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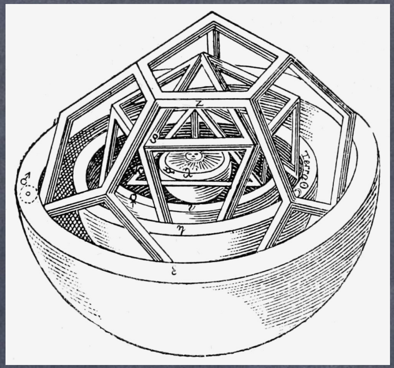


On tensor modes in solid inflation

w/ Jonghee Kang

(w/ Solomon Endlich and Junpu Wang, 2012)

Inflation: usual story

- The early universe: homogeneous and isotropic

- Usually modeled via $\varphi_a = \varphi_a(t)$

- Time-translations spontaneously broken

⇨ Goldstone excitation = adiabatic perturbations

- Systematic effective field theory

(Creminelli, Luty, Nicolis, Senatore 2006
Cheung, Creminelli, Fitzpatrick, Kaplan, Senatore 2007)

Solid inflation

(Endlich, Nicolis, Wang 2012)
(Gruzinov 2004)

- t-independent, x-dependent fields: $\varphi_a = \varphi_a(\vec{x})$
- time-translations unbroken
- spatial translations and rotations, broken

Apparently violates:

1. homogeneity and isotropy ⟹ internal symmetries
2. the need for a physical "clock" ⟹ gravity

Homogeneity and isotropy

- Ex: one scalar w/ vev $\langle\varphi\rangle = x$
- If it has a shift symmetry $\varphi \to \varphi + a$
- unbroken diagonal translation

$$\begin{cases} x \to x - a \\ \varphi \to \varphi + a \end{cases}$$

Rotations still broken $\Longrightarrow$ need 3 fields

3 scalars: $\phi^I(\vec{x}, t) \qquad I = 1, 2, 3$

vevs: $\langle \phi^I \rangle = x^I$

If internal symmetries:

