Exploring the Weak Gravity Conjecture

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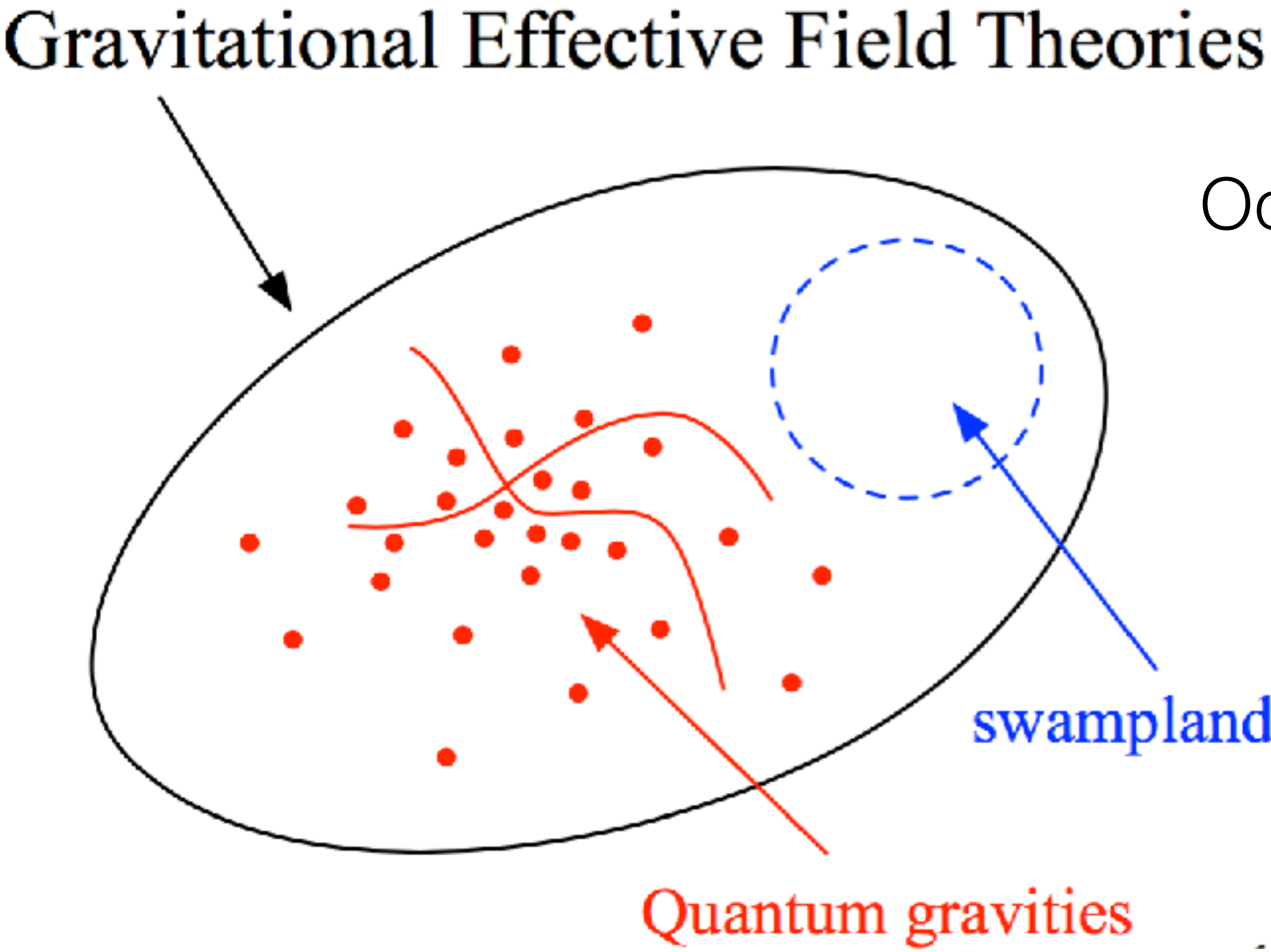
Based on 1506.03447, 1509.06374, 1605.05311, 1606.08437 with **Ben Heidenreich** and **Tom Rudelius.**
& 161n.nnnnn with **Grant Remmen, Thomas Roxlo, and Tom Rudelius**



Ooguri, Vafa 2005

Concretely: How good can approximate symmetries be?

Can an approx. global symmetry be "too good to be true" and put a theory in the swampland?

Approximate Symmetries

For large-field, natural inflation we might like to have a good approximate shift symmetry

$$\phi \to \phi + f, \quad f > M_{\rm Pl}$$

In effective field theory, nothing is wrong with this. In quantum gravity, it is dangerous. **Quantum gravity theories have no continuous global symmetries.**

Basic reason: throw charged stuff into a black hole. No hair, so it continues to evaporate down to the smallest sizes we trust GR for. True of **arbitrarily large charge ⇒ violate entropy bounds.**

(see Banks, Seiberg 1011.5120 and references therein)

The Power of Shift Symmetries

Scalar fields with good *approximate* shift symmetries can play a role in:

- driving expansion of the universe (now or in the past)
- solving the strong CP problem
- making up the dark matter
- breaking supersymmetry
- solving cosmological gravitino problems

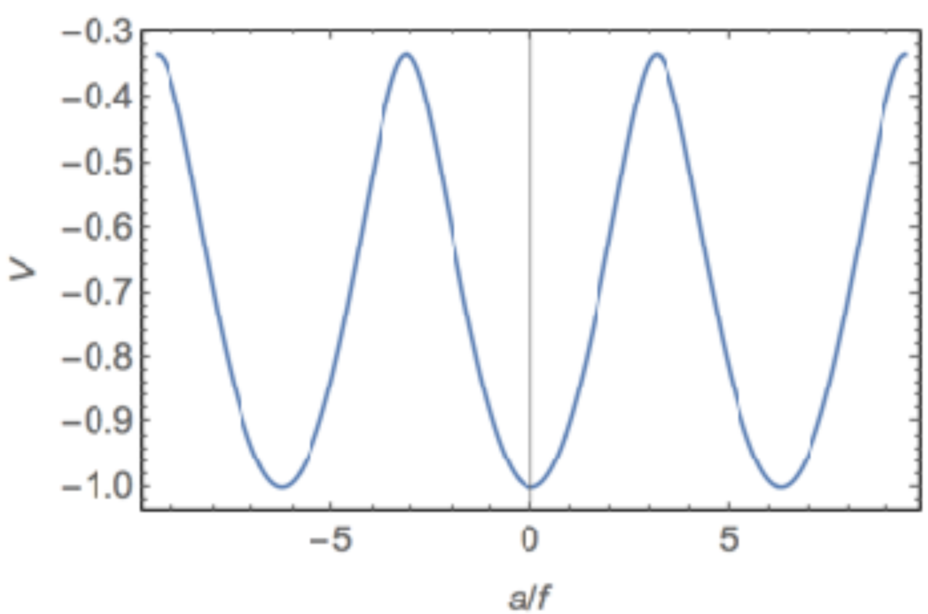
These are serious, real-world phenomenological questions!

