Leonardo Senatore (Stanford)

Inhomogeneous Anisotropic Cosmology:

Starting Inflation
in inhomogeneous universe

with East, Linde and Kleban **1511, JCAP 2016,**
with Kleban **1602, JCAP 2016**

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On the `Initial Patch Problem'

with East, Linde and Kleban **1511 JCAP 2016,** rejected by **PRL** and **PRD**
with Kleban **1602, JCAP 2016**

Introduction

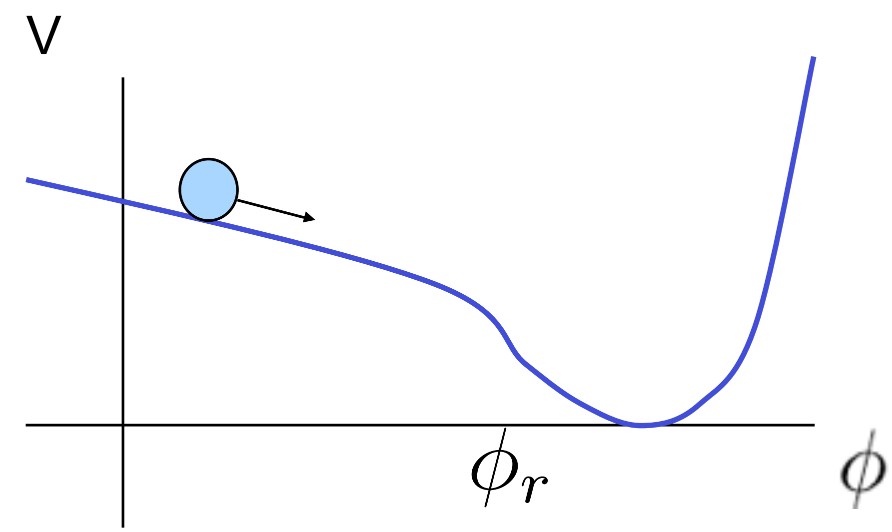
– Inflation is believed to have problems to start (`Initial Patch Problem').

– This talk will argue (prove) that one of the reasons of concerns is not sustained

  – it will do so by using some interesting mathematics,

    • used to answer a very very physical question

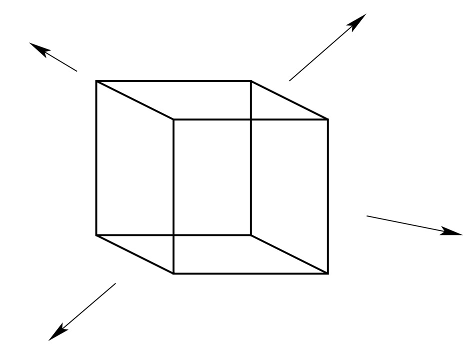
The Problem



– If we have the inflaton on top of his potential

– and the space is homogeneous on a H_I patch

– then inflation starts



• Question: how likely is to have an homogenous patch of this size?

– Here I will first present an apparently compelling argument (at least to me), that if

$$H_I \ll M_{\rm Pl} \quad \Rightarrow \quad \text{prob} \sim e^{-\frac{M_{\rm Pl}}{H_I}}$$

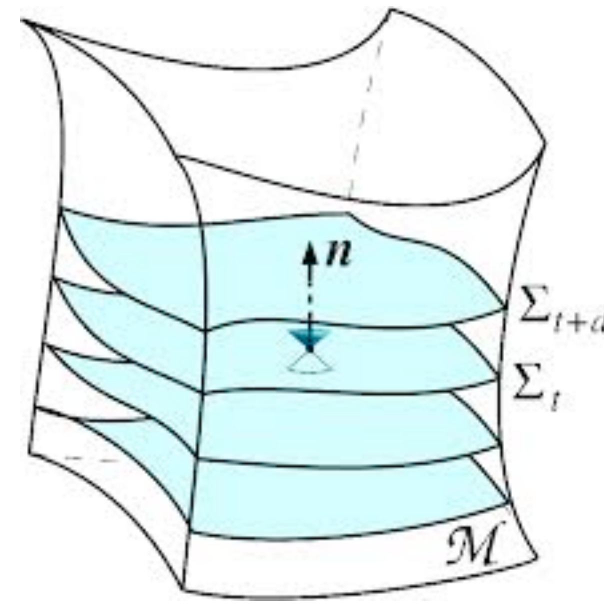
– this is the so-called `initial patch problem'

– and we will show that this argument is not really correct

– But first, let us open a parenthesis

FRW Cosmology

–Let us review FRW cosmology



$$ds^2 = -dt^2 + a(t)^2 d\Sigma^2$$

$$d\Sigma^2 = \frac{dr^2}{1-kr^2} + r^2 d\Omega_2^2 \;, \qquad d\Omega_2^2 = d\theta^2 + \sin(\theta)^2 d\phi^2$$

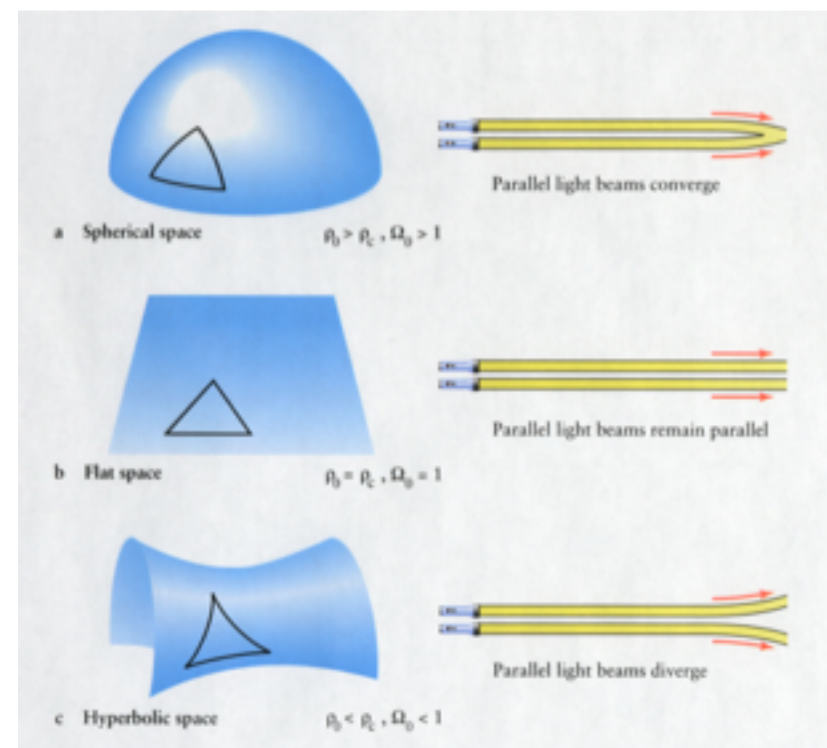
–where

$$k = +1 \;, \text{Sphere}$$

$$k = 0 \;, \text{Flat}$$

$$k = -1 \;, \text{Hiperboloid}$$

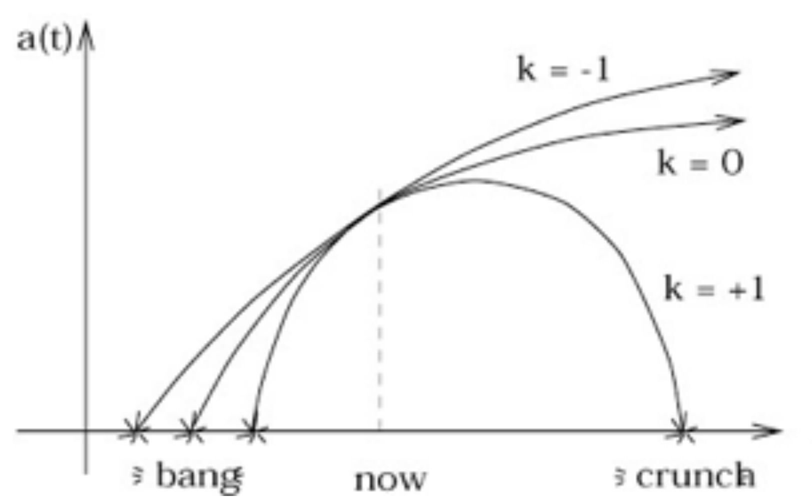
$$a(t) = \text{radius of curvature}$$



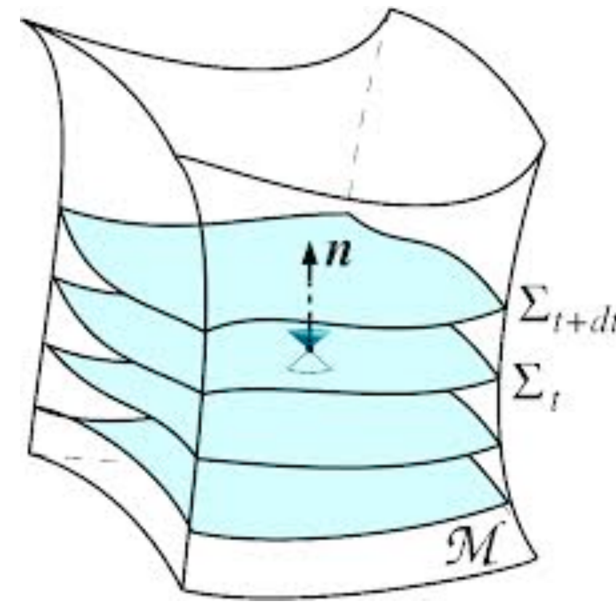
–Typical evolution

–closed universe: recollapses

–open and flat: expand forever

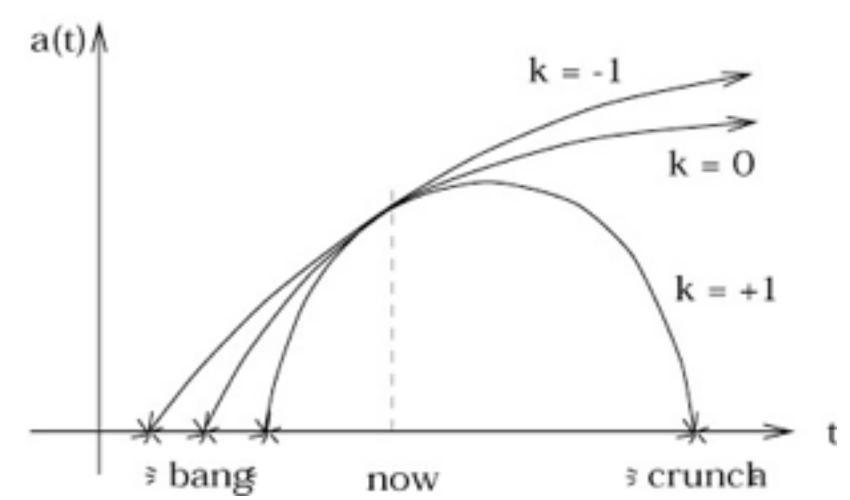


FRW Cosmology



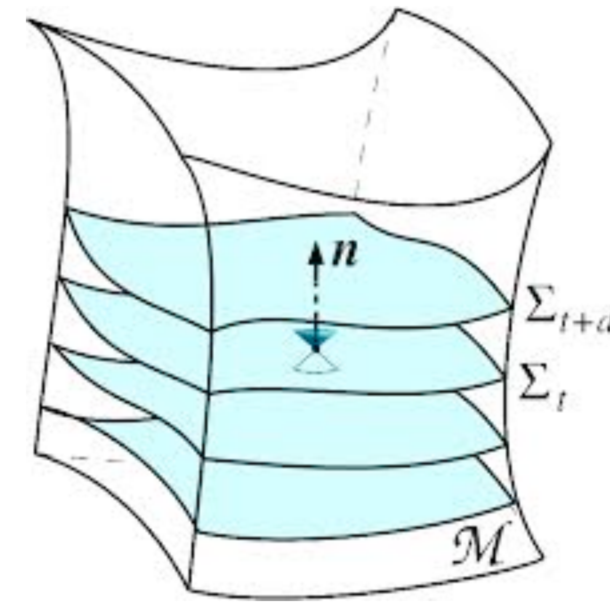
– Let us see that open & flat FRW universes cannot recollapse

- Consider a spatial surface Σ
  - normal vector $n_\mu = (1, \vec{0})$
  - induced metric $$h_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$$ $$\Rightarrow \quad h_{00} = h_{0i} = 0 \ , \quad h_{ij} = g_{ij}$$
  - Einstein Equations $$G_{\mu\nu} = 8\pi G\, T_{\mu\nu} - \Lambda g_{\mu\nu} \ ,$$ $$\Rightarrow \quad G_{\mu\nu} n^\mu n^\nu = (8\pi G\, T_{\mu\nu} - \Lambda g_{\mu\nu})\, n^\mu n^\nu$$ $$\Rightarrow \quad H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3}$$



  - If the universe starts expanding, and it turns back
    - $\Rightarrow$ there is a surface with $H = 0$
    - $\Rightarrow$ on this surface $\Rightarrow \quad \frac{k}{a^2} = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3}$
    - If $(\rho > 0 \,\&\, \Lambda > 0) \,\&\, k \leq 0 \quad \Rightarrow$ Impossible

FRW Cosmology



– Collapse for open and flat universe?

$$\Rightarrow \quad \frac{k}{a^2} = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3}$$