$$\phi^I \to \phi^I + a^I$$
$$\phi^I \to SO(3)\, \phi^I$$

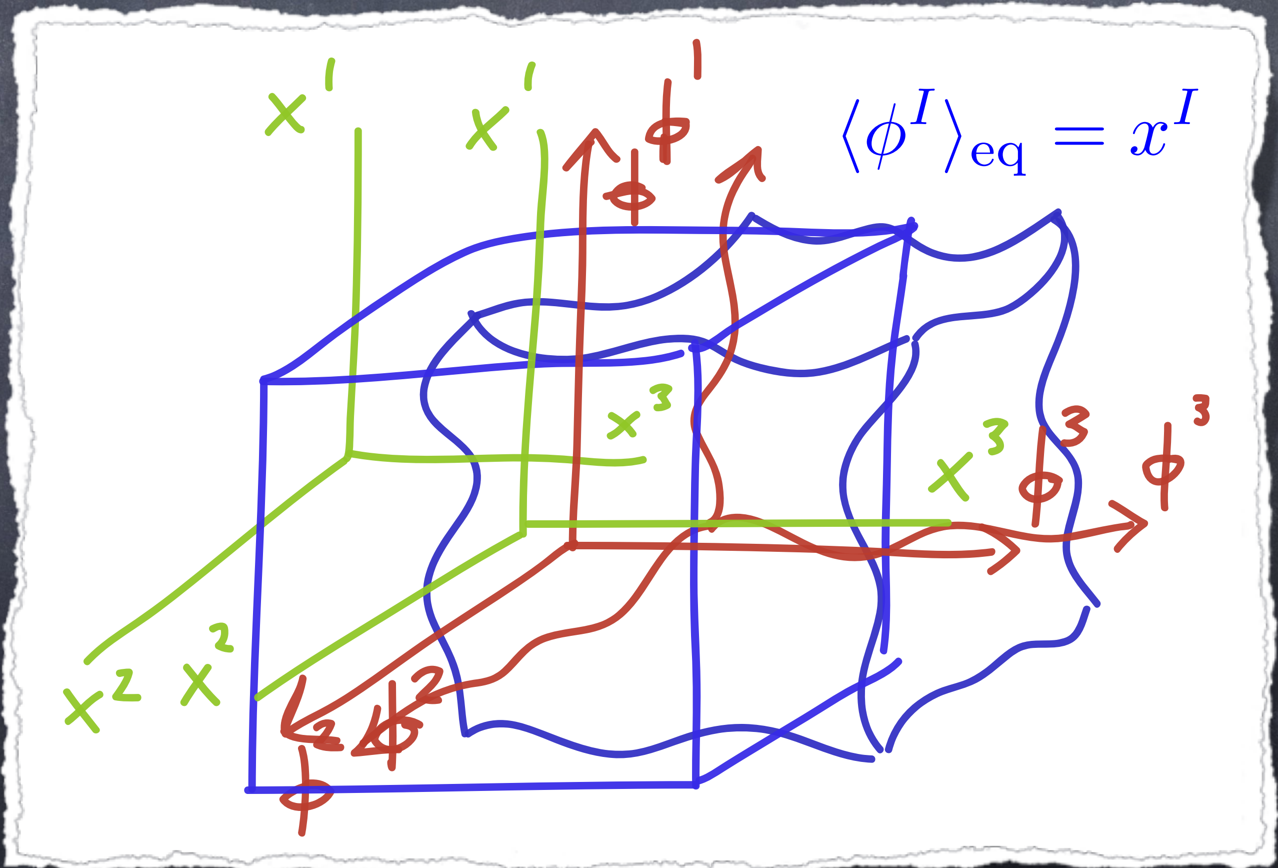
then unbroken diagonal subgroups

This is a solid

EFT for solids

Dof: volume elements' positions

$$\phi^I(\vec{x}, t) \qquad I = 1, 2, 3$$



Action

$$B^{IJ} \equiv \partial_\mu \phi^I \partial^\mu \phi^J$$

$$\mathcal{L} = F\Big([B], \frac{[B^2]}{[B]^2}, \frac{[B^3]}{[B]^3}\Big) + \dots \qquad\qquad [\dots] = \mathrm{Tr}(\dots)$$

$$(X, Y, Z)$$

(Dubovsky, Gregoire, Nicolis, Rattazzi 2006)
(Son 2005)

Problem


The universe is expanding.
Your closet is not.
Get more closet space: ManhattanMiniStorage.com
manhattan mini storage

Stress-energy tensor

$$T_{\mu\nu} \sim (F, F') \times (g_{\mu\nu}, \partial_\mu \phi^I \partial_\nu \phi^J) \times (\delta^{IJ}, B^{IJ}, B^{IK} B^{KJ})$$

On the background $\quad B^{IJ} = \delta^{IJ}$

$$T_{\mu\nu} \to \begin{cases} \rho = -F \\ \rho + p = -2\, X F_X \end{cases}$$

inflation ("slow roll") ⇨ small $\quad F_X = \mathcal{O}(\epsilon)$

⇨ Approximate internal scale invariance $\qquad \phi^I \to \lambda \phi^I$

Cosmological perturbations

$$\phi^I = x^I + \pi^I$$

$$g_{\mu\nu} = g_{\mu\nu}^{\rm FRW} + \delta g_{\mu\nu}$$

U.G.: $F(B^{IJ}) \to F(g^{IJ})$

Lorentz violating massive gravity

Very roughly:

$$\mathcal{L}_2 \sim F_X \cdot (\partial\pi)^2$$

$$\mathcal{L}_3 \sim F \cdot (\partial\pi)^3$$

$$\zeta \sim \vec{\nabla} \cdot \vec{\pi}$$

$\Rightarrow$

$$\langle\zeta\zeta\rangle \sim \frac{1}{\epsilon}\frac{1}{c_L^5}\frac{H^2}{M_{\rm Pl}^2}$$ (cf. $\frac{1}{\epsilon}\frac{1}{c_L}\frac{H^2}{M_{\rm Pl}^2}$)

$$\frac{\mathcal{L}_3}{\mathcal{L}_2} \sim \frac{1}{\epsilon}\frac{1}{c_L^2}\zeta$$ (cf. $\frac{1}{c_L^2}\zeta$)

$$S^{(2)} = S_\gamma^{(2)} + S_T^{(2)} + S_L^{(2)}$$

$$S_\gamma^{(2)} = \tfrac{1}{4} M_{\rm Pl}^2 \int dt\, d^3x\, a^3 \Big[\tfrac{1}{2}\dot\gamma_{ij}^2 - \tfrac{1}{2a^2} \big(\partial_m \gamma_{ij}\big)^2 + 2\dot H c_T^2\, \gamma_{ij}^2 \Big]$$

$$S_T^{(2)} = M_{\rm Pl}^2 \int dt \int_{\vec k} a^3 \bigg[\frac{k^2/4}{1 - k^2/4a^2 \dot H} \, \big|\dot\pi_T^i\big|^2 + \dot H c_T^2\, k^2 \big|\pi_T^i\big|^2 \bigg]$$

$$S_L^{(2)} = M_{\rm Pl}^2 \int dt \int_{\vec k} a^3 \bigg[\frac{k^2/3}{1 - k^2/3a^2 \dot H} \big|\dot\pi_L - (\dot H/H)\pi_L\big|^2 + \dot H c_L^2\, k^2 \big|\pi_L\big|^2 \bigg]$$

$$S_3 \simeq \int d^4x \left(- \tfrac{1}{243} F_Y \right) \cdot \Big\{ 16 \, [\partial\pi]^3 - 36 \, [\partial\pi]^2 \left([\partial\pi \cdot \partial\pi^T] + [(\partial\pi)^2] \right)$$
$$+ 18 \, [(\partial\pi)^3] + 18 \, [(\partial\pi)^2 \cdot \partial\pi^T] \Big\} \, ,$$