Example: QCD Axion

Weinberg/Wilczek/…. taught us to promote theta to field:

$$\frac{a}{f_a}\frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu}$$

instantons: shift-symmetric potential



di Cortona et al. 1511.02867

$$V(a) = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4 m_u m_d}{(m_u+m_d)^2}\sin^2\left(\frac{a}{2f_a}\right)}.$$

$$m_a = 5.70(6)(4)\,\mu\text{eV}\left(\frac{10^{12}\text{GeV}}{f_a}\right)$$

Important point: large field range, small spurion

$$m_a^2 = \frac{1}{f^2}\frac{m_\pi^2 f_\pi^2 (m_d/m_u)}{(1+(m_d/m_u))^2} \qquad \lambda = -\frac{1}{f^4}\frac{m_\pi^2 f_\pi^2 (m_d/m_u)((m_d/m_u)^2-(m_d/m_u)+1)}{(1+m_d/m_u)^4}$$

Example: QCD Axion

- Stellar cooling constraints: $f_a >\sim 10^9$ GeV
- This means the quartic coupling $<\sim 10^{-41}$. Any axion is ***very*** weakly coupled!
- Whole potential has an overall **exponentially small spurion e^{-S} in front**; *relative* terms f_a suppressed.
- Upper bound on f_a? Black hole super-radiance (Arvanitaki et al.) rules out a window near the Planck scale
- Weak Gravity Conjecture: $f_a <\sim M_{Pl}/S$. Links **how exponentially small the spurion is** with relative suppression between terms.

What is the Weak Gravity Conjecture?

The Weak Gravity Conjecture

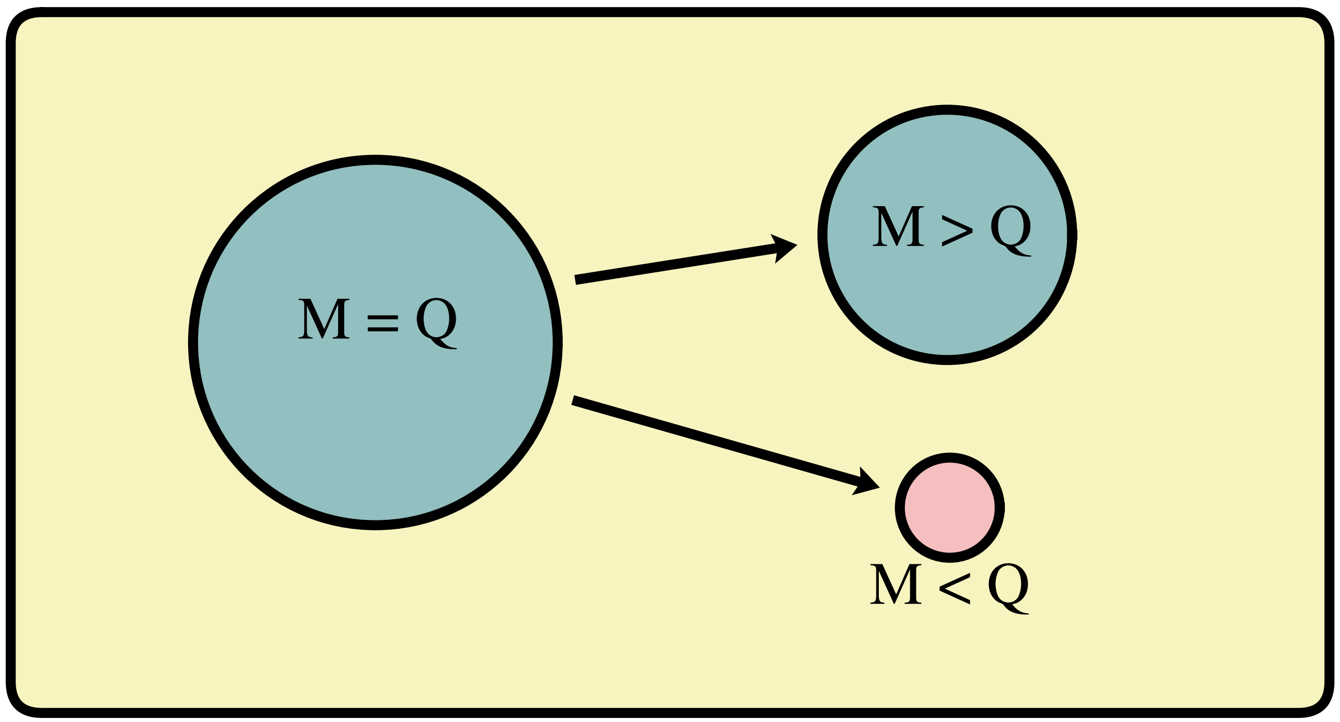
In 2006, Arkani-Hamed, Motl, Nicolis, and Vafa (AMNV) conjectured that **in any consistent gravitational theory with a U(1) gauge field—like electromagnetism—there is *at least one particle*** with

$$m \leq \sqrt{2} e Q M_{\mathrm{Pl}}$$

This is the **opposite** of the black hole extremality bound.

For such particles, gravity is weaker than E&M.

What is the Conjecture?



Arkani-Hamed, Motl, Nicolis, Vafa **("AMNV")** hep-th/0601001

Particle exists with M<Q (superextremal).