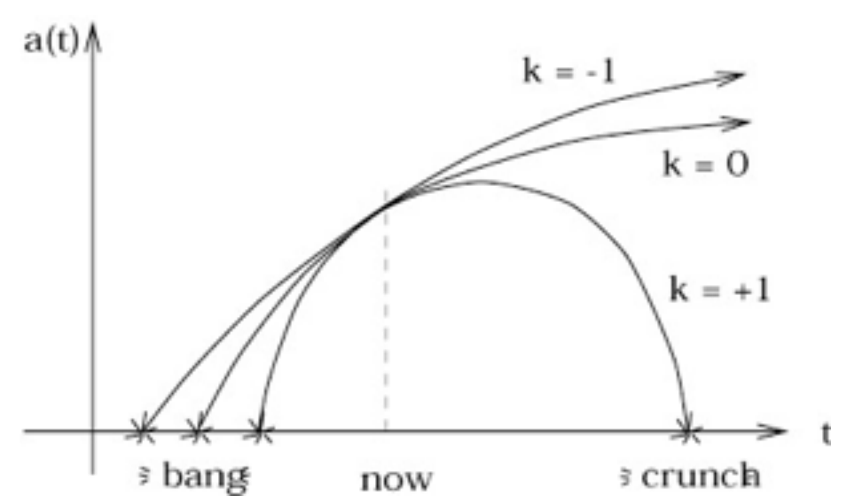
$$(\rho > 0 \,\&\, \Lambda > 0) \,\&\, k \leq 0 \quad \Rightarrow \quad \text{Impossible}$$

– no need of the explicit solution

- Also closed universe can not-recollapse: it needs to become larger than

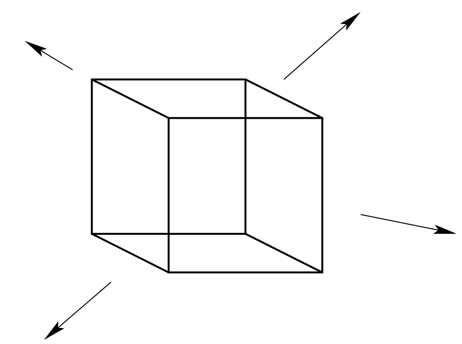
$$\Rightarrow \frac{k}{a^2_{\rm max}} \lesssim H_I^2 \sim \Lambda / M_{\rm Pl}^2$$

- i.e.: it needs to be large enough and have a large enough CC
  - Otherwise, it recollapses in an Hubble time



The Initial Patch Problem

– Imagine we start with a small inhomogeneous universe, not dominated by Inflaton potential. Say it expands in a decelerating way.

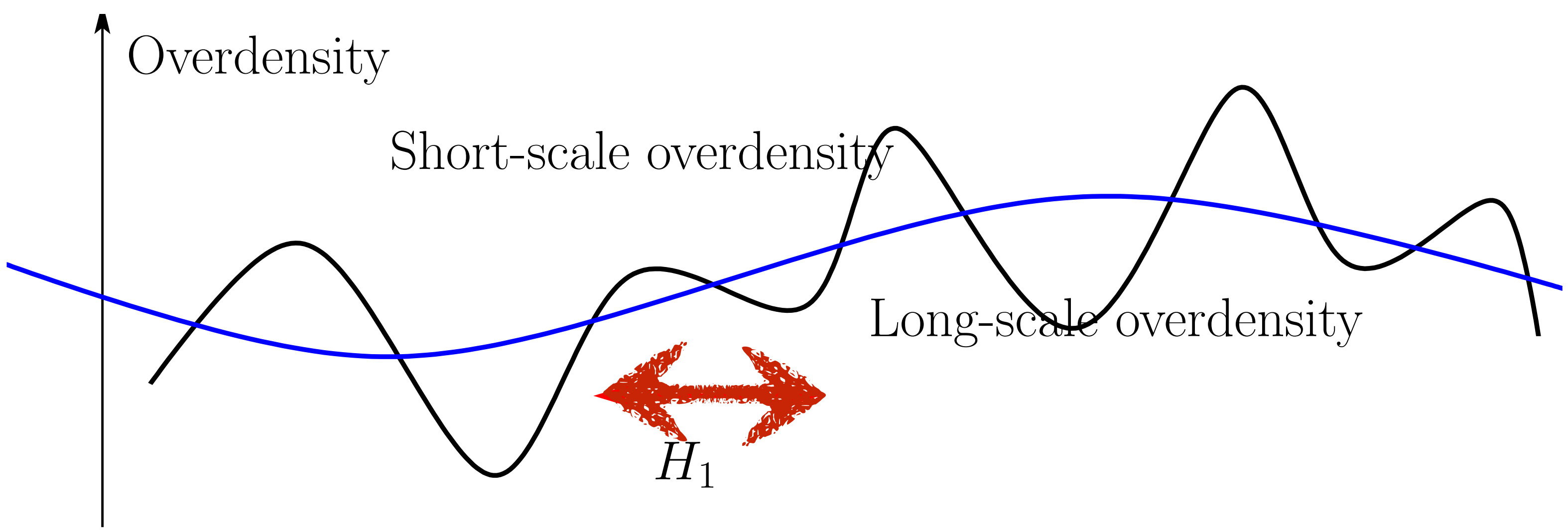


– $\Longrightarrow$ to start inflation, we need H to decrease to H_I

– During this time, many modes become shorten than H

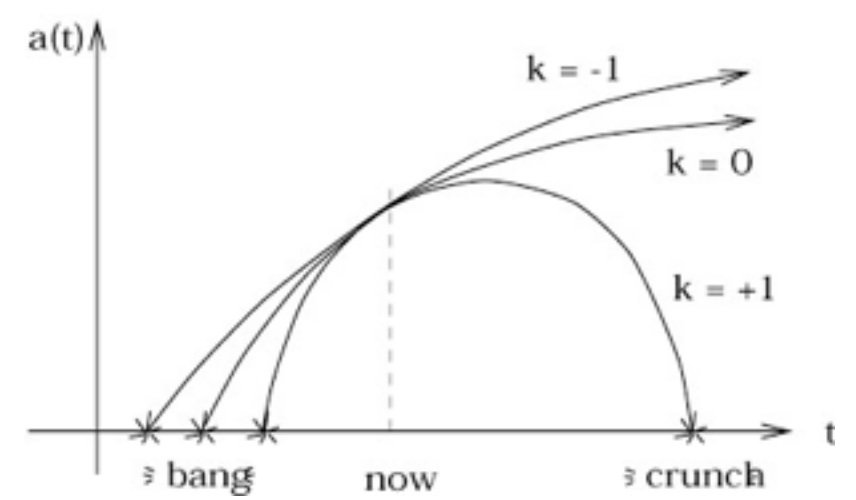
– Modes longer than H count as an effective Homogeneous density $\rho_{\rm FRW}$

– At each time step, $\rho_{\rm FRW}$ changes by order 1: $\delta\rho/\rho \sim 1 \quad \Rightarrow \quad \delta\rho_{\rm FRW}/\rho_{\rm FRW} \sim 1$



– since $\delta\rho/\rho \sim 1$ and H is independent,

– $\Rightarrow \dfrac{\delta\left(\frac{k}{a^2}\right)}{\frac{k}{a^2}} \sim 1$: the curvature jumps by order one » at each Hubble time.



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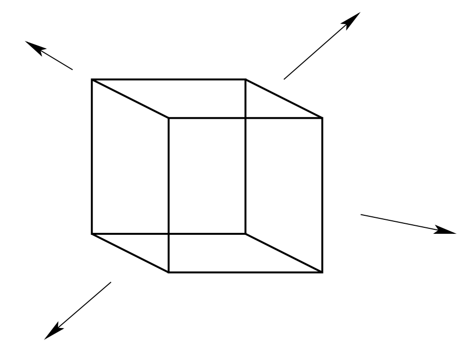

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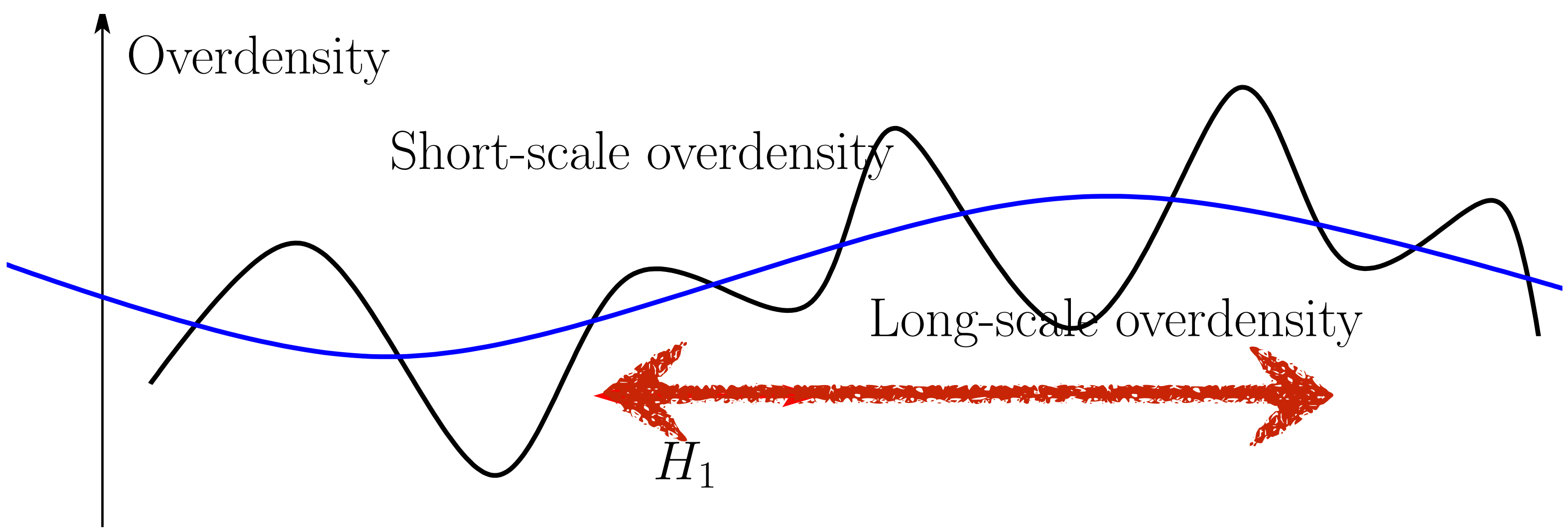

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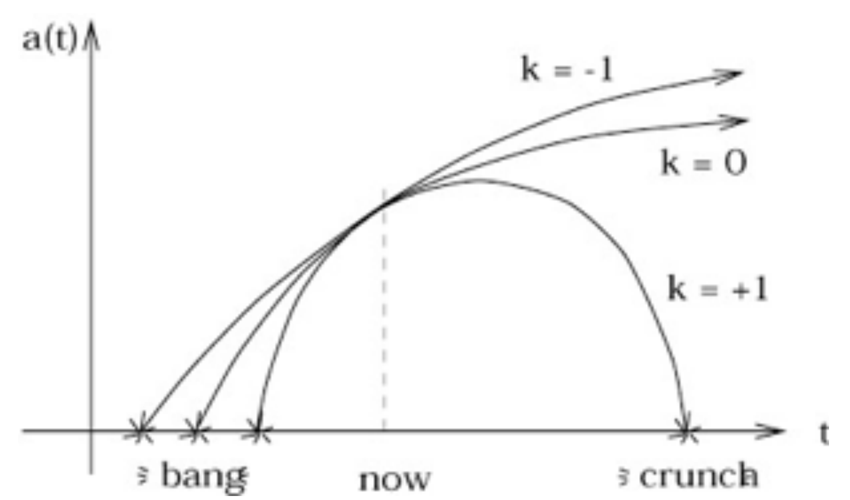


The Initial Patch Problem

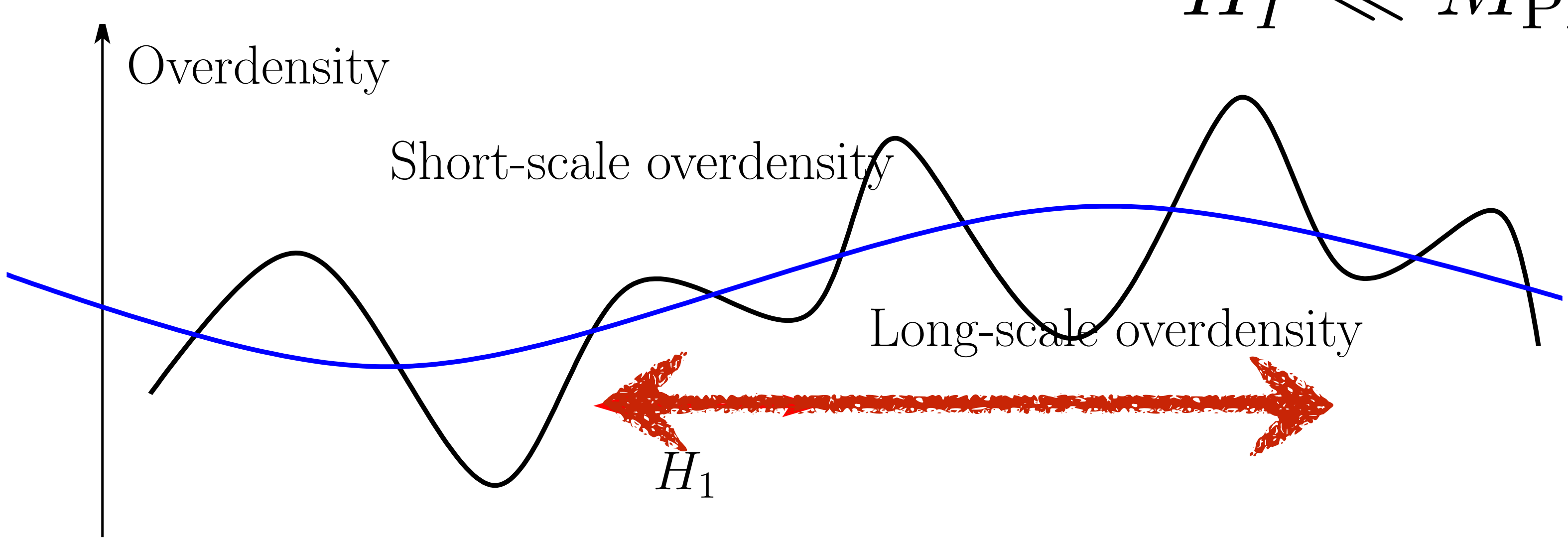
– At each time step $\Rightarrow \frac{\delta\left(\frac{k}{a^2}\right)}{\frac{k}{a^2}} \sim 1$

– If the universe ever becomes closed, it recollapses

– The number of modes entering the horizon before starting inflation is $\propto M_{\rm Pl}/H_I$

– If any mode is uncorrelated, the probability to survival and to start inflation is

$$H_I \ll M_{\rm Pl} \quad \Rightarrow \quad {\rm prob} \sim e^{-\frac{M_{\rm Pl}}{H_I}}$$



– Inflation seems to be exponentially unlikely

– so called `Intial patch problem' (many authors)

• We will see that this to-me compelling argument is incorrect

The Initial Patch Problem

– Several mechanism to avoid to this problem have been proposed in the years

  – Linde likes starting with a Planckian torus

  – I like starting out of false vacuum eternal inflation.