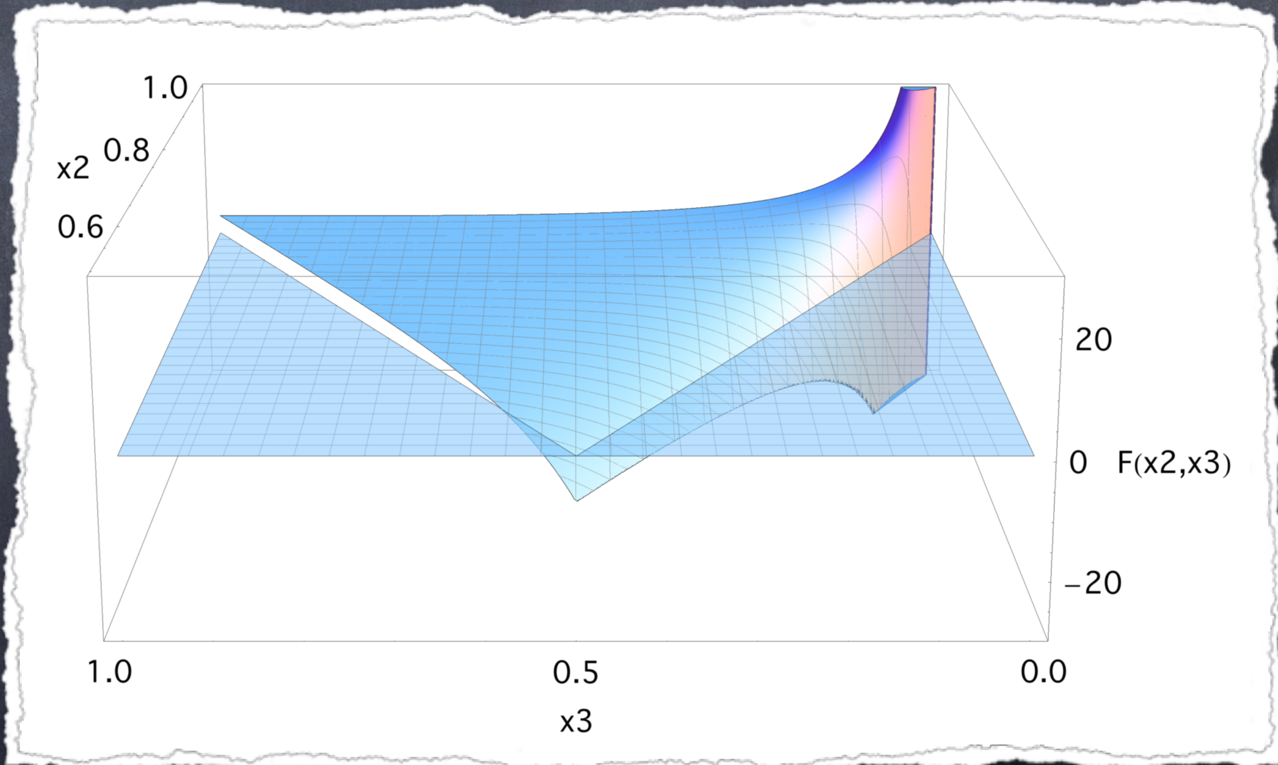
Observables

$$n_S - 1 = 2\epsilon\, c_L^2 - \eta - 5s$$

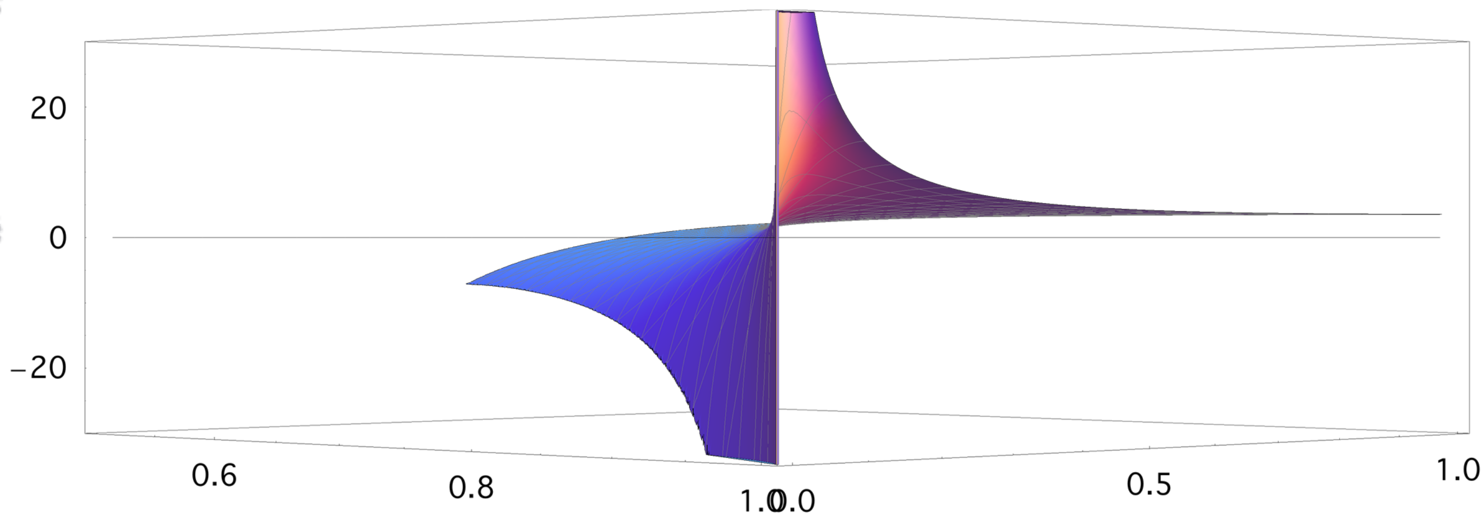
$$n_T - 1 = 2\epsilon\, c_L^2 \qquad \text{(mass term } \sim c_T^2 \text{)}$$