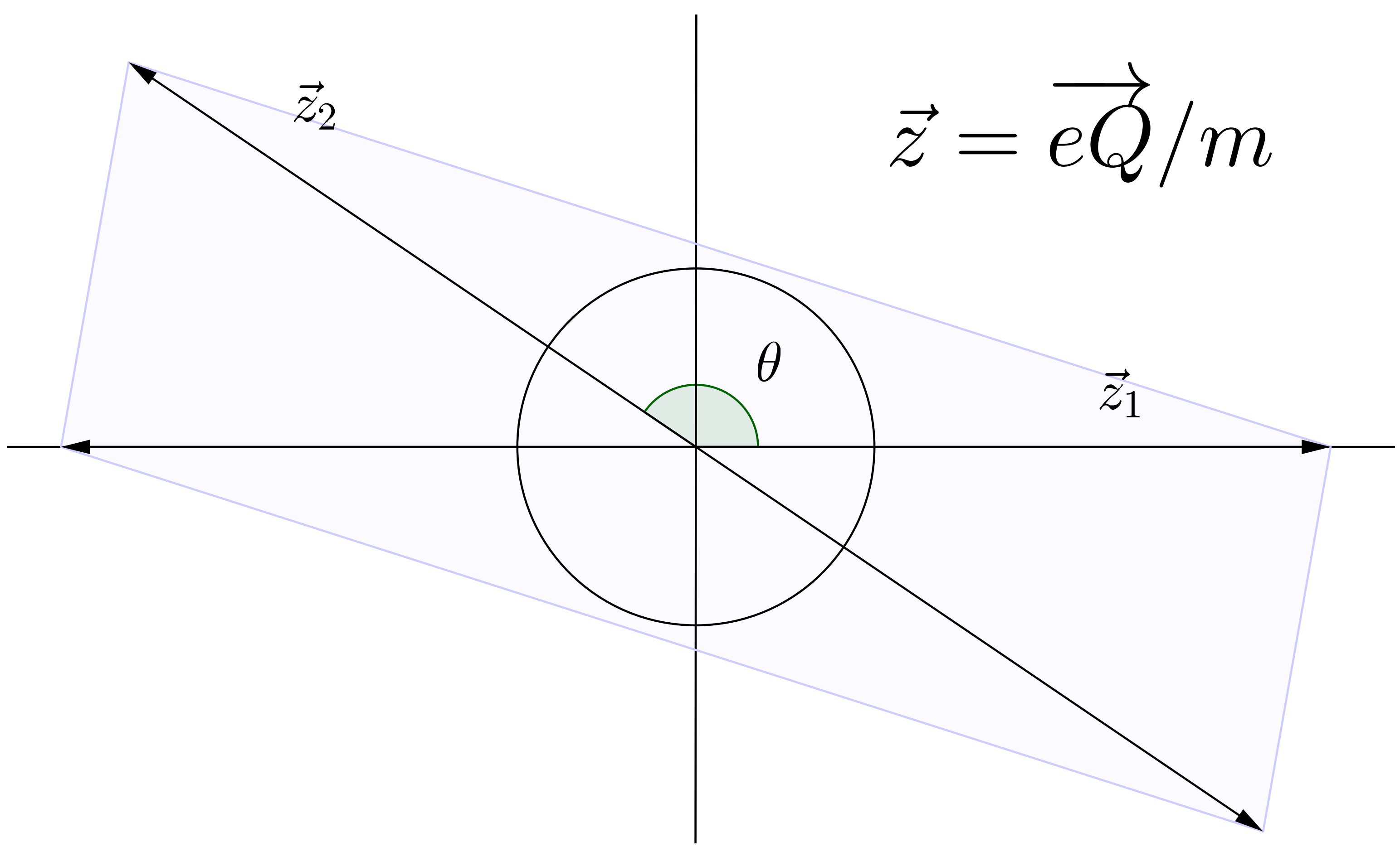
Why? Postulate that extremal black holes should decay.

Why? Else: nearly-extremal black holes spontaneously evolve toward extremality. Sequester information and entropy forever. Seems strange.

Convex Hull Condition

Multiple U(1) gauge groups: black hole extremality bound generically becomes

$$M_{\mathrm{BH}} > \sqrt{(e_1 Q_1)^2 + (e_2 Q_2)^2 + \ldots + (e_N Q_N)^2}\, M_{\mathrm{Pl}}$$

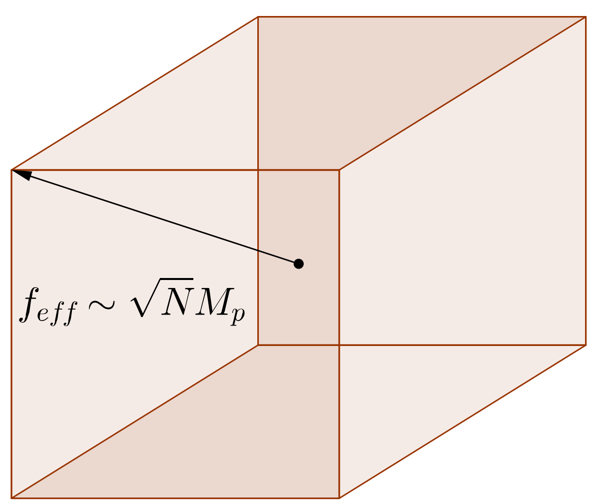


Stricter WGC than each U(1) individually! Cheung, Remmen 1402.2287

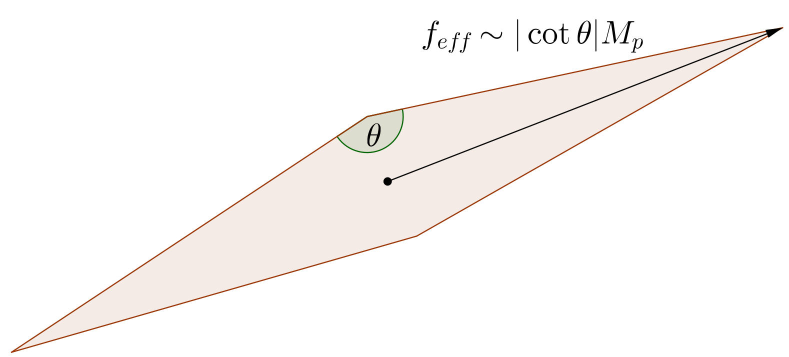
CHC: convex hull contains the unit ball

Super-Planckian Axions?

