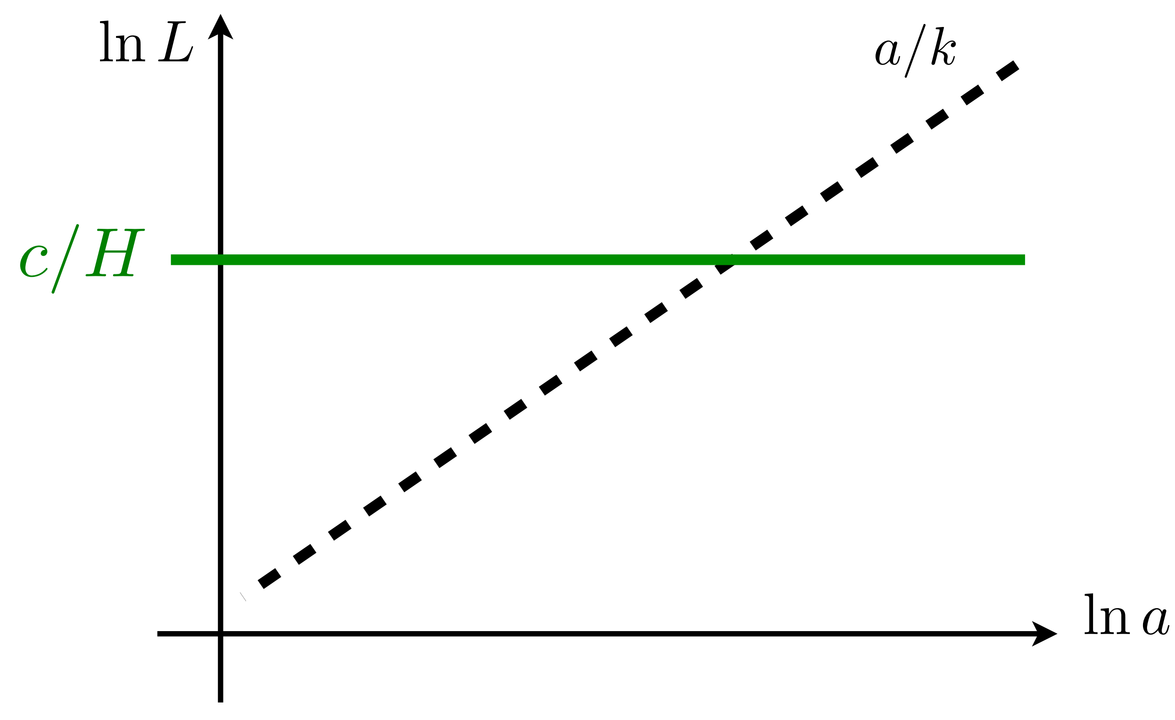

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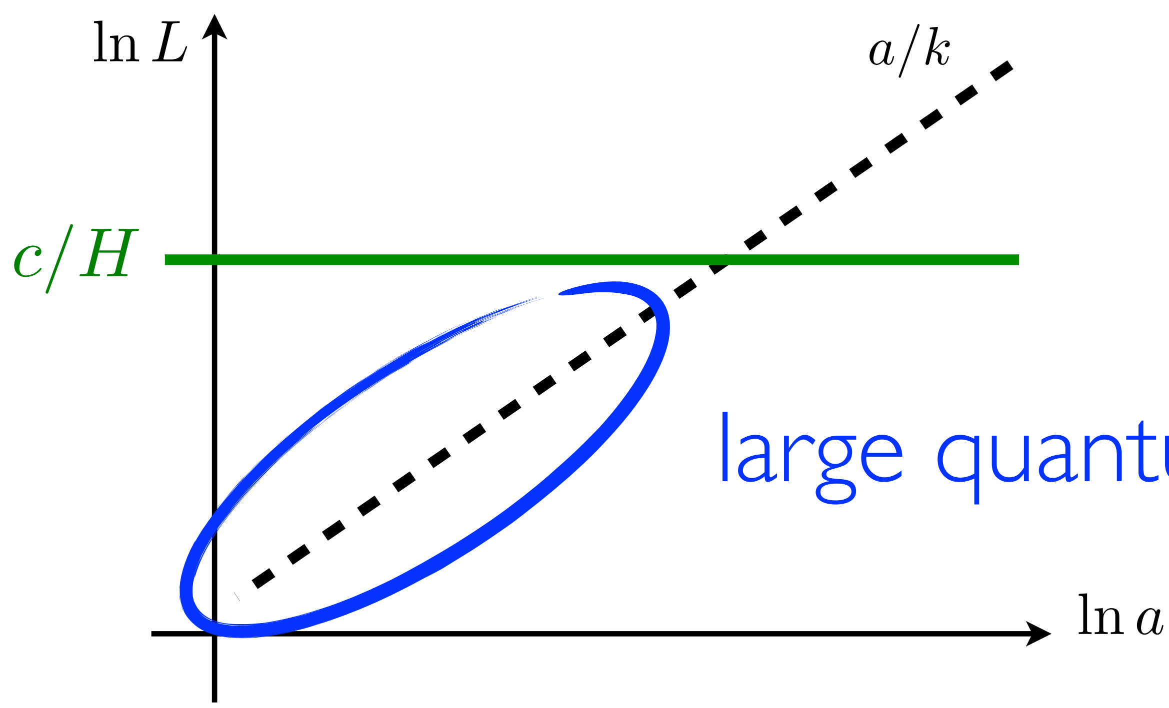