- but somehow we continue to talk about this problem

    - because these above were ad-hoc solutions

- The solution we present here is radically different

  – we will show that this problem is non-existent (but for a deep, non-trivial, reason)

Inhomogenous Cosmology

– Already Wald (1983), here, had shown that if the weak energy condition is preserved, all homogeneous but inosotropic universe (Bianchi universes) that are not `closed' (that is non-Bianchi-Type-IX universes) cannot recollapse.

– WEC: $T_{\mu\nu}t^{\mu}t^{\nu} \geq 0 \quad (\text{i.e.} ``\rho \geq 0, \rho + p > 0'') \; , \;\; \text{for any } t^{\mu} \text{ timelike}$

– But inhomogeneities are more challenging.

  – diff. equations become partial diff., and singularities form, geodesic cross, etc. It is a much less symmetric situation.

  – we will see that a sort of similar conclusion holds

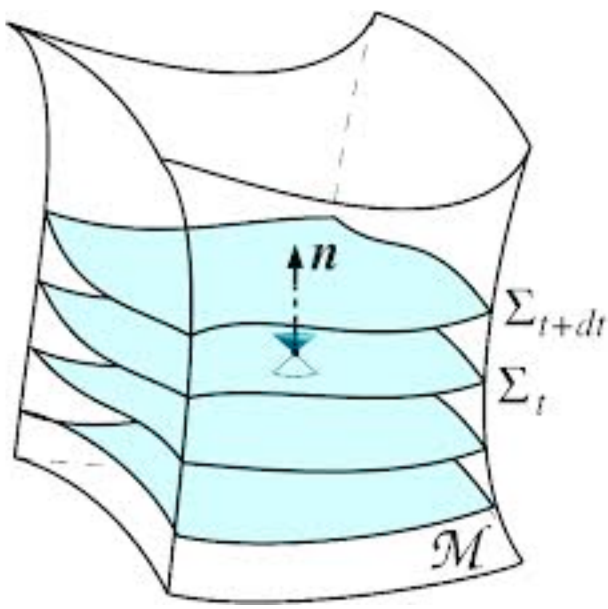
• Let us therefore consider general `cosmologies'.

A Cosmology

– *First Assumption*: we consider a cosmology:

  – a connected 3+1 dimensional spacetime with a compact Cauchy surface (will comment later on the non-compact case)

- This implies (Geroch, here, 1970):
  – the spacetime is topologically $R \times M$ where M is a 3-manifold
  – it can be foliated by a family of topologically identical Cauchy surfaces M_t



Theorem

with Kleban **JCAP2016**

– Under this and other assumptions that we will specify, we will prove:

There cannot exist a non-singular spacelike hypersurface with maximum volume: given any time slice, there is another with larger spatial volume. Furthermore, in an initially expanding universe there must be at least one expanding region on every timeslice, and if $\Lambda > 0$ the expansion rate in that region is bounded from below by that of de Sitter spacetime in the flat slicing.

For the first sentence, see also Barrow and Tippler **1985**

Theorem

with Kleban **JCAP2016**

– This implies that in a big bang cosmology, there cannot be a big crunch

  – *very strongly* suggesting that the cosmology will reach infinite volume, gradient energy will dilute, and inflation will eventually start, no matter what are the initial inhomogeneities and the scale of inflation

There cannot exist a non-singular spacelike hypersurface with maximum volume: given any time slice, there is another with larger spatial volume. Furthermore, in an initially expanding universe there must be at least one expanding region on every timeslice, and if $\Lambda > 0$ *the expansion rate in that region is bounded from below by that of de Sitter spacetime in the flat slicing.*

For the first sentence, see also Barrow and Tippler **1985**

Proof

–The proof is very simple. A regular surface of maximum volume has extrinsic curvature

$$K = 0 \quad \text{everywhere}$$

as $\mathcal{L}_n \log \sqrt{h} = K,$

–To satisfy Einstein equations, this implies

$$R^{(3)} \geq 0 \quad \text{everywhere}$$

–But topologically some manifolds require

$$R^{(3)} \leq 0 \quad \text{at least at one point}$$

–Therefore maximal surfaces cannot exist

Important Hypothesis

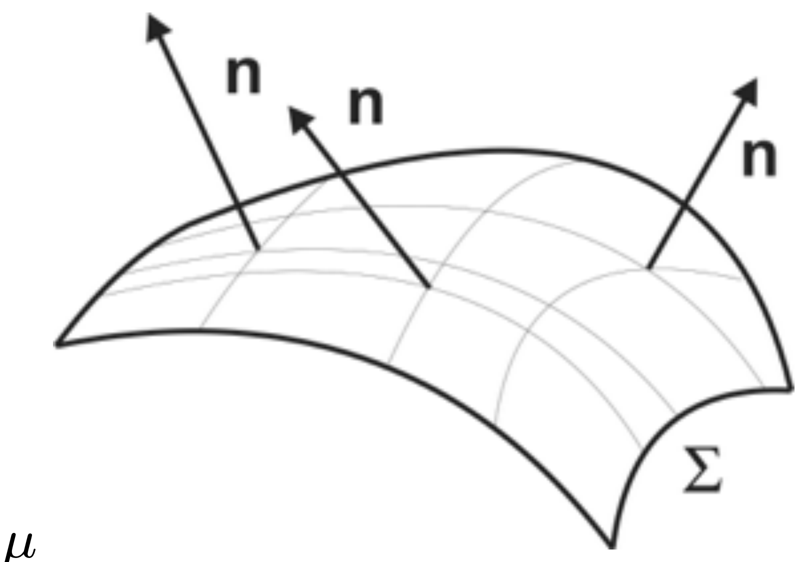
– A cosmology

– The spatial topology of M_t must not be `closed', i.e. it must not be of type (i) that we define below (roughly, M_t must not be a sphere)

– The weak energy condition holds (satisfied by matter, radiation and $V > 0$)

Notation

– n_μ is the orthonormal vector to M_t : $n_\mu n^\mu = -1$



– Spatial metric $h_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$

– Extrinsic curvature $K_{\mu\nu} = h_\mu{}^\rho \nabla_\rho n_\nu$, $K = \nabla_\mu n^\mu$

–how much the family of geodesics induced by n_μ deviates

–In particular $K = \nabla_\mu n^\mu$, $\sigma_{\mu\nu} = K_{\mu\nu} - \frac{1}{3} K h_{\mu\nu}$

–Notice $\mathcal{L}_n \log \sqrt{h} = K,$: rate of growth of volume

$$\Rightarrow \quad \sqrt{h} \sim \sqrt{h_0}\, e^{Kt}$$

Proof

– Similarly to FRW case, consider (Λ reabsorbed in stress tensor)

$$n^\mu n^\nu G_{\mu\nu} = 8\pi G_N \, T_{\mu\nu} \, n^\mu n^\nu$$

– From Gauss-Codazzi

$$n^\mu n^\nu G_{\mu\nu} = \frac{1}{2}\left\{R^{(3)} + \left(K^\mu_\mu\right)^2 - K_{\mu\nu}K^{\mu\nu}\right\} = \ 3 - \text{surface quantities}$$

– $\Longrightarrow$ we have

$$16\pi G_N T_{\mu\nu} n^\mu n^\nu = R^{(3)} + \frac{2}{3}K^2 - \sigma_{\mu\nu}\sigma^{\mu\nu}$$

– If a surface has extremal volume, the volume is stationary wrt any variations. Since

$$\mathcal{L}_n \log \sqrt{h} = K, \quad \Rightarrow \quad K = 0 \quad \text{everywhere}$$

– Then we have, on an extremal surface,

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– Then we have, on an extremal surface,

$$\underbrace{16\pi G_N \, T_{\mu\nu} n^\mu n^\nu}_{\geq 0 \text{ by WEC}} = R^{(3)} \underbrace{-\sigma^{\mu\nu}\sigma_{\mu\nu}}_{\leq 0}$$

Proof

– One Einstein eq. on extremal surfaces gives

$$\underbrace{16\pi G_N\, T_{\mu\nu} n^\mu n^\nu}_{\geq 0 \text{ by WEC}} = R^{(3)} \underbrace{-\sigma^{\mu\nu}\sigma_{\mu\nu}}_{\leq 0}$$

– If $R^{(3)} \leq 0 \quad \text{at least at one point}$, $\Longrightarrow$ this equation cannot be satisfied

– $\Longrightarrow$ an extremal surface cannot exist

– It turns out that for order-one fraction of topologies $R^{(3)} \leq 0 \quad \text{at least at one point}$

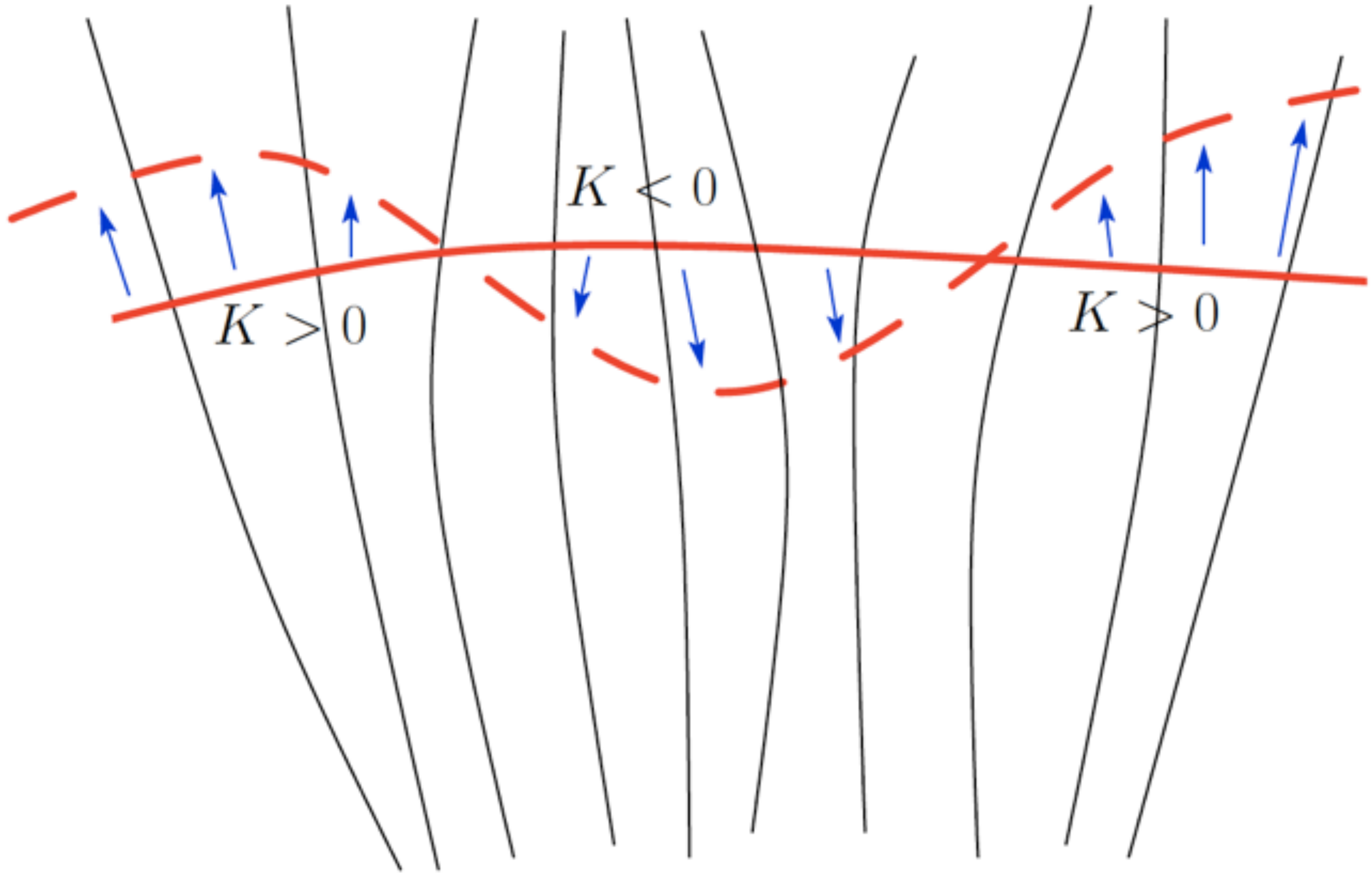
– $\Longrightarrow$ an extremal surface cannot exist on these topologies