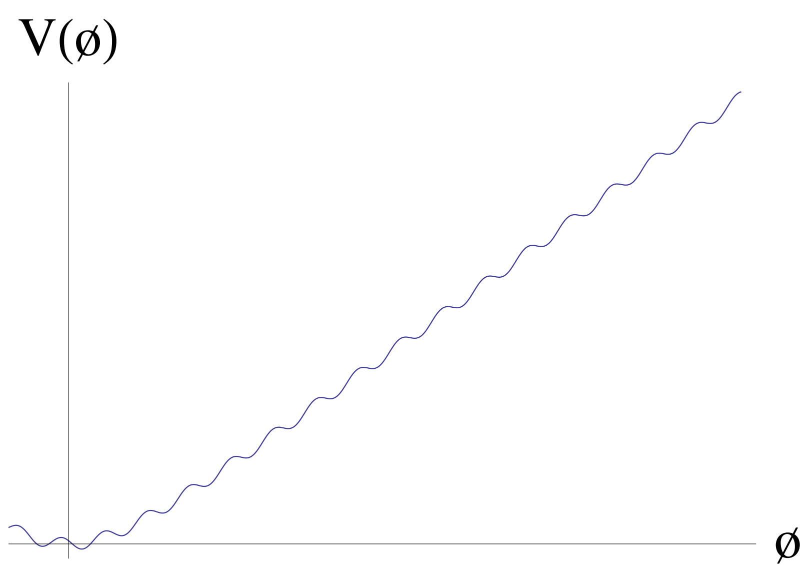
Axion fields in string theory are generally found to have ***f* < *M*$_{Pl}$**. (Banks, Dine, Fox, Gorbatov hep-th/0303252)
So can't immediately use one for large-field inflation.
People have tried **clever model-building** to improve:



N-flation
(Dimopoulos, Kachru, McGreevy, Wacker '05)



Alignment
(Kim, Nilles, Peloso '04)



Monodromy
(Silverstein, Westphal '08; McAllister, Flauger, ….)

WGC provides a handle in the case $a = \oint A_y dy$

Inflation vs. WGC: Magnetic

Also see electric arguments: AMNV; Rudelius; Brown, Cottrell, Shiu, Soler

AMNV pointed out that magnetic monopoles have a Weak Gravity bound that looks like

$$m_{\rm mag} < (q_{\rm mag}/e) M_{\rm Pl}$$

If we interpret the classical monopole radius $\int_{r_{\rm cl}} \frac{1}{e^2}\vec{B}^2 = m_{\rm mag}$ as a cutoff length scale for EFT, we have

$$\Lambda = r_{\rm cl}^{-1} < e M_{\rm Pl}$$

(also follows from Lattice WGC because tower of states)

If our 5D radius should be bigger than this cutoff, we have

$$R > r_{\rm cl} > \frac{1}{e M_{\rm Pl}}$$

implying again that $f < M_{\rm Pl}$

(de la Fuente, Saraswat, Sundrum 1412.3457)

Inflation vs WGC: Multi-Field

WGC, then, tells us **decay constants are less than the Planck scale**. But **N-flation, alignment,** and **monodromy** are explicitly trying to get super-Planckian field range out of sub-Planckian decay constants. Does WGC help here?

Start with ***N-flation***. N axions from N U(1) gauge groups. Try to move along diagonal to get

$$f_{\text{eff}}^2 = f_1^2 + f_2^2 + \ldots + f_N^2 = \left(\frac{1}{2\pi R}\right)^2 \left(\frac{1}{e_1^2} + \ldots + \frac{1}{e_N^2}\right).$$

Magnetically charged black holes obey

$$Q_{\text{eff}} \equiv \sqrt{\frac{Q_1^2}{e_1^2} + \frac{Q_2^2}{e_2^2} + \ldots + \frac{Q_N^2}{e_N^2}} < M_{\text{BH}}/M_{\text{Pl}}$$

Key fact: the same quadratic form appears in the kinetic term of the axion (hence *f_{eff}*) and in the extremality bound of the black hole!

With a little work, derive that the magnetic WGC excludes this kind of simple N-flation.

$$f_{\mathrm{eff}}^2 = f_1^2 + f_2^2 + \ldots + f_N^2 = \left(\frac{1}{2\pi R}\right)^2 \left(\frac{1}{e_1^2} + \ldots + \frac{1}{e_N^2}\right).$$

Magnetically charged black holes obey

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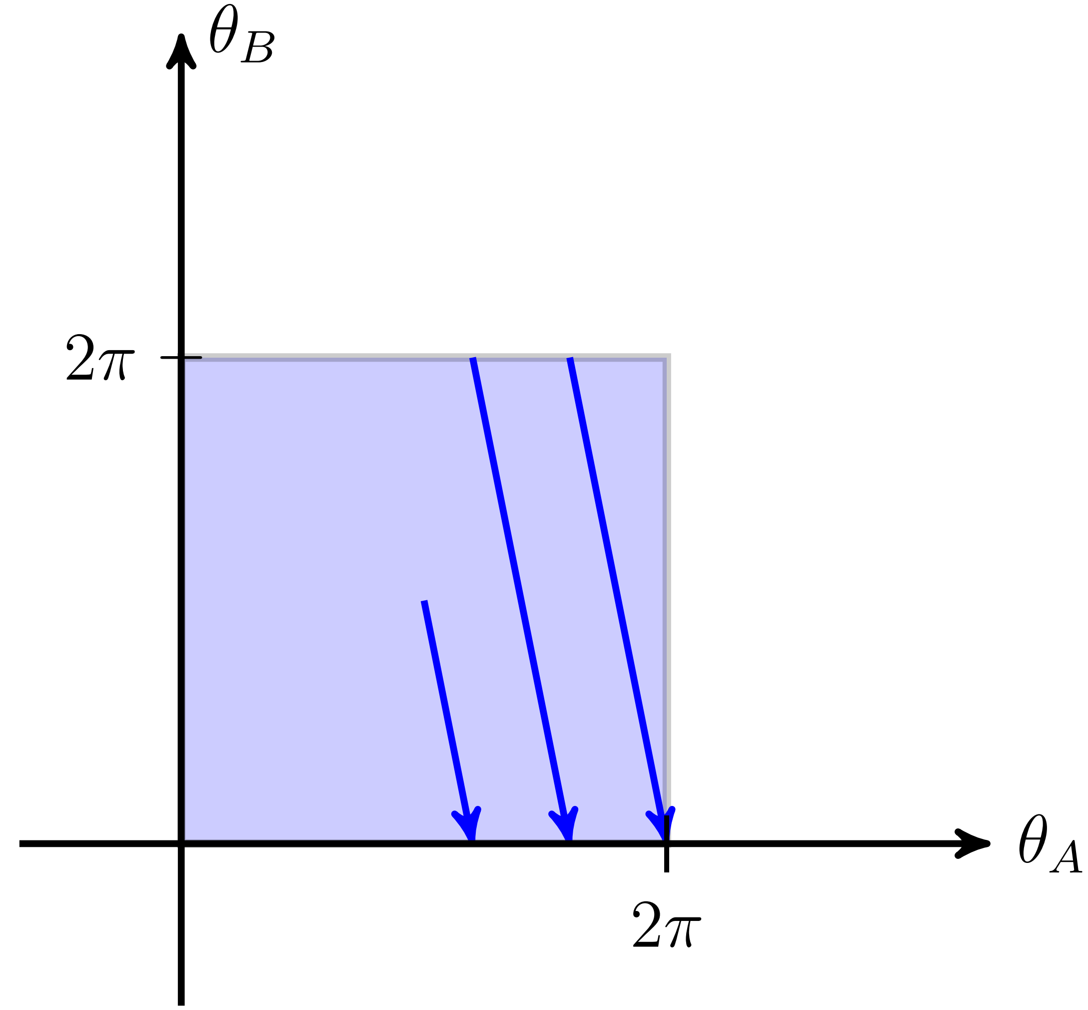
An Alignment Model