large quantum fluc. for $c \ll 1$

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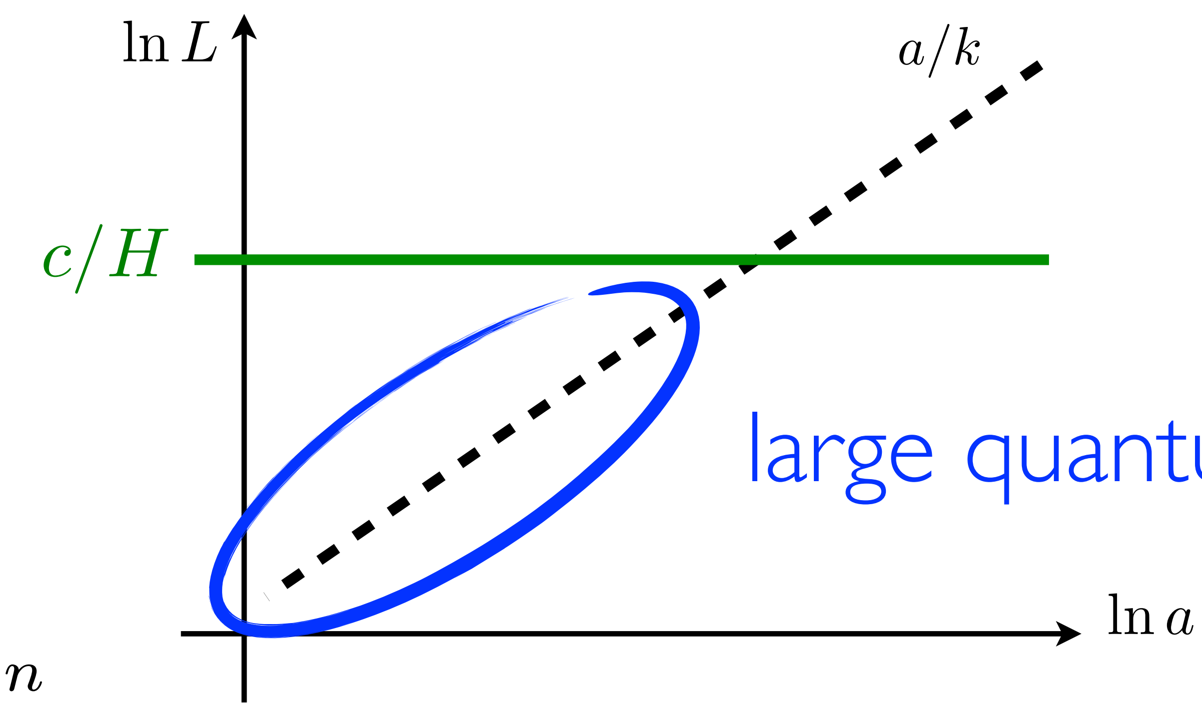