$$r = 16\,\epsilon c_L^5$$

$$\langle \zeta\zeta\zeta \rangle \propto$$



Quadrupolar "squeezed limit"



$$\langle\zeta\zeta\zeta\rangle \to f_{NL} \times \langle\zeta\zeta\rangle\langle\zeta\zeta\rangle \times (1 - 3\cos^2\theta)$$

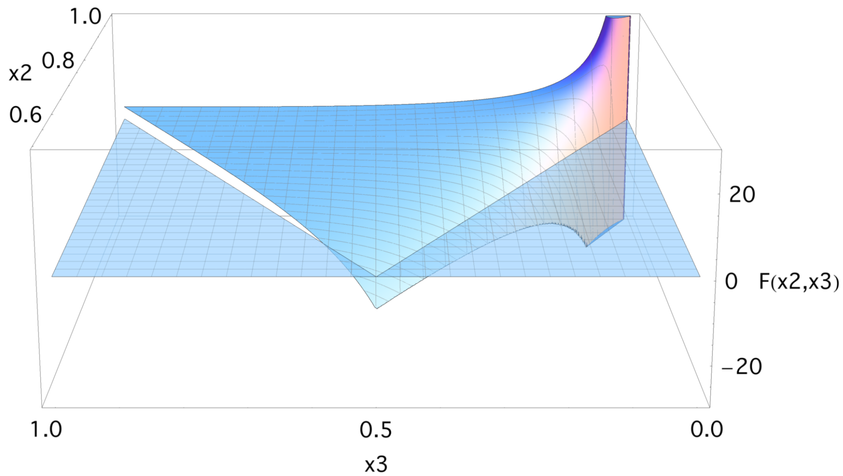
$$f_{NL} \sim \frac{1}{\epsilon}\frac{1}{c_L^2}$$

2% overlap w/ "local" shape

39% w/ "equilateral"

32% w/ "orthogonal"

(see also Shiraishi et al. 2012, Barnaby et al. 2012, Bartolo et al. 2013)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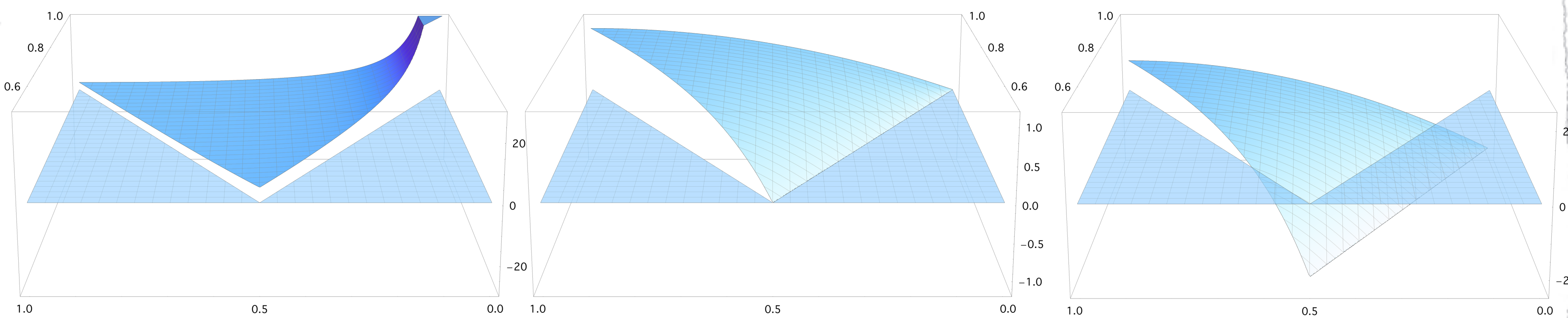
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Anisotropic generalizations

$$\phi^I \to \phi^I + a^I$$
$$\phi^I \to \cancel{SO(3)}\,\phi^I$$

discrete rotations

Yet, we want:

- isotropic background
- isotropic scalar spectrum

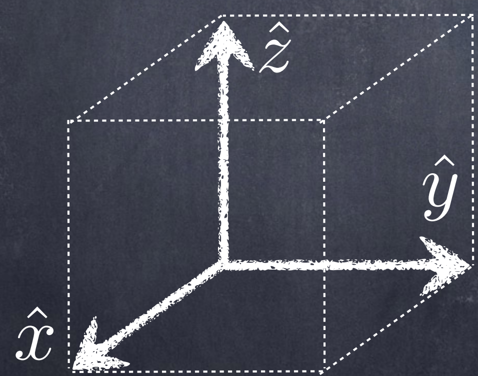
Background

T_{00}

$T_{ij} \propto \delta_{ij}$

Discrete subgroup of SO(3) with isotropic 2-index tensors?