If magnetic and electric charges are **not simultaneously simple**, can evade WGC.

de la Fuente, Saraswat, Sundrum:

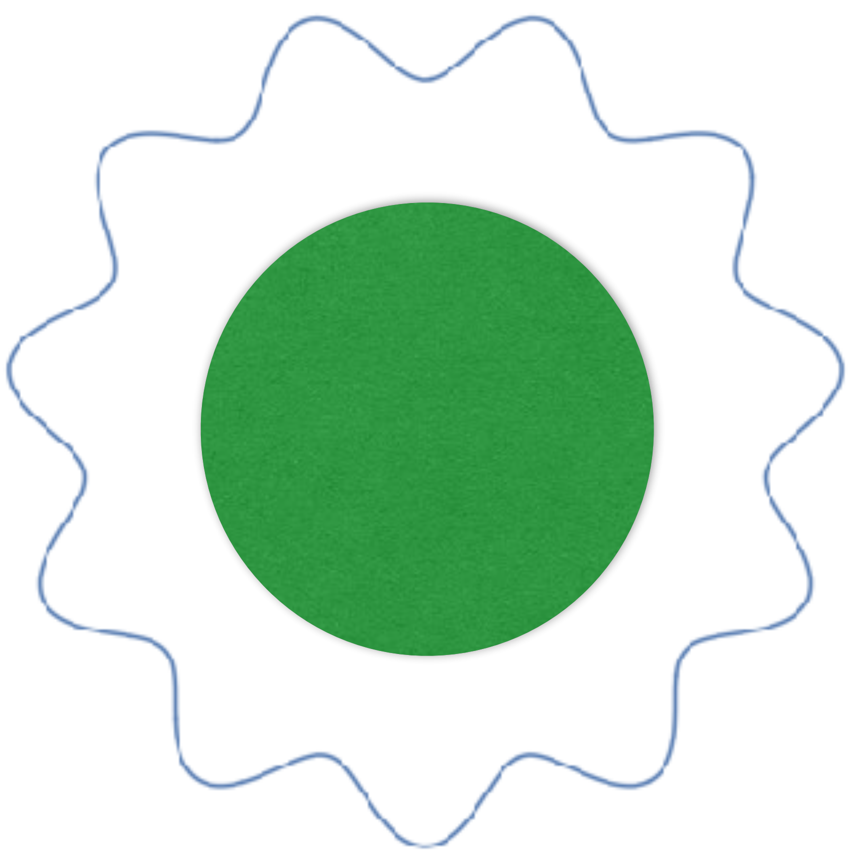
- two axions, *A* and *B*, from 5D gauge fields
- no kinetic mixing
- electrically charged particles (1,0) and (N,1)
- assume $f_A < M_{Pl}$, $f_B < M_{Pl}$



i.e. *implicitly* assumes magnetic WGC with (1,0) and (0,1)

Monopoles and Cutoffs

This example satisfies minimal versions of WGC. But it has strange features. The electrically charged particle of charge (N,1) and the magnetic monopole of charge (1,0) have a **nonminimal Dirac quantization**—especially at large N.



Is the classical monopole radius really the right cutoff?

$$\vec{L} = \int d^3\vec{r}\,\vec{r} \times \vec{E} \times \vec{B} \sim N$$

Charged particle wavefunction probes length scales shorter by *N*.

KK Modes and Cutoffs

The shift symmetry of the axion is satisfied in the presence of KK modes by a **monodromy** in the spectrum.

5D coupling $\int d^5x\sqrt{-g}\frac{q\theta}{2\pi R}\bar{\Psi}\Gamma^5\Psi$ leads to 4D KK theory

$$\mathcal{L}_{\rm eff} = \sum_{n=-\infty}^{\infty}\left(i\bar{\psi}_n\gamma^\mu D_\mu\psi_n + m_5\bar{\psi}_n\psi_n + i\frac{n-\frac{q\theta}{2\pi}}{R}\bar{\psi}_n\gamma^5\psi_n + \frac{c}{\Lambda}\left|\frac{n-\frac{q\theta}{2\pi}}{R}\right|^2\bar{\psi}_n\psi_n + \ldots\right).$$

In the presence of large charges, modes that are light in one part of the inflaton's trajectory can be **above the cutoff** in other places, unless we invoke the stronger constraint

$$R > N r_{\rm cl} \quad \text{rather than} \quad R > r_{\rm cl}$$

That implies $f_{\rm eff} < M_{\rm Pl}$.

Lesson

The case that is hardest to exclude with the Weak Gravity Conjecture is one that involves **venturing far enough out in moduli space** that modes can descend from the cutoff to become light.

Is this a problem? Seems to be no clear consensus on this question.

I think that charge/monopole scattering has the potential to provide a sharp argument that it is.

These applications of the WGC ***assume*** that just a few particles satisfy the Convex Hull Condition. Is that safe?