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For $c \propto a^{-n}$,

$$\mathcal{P}_B \sim \frac{k^4}{2\pi^2 a^4 c_\star} \qquad \mathcal{P}_E \sim \frac{c_\star k^4}{2\pi^2 a^4} \left(\frac{a}{a_\star} \right)^{-4(n+\frac{1}{2})}$$

$\star$: at sound horizon exit

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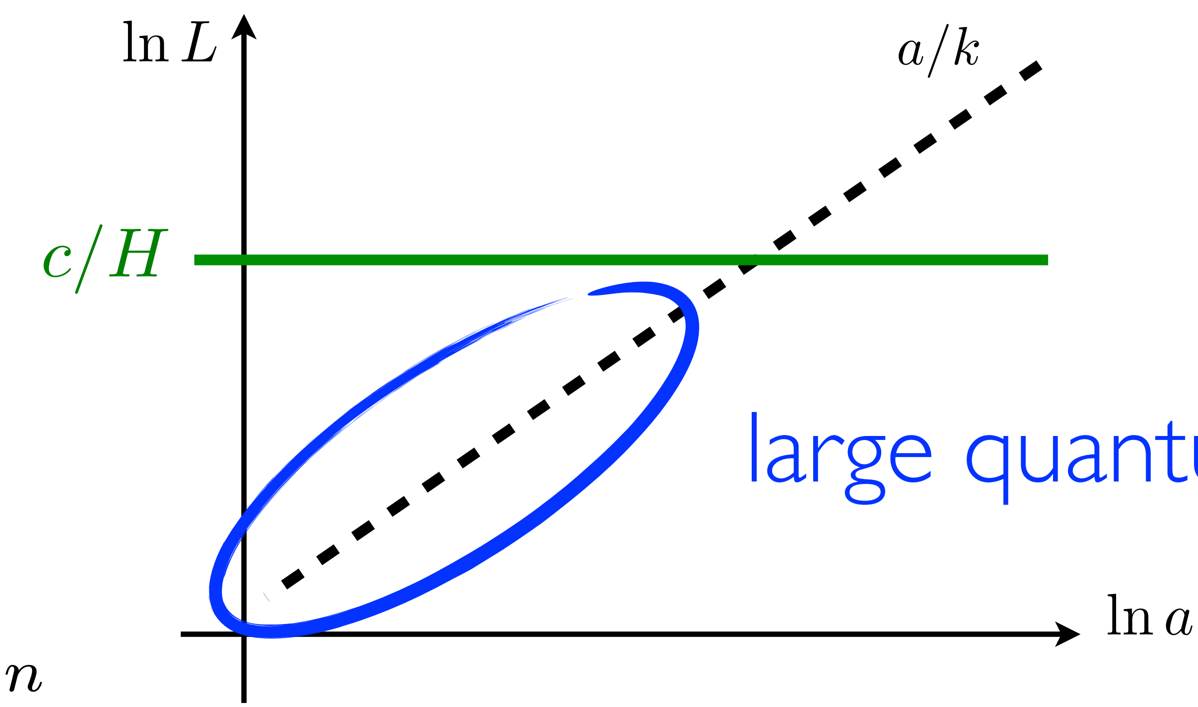

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However, c cannot be arbitrarily small.