– This means that in a spacetime with this topology, and with a big-bang or big-crunch, given a surface, we can always find a surface with larger volume (either in the future or in the past)

– This surface can be found with the following procedure

Mean Curvature Flow

– Take a surface, and deform it forward or backward according to sign of K



– The change of volume: $\frac{\partial V}{\partial \lambda} = \int d^3x K^2 \sqrt{h} \equiv \langle K^2 \rangle \geq 0$

– So this procedure either converges to an extremal surface, if it can exist, with $K = 0$ everywhere

– or it gives a surface of larger volume indefinitely

Thorston Geometrization Conjecture

Thorston, Hamilton, Perelman

– To determine which manifolds must have $R^{(3)} \leq 0$ at least at one point, consider that all compact oriented 3-manifold fall into one of these three classes

– (i) ``Closed'': any function on M_t can be the $R^{(3)}$ of a smooth metric on M_t

- ex: $S^3,\ S^2 \times S^1,\ S^3/\Gamma(\text{with } \Gamma \in SO(4)), RP^3$

– (ii) ``Flat'': any function on M_t can be the $R^{(3)}$ of a smooth metric on M_t if it is negative somewhere or zero everywhere

- ex: $R^3/\Gamma(\text{ with } \Gamma \text{ an isometryof } R^3)$

– (iii) ``Open'': any function on M_t can be the $R^{(3)}$ of a smooth metric on M_t if it is negative somewhere

- ex: $H^3/\Gamma,\ H^2 \times R, nil, sol, \widetilde{SL}(2,R)$

– Any connected sum of (i) and (ii) with a factor of (iii) is of kind (iii)

Thorston Geometrization ~~Conjecture~~ Classification

Thorston, Hamilton, Perelman

- To determine which manifolds must have $R^{(3)} \leq 0$ $\text{at least at one point}$, consider that all compact oriented 3-manifold fall into one of these three classes
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      - and connected sums
  - (ii) ``Flat'': any function on M_t can be the $R^{(3)}$ of a smooth metric on M_t if it is negative somewhere or zero everywhere
    - ex: $R^3/\Gamma(\text{ with } \Gamma \text{ an isometryof } R^3)$
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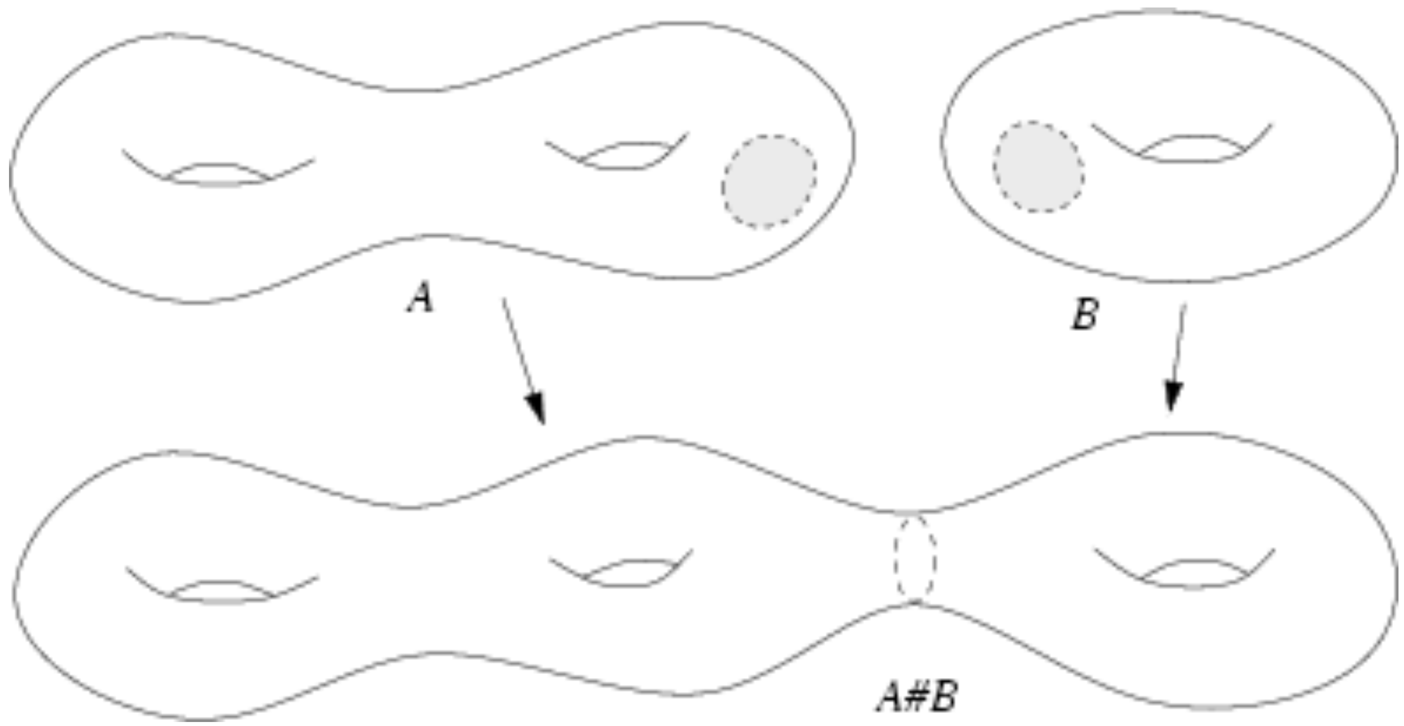
    - ex: $H^3/\Gamma,\ H^2 \times R, nil, sol, \widetilde{SL}(2,R)$
  - Any connected sum of (i) and (ii) with a factor of (iii) is of kind (iii)

Thorston Geometrization Classification

Thorston, Hamilton, Perelman

– A basis of manifolds is given by the connected sums of the examples in (i), (ii), (iii).

– Any connected sum of (i) and (ii) with a factor of (iii) is of kind (iii)



– So, the number of compact manifolds is countable (infinite), it can be identified with an integer. In FRW, we had just 3 integers.

Thorston Geometrization Classification

Thorston, Hamilton, Perelman

–For type (iii), the theorem is established.

–For type (ii), it could be that $R^{(3)} = 0$ everywhere

– if extremal surface exists, we have:

$$\underbrace{16\pi G_N T_{\mu\nu} n^\mu n^\nu}_{\geq 0 \text{ by WEC}} = R^{(3)} \underbrace{-\sigma^{\mu\nu}\sigma_{\mu\nu}}_{\leq 0}$$

$$\Rightarrow \sigma_{\mu\nu} = 0 \;\&\; T_{\mu\nu} n^\mu n^\nu = 0, \quad \Rightarrow \quad T_{\mu\nu} = 0 \text{ (with dominant energy condition)}$$

$$\text{DEC} : T_{\mu\nu} t^\nu \text{ is past directed for all future directed timelike } t^\mu, \; \text{``}\rho > |p|\text{''}$$

– This is an uninteresting empty universe

``Open'' universes

–In all cases where $R^{(3)} \leq 0$ at least at one point, at that point we have

$$16\pi G_N T_{\mu\nu} n^\mu n^\nu = R^{(3)} + \frac{2}{3} K^2 - \sigma_{\mu\nu}\sigma^{\mu\nu}$$

$$\Rightarrow \quad |K| \geq K_\star \equiv \sqrt{24\pi G_N T_{\mu\nu} n^\mu n^\nu}.$$

–If we add that the universe is initially expanding, than we have that at that point

$$K \geq K_\star$$

``Open" universes

– We have at one point on any surface $|K| \geq K_\star \equiv \sqrt{24\pi G_N T_{\mu\nu} n^\mu n^\nu}.$

– if the universe is initial expanding

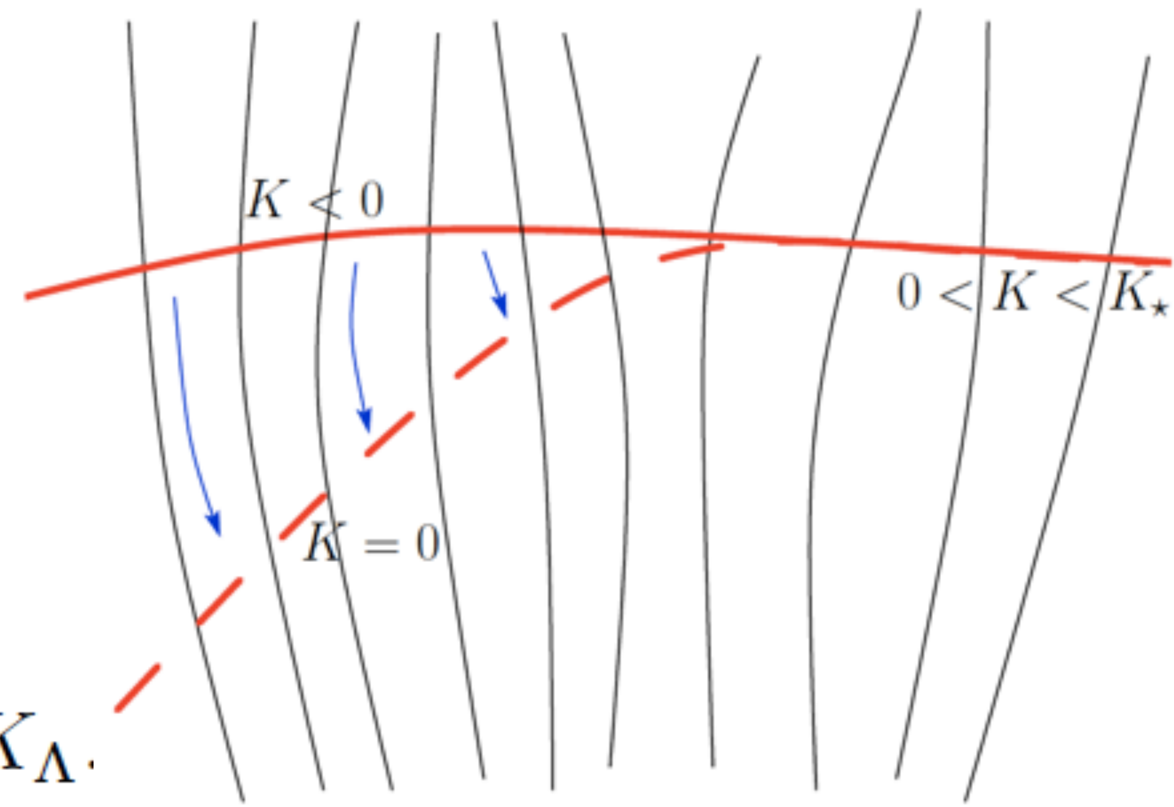
$$\Rightarrow K \geq K_\star$$ at that point

– Indeed: Assume there is a surface where $K < K_\star$ everywhere

– can pull back the surface to obtain a surface where $|K| < K_\star$ everywhere

- contradiction
- A more formal proof follows from

Gerhardt **1983**



- Therefore, in any slice there must be a point that
- expands with $K \geq K_\star$

– Suppose now there is a positive CC

– (as in inflation), then $K \geq \sqrt{24\pi G_N \Lambda} \equiv K_\Lambda.$

– a region expands always faster than in inflation

Subtleties

– We have given a procedure to create surfaces of ever grow. volume (Mean Curv. Flow)

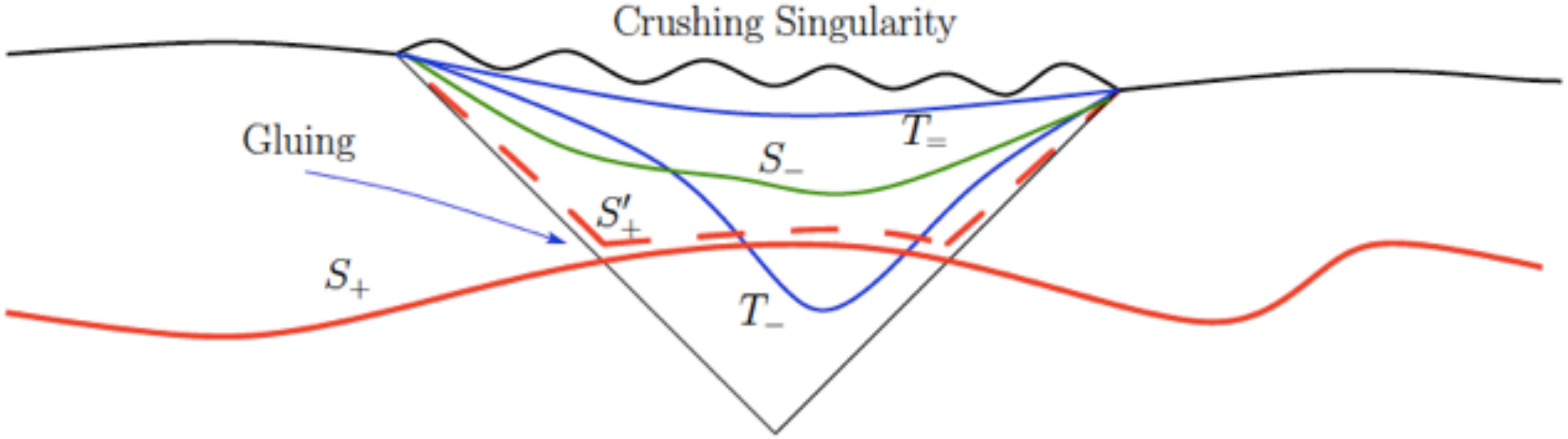
– We need to assume the absence of finite volume singularities