What is the Weak Gravity Conjecture *really*?
Some particle has:

$$m \leq \sqrt{2} e Q M_{\mathrm{Pl}}$$

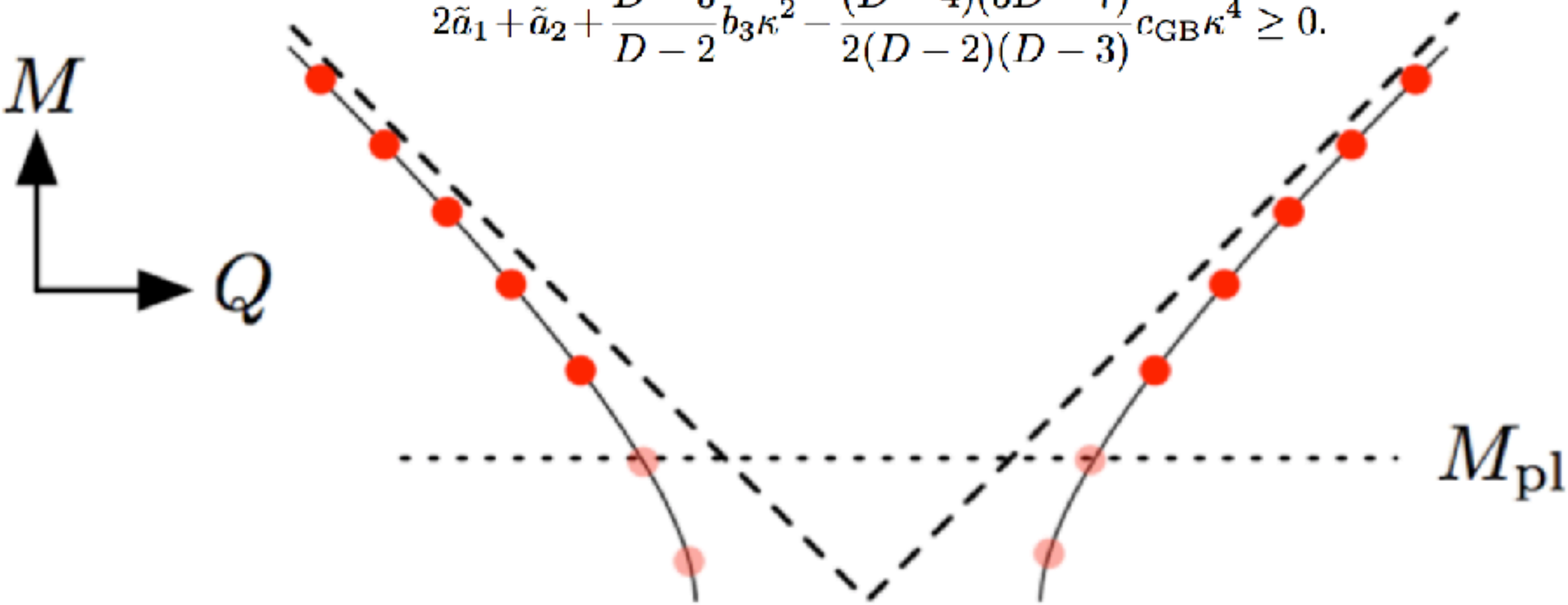
But what is a "particle"? Would a black hole count?

Black hole extremality, corrected

$$\sqrt{-g}\left(\frac{R}{2\kappa^2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + c_{\rm GB}\mathcal{O}_{\rm GB} + \tilde{a}_1(F_{\mu\nu}F^{\mu\nu})^2 + \tilde{a}_2 F_{\mu\nu}F^{\nu\rho}F_{\rho\lambda}F^{\lambda\mu} + b_3 R^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}\right).$$

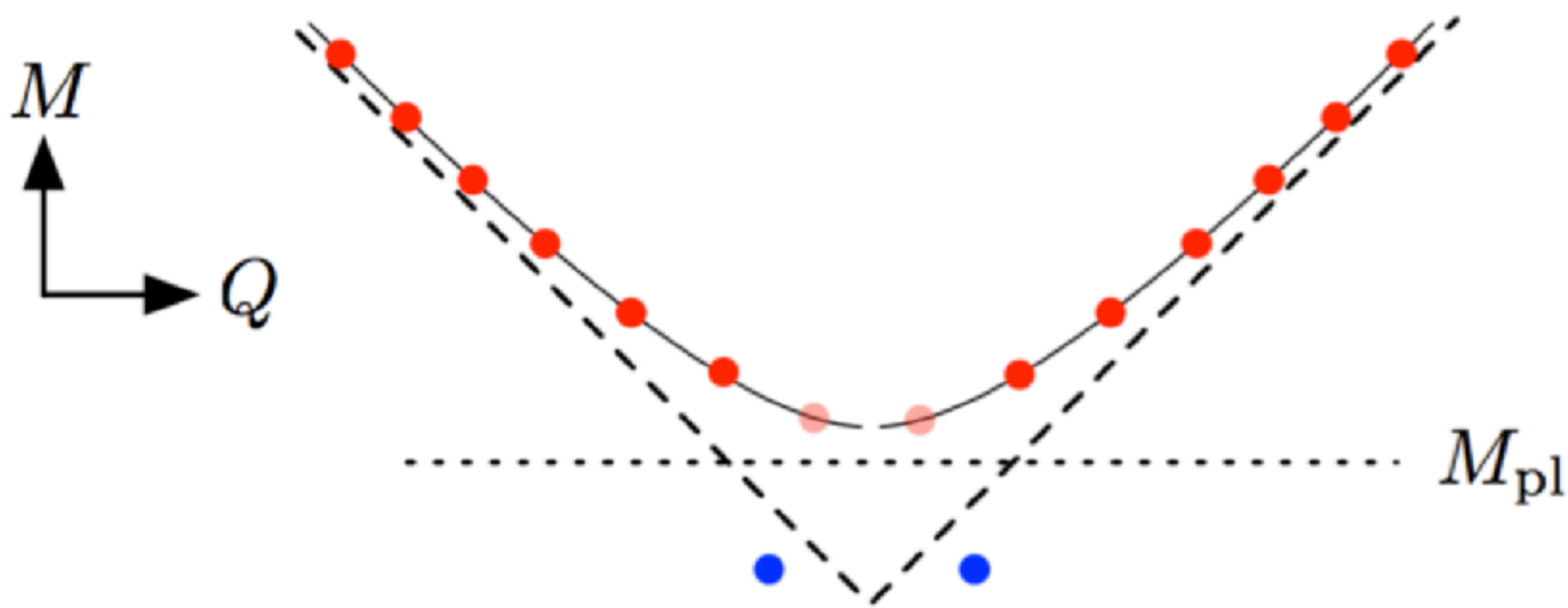
Kats, Motl, Padi '06: **black holes obey WGC if**

$$2\tilde{a}_1 + \tilde{a}_2 + \frac{D-3}{D-2}b_3\kappa^2 - \frac{(D-4)(3D-7)}{2(D-2)(D-3)}c_{\rm GB}\kappa^4 \geq 0.$$