CONSTRAINTS ON GENERAL QUANTUM MECHANICAL SCENARIOS

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uncertainty principle

$$[A_i(\tau, \boldsymbol{x}), \Pi_j(\tau, \boldsymbol{y})] = i\delta^{(3)}(\boldsymbol{x} - \boldsymbol{y})\delta_{ij}$$

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$$\rightarrow \quad \mathcal{P}_{B0}(k) \lesssim (10^{-43}\,{\rm G})^2 \left(\frac{k}{a_0}\,{\rm Mpc} \right)^4 \left(\frac{10^{-23}\,{\rm GeV}}{H_{\rm inf}} \right)^{2/3}$$

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Quantum Mechanical Phase

uncertainty principle

$$[A_i(\tau, \boldsymbol{x}), \Pi_j(\tau, \boldsymbol{y})] = i\delta^{(3)}(\boldsymbol{x} - \boldsymbol{y})\delta_{ij}$$

Classical Phase

kinetic energy constraint

$$\rho_{\rm kin} \sim \left\langle \frac{I^2}{a^4} A_i' A_i' \right\rangle < \rho_{\rm inf}$$

If the transition is smooth,

$c_{\rm p} = \frac{\omega_k}{k}$ at classicalization is bounded from below.

$$\rightarrow \quad \mathcal{P}_{B0}(k) \lesssim (10^{-43}\,\mathrm{G})^2 \left(\frac{k}{a_0}\,\mathrm{Mpc} \right)^4 \left(\frac{10^{-23}\,\mathrm{GeV}}{H_{\rm inf}} \right)^{2/3}$$

CONSTRAINTS ON GENERAL QUANTUM MECHANICAL SCENARIOS

Cases with non-smooth quantum to classical transitions (e.g. resonant production) can also be constrained.

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Under rather generic assumptions, a bound similar to that on classical scenarios can be obtained:

→ $T_{\rm reh} \lesssim 10^2\,{\rm MeV}$ for 10^{-15} G on Mpc scales or larger

SUMMARY OF QUANTUM MECHANICAL SCENARIOS

- Bounds are at least as strong as for classical scenarios.

- With smooth quantum to classical transitions, $B \lesssim 10^{-43}$ G on Mpc scales.

WHAT ELSE TO TRY?

- time kinetic terms with less/more than two derivatives
- live with strong couplings (how to shut off Schwinger effect?)
- non-Bunch-Davies vacua
- magnetic field evolution after inflation
  (e.g. IR cascade of helical fields)
- magnetogenesis in non-inflationary universe