Ex: cubic group



$$O^{(2)}_{ij} = \hat{x}_i \hat{x}_j + \hat{y}_i \hat{y}_j + \hat{z}_i \hat{z}_j = \delta_{ij}$$

accidentally isotropic!

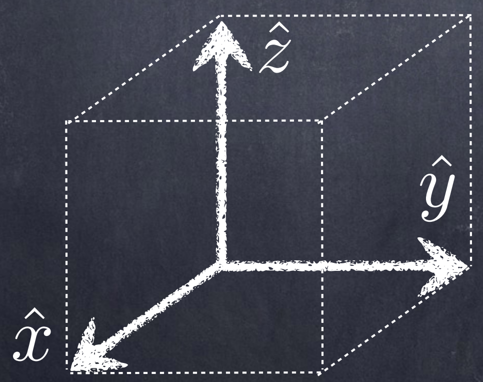
Scalar spectrum

$$\phi^I = x^I + \pi^I$$

$$\mathcal{L}_2 = O^{(2)}_{ij} \cdot \dot{\pi}_i \dot{\pi}_j + O^{(4)}_{ijkl} \cdot \partial_i \pi_j \partial_k \pi_l$$

Discrete subgroup of SO(3) with isotropic 4-index tensors?

Ex: cubic group



$$O^{(4)}_{ijkl} = \quad \delta_{ij}\delta_{kl}$$

$$\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}$$

$$\hat{x}_i \hat{x}_j \hat{x}_k \hat{x}_l + (\hat{x} \to \hat{y}, \hat{z})$$

not isotropic!

Scalar 3-pt function:

$$\mathcal{L}_3 \supset O^{(6)}_{ijklmn} \cdot \partial_i \pi_j \, \partial_k \pi_l \, \partial_m \pi_n$$

Tensor spectrum:

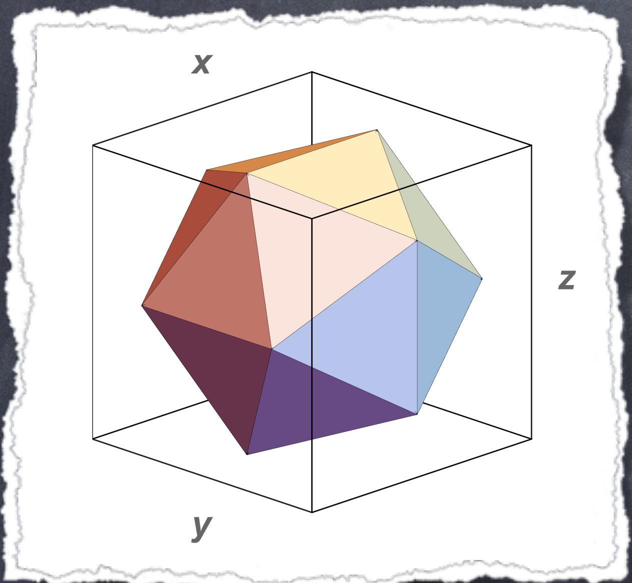
$$\mathcal{L}_2 = O^{(4)}_{ijkl} \cdot \dot{\gamma}_{ij} \, \dot{\gamma}_{kl} + O^{(6)}_{ijklmn} \cdot \partial_i \gamma_{jk} \, \partial_l \gamma_{mn}$$

Looking for a discrete subgroup of SO(3) w/

- Isotropic $O^{(2)}$
- Isotropic $O^{(4)}$
- Anisotropic $O^{(6)}$

$\Rightarrow$ { isotropic background, scalar spectrum
anisotropic scalar 3-pt function, tensor spectrum

Only one possibility: icosahedral group



$$O^{(2)}_{ij} = \delta_{ij}$$

$$O^{(4)}_{ijkl} = \quad \delta_{ij}\delta_{kl}$$
$$\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}$$

$$\begin{aligned} O^{(6)}_{ijklmn} = \quad & 2(\gamma+2)\delta_{ijklmn} \\ & + (\gamma+1)(\delta_{ijkl}\delta_{mn}\delta_{m,i+1} + \dots) \\ & + (\delta_{ijkl}\delta_{mn}\delta_{m,i-1} + \dots) \end{aligned}$$