  – the universe could stop expanding all of a sudden

  – these singularities are conjectured to be removed with some mild assumptions of regularity of stress tensor   Barrow and Tippler **1985**

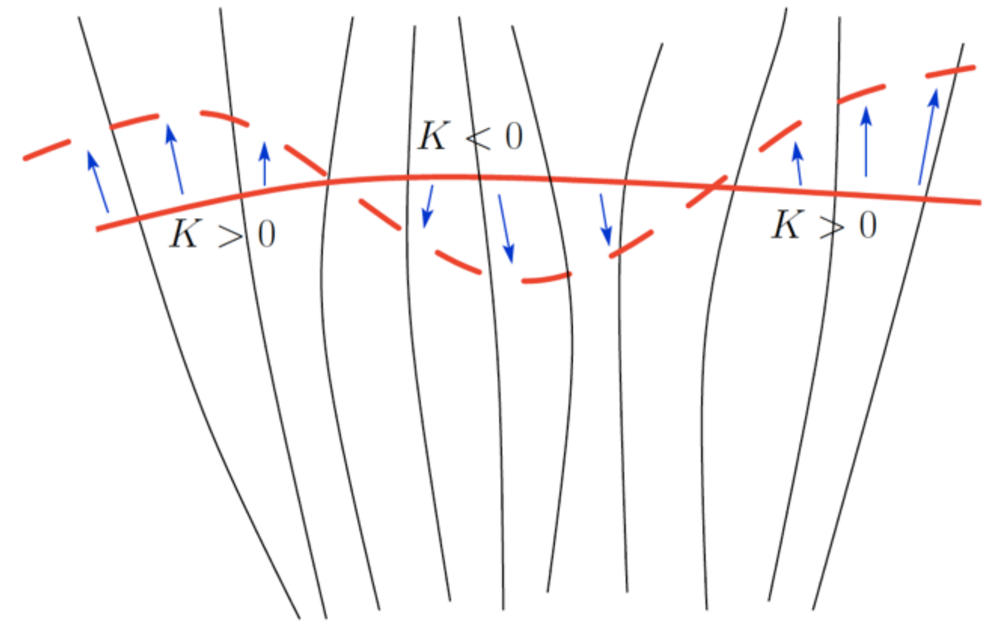
  – these singularities sound quite unphysical to us

- Our spacetime will develop localized black holes and other singularities: no problem.
- In the JCAP version, we have proven the quite intuitive fact that mean curvature flow will not enter and hit crushing singularities, such as the ones that form inside Black Holes. Idea: close to crushing singularity, surfaces have *arbitrary* negative extrinsic curvature.



Does the universe reach infinite volume?

– It is tempting to conclude that the slices obtained with mean curvature flow reach infinite volume



– But it could be that the overall volume keeps

  – growing reaching an asymptote

– This is very unlikely:

  – there is always a region that expands fast

  – therefore this region should shrink, becoming singular

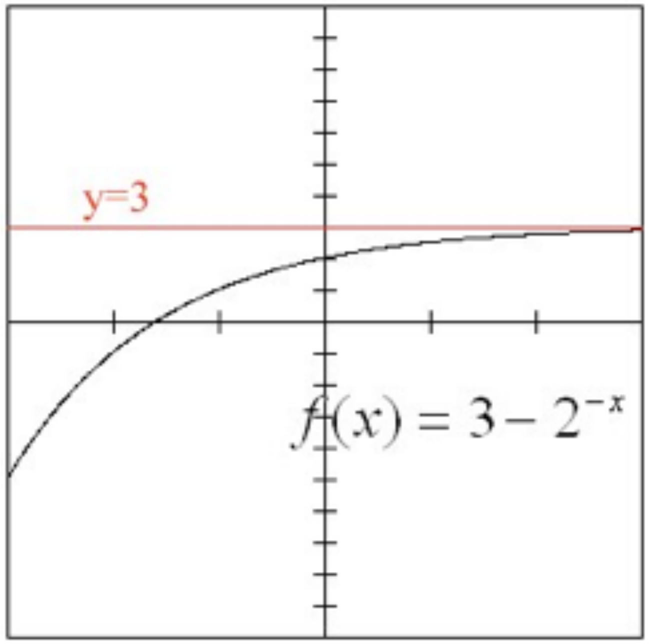
  – Theorem: this can only happen if the spacetime becomes singular at that point or if surface embedding reaches infinite time

    • which it must as otherwise it would reach extremality

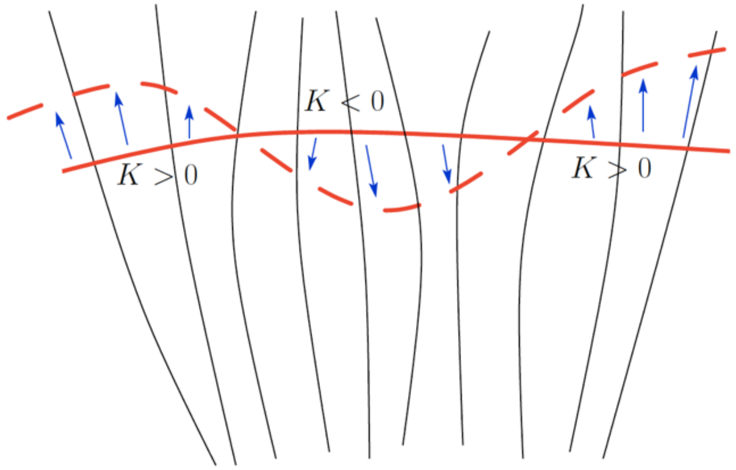
  – Then, unless the surface is null, it has infinite volume

    • (this is not proven yet: we are trying to prove this)

  – Physically: hard to imagine to reach a static universe



Implications for inflation



– Modulo these concerns, these surfaces will reach

– infinite volume, so that the vacuum energy will dominate

– The probability to start inflation is therefore reduced to estimating the probability for these topologies (clearly order one), as well as for the inflaton of being on top of the potential (see later, but I will not make a judgement on this)

– The problem of initial homogeneity seems to have been resolved (but showing it does not hold for topological reasons)

  – the idea of the random walk of $\delta\rho/\rho$ did not hold because topology induced a correlation among modes.

    • Other papers were mis-interpreted because we show that inflation is a huge attractor

More `practical' considerations

with East, Linde and Kleban **1511**

– This is as far as pure Mathematics has brought us (at least so far).

- There is a strong suggestion that inflation will always start for these topologies.
- We can check this by performing numerical simulations (actually historically we have done things in reverse order).

- Approach: start with highly inhomogenous universe, and simulate its evolution.

  – need a code that can handle singularities, batch holes and horizons.

  – Solution: make contact with BH mergers community and the problem is solved

    - for us, this amounted to contacting William East

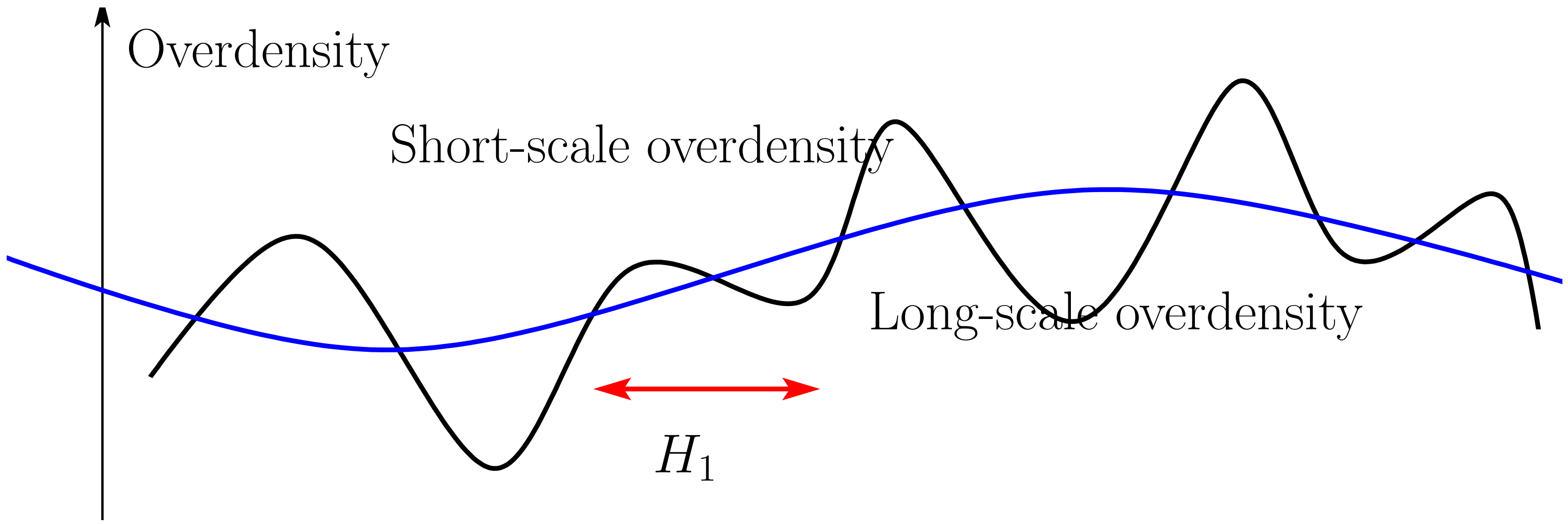
Strategy of simulations

with East, Linde and Kleban **1511**

– Consider an initially expanding highly inhomogenous universe.

– Define a local hubble rate by the extrinsic curvature: H_0

–There will be modes longer and shorter than H_0

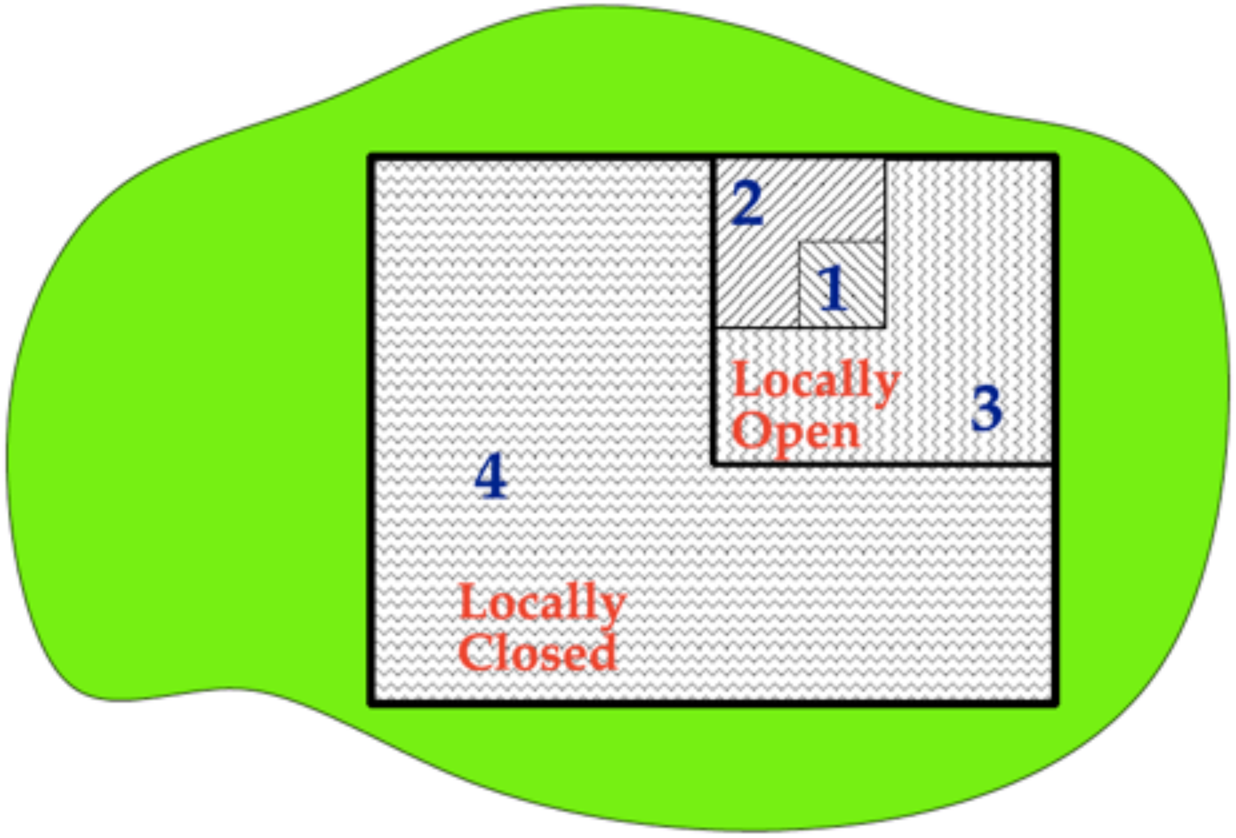


Strategy of simulations

with East, Linde and Kleban **1511**

- The effect of long modes $k/H_i \ll 1$ can be understood analytically.
  - As long as a mode is longer than Hubble, its effect is simply to renormalize the energy density: it induces a local homogenous anisotropic (Bianchi) universe
  - Their evolution is well understood (Wald 1983): only closed universes can recollapse
  - If a region of order H_I is open on average (meaning that the zero mode of the energy is such that it makes it of open-Bianchi type), there must be a region where the effect of long modes is to keep it open at all times