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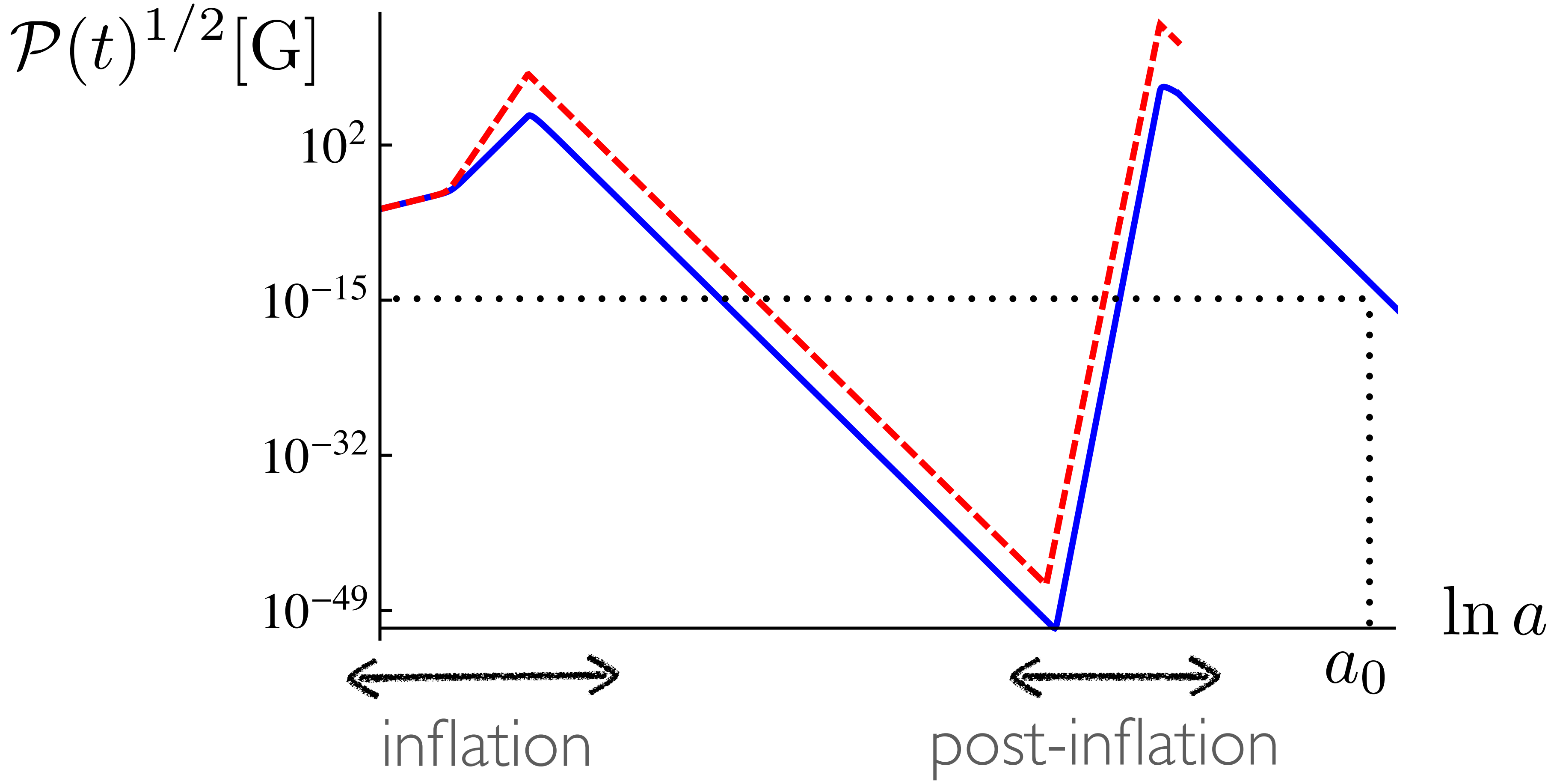
Magnetogenesis in the Post-Inflationary Universe

arXiv:1403.5168

POST-INFLATIONARY MAGNETOGENESIS

TK '14

By breaking conformal symmetry after inflation, magnetic fields can be generated up until reheating.



Conformal symmetry breaking from couplings with inflaton/spectator field that is rolling/oscillating.