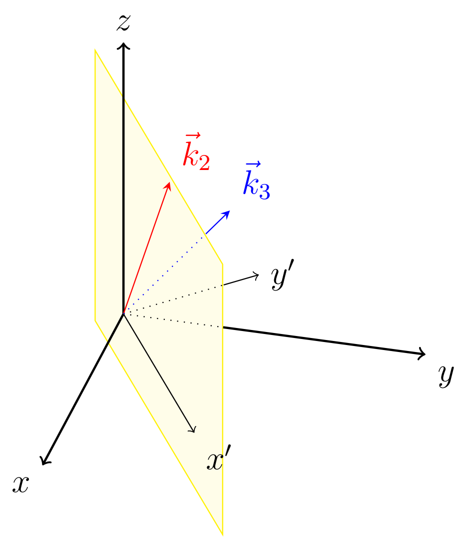
$$\gamma = (1+\sqrt{5})/2$$

Scalar 3-pt function

Messy expression — depends on vectors k2, k3

Two independent parameters α, β

Anisotropies $\propto (\beta - 9/2)$



Overlap with standard shapes $\propto (\beta - 8)$

$\Rightarrow \beta = 8$ completely anisotropic case

$$\bar{f}_{\rm NL}(\theta_2) = -\frac{\alpha}{\epsilon c_L^2}\left[\frac{19415}{378}(\beta - 8) + \frac{104135}{6048}(2\beta - 9)P_6(\cos\theta_2)\right]$$

Tensor spectrum

No anisotropy to lowest-order in derivatives:

$$F(B^{IJ}) = F(g^{\mu\nu}\partial_\mu\phi^I\partial_\nu\phi^J) \quad \not\to \quad \partial\gamma\partial\gamma$$

$$\to \quad O^{(4)}_{ijkl}\cdot\gamma_{ij}\,\gamma_{kl}$$

Needs higher-derivative couplings — e.g.:

$$\mathcal{L} \supset \frac{1}{M^2}\,(R^{\mu\nu\rho\sigma}\partial_\mu\phi^I\partial_\nu\phi^J\partial_\rho\phi^K\partial_\sigma\phi^L)^3\cdot T_{\rm aniso}$$

$$R_{\mu\nu\rho\sigma} \sim H^2 + \underline{H\partial\gamma} + \partial\gamma\partial\gamma + \partial\partial\gamma$$

$$\Longrightarrow \qquad \delta c_\gamma^2 = \frac{H^4}{M^2 M_{\rm Pl}^2}$$

However:

$$R_{\mu\nu\rho\sigma} \sim H^2 + H\partial\gamma + \partial\gamma\partial\gamma + \underline{\partial\partial\gamma}$$

⇒ ghosts at $E_* \sim H/\delta c_\gamma$

⇒ effect is perturbative: $\delta c_\gamma \ll 1$

Same conclusion for other higher derivative operators
E.g.

$$\nabla\nabla\phi \ldots \nabla\nabla\phi \cdot T_{\text{aniso}}$$

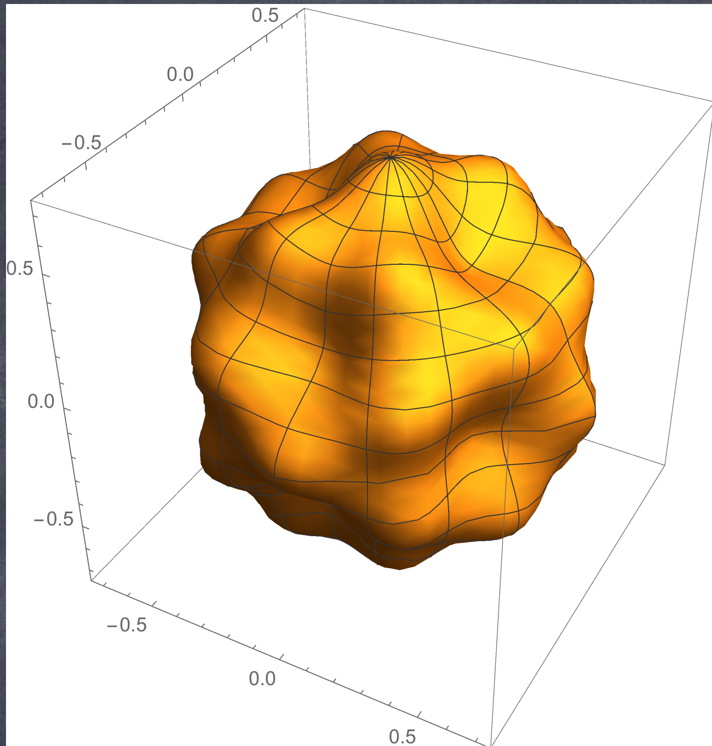
$$\nabla\nabla\phi = (H + \underline{\partial\gamma})(1 + \partial\pi) + \underline{\partial\partial\pi}$$

Regardless of where it comes from:

$$\mathcal{L}_\gamma \propto \dot{\gamma}_{ij}^2 - (\partial_k \gamma_{ij})^2 - \delta c_\gamma^2 \, T_{\rm aniso}^{ijklmn} \, \partial_i \gamma_{jk} \, \partial_l \gamma_{mn}$$

⇒ spectra: $\langle \gamma_+ \gamma_+ \rangle = \langle \gamma_- \gamma_- \rangle$