Black hole extremality, corrected

$$\sqrt{-g}\left(\frac{R}{2\kappa^2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + c_{\mathrm{GB}}\mathcal{O}_{\mathrm{GB}} + \tilde{a}_1(F_{\mu\nu}F^{\mu\nu})^2 \right.$$
$$\left. + \tilde{a}_2 F_{\mu\nu}F^{\nu\rho}F_{\rho\lambda}F^{\lambda\mu} + b_3 R^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}\right).$$



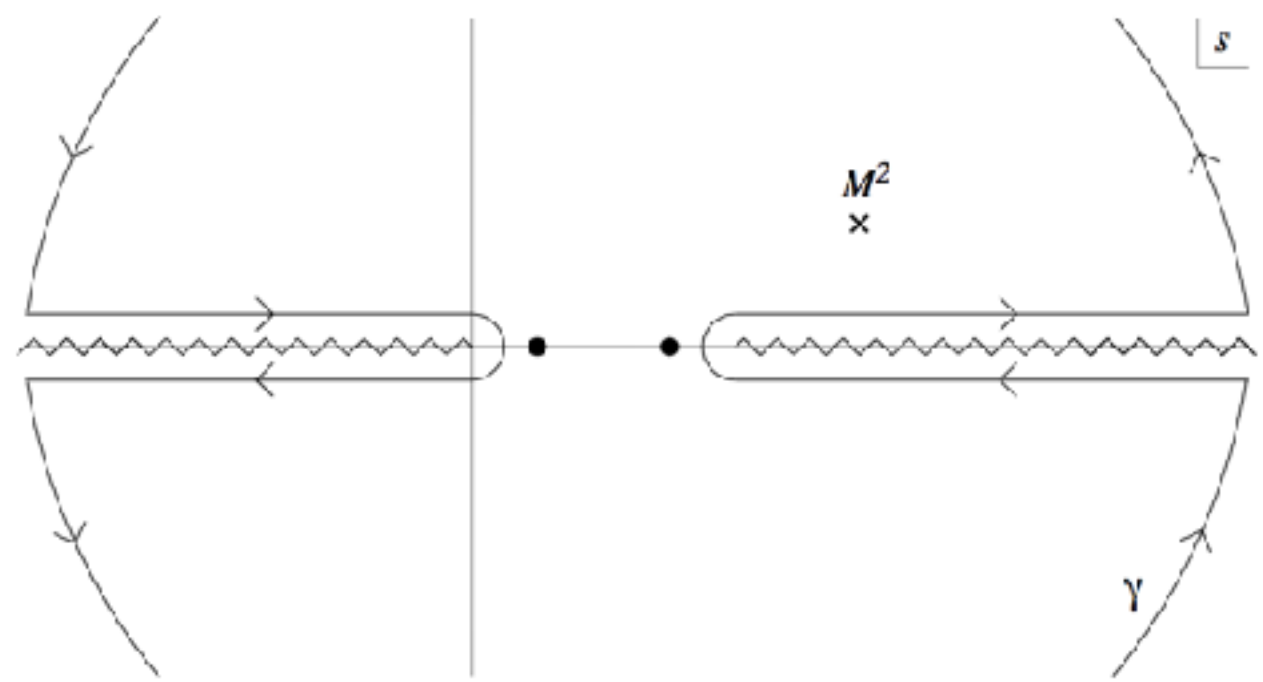
With the opposite inequality, would need light charged particles (not black holes) for WGC to be true

Black hole superextremality from unitarity

MR, G. Remmen, T. Roxlo, T. Rudelius, to appear

Argument along the lines of Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi hep-th/0602178

Prove positivity of coefficient in low-energy amplitude using dispersion relation to relate to integral over branch cut, related to total cross section



$$\oint \frac{\mathcal{M}(s, t \to 0)}{s^3} ds$$ to extract s^2 coeff.

Black hole superextremality from unitarity

MR, G. Remmen, T. Roxlo, T. Rudelius, to appear

Subtleties: $-Gs^2/t$ from graviton in t-channel makes forward limit divergent. Subleading terms tricky too, e.g.:

$$\mathcal{M}^{(4\partial)}_{\gamma g \gamma g}(s, t \to 0) = 2b_3\kappa^2 s^2 \left[-(\epsilon_1 \cdot \epsilon_2 \cdot \epsilon_4 \cdot \epsilon_3) - (\epsilon_1 \cdot \epsilon_4 \cdot \epsilon_2 \cdot \epsilon_3) + 2\frac{(q \cdot \epsilon_1)(q \cdot \epsilon_3)}{q^2}\mathrm{Tr}(\epsilon_2 \cdot \epsilon_4)\right]$$
$$+ 4c_{\mathrm{GB}}\kappa^4 s^2 \left[(\epsilon_1 \cdot \epsilon_3)\mathrm{Tr}(\epsilon_2 \cdot \epsilon_4) - \frac{2q \cdot \epsilon_4 \cdot \epsilon_2 \cdot q}{q^2}\epsilon_1 \cdot \epsilon_3\right].$$

Four-derivative contributions contain $q^\mu q^\nu / q^2$; amplitude is not really a function of momenta but depends on how forward limit is taken. Naive arguments can prove false things like negativity of Gauss-Bonnet terms (Bellazzini, Cheung, Remmen).

Unitarity Bounds on Photon/Graviton Scattering

MR, G. Remmen, T. Roxlo, T. Rudelius, to appear

Work in progress

Preliminary results interesting, but I'll hide the details because we still have some confusions.