POST-INFLATIONARY MAGNETOGENESIS

TK '14

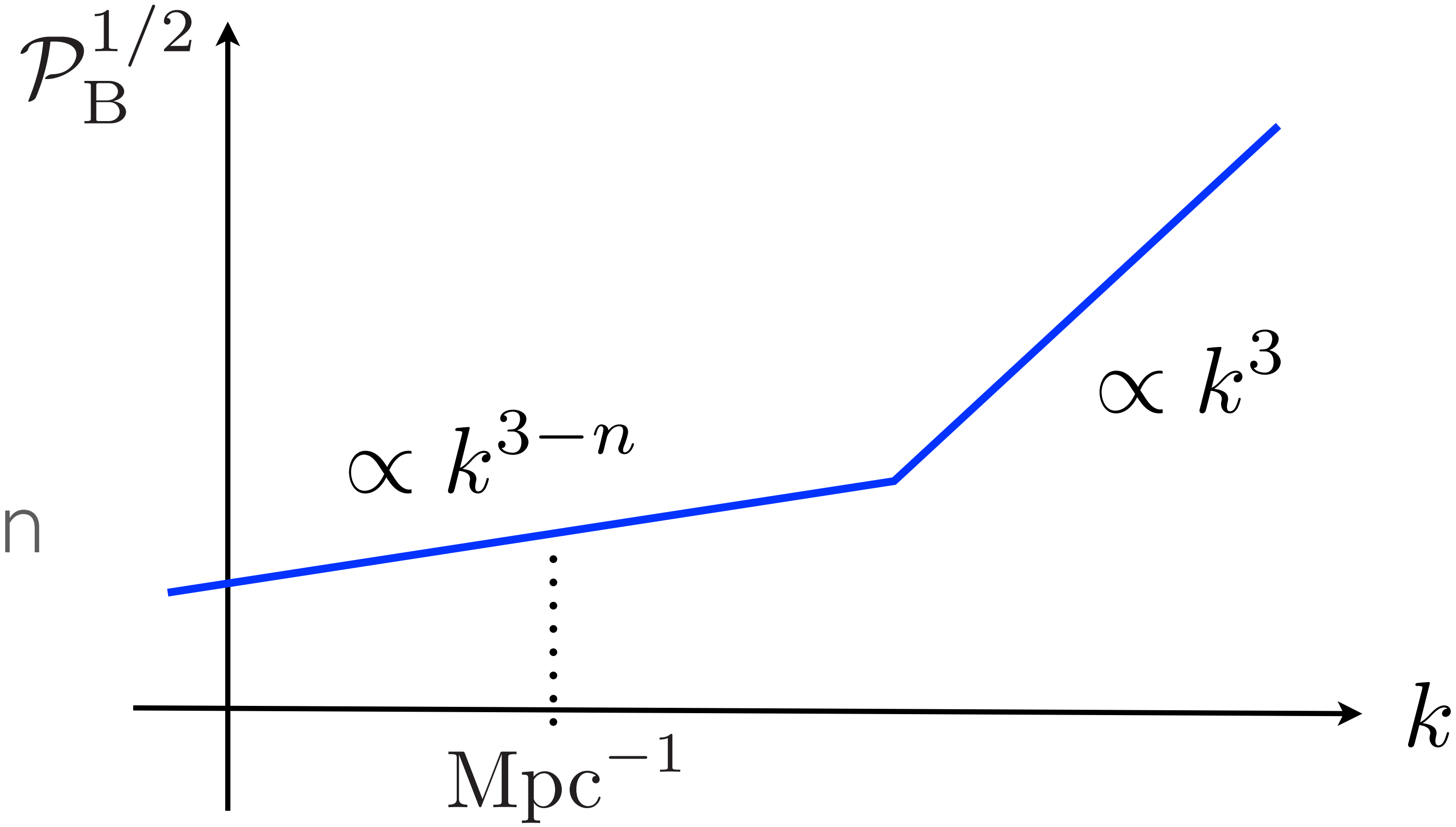
- combined inflationary/post-inflationary magnetogenesis can produce $B \geq 10^{-15}\mathrm{G}$ on Mpc scales

- free of electric energy domination, strong couplings, spoiling density perturbations