$\langle \gamma_+ \gamma_- \rangle \neq 0$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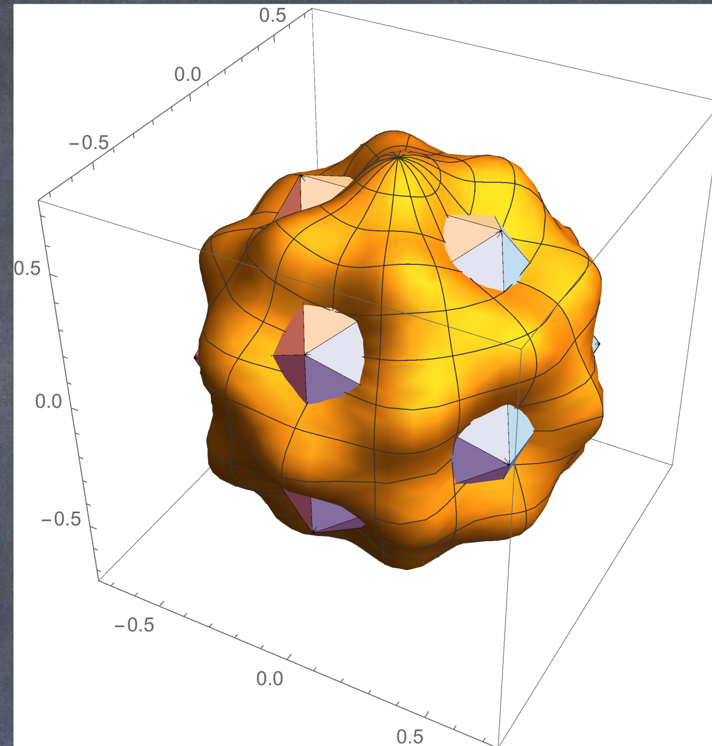
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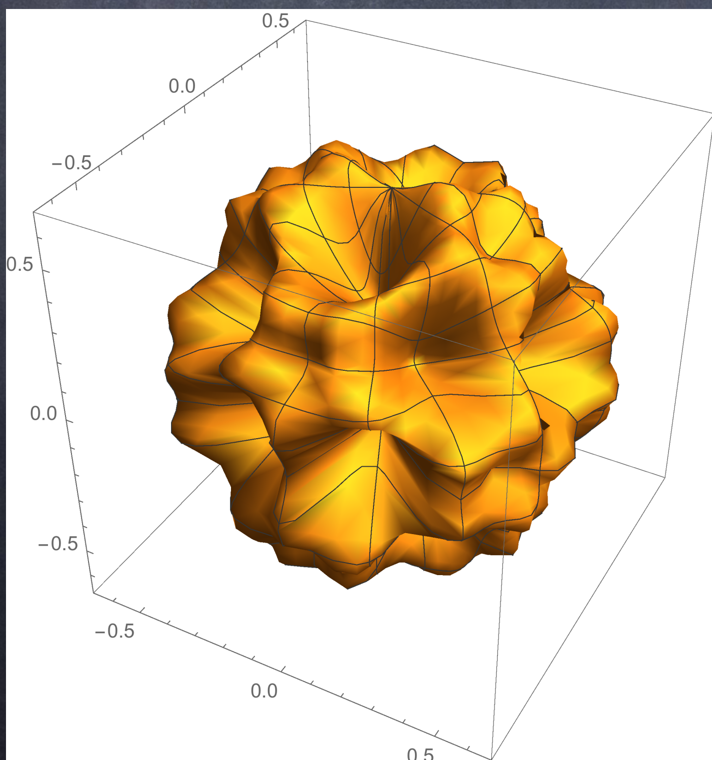
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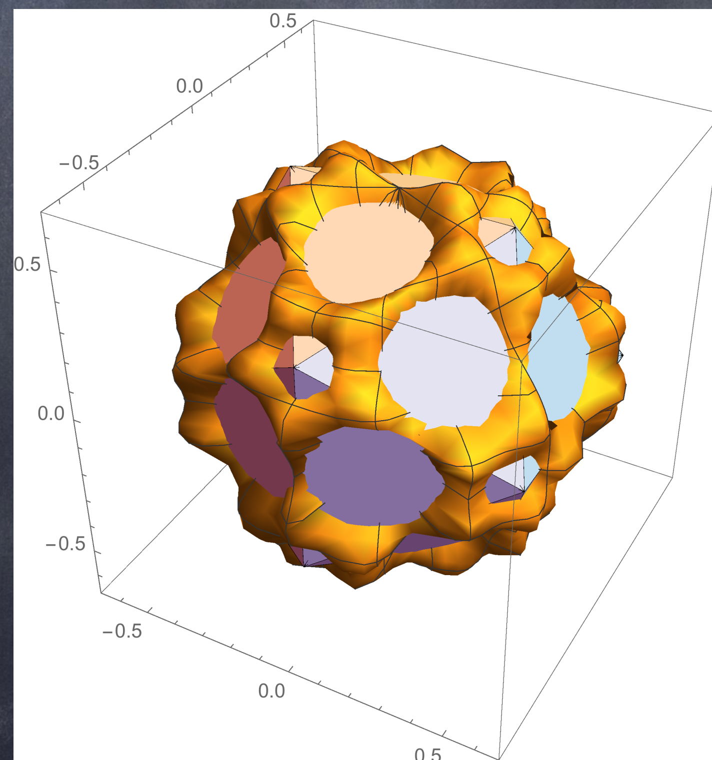
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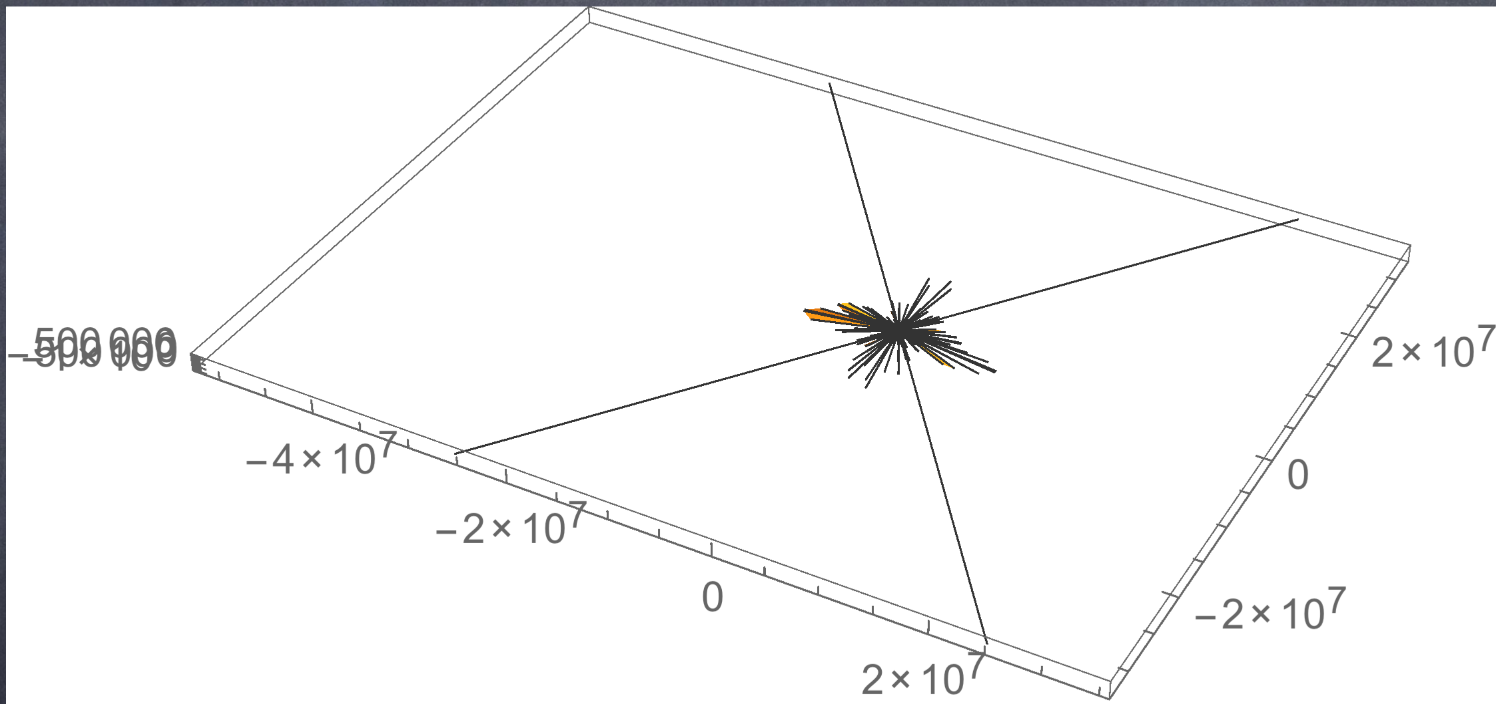
Instabilities