Hope: conclude that unitarity means black holes are superextremal

On the one hand, this means that the most mild form of the WGC—where we don't specify if the charged objects are particles or black holes—***follows from unitarity*** (up to plausible assumptions and fine print about analytic properties in theories with gravity)

On the other, if tiny corrections to extremal black holes lead to WGC, it seems kind of uninspiring. We were hoping for **particles**, things light enough to matter in effective field theory.

Do we give up? No. Some internal consistency arguments plus many examples in string theory make us think that **the right version of the Weak Gravity Conjecture is *much* stronger than what AMNV proposed.**

Lattice Weak Gravity Conjecture: A (Tentative) Guess

For **any set of charges allowed by Dirac quantization**, there exists a particle **with those charges** that has

$$m < \left|\vec{Q}\right| M_{\mathrm{Pl}}$$

where the precise meaning of $\left|\vec{Q}\right|$ can be read off from the black hole solutions.

Infinitely many states satisfy a bound; **some are particles, some are black holes (interpolate between).**

The LWGC predicts that in weakly coupled theories there is an infinite ***tower of states*** that becomes light as the coupling goes to zero.

Applies to nonabelian gauge theories as well as U(1).

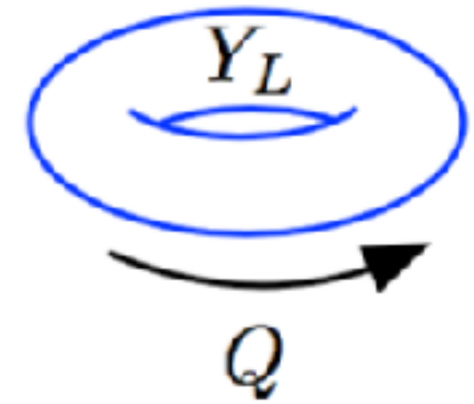
Very weakly coupled theories must have a **very low cutoff.**

Tower at eM_{Pl}.
Strong gravity at $\Lambda_* < e^{1/3} M_{\text{Pl}}$.

Consistent with behavior of Kaluza-Klein towers, string towers, etc.

Is this too good to be true? Checks out in many examples:

Heterotic S.T. spectrum

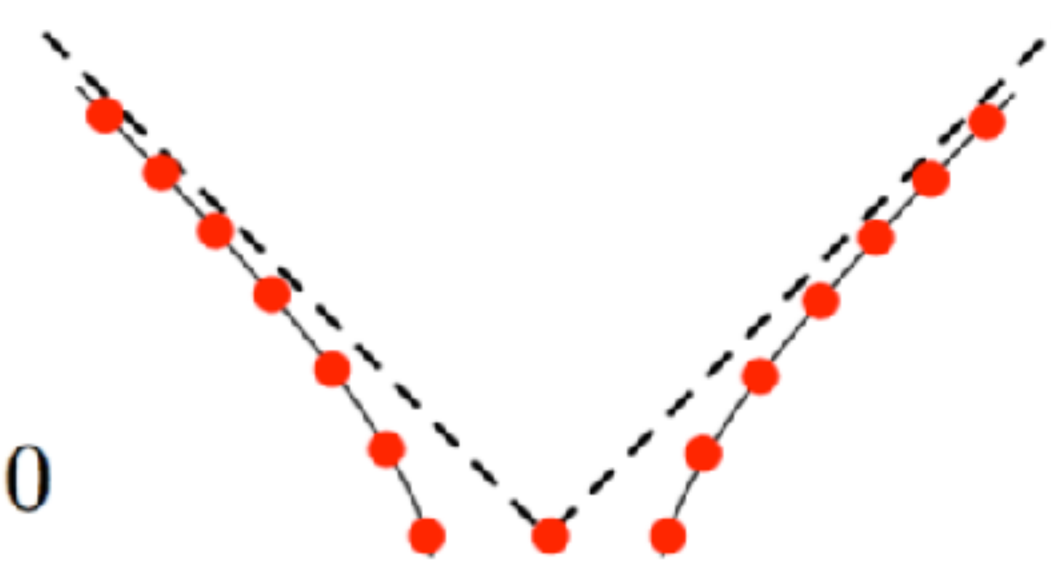


$$\frac{\alpha'}{4}m^2 = N + \frac{1}{2}Q^2 - 1 = \tilde{N} + \frac{1}{2}r^2 - \frac{1}{2}$$

$$Q \in \Gamma_8^{0s} \otimes \Gamma_8^{0s} \text{ or } Q \in \Gamma_{16}^{0s}, \quad r \in \Gamma_4^{vc}$$

$$Q^2 \in 2\mathbb{Z}, \;\; r^2 \in 2\mathbb{Z}+1$$

$$\frac{\alpha'}{4}m^2 = \frac{1}{2}Q^2 - 1, \;\; Q \neq 0$$



LWGC ok!

Lattice is *Slightly* Too Strong

Consider a toroidal compactification on T^3 with coordinates *x*, *y*, *z*. Quotient by **freely acting Z/2 x Z/2 group:**

$$\mathbb{Z}_2: \quad x \mapsto x + \frac{1}{2}R_x,\, y \mapsto y + \frac{1}{2}R_y,$$
$$\mathbb{Z}_2': \quad x \mapsto -x,\, z \mapsto z + \frac{1}{2}R_z.$$

Leaves unbroken KK U(1)s in *y*, *z* directions

First **Z/2** guarantees $\mathbf{n_x + n_y}$ is an **even number.**
Second **Z/2** removes U(1) in *x* direction.

Result: **sites of lattice with *odd*** $\boldsymbol{n_y}$ have no superextremal states (extra contribution to **mass** from n_x but **no charge**).

Modular invariance implies, for closed string U(1)s in perturbative string theories, that least a **sublattice** of states obey WGC. *(Also Montero, Shiu, Soler on AdS3/CFT2.)*

In practice, it seems that the fraction of lattice sites that fail WGC is never large.

We are building up a large collection of examples.

Next step: **what does this imply for inflation?** Older arguments need to be revisited.

Conclusions

AMNV's conjecture has implications for inflation and other real-world physics.

We have noticed that in every example we understand an **infinitely stronger statement is true**.

We are trying to find the **right statement consistent with all the examples**, strong enough but not too strong.

Implications for mathematics, cosmology, and particle physics. **Many concrete, tractable paths for further progress.**