- may also evade the Schwinger constraint

BLUE-TILTED B SPECTRUM

case with
$I \propto a^{-n}$
during inflation



BLUE-TILTED B SPECTRUM

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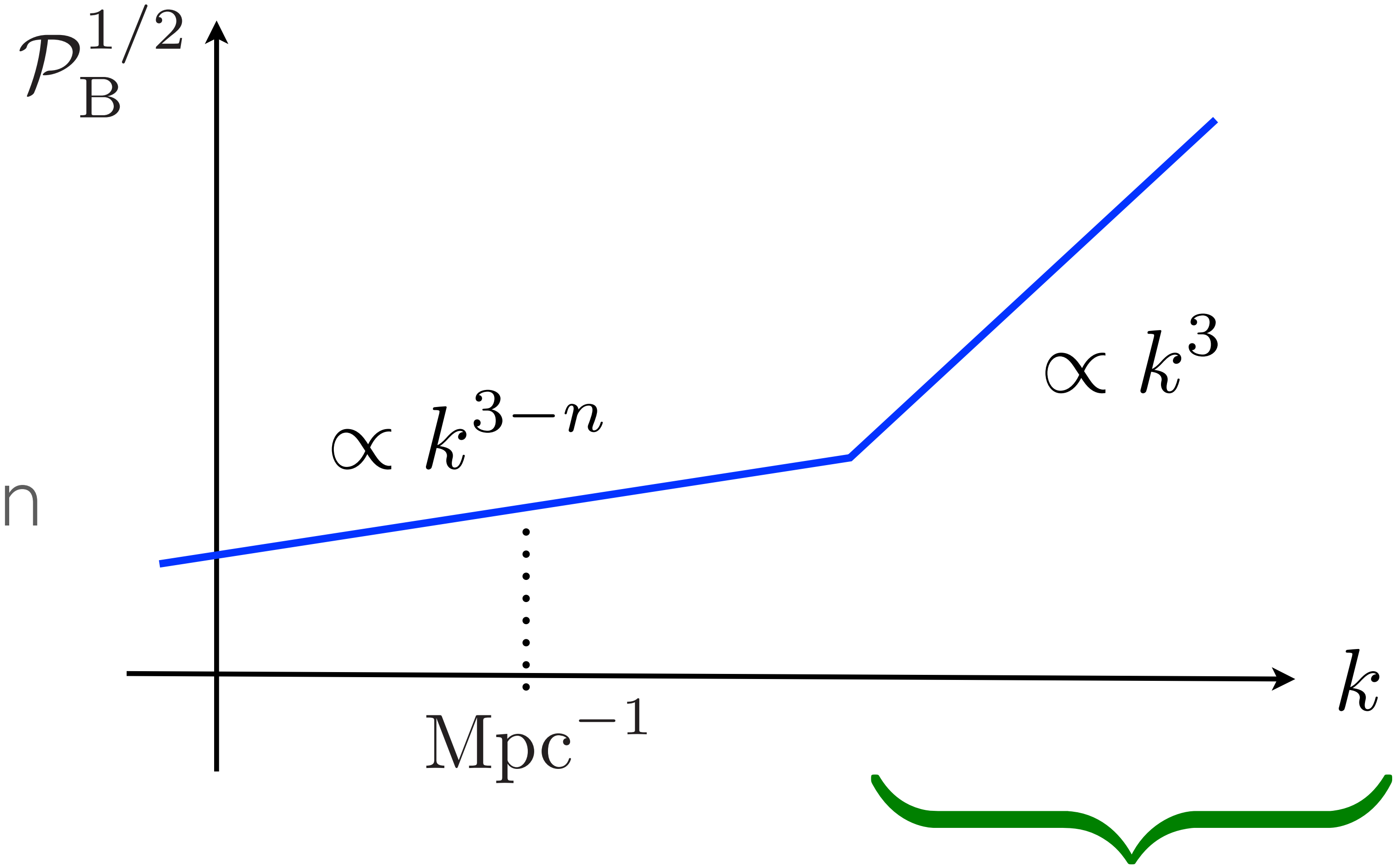