-0.3

Large (isotropic) tensors?

Can we have:

$$\mathcal{L}_\gamma \propto \dot{\gamma}_{ij}^2 - c_\gamma^2 (\partial_k \gamma_{ij})^2$$

with $c_\gamma \ll 1$?

That is, can we have $r \gg 16\epsilon$?

Absence of ghosts $E < H$ ⟹ at most linear in $R_{\mu\nu\rho\sigma}$

(cf. Creminelli et al., 2014)

Try:

$$\mathcal{L} = F(X,Y,Z) + \frac{1}{2}M_{\rm Pl}^2\big(R + \alpha\, R^{\mu\nu}\,\partial_\mu\phi^I\partial_\nu\phi^J\, B^{-1}_{IJ}\big)$$

⟹ $$c_\gamma^2 = 1+\alpha$$

But also $$c_T^2 \simeq \frac{3}{4}\Big(\frac{1}{1+\alpha} + c_L^2\Big)$$

$$\zeta = \frac{1+\frac{4}{3}c_T^2\alpha(1+\alpha)}{1+\alpha}\vec{\nabla}\cdot\vec{\pi}$$

...

⟹ $$r = 16\epsilon c_L^5 \times \frac{(1+\alpha)^{3/2}}{(1+\alpha c_L^2)^2} \ll 16\epsilon \qquad (0 < c_{L,T}^2 < 1)$$

Conclusions

- Observed isotropy of the universe could be accidental
- Potentially anisotropic non-gaussianity
- Potentially anisotropic tensor modes
- No large tensors for now.. fundamental reason?