$$\sim \int_V (\rho - M_{\rm Pl}^2 H^2) \leq 0$$



Strategy of simulations

with East, Linde and Kleban **1511**

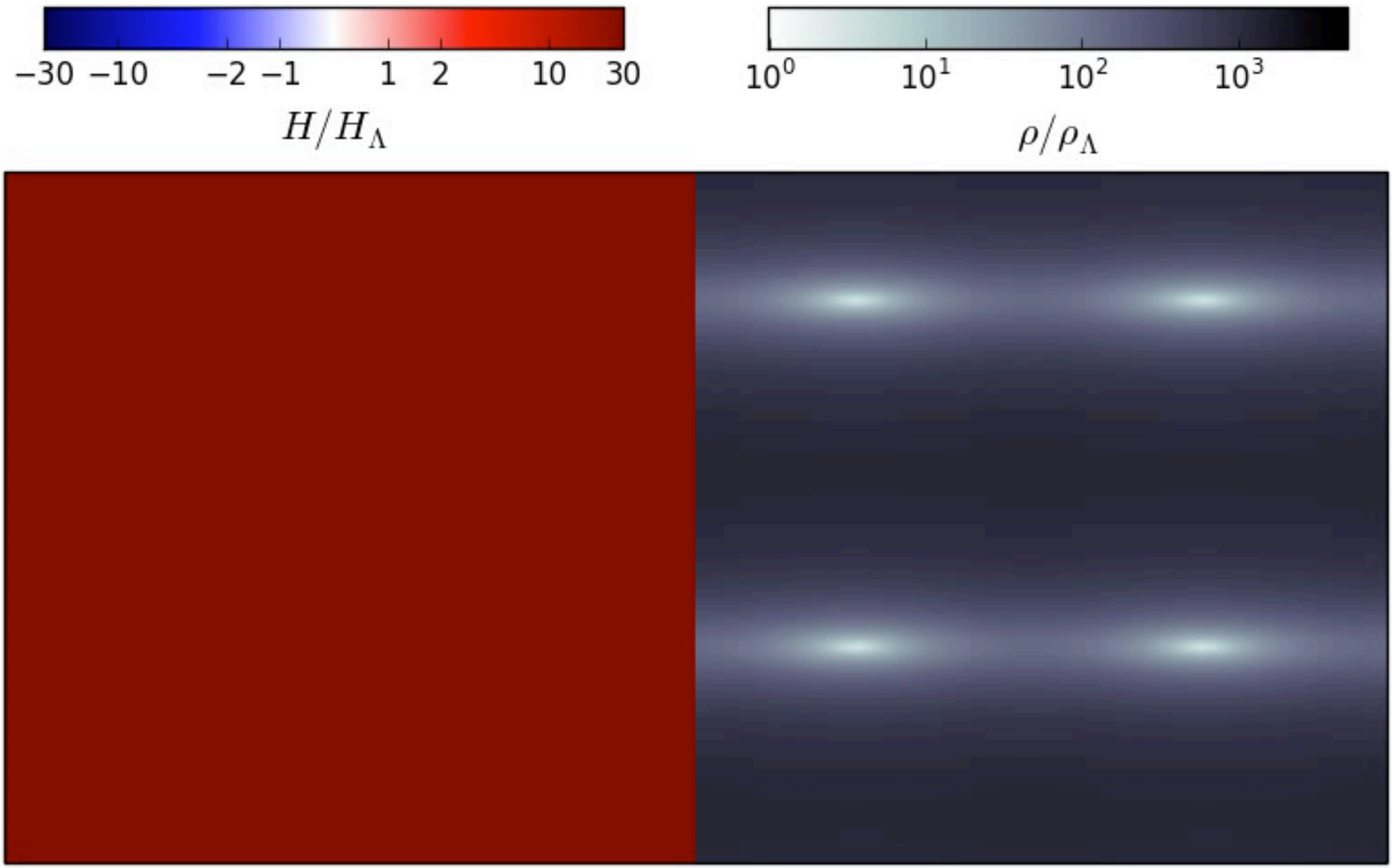
- The effect of short modes $k/H_i \gtrsim 1$ have strong dynamical effect
  - they will form overdensity that end in Black Holes
    - but, by conservation of energy, empty regions will form, and in those locations inflation will start (this is at least the idea)
- We start with simulation with domination of gradient energy:

$$\phi(t=0,\mathbf{x}) = \phi_0 + \delta\phi \left[\sum_{1 \le |\mathbf{k}L/2\pi|^2 \le N} \cos(\mathbf{k}\cdot\mathbf{x} + \theta_{\mathbf{k}}) \right],$$