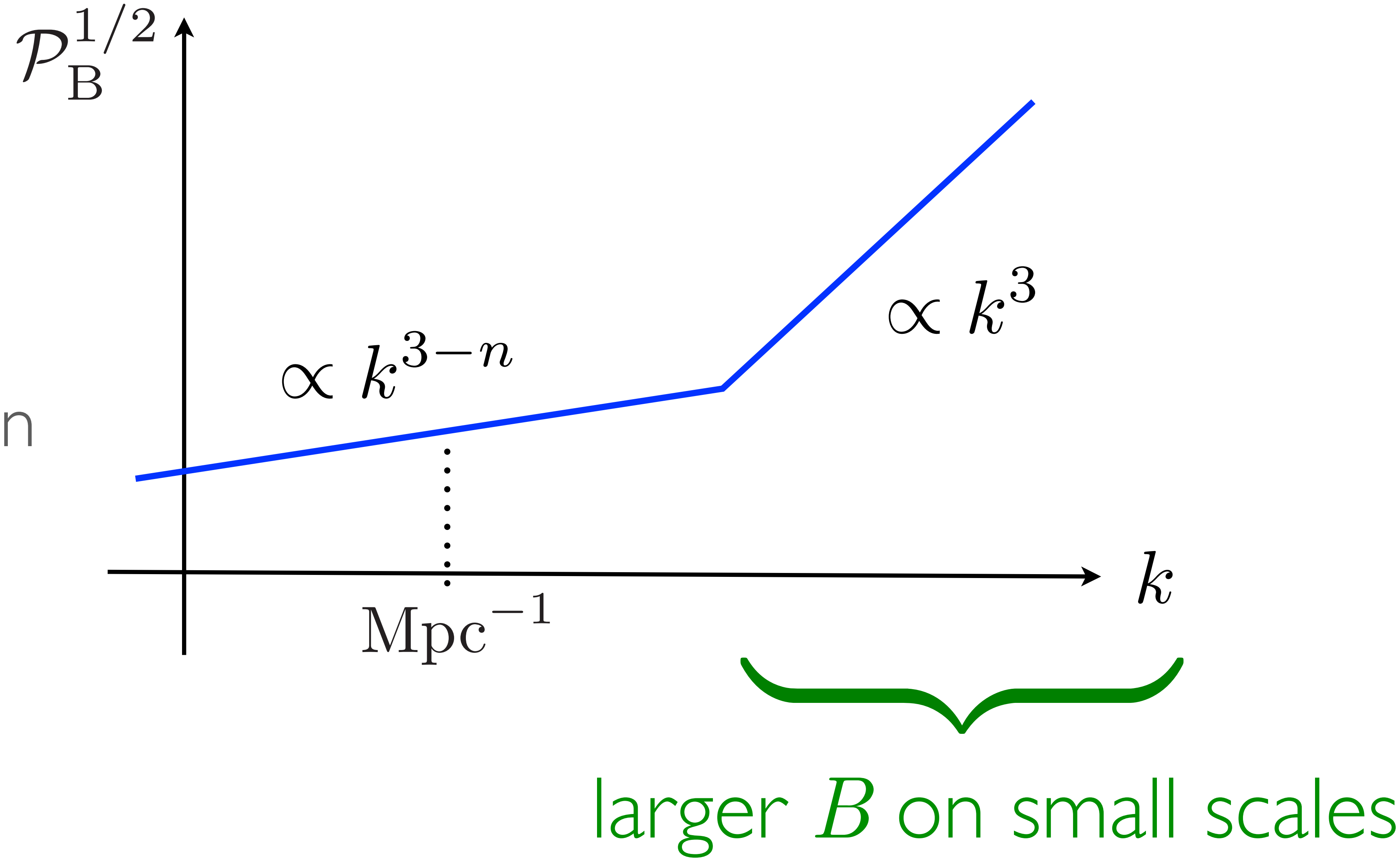
larger B on small scales

BLUE-TILTED B SPECTRUM

case with

$I \propto a^{-n}$

during inflation



B measurement on other scales are welcome!

e.g., with 21-cm observations (Venumadhav, Oklopcic, Gluscevic, Mishra, Hirata '14)

SUMMARY

- Inflationary magnetogenesis is highly constrained.
  - ✓ Classical scenarios require $T_{\rm reh} \lesssim 10^2$ MeV for producing 10^{-15} G on Mpc scales or larger.
  - ✓ Schwinger effect makes it worse: $B < 10^{-30}$ G for I^2FF models.
  - ✓ Stronger constraints for quantum mechanical scenarios.
- Post-inflationary magentogenesis can produce $B \geq 10^{-15}$ G on Mpc scales.
- Further investigation of cosmological magnetic fields may provide new insights into the very early universe!