Simulation

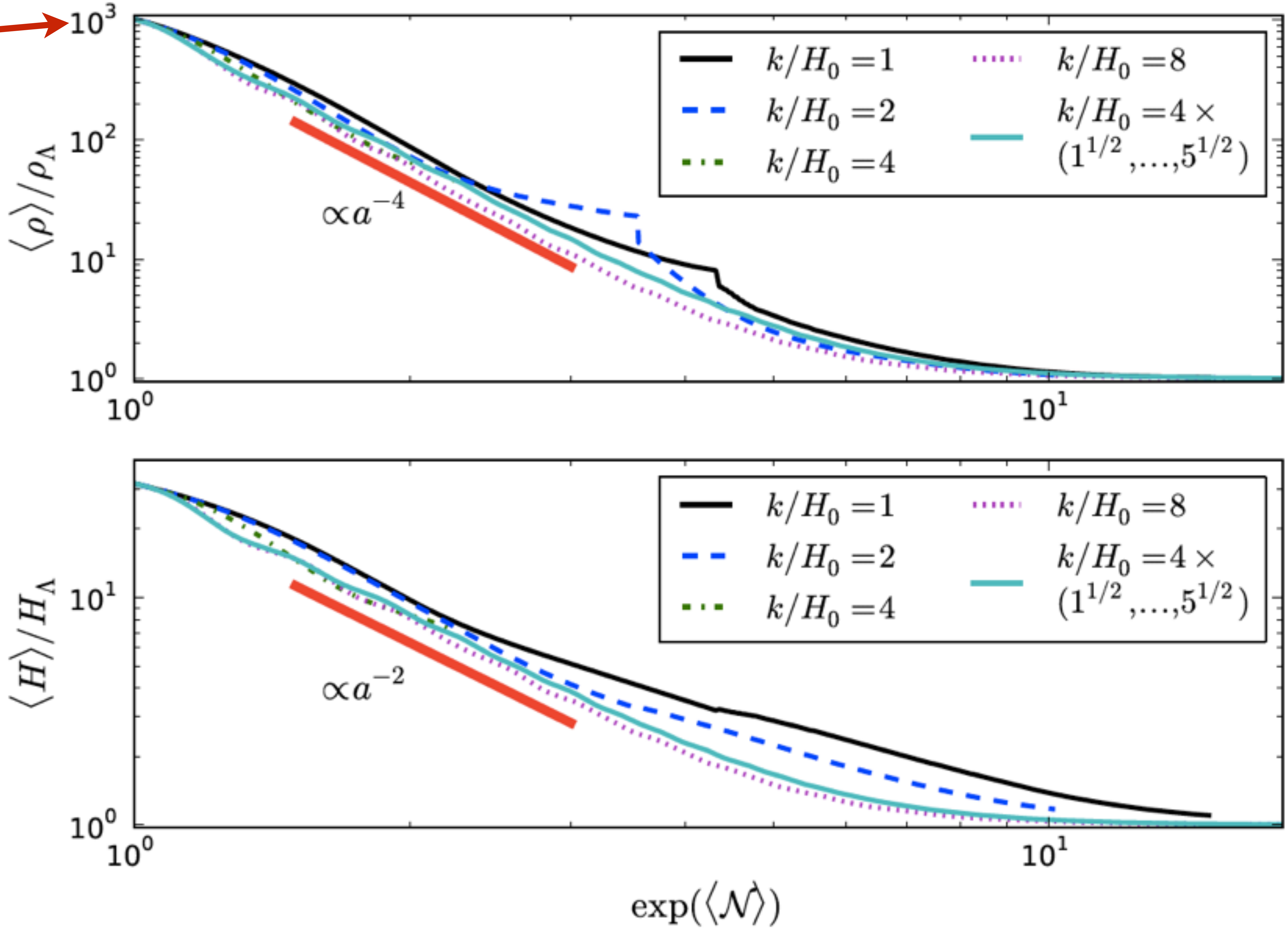
– Inflation starts

– BH emit GW



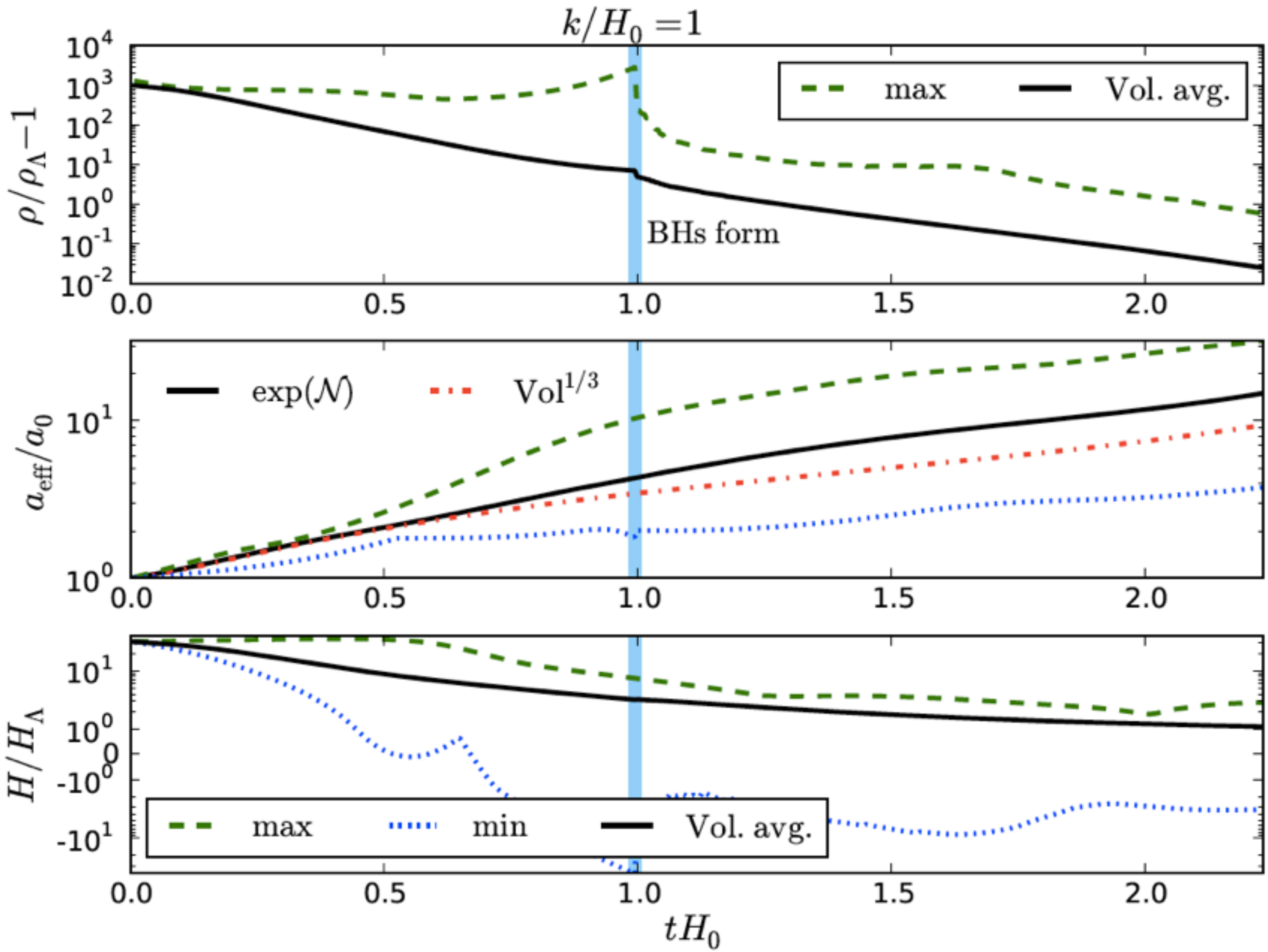
Approaching Inflation

High initial density



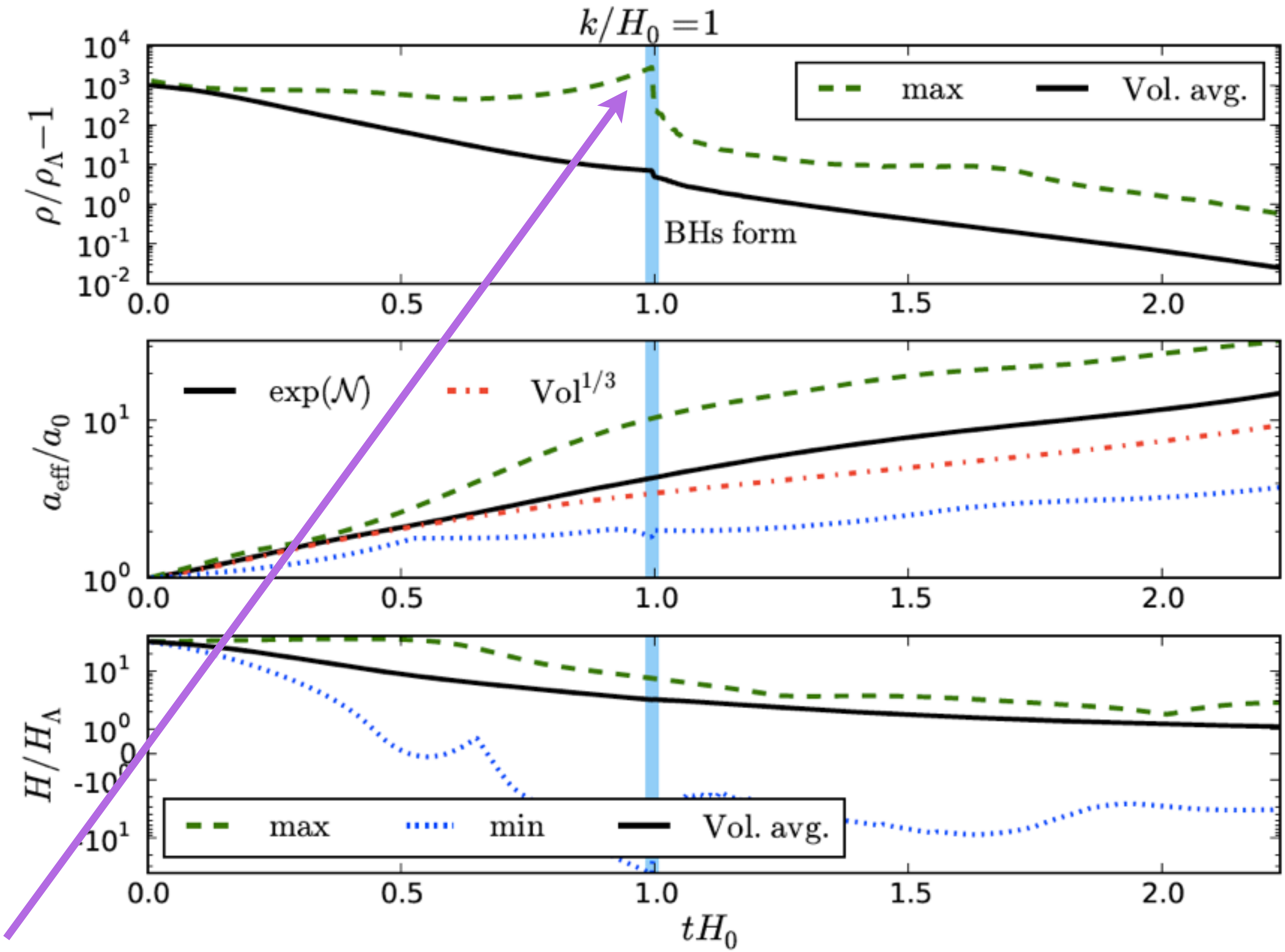
– Universe on average expands like radiation and then as CC

Approaching Inflation



– we start with high gradient densities.

Approaching Inflation



– BH form but dilute, universe homogenizes, inflation starts

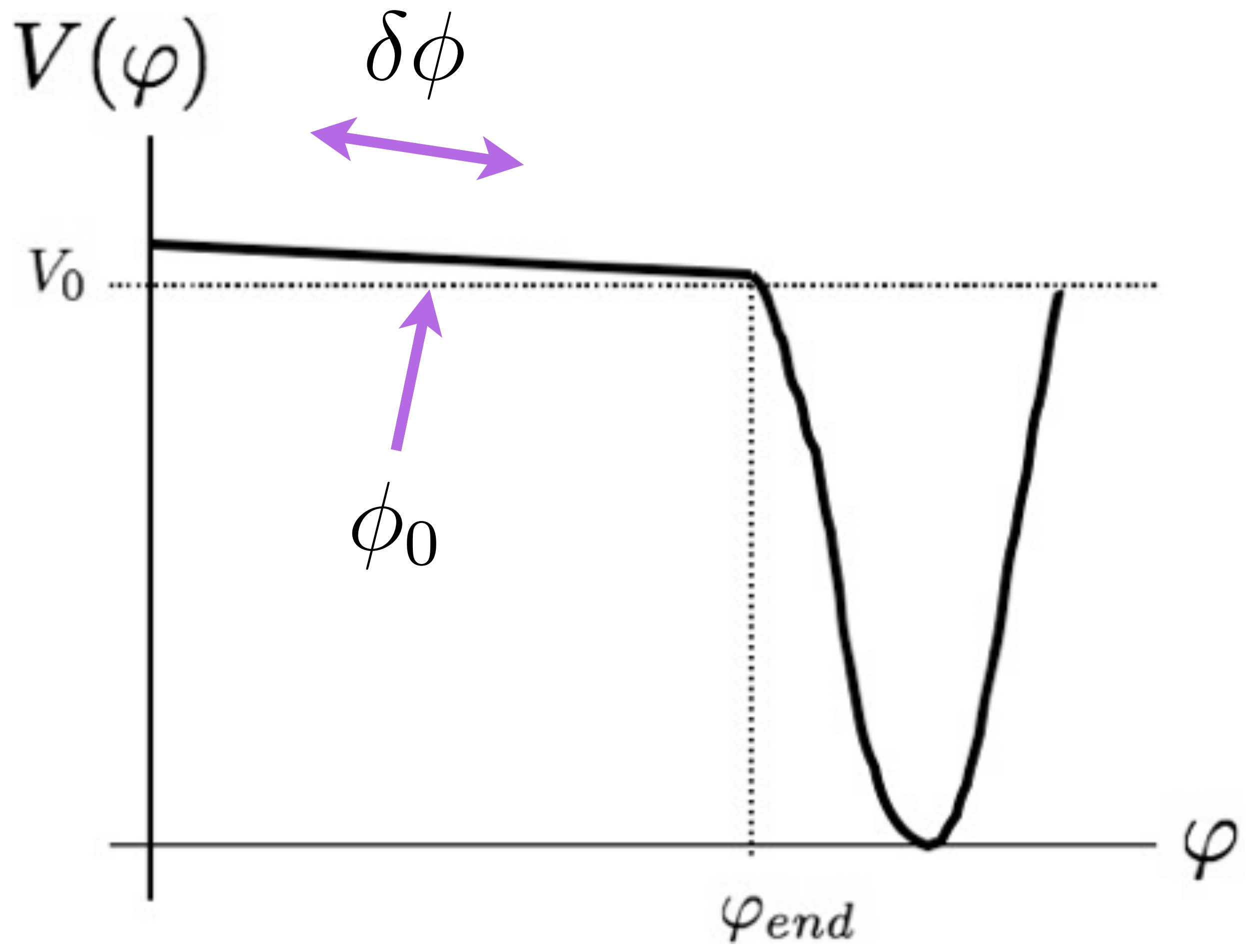
Slight Issue for Inflation

- It looks like there is no need to impose initial homogeneity to have inflation somewhere.
  - just estimate probability of topology (or of zero mode)

- But we require the field to be on top of the potential.
  - Why?

Slight `Novel' Issue for Inflation

– Consider an inflaton potential with a very perturbed initial field configuration.



– The fluctuations can be very large, going beyond the plateau

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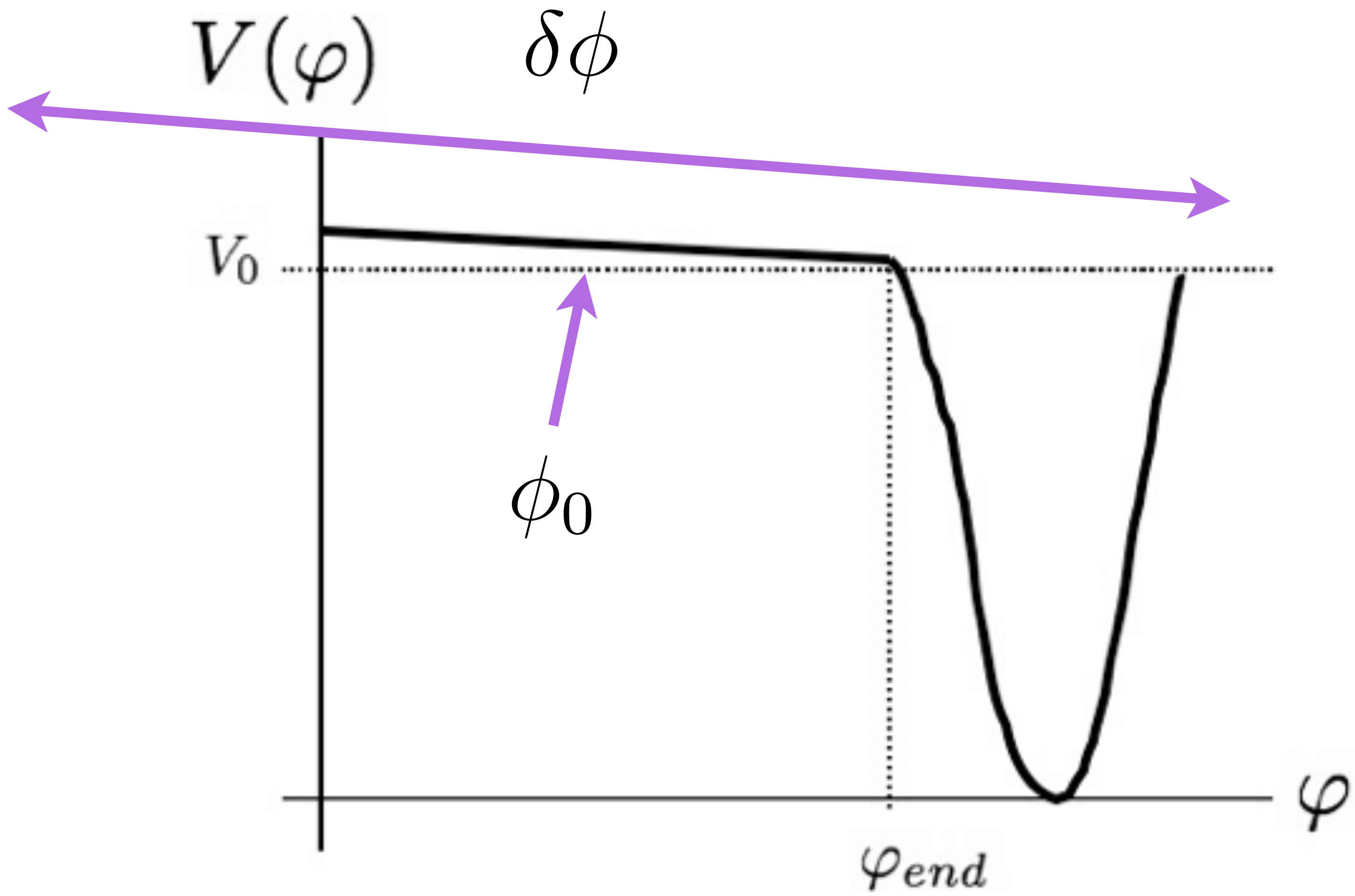


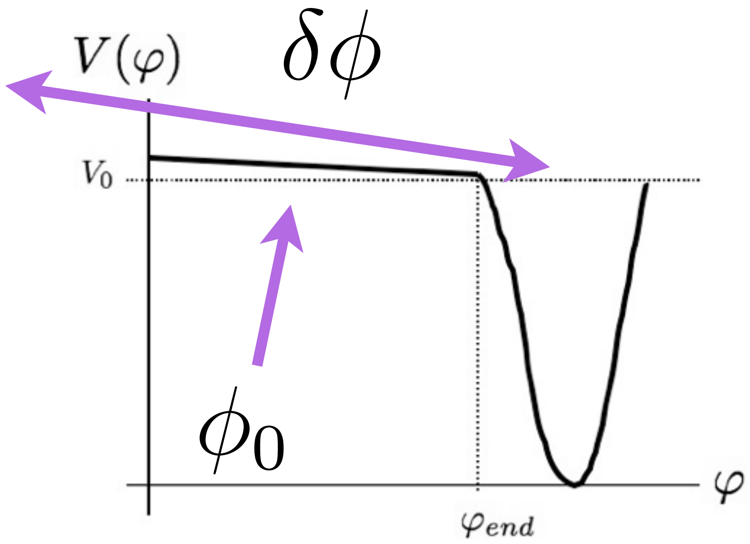
– The fluctuations can be very large, going beyond the plateau

Slight Issue for Inflation

– The dynamics of the average field is given by

– The average of the eq. of motion

$$\langle V'(\phi)\rangle \gg V'(\langle\phi\rangle)$$



– From an EFT point of view: integrate out the fast moving classical fluctuations to keep only the homogeneous mode. Since the classical fluctuations probe the high gradient region, the effective potential for the zero mode looks more something like this:

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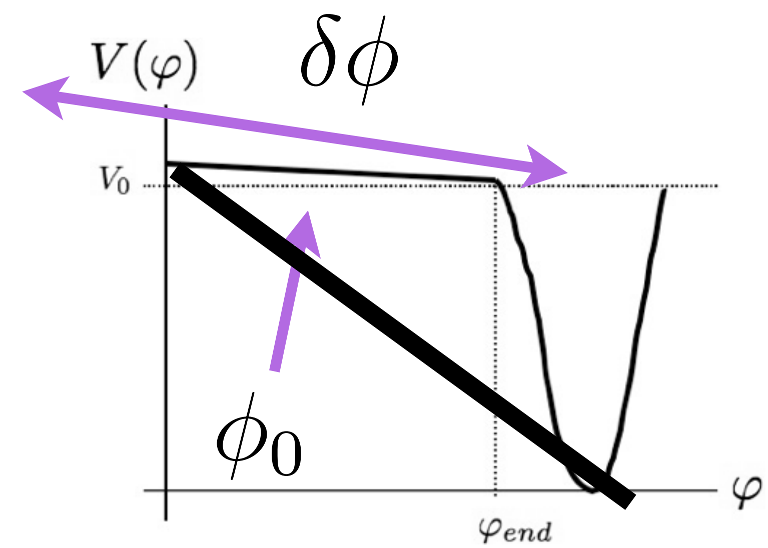


– From an EFT point of view: integrate out the fast moving classical fluctuations to keep only the homogeneous mode. Since the classical fluctuations probe the high gradient region, the effective potential for the zero mode looks more something like this:

–a non-inflating potential: in fact, we find no inflation in simulations

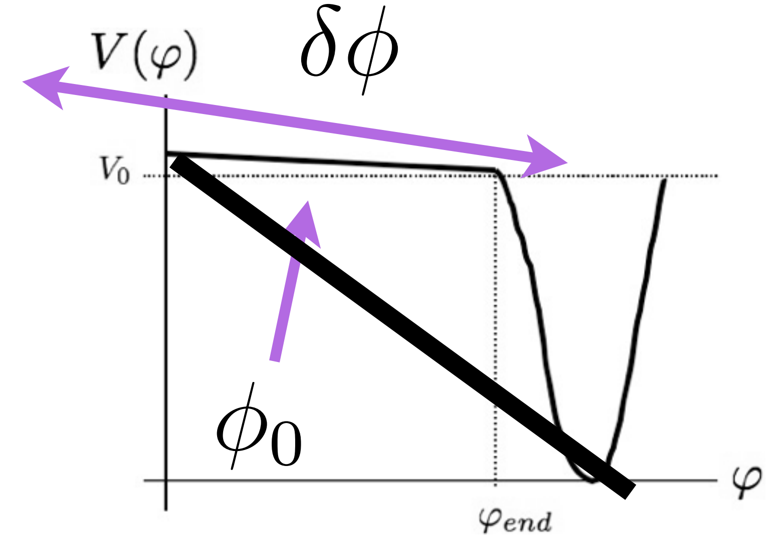
–In particular: $\langle V'(\phi)\rangle \sim \frac{V(\langle\phi\rangle)}{\langle\phi\rangle} = \left(\frac{M_P}{\langle\phi\rangle}\right)\left(\frac{V'(\langle\phi\rangle)}{\sqrt{\epsilon}}\right) > V'(\langle\phi\rangle)$

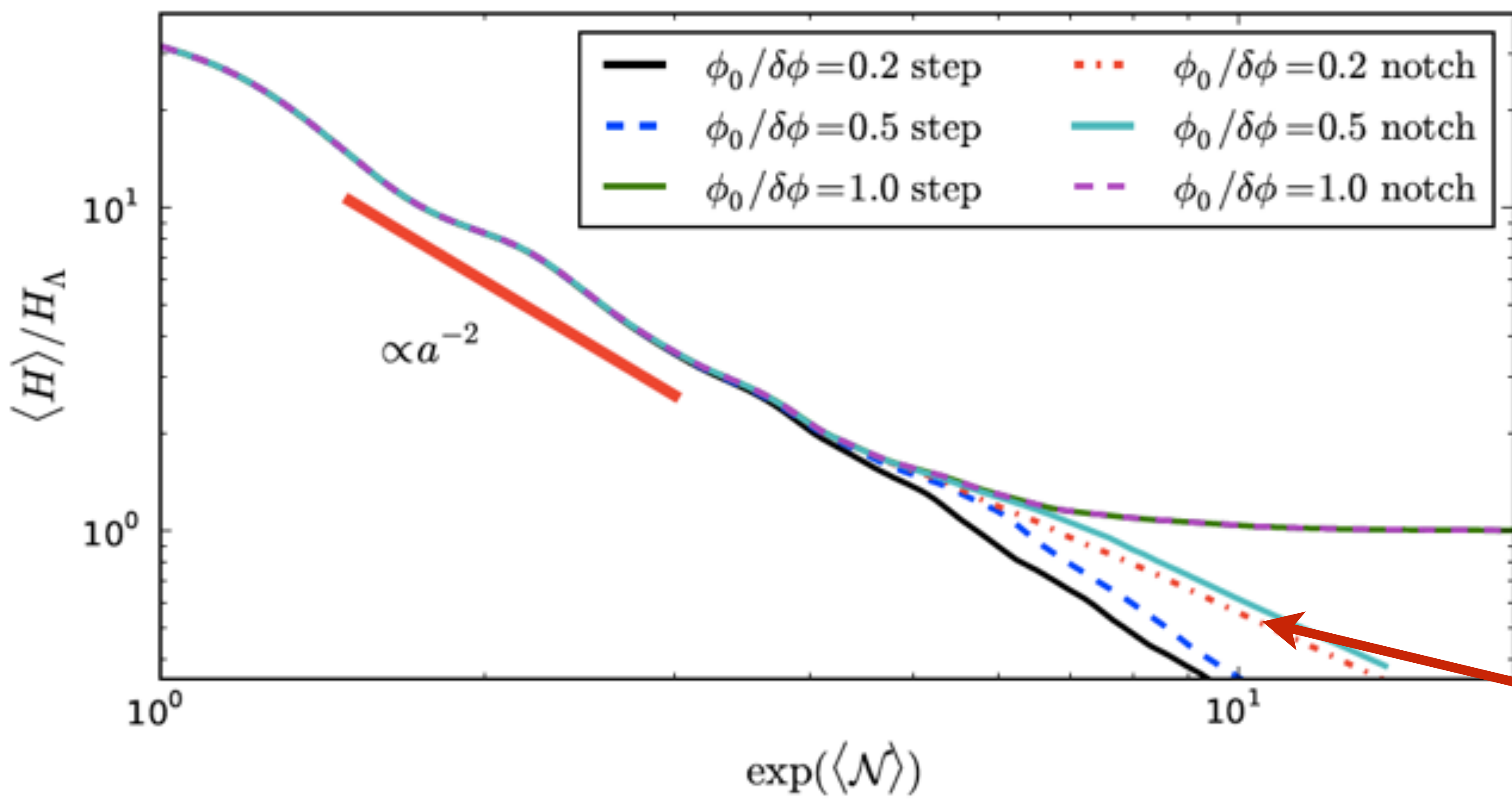
unless $\langle\phi\rangle > M_P/\sqrt{\epsilon}$

- Disclaimer: this is different than the former homogeneity problem
  - and quantitatively is highly model dependent

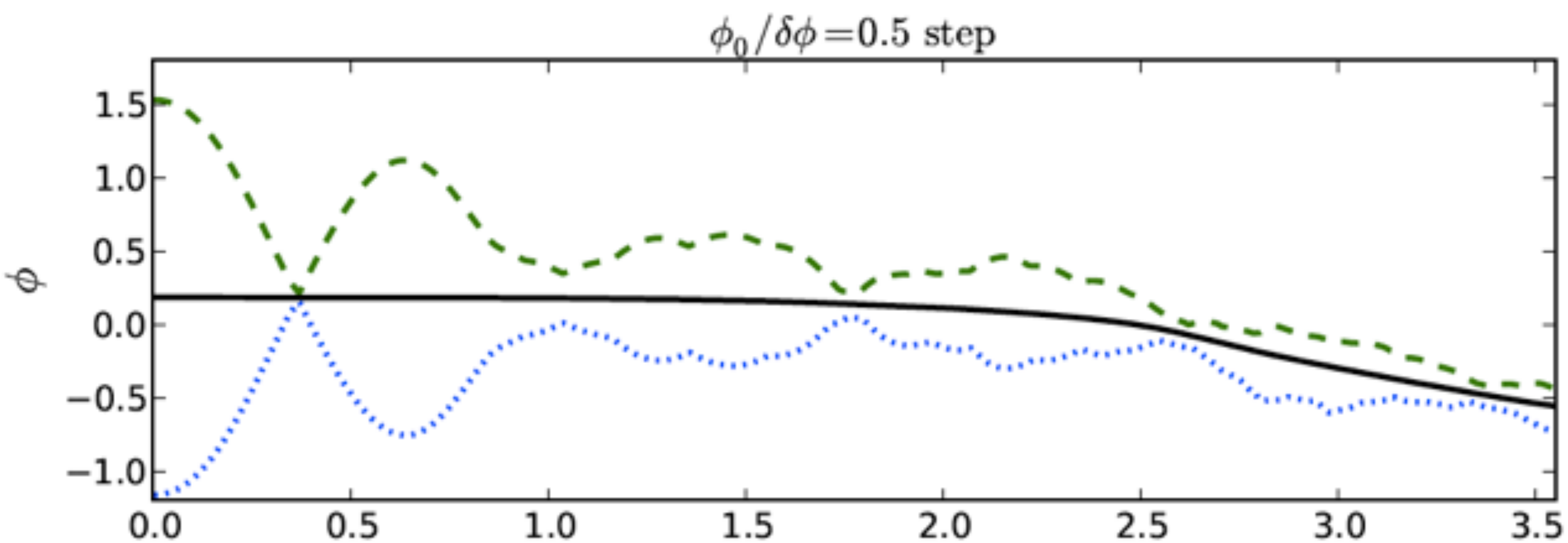
Non-inflating with steep `effective' Potential

– Numerical results





Potential never dominates



Inflaton is dragged to the bottom

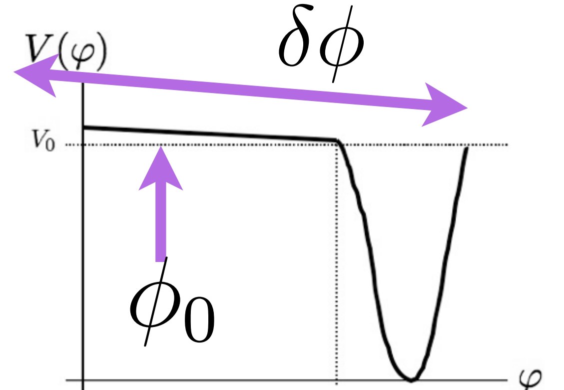
Comments on subsequent Literature

– Recently, a group from Austin and Imperial College, Claugh et al **2016**, produced a paper that nicely confirms our numerical results (the wording is to us a bit misleading: to us, they exactly confirm us both in the case of large *and* small field inflation, with the effect that I just described, so, please read beyond words)

– Furthermore, they point out the existence of ancient literature, Laguna et al, **1991**; Kurki-Suonio, **1993** where our results were anticipated. We disagree with this: the former literature was actually simulating only homogenous cosmology (if you read the paper), where the attractor nature of the inflationary solution made it clear what to expect.

– It was indeed impossible to simulate truly inhomogenous, and therefore non-trivial, cosmologies because of the formation of Black Holes. It was only the breakthrough of the last decade that now we can apply this to the early universe.

– In fact, to our knowledge ours was the first application of codes that can handle singularities to cosmology



Conclusions

- It is widely believed that in order to start inflation, we need an homogenous patch of order H_I . This seemed to required fine tuning: `initial patch problem'
- By using numerical GR simulations that are able to handle singularities and horizons
- & less-usual mathematical techniques such as mean curvature flow and topology
- we have shown that this does not seem to be necessary:
  - even by starting with an highly inhomogeneous universe,
  - if we condition on the topology of the manifold (or on the zero mode of the fluctuations),
  - there will be a (potentially small in terms of initial coordinates) region were inflation will start with certainty.

• Both the numerical approach and the analytical approach are relatively new and unfamiliar. It looks like we just scratched the surface of a new attack to the study of the initial conditions of the universe.

On non-compact case

– We always expect in physics that if the manifold is large enough, the boundary conditions do not matter, so what we have for a non-compact manifold is what we have for a torus.

– Furthermore, it is not true the crucial condition that a `open' non-compact manifold has

$$R^{(3)} < 0 \text{ somewhere} \quad \text{ex} : S^2 \times R$$

– But it is true that is must be that one eigenvalue λ_i of $R^{(3)}_{ij}$ must be negative

– But it would seam very peculiar for a non-compact manifold to have

$$\lambda_1 < 0 \text{ everywhere}$$

$$R^{(3)} = \sum_i \lambda_i > 0 \text{ everywhere}$$

– As soon as we guarantee that $R^{(3)} < 0$ at one point on any slice, no recollapse follows.